There is a distribution of agents with wealth ω given by $H(\omega)$

$$Max\ \ln c_t + \beta E \ln c_{t+1}$$

Intermediation cost is α :If agents pay cost α they can pool rescources and use intermediation to invest in an asset paying

$$r' = R + \varepsilon, \qquad \varepsilon\ is\ \ iid, \quad E(\varepsilon) = 0$$

The safe asset pays r. Investment is indivisable: all wealth must be invested in one asset or the other. Intermediation is necessary because r' is not observed. Only with financial intermediation there is risk-sharing. However agents incur per person financial costs a. They cannot pool to pay a single α : They have individual accounts with the intermediary.

$$Max\ \ln(c) + \beta E \ln R(\omega - c)$$

FOC

$$c^{-1} = \beta(\omega - c)^{-1}$$

$$c = \omega - \beta c$$

$$c = \frac{1}{1+\beta}\omega$$

$$w - c = \frac{\beta}{1+\beta}\omega$$

If support of $\varepsilon < -R,$ without intermediation all agents invest in safe asset because of log utility

Agents who invest in safe asset get $r < R$.

$$V_s = \ln \frac{1}{1+\beta}\omega + \beta \ln\left(\frac{\beta r}{1+\beta}\omega\right)$$

$$\frac{dV_s}{d\omega} = \frac{1}{\omega} + \beta\frac{1}{\omega} = \frac{1+\beta}{\omega}$$

Agents who invest in unsafe asset get, (with pooled rescorces guaranteeing a return R)

$$V_u = \ln \frac{1}{1+\beta}(\omega - \alpha) + \beta \ln\left(\frac{\beta R}{1+\beta}(\omega - \alpha)\right)$$

$$\frac{dV_u}{d\omega} = \frac{1}{\omega - \alpha} + \beta\frac{1}{\omega - \alpha} = \frac{1+\beta}{\omega - \alpha} > \frac{1+\beta}{\omega}$$

Cutoff $\bar{\omega}$:

$$V_s = V_u$$

$$V_s = \ln \frac{1}{1+\beta}\bar{\omega} + \beta \ln\left(\frac{\beta r}{1+\beta}\bar{\omega}\right)$$

$$= \ln \frac{1}{1+\beta}(\omega - \alpha) + \beta \ln\left(\frac{\beta R}{1+\beta}(\omega - \alpha)\right) = V_u$$

$$\ln \frac{1}{1+\beta} + \ln \bar{\omega} + \beta \ln\left(\frac{\beta r}{1+\beta}\right) + \beta \ln(\bar{\omega})$$

$$=$$

$$\ln \frac{1}{1+\beta} + \ln(\bar{\omega} - \alpha) + \beta \ln\left(\frac{\beta R}{1+\beta}\right) + \beta \ln(\bar{\omega} - \alpha)$$

$$(1+\beta)\ln\bar{\omega} + \beta\ln\left(\frac{\beta r}{1+\beta}\right)$$

$$= (1+\beta)\ln(\bar{\omega}-\alpha) + \beta\ln\left(\frac{\beta R}{1+\beta}\right)$$

$$(1+\beta)(\ln\bar{\omega} - \ln(\bar{\omega}-\alpha))$$

$$= \beta\left(\ln\left(\frac{\beta R}{1+\beta}\right) - \ln\left(\frac{\beta r}{1+\beta}\right)\right)$$

$$\ln\frac{\bar{\omega}}{\bar{\omega}-\alpha} = \frac{\beta}{(1+\beta)}\ln\left(\frac{R}{r}\right)$$

$$\bar{\omega} = \left(\frac{R}{r}\right)^{\frac{\beta}{(1+\beta)}}(\bar{\omega}-\alpha)$$

$$\bar{\omega}\left(\left(\frac{R}{r}\right)^{\frac{\beta}{(1+\beta)}} - 1\right) = \left(\frac{R}{r}\right)^{\frac{\beta}{(1+\beta)}}\alpha$$

$$\bar{\omega} = \left(\left(\frac{R}{r}\right)^{\frac{\beta}{(1+\beta)}} - 1\right)^{-1}\left(\frac{R}{r}\right)^{\frac{\beta}{(1+\beta)}}\alpha > 0$$

Implications:

1: Initially, widening income inequality: rich invest in risky asset and earn higher return though pooling

2. Eventually stabilize if

$$\frac{\beta r}{1-\beta} > 1$$

since the wealth of those holding safe assets grow.

3. Over time, financial intermediation (financial institutions) will grow.