Howitt-McAffee QJE 1988

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- We postulate a model with a large number of identical firms, and an even larger number of identical households, all risk neutral and all with the same constant rate of pure time preference.
- There are three tradable objects: output, homogeneous labor services, and money.The money is a pure accounting device.
- A firm's receipts are instantaneously transferred to its workers and owners and must beused for purchasing output from other firms during the current period.
- No credit market is assumed to exist, but that is no restriction in a world of risk neutrality and identical rates of time-preference.
- The output market is perfectly competitive, in that all firms and households perceive a perfectly elastic demand schedule, and aggregate demand always equal aggregate output.

- Firms incur a transaction cost to operate in the market. This transaction cost takes the form of output used up in the selling process.
- Thus, a firm employing n units of labor will have a gross revenue of $f(n)$, where f is its production function, and will pay a total transaction cost of $\sigma(n, \bar{n})$, where $\bar{n}$ is aggregate employment (per firm).
- This transaction cost depends upon the firm's own employment because the more it sells the greater the required cost. It depends upon aggregate employment (per firm) $\bar{n}$ because of the trade externality.
- The larger is $\bar{n}$, the greater is the equilibrium level of aggregate demand, and by assumption, the less the cost of selling a given quantity, $\sigma_2 < 0$. (Aggregate demand externality)

- The typical firm's wage bill will be $w(n, \bar{n})$.
- This function can be derived in a number of different ways: for example, if the labor market is perfectly competitive when $w(n, \bar{n}) = w^s(\bar{n}).n$, where w^S is the supply price of $\bar{n}$ units of labor (per firm), $w_1 > 0,\ w_2 > 0$.
- Could be result of bargaining.
- The firm also faces hiring-costs which it incurs in the form of output used up.
- The costs are given by the function $\gamma(\dot{n} + \delta n,\bar{n})$, where δ can be interpreted as either the death rate of workers or the exogenous rate at which job separations occur for noneconomic reasons.
- As $\bar{n}$ creases, the size of the pool from which the typical firm draws its new recruits is thereby reduced, so $\gamma_2 > 0$, and $\gamma_{12} > 0$.

Profit flows

$$L = f(n) - \sigma(n, \bar{n}) - w(n, \bar{n}) - \gamma(\dot{n} + \delta n, \bar{n}) \equiv \Pi(n, \bar{n}) - \gamma(\dot{n} + \delta n, \bar{n})$$

- Assume L is twice differentiable, and for any $\bar{n}$ concave in $(n, \bar{n})$, $L_{11} = -\gamma_{11} < 0$.
- If r is the time discount, stationary (steady state) equilibria depend on

$$\begin{aligned} g(n) &= L_2(0, n, n) - rL_1(0, n, n) \\ &= f'(n) - \sigma_1(n, \bar{n}) - w_1(n, n) - (r + \delta)\gamma_1(\delta n, n) \end{aligned}$$

- This function describes the steady state marginal profit of employment; i.e., the marginal contribution of a unit of labor to the stationary flow of profits: $L_2 = \Pi_1 - -\delta\gamma_1$, minus the interest cost of the initial recruiting outlay, $-rL_1 = \gamma r_1$.

- Assume $g(n) < 0$ for $n \in (0, n_1)$ and $n \in (n_2, 0)$, $g(n) > 0$ for $n \in (n_1, n_2)$

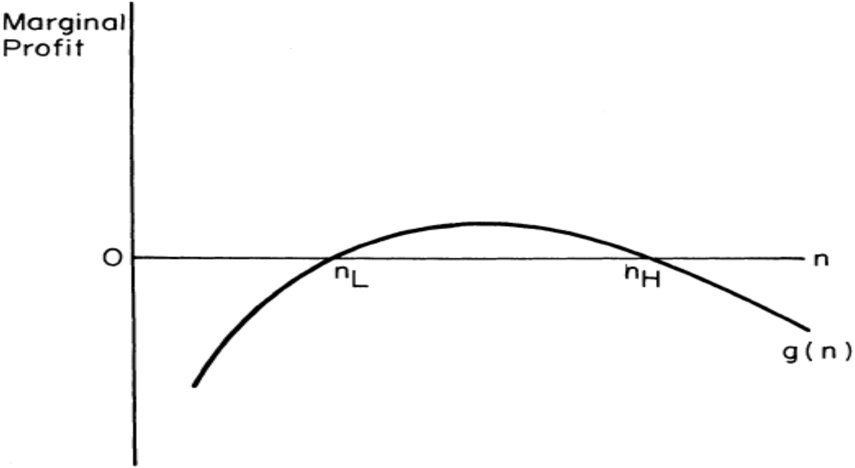

FIGURE I
Multiple Equilibria

$$max \int_0^{\infty} e^{-rt} L(\dot{n}, n, \bar{n}) dt$$

$$L_2(\dot{n}, n, \bar{n}) - rL_1(\dot{n}, n, \bar{n}) = \frac{d}{dt} L_1(\dot{n}, n, \bar{n})$$

which is the optimal solution if transversality holds, $\lim_{t\to\infty} e^{-rt} L_1(\dot{n}, n, \bar{n}) = 0$.Linearizing and evaluating at steady state n^* :

$$\begin{aligned} 0 &= L_{11}\ddot{n} + (L_{13} - rL_{11})\dot{n} - (L_{22} + L_{23} + rL_{12} + rL_{13}) * (n - n^*) \\ &= -\gamma_{11}\ddot{n} + (r\gamma_{11} - \gamma_{12})\dot{n} - g'(n^*)(n - n^*) \end{aligned}$$

Roots

The roots of this system, (λ_1, λ_2) must satisfy.

$$\begin{aligned} \lambda_1 + \lambda_2 &= r - \gamma_{12}/\gamma_{11} \\ \lambda_1 \lambda_2 &= g'(n^*)/\gamma_{11} \end{aligned}$$

- Note: if $\gamma_{12} = 0,\ \lambda_1 + \lambda_2 = r > 0$; if $g'(n^*) < 0,\ \lambda_1\lambda_2 < 0$: no externality→saddle: Given initial n,single choice of $\dot{n}$ to converge to n^*.
- But if $\lambda_1 + \lambda_2 = r - \gamma_{12}/\gamma_{11}$ is negative (big enough γ_{12}/γ_{11}) and $g'(n^*) > 0$ both roots of the same sign: can be sink, or source surrounded by cycle (Diamond-Fudenberg).

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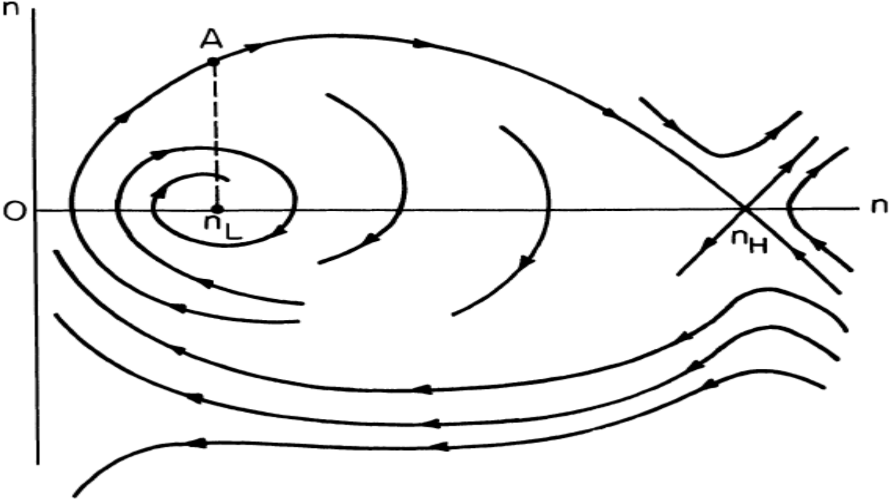


FIGURE II
Local Stability

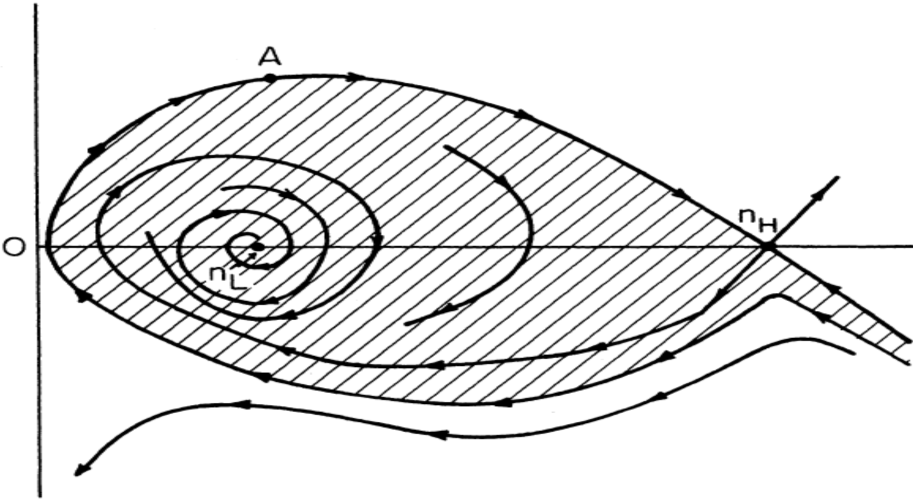
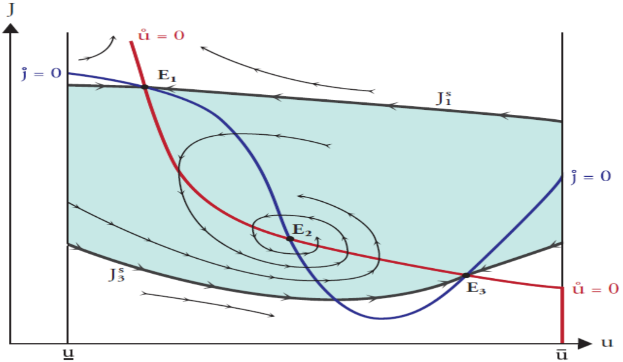


FIGURE III
Perfect Foresight Paths

Kaplan and Menzio



J is value of employee to firm, u is unemployment. Initial u_{-1} is given