Financial Markets, the Real Economy, and Self-fulfilling Uncertainties

Jess Benhabib*  Xuewen Liu†  Pengfei Wang‡

This version: February 2017

Abstract

Uncertainty in both financial markets and the real economy rises sharply during recessions. We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production (acquisition) and aggregate economic activities to explain this intriguing empirical fact. We argue that there exists mutual learning between financial markets and the real economy. Their joint information productions determine both the real production efficiency in the real sector and the price efficiency in the financial sector. The mutual learning makes information production in the financial sector and that in the real sector a strategic complementarity. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sectors produce little information in anticipation of the other sector to do so. At the same time, aggregate output falls as the real production efficiency deteriorates. Our model has other implications on aggregate economic activities.

JEL classification: G01, G20, E2, E44

Keywords: Information production, Strategic complementarity, Real economy, Two-way learning, Economic uncertainty

*Department of Economics, New York University, 19 West 4th Street, New York, NY 10012, USA. Office: (212) 998-8971, Fax: (212) 995-4186. Email: jess.benhabib@nyu.edu.
†Department of Finance, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7679. Email: xuewenliu@ust.hk.
‡Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7612. Email: pfwang@ust.hk.
1 Introduction

Uncertainty in both financial markets and the real economy rises sharply during recessions. The recent financial crisis of 2007-2009 presented one of the most striking episodes of such heightened uncertainty. The financial market uncertainty, measured by the VIX index, jumped by an astonishing 313% in the Great Recession. The increase in measured real uncertainty was equally impressive. For instance, the macroeconomic uncertainty measured in Jurado et al. (2015) almost doubled, and Bloom et al. (2012) reports a 152% increase in the micro-level real uncertainty measured by the firm-level dispersion of output. What causes such sudden spikes in uncertainty? Why do financial uncertainty and real uncertainty move together? Why do they rise sharply in recessions? These challenging questions are of central importance for understanding the interaction between financial markets and the real economy. Despite a large and flourishing literature on uncertainty since the pioneering work of Bloom (2009), these questions are still largely unanswered. The purpose of this paper is to provide a theoretical framework to address these questions.

We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production (or acquisition) and aggregate economic activities. As the starting point of our theory, we argue that there exists mutual learning between financial markets and the real economy. Their joint information productions determine both the real production efficiency in the real sector and the price efficiency in the financial sector. As an example, oil producing companies scrutinize oil future prices when they make their production decisions, while the financial market studies the financial reports from these producing companies to learn information when trading on oil futures. This mutual learning makes information production in the financial sector and that in the real sector a strategic complementarity. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sides produce little information in anticipation that the other side will do so. At the same time, aggregate output falls as the real production efficiency deteriorates.

We formalize the idea in an extended Grossman-Stiglitz (1980) model. Our key innovation is that we introduce a real sector along the line of Dixit-Stiglitz (1980) — in our framework, firms have to make investment decisions under imperfect information about two dimensions of uncertainty: their idiosyncratic productivity and demand shocks. We start with one firm and one financial market in our baseline partial equilibrium model for a given aggregate output. To reduce uncertainty, the firm can learn about its idiosyncratic productivity shock by incurring a cost, but it has to infer its demand shock from the information provided in the financial market where speculators (or traders) have a comparative advantage in acquiring information about the demand shock. In this context, the financial price is jointly determined by the firm’s information production and thereby its disclosure and the demand information produced by financial market
speculators. To understand strategic complementarity in these two sources of information, first consider that the firm makes more accurate information disclosure about its productivity shock. The reduced uncertainty about the productivity shock attracts more informed traders and induces more aggressively trading. Hence, information production on the demand shock in the financial market increases. Conversely, suppose that the financial market becomes more informative about the demand shock for some reason. The reduced uncertainty regarding the demand implies that the stake is higher for better investment that is more closely aligned with the true realized productivity shock. Hence, the firm has stronger incentives to acquire precise information about its productivity shock.

As the marginal benefit of acquiring information for the firm depends on aggregate output (besides financial price informativeness), the nature of equilibrium also crucially depends on the level of aggregate output. When the aggregate output is sufficiently high, the resulting equilibrium is unique in which the firm produces and discloses more precise information and the financial market generates a more informative price signal. As a result, both financial and real uncertainty are low. When the aggregate output is sufficiently low, not acquiring information is a dominant strategy for the firm. Anticipating this, speculators in the financial market also have little incentive to acquire information about the firm’s demand shock. The equilibrium is hence also unique. However, when the aggregate output is in the intermediate range, the economy has two self-fulfilling equilibria. The information produced by the firm and the information generated by financial market in one equilibrium (the “good” equilibrium) are much more precise than those in the other equilibrium (the “bad” equilibrium). Consequently, a sudden outburst of uncertainty can arise as a self-fulfilling equilibrium phenomenon.

We then extend the baseline model to a macroeconomic general-equilibrium framework with aggregate production to endogenize the aggregate output. The final consumption good is produced with the inputs of a continuum of intermediate capital goods according to a Dixit-Stiglitz production function. Each intermediate capital good is produced by one firm located on an island in the spirit of Lucas (1972). We show that complementarity in information production exists between the financial sector and the real sector within an island and between islands. The complementarity between two islands arises from strategic complementarity in production between firms. When information signals on some islands become noisier, the real investment decisions on those islands become less efficient and consequently the aggregate output declines. This causes the aggregate demand faced by other islands to drop. Thus, incentives to acquire information in the real sector on those other islands are also reduced, which decreases information acquisition in their financial sectors as well. The aggregate output hence declines further, which in turn affects those islands experiencing the original shock. Similar to the partial equilibrium model, the economy may feature two equilibria: a “good” equilibrium with more information production and higher real economic
activities, and a “bad” equilibrium with the opposite features.

Although our model is too stylized to be confronted with the actual data, it does explain, qualitatively, several prominent features of macroeconomic fluctuations. First, our quantitative exercise shows that with reasonable parameter values our model can generate a very large drop in the aggregate-level output and investment in the absence of any aggregate shock. Second, the measured uncertainty in both financial markets and the real economy has a sizable spike in recessions. The residual uncertainty faced by firms and by financial investors increases substantially when the economy shifts from the “good” to the “bad” equilibrium. Third, our model implies that financial panics seemingly start with some gradual deterioration in economic fundamentals. When the deterioration reaches a tipping point, uncertainty surges and the real economic activities collapse.¹

Our model has additional implications for aggregate economic activities. First, our model highlights how information frictions can be an important reason for resource misallocation. Efficient allocation requires more resources to be allocated to firms with higher realized productivity and stronger demand shocks. Higher residual uncertainty under less information leads to a higher degree of resource misallocation. In our model, the endogenous aggregate total factor productivity (TFP), which maps the degree of resource misallocation, is an increasing function of information precision. As information precision is procyclical, our model is consistent with an established empirical fact that aggregate productivity is procyclical. Second, a small shock to the financial sector that impairs its ability to perform price discovery can have a large impact on the aggregate economy due to the compound feedback loops. In fact, both aggregate investment and the endogenous aggregate TFP are decreasing in information precision. Hence, a small shock to the financial sector can have a large impact on all three quantities (aggregate investment, endogenous aggregate TFP, and aggregate output) in the same direction. Third, our model provides an information contagion channel, where a shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. This is consistent with some evidence that idiosyncratic firm-level shocks can be the origin of aggregate fluctuations (see Gabaix (2011)).

Related literature. A burgeoning literature in finance studies the informational feedback effects of financial markets (see Bond, Edmans and Goldstein (2012) for an extensive survey of this literature). This literature argues that firm managers on the real side of the economy learn from financial prices. Among others, ² Goldstein, Ozdenoren and Yuan (2013) and Sockin and Xiong

¹The Delinquency Rate on Single-Family Residential Mortgages had begun a steady climb in August 2006 before it led to the collapse of several large financial institutions and the subsequent financial crisis in 2008-2010.
(2015) develop clean model frameworks showing how prices in the secondary financial market can aggregate dispersed information of speculators and guide firm managers or investors to make better real investment decisions. The learning in this literature is one way — the real sector learns from financial markets. On the other hand, the accounting literature emphasizes the opposite direction of learning — arguing that firm managers typically know more than financial market participants — and focuses on studying how firm managers (i.e., insiders) disclose information to the capital market, based on which financial speculators trade and security prices are formed. Our paper advances these two bodies of literature by introducing and studying \textit{mutual (two-way) learning} between the real sector and financial markets. The two-way learning mechanism sheds light on important questions, such as how a financial price is formed, where the information comes from, and how the sources of information interact.

The finance literature pioneered by Grossman and Stiglitz (1980) and Verrecchia (1982) studies information production (or acquisition) in financial markets. The recent work of Goldstein and Yang (2015) analyzes a model where two different groups of financial traders are informed of different fundamentals of a security. They show that trading as well as information acquisition by these two groups of financial traders exhibit strategic complementarities. Ganguli and Yang (2009) study a model where traders can obtain private information about the supply of a stock in addition to that about its payoff. They show complementarity in information acquisition and the existence of multiple equilibria. Our model introduces the real sector and aggregate production into a Grossman-Stiglitz-type model; information acquisition in our model takes place both in the real sector and in financial markets. Adding to this literature, our paper shows that complementarity in information production exists between the real sector and the financial sector, which has important macroeconomic implications.\textsuperscript{3}

A large literature in macroeconomics documents robust evidence of countercyclical uncertainty — both real uncertainty and financial uncertainty increase during recessions. Real uncertainty is often proxied by firm-level dispersion in earnings, productivity and output, and the volatility of aggregate output forecast error, while financial uncertainty is often measured by financial market volatility and the VIX index (Bloom (2009), Bloom et al. (2012), Jurado et al. (2015)). An ongoing heated debate in this literature concerns the question of causality, i.e., whether uncertainty is a cause or merely a response to recessions and where uncertainty comes from (see, e.g., Bachmann and Bayer (2013, 2014)). Interestingly, a recent paper by Ludvigson et al. (2017) empirically identifies that sharply higher real uncertainty in recessions is most often an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is

\textsuperscript{3}Goldstein and Yang (2016) analyze a model where there are multiple dimensions of uncertainty and market prices convey information to real decision-makers. They focus on studying the effect of disclosing public information on real efficiency. There is no information production (acquisition) in their model, while studying the interaction of information production in the real sector and that in the financial sector is a key emphasis of our model.
a likely source of the fluctuations. Our paper contributes to the debate by providing a theoretical framework that is able to address the three variables simultaneously — real uncertainty, financial uncertainty, and aggregate economic activities — and show how they are related.4

Bacchetta, Tille and Wincoop (2012) also study the self-fulfilling nature of uncertainty. They construct an interesting endowment economy in which agents have mean-variance preferences so that the equilibrium asset prices are negatively linked to the perceived risk of future prices. If the agents believe that pure sunspots matter for asset prices, then the perceived risk of future prices increases. As a result, the current asset prices will indeed be affected. The uncertainty is self-fulfilling because there also exists another equilibrium in which the asset prices are certain and hence bear zero risk. Fajgelbaum, Schaal and Taschereau-Dumouchel (2016) propose a theory of self-reinforcing episodes of high uncertainty and low activity, through the mechanism of the “wait-and-see” effect together with agents learning from the actions of others. In contrast to these contributions, self-fulfilling uncertainty in our model comes from the information interdependence between financial markets and the real economy. This information interplay also allows us to study the impact of uncertainties on real economic activities.

Finally, our model is related to a small body of macroeconomics literature that studies how financial markets affect business cycle fluctuations through information channels.5 Angeletos, Lorenzoni and Pavan (2010) build a two-stage feedback model where financial markets in the second stage learn from the volume of asset selling of entrepreneurs in the first stage, which generates strategic complementarity in investment that amplifies non-fundamental shocks at that stage. Benhabib, Liu and Wang (2016a) present a self-fulfilling business cycle model, where financial market sentiments affect the price of capital, which signals the fundamentals of the economy to the real side and consequently leads to real output that confirms the sentiments. David, Hopenhayn and Venkateswaran (2016) conduct a quantitative study that links imperfect information and resource misallocation, where firms learn from both private sources and imperfectly informative stock market prices about one dimension of fundamental uncertainty. The information is exogenous in David, Hopenhayn and Venkateswaran, and they conclude that firms turn primarily to internal sources for information, rather than to financial markets.6 Compared with the aforementioned studies, ours shows that the amount of information in the economy is endogenous, and that there is feedback between the level of economic activity and the amount of information, amplified through the mutual learning

4Benhabib, Liu and Wang (2016b) study endogenous information acquisition of firms that links real uncertainty and economic activities. However, there is no financial market in the model there and that paper does not touch upon financial uncertainty. Veldkamp (2005) and Nieuwerburgh and Veldkamp (2006) study learning asymmetries in business cycles, without involving financial markets.
6Because David, Hopenhayn and Venkateswaran (2016) consider neither endogenous information acquisition nor the feedback on information production between the real side and financial markets, incorporating our mechanism in their quantitative study may bring new results; see our numerical calibration in Section 5.
between firms and financial markets.

The paper is organized as follows. In Section 2, we present the simple baseline model. In Section 3, we extend our baseline model to study endogenous information. In Section 4, we further extend the model to a macroeconomic framework. Section 5 provides quantitative analysis of our model. Section 6 concludes.

2 The Baseline Model

In this section, we present a simple baseline model with one firm, one financial market, and with exogenous information. The firm faces two uncertainties: demand shocks and supply (or productivity) shocks. The firm has some information about the supply shocks while the financial market has some information about the demand shocks. We show that there exists two-way learning between the firm and the financial market.

2.1 Setup

There are two types of agents: firm \( j \) and a group of financial market traders (speculators). There are two types of goods: an intermediate capital good and a final consumption good. The price of the consumption good is normalized as the numeraire, \( P = 1 \).

**Intermediate Goods Firm** Firm \( j \) is an intermediate goods firm. It produces the intermediate capital good \( Y_j \) using the input of the final consumption good according to the production function

\[
Y_j = Z A_j K_j^\eta \quad \text{for } 0 < \eta < 1,
\]

where \( Z \) is the common productivity shock to the whole economy (regarded as a constant in the baseline model), \( A_j \) is Firm \( j \)'s productivity, and \( K_j \) is the investment input of the final consumption good.\(^7\) We will show that firm \( j \) borrows the investment input at interest rate \( R_f = 1 \).

The market demand function of the intermediate capital good \( Y_j \) is assumed to be

\[
Y_j = \left( \frac{1}{P_j} \right)^\theta \epsilon_j Y,
\]

where \( P_j \) is the price of the capital good \( j \) (in terms of the final consumption good), and \( \epsilon_j \) measures the idiosyncratic demand shock to good \( j \). Moreover, in the baseline model \( Y \) is an exogenous constant, which corresponds to the aggregate output (real GDP) (denote \( y \equiv \log Y \)), whereas parameter \( \theta \) measures the price elasticity of demand.

\( ^7 \)The input \( K_j \) will fully depreciate after production.
Financial Market and Traders (Speculators) A financial market exists, where speculators trade a financial asset (a derivative) contingent on the firm’s asset value or firm value (also its total income):\(^8\)

\[ V_j = Y_j P_j. \]  

(3)

Specifically, we assume that the payoff of the financial derivative contract takes the form of

\[ v_j = \log V_j, \]

where \( v_j \) also corresponds to the growth of the firm value. Denote the market trading price of the financial derivative contract by \( q_j \). That is, the long position of one unit of the financial asset (derivative) incurs an initial outlay of \( q_j \) and entitles to having the risky payoff \( v_j \) later.

The utility function of speculators is assumed to be

\[ U(W^i) = -\exp(-\gamma W^i), \]

where \( W^i \) is the end-of-period wealth for speculator \( i \), and \( \gamma \) is risk aversion (CARA) coefficient. The initial wealth for a speculator is assumed to be \( W_0 \) and the risk-free (gross) interest rate is \( R_f \equiv 1 \). This means that if a speculator takes a position of \( d \) units of the financial asset, his end-of-period wealth would be

\[ W^i = (W_0 - d q_j) R_f + d v_j = W_0 + d(v_j - q_j). \]

The assumption that speculators trade a derivative contract contingent on the firm’s asset value, \( V_j \), is made for tractability. This is along the line of the assumption in the literature that a firm’s asset value or sales revenue follows a geometric Brownian motion. The financial derivative can also be contingent on the firm’s product price, \( P_j \) (that is, \( v_j \) takes the form of \( v_j = \log P_j \)). In the latter case, the financial market can be interpreted as a commodity financial futures market that specializes in trading financial futures regarding the intermediate capital good \( Y_j \), in the spirit of Sockin and Xiong (2015). Assuming that the underlying asset of the derivative is either \( V_j \) or \( P_j \) is to ensure that the payoff of the underlying asset follows a log-normal distribution and thus to achieve tractability. This parallels the modeling device that assumes a specific function form of noisy trading (or asset supply) as in Goldstein, Ozdenoren and Yuan (2013), Sockin and Xiong (2015), and Goldstein and Yang (2016).

The net aggregate supply of the financial asset (i.e., derivative) is assumed to be 0. The demand

---

\(^8\)In the balance sheet of the firm at the end of the period, the asset value of the firm is its total income; the debt value is the investment cost; and the equity value is the net profit. This is consistent with the literature where the asset value, not the equity value, is assumed to follow a geometric Brownian motion (a log-normal distribution).
of noise/liquidity traders in the financial market is \( n_j \), where \( n_j \) follows distribution \( n_j \sim N(0, \sigma_n^2) \).

**Uncertainties and Information** The firm faces two uncertainties: productivity (or supply) shock \( A_j \) and demand shock \( \epsilon_j \). Their prior distributions are \( \log A_j \equiv a_j \sim N(-\frac{1}{2}\sigma_a^2, \sigma_a^2) \) and \( \log \epsilon_j \equiv \epsilon_j \sim N(-\frac{1}{2}\sigma_\epsilon^2, \sigma_\epsilon^2) \) (denote \( \tau_a = 1/\sigma_a^2 \) and \( \tau_\epsilon = 1/\sigma_\epsilon^2 \)). \( a_j \) and \( \epsilon_j \) are independent. The common productivity shock \( Z \) is public information (denote \( z = \log Z \)).

In the baseline model, we assume that the firm and the financial market have some exogenous (imperfect) information about \( a_j \) and \( \epsilon_j \), respectively. Specifically, the firm possesses or is endowed with a noisy signal about its own productivity:

\[
s_j = a_j + \epsilon_j,
\]

where \( \epsilon_j \sim N(0, \sigma_\epsilon^2) \) (denote the precision of the signal by \( \tau_\epsilon = \frac{1}{\sigma_\epsilon^2} \)). Firm \( j \) will disclose its signal \( s_j \) to the financial market.\(^9\) For simplicity, we assume that the firm has no private information about the demand shock, \( \epsilon_j \).

In the financial market, as in Grossman and Stiglitz (1980), there are two types of traders: informed and uninformed traders. An informed trader \( i \) has a noisy private signal

\[
x^i_j = \epsilon_j + \theta^i_j,
\]

where \( \theta^i_j \sim N(0, \sigma_{\theta}^2) \) and \( \theta^i_j \) is independent across informed traders (denote \( \tau_{\theta} = \frac{1}{\sigma_{\theta}^2} \)). An uninformed trader has no private signal regarding \( \epsilon_j \). The proportion of informed traders is \( \lambda \), which is exogenous in the baseline model. Note that for the extreme case of \( \sigma_{\theta}^2 = 0 \), an informed trader’s signal represents perfect information about \( \epsilon_j \).

**Timeline** The sequence of events in the baseline model is as follows:

- **T\(_1\)**: Firm \( j \) discloses its signal \( s_j \) to the financial market.
- **T\(_2\)**: Financial market trading takes place, and financial price \( q_j \) is realized.
- **T\(_3\)**: Firm \( j \) makes its investment decision, \( K_j \), based on information \( \{s_j, q_j\} \).
- **T\(_4\)**: The income or asset value, \( V_j \), is realized. The payoff of the financial contract is delivered.

### 2.2 Equilibrium

The equilibrium consists of a financial market equilibrium at \( T_2 \) and the firm’s investment decision at \( T_3 \). We conduct analysis by backward induction.

\(^9\)As will become clear later, a firm has incentives to disclose its information to the financial market because the disclosure can “attract” more information from the financial market, which can guide the firm to make better investment decisions.
Firm $j$’s Investment Decision at $T_3$  Firm $j$ maximizes its expected profit:

$$K_j \equiv K(s_j, q_j) = \arg \max_{K_j} \mathbb{E}[P_j Y_j - R_j K_j | s_j, q_j]$$

(4)

with constraints (2) and (1). Here $\mathbb{E}(\cdot | s_j, q_j)$ is the conditional expectation operator over $a_j$ and $\varepsilon_j$.

Financial Market Trading at $T_2$  In the financial market, the information set of informed speculators is $\{s_j, q_j, x_j^i\}$ while that of uninformed speculators is $\{s_j, q_j\}$.

An informed speculator chooses his risky asset holdings, $d^{Ii}$, to maximize his utility:

$$d^{Ii}(s_j, q_j, x_j^i) = \arg \max_{d^{Ii}} \mathbb{E}[U(W^{Ii}) | s_j, q_j, x_j^i],$$

(5)

where $W^{Ii} = (W_0 - c) + d^{Ii}(v_j - q_j)$ and $c$ denotes a constant expense, to be explained later. An uninformed speculator chooses his risky asset holdings, $d^{Ui}$, to maximize his utility:

$$d^{Ui}(s_j, q_j) = \arg \max_{d^{Ui}} \mathbb{E}[U(W^{Ui}) | s_j, q_j],$$

(6)

where $W^{Ui} = W_0 + d^{Ui}(v_j - q_j)$. In (5) and (6), $\mathbb{E}(\cdot)$ is the expectation operator over $v_j$.

The equilibrium of our baseline model is formally defined as follows.

Definition 1 An equilibrium consists of the financial price function $q_j = q(s_j, \varepsilon_j, n_j)$ and the firm’s investment decision function $K_j = K(s_j, q_j)$, such that

1. Price $q(s_j, \varepsilon_j, n_j)$ clears the financial market at $T_2$:

$$\lambda \int d^{Ii} + (1 - \lambda) \int d^{Ui} + n_j = 0,$$

(7)

where, for given $K_j = K(s_j, q_j)$, $d^{Ii}$ and $d^{Ui}$ solve (5) and (6), respectively.

2. Given price $q(s_j, \varepsilon_j, n_j)$, investment decision $K(s_j, q_j)$ solves the firm’s problem (4).

The equilibrium defined in Definition 1 highlights the two-way feedback (i.e., a fixed-point problem) between the financial market and the real economy. On the one hand, the financial price at $T_2$ should reflect the (forward-looking) investment decision at $T_3$ (and thereby the financial asset’s fundamentals at $T_4$). On the other hand, the financial price at $T_2$ influences and guides the investment decision on the real side of the economy at $T_3$. 
2.3 Characterization of Equilibrium

First, we characterize the financial market equilibrium. We conjecture that \( \log K_j \equiv k_j = k(s_j, q_j) \) is a linear function in Definition 1. Plugging (2) and (1) into (3) yields

\[
v_j = \frac{1}{\theta} \varepsilon_j + \left( 1 - \frac{1}{\theta} \right) (z + a_j) + \eta \left( 1 - \frac{1}{\theta} \right) k_j + \frac{1}{\theta} y,
\]

(8)

which depends on \( \varepsilon_j, a_j \) and \( k_j \). However, speculators are certain about \( k_j \) but not \( \varepsilon_j \) and \( a_j \), because \( k_j \) is a function of signals \( s_j \) and \( q_j \) and thus speculators perfectly foresee the investment decision of the firm.

In solving (5), we find that

\[
d^{Ii} = \frac{E[v_j|s_j, q_j, x^i_j] - q_j}{\gamma \text{Var}[v_j|s_j, q_j, x^i_j]}.
\]

(9)

Similarly, (6) gives

\[
d^{Ui} = \frac{E[v_j|s_j, q_j] - q_j}{\gamma \text{Var}[v_j|s_j, q_j]}
\]

(10)

We also conjecture a linear price function:

\[
q_j = \beta_0 + \beta_1 (\varepsilon_j + \beta_2 s_j + \beta_3 n_j),
\]

(11)

where \( \beta_0, \beta_1, \beta_2 \) and \( \beta_3 \) are coefficients. Combined with \( s_j \), price \( q_j \) can be converted into another piece of public information about \( \varepsilon_j \):

\[
\tilde{q}_j(q_j, s_j) = \frac{q_j - \beta_0 - \beta_1 \beta_2 s_j}{\beta_1} = \varepsilon_j + \beta_3 n_j \equiv \varepsilon_j + q^q_j,
\]

(12)

where \( q^q_j \sim N(0, \sigma^2_q) \) with \( \sigma^2_q = \beta_3^2 \sigma^2_n \) (written as \( \tau_q \equiv \frac{1}{\sigma^2_q} \) representing the precision of the price signal). Information set \( \{s_j, q_j\} \) is a one-to-one mapping into \( \{s_j, \tilde{q}_j\} \). In particular, the informativeness of price \( q_j \) (together with signal \( s_j \)) about \( \varepsilon_j \) is fully captured by the term \( q^q_j \). In other words, given \( \sigma^2_n \) (and \( s_j \)), \( \beta_3 \) measures price informativeness of \( q_j \) about \( \varepsilon_j \).

Plugging (8) and (11) into (9) and (10), together with (7), yields the financial market equilibrium. We have Lemma 1.

**Lemma 1** In the equilibrium of the financial market, for a given \( \lambda \), \( \tau_q \) is an increasing function of \( \tau_e \), i.e., \( \frac{\partial \tau_q}{\partial \tau_e} > 0 \).

**Proof.** See Appendix.

Lemma 1 states that when the precision of the firm’s disclosed information about \( a_j \) increases, the informativeness of the financial price about \( \varepsilon_j \) also increases. The intuition is as follows. The
total uncertainty over $v_j$ is the sum of uncertainties over $a_j$ and $\varepsilon_j$. When uncertainty over $a_j$ decreases under a higher $\tau_e$, informed traders have incentives to trade more aggressively, which overwhelms the trading of noise/liquidity traders, thus increasing the informativeness of the financial price.

Lemma 1 shows that the financial price comes partially from information disclosure in the real sector and partially from price discovery in the financial market. These two sources of information interact. This is a novel insight of our paper.

**Illustration** We use the extreme case of $\sigma_\theta^2 = 0$ (in which case informed traders are perfectly informed of $\varepsilon_j$) to provide explicit solutions to the financial market equilibrium. For an informed trader,

$$\mathbb{E}[v_j|s_j, q_j, \varepsilon_j] = \frac{1}{\theta} \varepsilon_j + \eta \left(1 - \frac{1}{\theta}\right) k(s_j, q_j) + \frac{1}{\theta} y + \left(1 - \frac{1}{\theta}\right) \left[z + \frac{\tau_a}{\tau_a + \tau_e} \left(-\frac{1}{2} \sigma_a^2\right) + \frac{\tau_e}{\tau_a + \tau_e} s_j\right]$$

and

$$\text{Var}[v_j|s_j, q_j, \varepsilon_j] = \left(1 - \frac{1}{\theta}\right) \frac{1}{\tau_a + \tau_e}.$$

For an uninformed trader,

$$\mathbb{E}[v_j|s_j, q_j] = \frac{1}{\theta} \mathbb{E}[\varepsilon_j|s_j, q_j] + \eta \left(1 - \frac{1}{\theta}\right) k(s_j, q_j) + \frac{1}{\theta} y + \left(1 - \frac{1}{\theta}\right) \left[z + \frac{\tau_a}{\tau_a + \tau_e} \left(-\frac{1}{2} \sigma_a^2\right) + \frac{\tau_e}{\tau_a + \tau_e} s_j\right]$$

and

$$\text{Var}[v_j|s_j, q_j] = \left(1 - \frac{1}{\theta}\right) \text{Var}[\varepsilon_j|s_j, q_j] + \left(1 - \frac{1}{\theta}\right) \frac{1}{\tau_a + \tau_e}.$$

Therefore, the market clearing condition, (7), implies

$$0 = n_j + \lambda \frac{1}{\theta} \varepsilon_j + \eta \left(1 - \frac{1}{\theta}\right) k(s_j, q_j) + \frac{1}{\theta} y + \left(1 - \frac{1}{\theta}\right) \left[z + \frac{\tau_a}{\tau_a + \tau_e} \left(-\frac{1}{2} \sigma_a^2\right) + \frac{\tau_e}{\tau_a + \tau_e} s_j\right]$$

$$\gamma \left(1 - \frac{1}{\theta}\right) \frac{1}{\tau_a + \tau_e}$$

$$+ (1 - \lambda) \left[\frac{1}{\theta} \mathbb{E}[\varepsilon_j|s_j, q_j] + \eta \left(1 - \frac{1}{\theta}\right) k(s_j, q_j) + \frac{1}{\theta} y + \left(1 - \frac{1}{\theta}\right) \left[z + \frac{\tau_a}{\tau_a + \tau_e} \left(-\frac{1}{2} \sigma_a^2\right) + \frac{\tau_e}{\tau_a + \tau_e} s_j\right] \right]$$

$$\gamma \left(\frac{1}{\theta}\right)^2 \text{Var}(\varepsilon_j|s_j, q_j) + \left(1 - \frac{1}{\theta}\right) \frac{1}{\tau_a + \tau_e}.$$

(13)

It is straightforward to see that (13) can be transformed to

$$f(s_j, q_j, y, z) + \lambda \frac{1}{\theta} \varepsilon_j + \frac{1}{\gamma \left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_e}} n_j = 0,$$

(14)
where \( f(s_j, q_j, y, z) \) is a linear function of \( s_j, q_j \) and \( y \). Hence,

\[
\beta_3 = \frac{\gamma \left(1 - \frac{1}{\theta} \right)^2}{\lambda^2} \frac{1}{\tau_a + \tau_e} \quad \text{or} \quad \tau_q = \frac{\lambda^2 \frac{1}{\theta^2}}{\gamma^2 \left(1 - \frac{1}{\theta} \right)^4} \frac{(\tau_a + \tau_e)^2}{\sigma_n^2}.
\] (15)

From (15), it is easy to see that \( \tau_q \) is an increasing function of \( \tau_e \) for a given \( \lambda \). In fact, as the precision of the firm’s disclosed information increases, informed speculators trade more aggressively (see (14)) and so the price becomes more informative.

Next, we move to characterize firm \( j \)’s investment decision at \( T_3 \). As shown in (12), firm \( j \)’s information set \( \{s_j, q_j\} \) at \( T_3 \) is equivalent to the information set \( \{s_j, \bar{q}_j\} \). The first-order condition of (4) implies

\[
K_j = K(s_j, \bar{q}_j) = \left[ \eta \left(1 - \frac{1}{\theta} \right) Y \hat{\gamma} Z^{1 - \frac{1}{\theta}} \right]^{\Theta} \left( \mathbb{E} \left[ e_j^\frac{1}{\theta} A_j^{1 - \frac{1}{\theta}} | s_j, \bar{q}_j \right] \right)^{\Theta},
\] (16)

where \( \Theta = -\frac{1}{\eta(1 - \frac{1}{\theta}) - 1} \in (1, \theta) \). We find that

\[
k_j = k(s_j, \bar{q}_j) = \phi'_0 + \Theta(1 - \frac{1}{\theta}) \frac{\tau_e}{\tau_a + \tau_e} s_j + \frac{\Theta}{\theta} \frac{\tau_q}{\tau_e + \tau_q} \bar{q}_j,
\] (17)

where the constant coefficient \( \phi'_0 \) is provided in Appendix. Because \( \bar{q}_j \) is a linear function of \( s_j \) and \( q_j \) by (12), (17) implies that \( k_j \) is also a linear function of \( s_j \) and \( q_j \), which confirms the earlier conjecture.

**Lemma 2** The firm’s investment decision at \( T_3 \), \( K(s_j, q_j) \), is given by (17) (together with (12)).

**Proof.** See Appendix. 

The realized profit for firm \( j \) at \( T_4 \) is \( \pi(a_j, \varepsilon_j, s_j, \bar{q}_j) = P_j (\varepsilon_j, Y_j) Y_j (a_j, K_j) - K_j(s_j, \bar{q}_j) \). Hence, the expected profit perceived at the stage of investment at \( T_3 \) is

\[
\mathbb{E} \left[ \pi(a_j, \varepsilon_j, s_j, \bar{q}_j) | s_j, \bar{q}_j \right] = \left[ 1 - \eta \left(1 - \frac{1}{\theta} \right) \right] \left[ \eta \left(1 - \frac{1}{\theta} \right) \right]^{\Theta - 1} \left( Y^{\frac{1}{\theta}} Z^{1 - \frac{1}{\theta}} \right)^{\Theta} \mathbb{E} \left[ e_j^\frac{1}{\theta} A_j^{1 - \frac{1}{\theta}} | s_j, \bar{q}_j \right]^{\Theta}.
\]

Exploiting the law of iterated expectations, we find that the ex ante expected profit of firm \( j \) perceived at \( T_0 \) is

\[
\mathbb{P}(\tau_e, \tau_q; Y, Z) = \mathbb{E} \mathbb{E} \left[ \pi(a_j, \varepsilon_j, s_j, \bar{q}_j) | s_j, \bar{q}_j \right] = \left[ 1 - \eta \left(1 - \frac{1}{\theta} \right) \right] \left[ \eta \left(1 - \frac{1}{\theta} \right) \right]^{\Theta - 1} \left( Y^{\frac{1}{\theta}} Z^{1 - \frac{1}{\theta}} \right)^{\Theta} \mathbb{E} \left( \mathbb{E} \left[ A_j^{1 - \frac{1}{\theta}} e_j^{\frac{1}{\theta}} | s_j, \bar{q}_j \right] \right)^{\Theta},
\]

12
where
\[
\mathbb{E} \left( \mathbb{E} \left( A_j \frac{1}{\theta} \epsilon_j \epsilon_j \mid s_j, \tilde{q}_j \right) \right)^{\Theta} = \exp \left\{ \frac{1}{2} \left\{ \left( \Theta (1 - \frac{1}{\theta}) \right)^2 - \Theta (1 - \frac{1}{\theta}) \right\} \frac{1}{\tau_{a}} + \frac{1}{2} \left\{ \left( \Theta \frac{1}{\theta} \right)^2 - \Theta 1 \right\} \frac{1}{\tau_{e}} \right\},
\]
by noting that the outer \( \mathbb{E}(\cdot) \) is the unconditional expectation operator over \( s_j \) and \( \tilde{q}_j \).

It is easy to show that
\[
\frac{\partial \Pi}{\partial \tau_e} > 0 \quad \text{and} \quad \frac{\partial \Pi}{\partial \tau_q} > 0.
\]
(19)
The intuition behind the above comparative statics is easy to understand. When the firm has a more precise signal about \( a_j \) or \( \varepsilon_j \), it makes a better investment decision because its investment can be more closely aligned with the realized productivity or demand shock. Moreover,
\[
\frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0,
\]
(20)
which means that the precisions of signals \( a_j \) and \( \varepsilon_j \) are complementary in affecting the firm’s ex ante profit.

A key insight of the baseline model is that the learning between the financial market and the real economy occurs both ways. The financial market learns information from a firm’s disclosure in trading, and conversely, the firm learns information from the financial price in making its real (investment) decision.

### 3 The Model with Endogenous Information

In this section, we study the model with acquisition of endogenous information. The purpose is to understand the information acquisition of the firm and that of the financial market, and how they interact.

We add \( T_0 \) to the timeline. At \( T_0 \), after the common productivity shock \( Z \) is realized (which becomes public information), the firm and the financial market simultaneously make their information acquisition decisions.

#### Setup
By paying an information acquisition cost \( b > 0 \) (in terms of the final consumption good), the firm receives a signal \( s_j = a_j + e_j \) with \( e_j \sim N(0, \sigma_e^2) \); otherwise, it receives a less precise signal \( s_j = a_j + e_j \) with \( e_j \sim N(0, \sigma_e^2) \), where \( \sigma_e^2 > \sigma_e^2 \). That \( \sigma_e^2 = \infty \) corresponds to the extreme case where the firm receives a useless signal. In short, \( \sigma_e^2 \in \{ \sigma_e^2, \sigma_e^2 \} \). Also denote \( \tau_e \equiv 1/\sigma_e^2 \) and \( \tau_e \equiv 1/\sigma_e^2 \). In addition, in the spirit of the classic moral hazard problem (concerning hidden actions), we assume that a firm’s choice of information precision, \( \tau_e \in \{ \tau_e, \tau_e \} \), is private information (i.e., unobservable by outsiders including financial market participants).
In the financial market, a trader can choose to be informed or uninformed. By paying an information acquisition cost $c > 0$ (in terms of the final consumption good), a trader receives a private signal $x_j^i = \varepsilon_j + g_j^i$ with $g_j^i \sim N(0, \sigma^2_j)$, as specified in the baseline model; otherwise, it receives no signal (or equivalently a useless signal). The proportion of informed speculators, $\lambda$, is endogenous.

Our assumption that the firm and financial markets have comparative advantages in acquiring information on different uncertainties is realistic. For example, financial analysts in major investment banks specializing in different regional or sectoral submarkets can on aggregate be better informed about the demand for the firm’s product than the firm itself. We can relax the assumption and alternatively assume that the firm can also obtain a noisy signal on the demand shock in addition to its signal on the productivity shock, under which our main results do not change (see Appendix B). In fact, as long as the firm learns information (about the demand shock) from the financial price, our result does not change qualitatively, no matter whether the firm itself also has some additional information about the demand shock.

**Information Acquisition Decision of Speculators**

Proportion $\lambda$ is determined such that an uninformed speculator and an informed one have the same ex ante utility:

$$\frac{EV(W^{li})}{EV(W^{ui})} = 1,$$

where $EV(W^i) \equiv \mathbb{E}[U(W^i)|s_j, q_j]$. We have the following result.

**Proposition 1** In the equilibrium of the financial market with endogenous $\lambda$, $\tau_q$ is a function of $\tau_e$ and $c$, written as $\tau_q = \tau_q(\tau_e; c)$. We have the comparative statics $\frac{\partial \tau_q}{\partial \tau_e} > 0$ and $\frac{\partial \tau_q}{\partial c} < 0$.

**Proof.** See Appendix. □

Proposition 1 states that with taking endogenous $\lambda$ into account, the informativeness of the financial price about $\varepsilon_j$ increases as the precision of the firm’s information about $a_j$ increases. The intuition behind the comparative statics is as follows. There are two driving forces under the comparative statics $\frac{\partial \tau_q}{\partial \tau_e} > 0$. First, as in the earlier discussion of Lemma 1, when uncertainty over $a_j$ decreases, informed traders trade more aggressively, increasing the informativeness of the financial price. Second, in the spirit of Grossman and Stiglitz (1980), (21) implies $e^{\gamma c} = \sqrt{\frac{\text{Var}[\varepsilon_j|s_j, q_j]}{\text{Var}[\varepsilon_j|s_j, q_j, x_j^i]}}$ (that is, $e^{\gamma c} = \sqrt{\frac{\text{Var}[(1 - \frac{1}{\theta}) a_j|s_j]}{\text{Var}[(1 - \frac{1}{\theta}) a_j|s_j] + \text{Var}(\frac{1}{\theta} \varepsilon_j|s_j, q_j, \theta)}$), the RHS of which, written as function $G(\tau_q; \tau_e)$, is the gain in information advantage for an informed speculator over an uninformed one. When one dimension of uncertainty (the term $\text{Var}[(1 - \frac{1}{\theta}) a_j|s_j]$) is reduced, the information advantage on the other dimension of uncertainty (the term $\text{Var}(\frac{1}{\theta} \varepsilon_j; q_j)$) becomes more useful. For example, in the extreme case when $\text{Var}[(1 - \frac{1}{\theta}) a_j|s_j]$ is very large, being informed has little advantage over
being uninformed. Hence, when the precision of signal \( s_j \) increases and thus \( \text{Var}(a_j|s_j) \) decreases, an uninformed speculator has incentives to *switch* to being informed by paying a cost \( c \). When more speculators acquire information, price informativeness also improves. As for comparative statics \( \frac{\partial \tau_q}{\partial c} < 0 \), a lower \( c \) induces more traders to become informed causing price informativeness to improve.

**Illustration**  We continue to use the extreme case of \( \sigma_e^2 = 0 \) (in which case informed traders are perfectly informed of \( \varepsilon_j \)) to provide explicit solutions to the financial market equilibrium. After taking endogenous \( \lambda \) into account, we have

\[
\sqrt{\frac{\text{Var}(a_j|s_j) + \text{Var}(\varepsilon_j|s_j,q)}{\text{Var}(a_j|s_j) + \text{Var}(\varepsilon_j|s_j,q_j,\varepsilon_j)}} = e^{\gamma c} \quad \Rightarrow \quad \sqrt{\frac{(1 - \frac{1}{\bar{\gamma}e + \tau_e})^2 \frac{1}{\bar{\gamma}e + \tau_e} + (\frac{1}{\bar{\gamma}e})^2 \frac{1}{\bar{\gamma}e + \tau_q}}{(1 - \frac{1}{\bar{\gamma}e})^2 \frac{1}{\bar{\gamma}e + \tau_q}} = e^{\gamma c},
\]

which implies that \( \tau_q \) is increasing in \( \tau_e \) and decreasing in \( c \). Recall that the LHS of equation (22) is written as function \( G(\tau_q; \tau_e) \). Figure 1 illustrates the solution (denoted by \( \tau_q = \tau_p \tau_e; c \)) to equation (22).

![Figure 1](image)

**Figure 1:** A graphical illustration of equilibrium \( \tau_q \) as a function of \( \tau_e \) and \( c \)

**Information Acquisition Decision of the Firm**  Considering that profit function \( \Pi(\tau_e, \tau_p; Y, Z) \) given in (18) has the properties of \( \frac{\partial \Pi}{\partial \tau_e} > 0 \) and \( \frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0 \) shown in (19) and (20), we can obtain firm \( j \)'s optimal information acquisition decision at \( T_0 \):

\[
\tau_e = \begin{cases} 
\tilde{\tau}_e & \text{if } \tau_q \geq \hat{\tau}_q(Y, Z, b) \\
\tau_e & \text{otherwise}
\end{cases}
\]
where threshold $\hat{\tau}_q \equiv \hat{\tau}_q(Y, Z, b)$ is defined as the unique root to the equation

$$\Pi (\tau_e = \hat{\tau}_e; \hat{\tau}_q, Y, Z) - \Pi (\tau_e = \tau_e; \hat{\tau}_q, Y, Z) = b.$$  \hfill (24)

It is easy to show that $\frac{\partial \tau_q(Y, Z, b)}{\partial Y} < 0$, $\frac{\partial \tau_q(Y, Z, b)}{\partial Z} < 0$, and $\frac{\partial \tau_q(Y, Z, b)}{\partial b} > 0$. We have the following result.

**Proposition 2** The optimal information acquisition decision of the firm at $T_0$, $\tau_e (\tau_q; Y, Z, b)$, is given by (23).

*Proof.* See Appendix. ■

Proposition 2 states that if and only if the firm expects the financial efficiency $\tau_q$ to exceed the threshold value $\hat{\tau}_q(Y, Z, b)$, would it choose a high precision $\tau_e = \hat{\tau}_e$. This is because precisions of signals $a_j$ and $\varepsilon_j$ are complementary in affecting the firm’s ex ante profit. Intuitively, when the uncertainty over $\varepsilon_j$ is reduced, knowing more about $a_j$ would better enable the firm to maximize its expected profit. The equilibrium $\tau_e$ also depends on $Y$ and $Z$; that is, when $Y$ or $Z$ increases, the marginal benefit of increasing the signal precision also increases for the firm, and so the firm is more likely to acquire more precise information.

Proposition 2 is a novel result of our model. Earlier work in the literature such as Goldstein and Yang (2015) has shown information production complementarity *within* the financial market. Our paper shows information production complementarity *between* the real side of the economy and the financial side.

**Full Equilibrium** With both Proposition 1 and Proposition 2, we are now able to characterize the full equilibrium. Proposition 1 gives the reaction function $\tau_q(\tau_e; c)$ while Proposition 2 gives the reaction function $\tau_e(\tau_q; Y, Z, b)$. Let

$$\tau_q^* \equiv \tau_q(\tau_e = \tau_e; c)$$

and

$$\tau_q^{**} \equiv \tau_q(\tau_e = \hat{\tau}_e; c);$$

clearly $\tau_q^{**} > \tau_q^*$ by Proposition 1. Proposition 3 follows.

**Proposition 3** The rational expectations equilibrium has three cases:

i) Case 1: $\tau_q^* > \hat{\tau}_q$ There is a unique equilibrium: $(\tau_e, \tau_q) = (\hat{\tau}_e, \tau_q^{**});$

ii) Case 2: $\hat{\tau}_q \in [\tau_q^*, \tau_q^{**}]$ There are multiple (two) equilibria: $(\tau_e, \tau_q) = (\hat{\tau}_e, \tau_q^{**})$ or $(\tau_e, \tau_q^{*});$

iii) Case 3: $\tau_q^{**} < \hat{\tau}_q$ There is a unique equilibrium: $(\tau_e, \tau_q) = (\tau_e, \tau_q^*),$

where threshold $\hat{\tau}_q = \hat{\tau}_q(Y, Z, b)$ is given by (24).
Proof. See Appendix. ■

The intuition behind Proposition 3 is as follows. If aggregate output \( Y \) is so high (and hence the threshold \( \tilde{\tau}_q(Y, Z, b) \) is so low by (24)) such that acquiring information is a dominant strategy for the firm (regardless of financial price informativeness \( \tau_q \)), then a unique equilibrium exists in which the real side acquires information and the financial efficiency is also in a higher level. This is case 1. Conversely, if aggregate output \( Y \) is so low (and hence the threshold \( \tilde{\tau}_q(Y, Z, b) \) is so high) such that not acquiring information is a dominant strategy for the firm, then a unique equilibrium exists in which the real side does not acquire information and the financial efficiency is also in a lower level. This is case 3. Between these two extreme cases, there are self-fulfilling multiple equilibria, which is case 2.

Figure 2 illustrates the three equilibrium cases, corresponding to different levels of aggregate output (i.e., \( Y_H > Y_M > Y_L \)). Recall that function \( G(\tau_q; \tau_c) \) measures the gain in information advantage for an informed speculator over an uninformed one, whereas the utility cost of information acquisition for an informed speculator is \( e^{\gamma c} \). Similarly, it is easy to see that when \( Y \) is kept constant, a change in \( b \) or \( c \) or \( Z \) (where a change in \( c \) corresponding to a vertical shift in the horizontal line \( e^{\gamma c} \) in Figure 2) also leads to different cases of equilibrium.
Figure 2: Three Cases of Equilibrium with $Y_H > Y_M > Y_L$ (Case 1: Top; Case 2: Middle; Case 3: Bottom)

4 The Macroeconomic Model

In this section, we extend the model to a macroeconomic framework. The extended model provides a macroeconomic background of the baseline model and endogenizes various exogenous specifications and variables of the baseline model. In particular, the aggregate output (i.e., real GDP), $Y$, is endogenized, which gives a number of new implications.
4.1 Setup

Final goods firms The final consumption good is produced with inputs of a continuum of capital goods according to a Dixit-Stiglitz production function

$$Y = \left[ \int \epsilon_j^\frac{1}{\sigma} Y_j^\frac{\theta - 1}{\sigma} \, dj \right]^\frac{1}{\theta - 1},$$

(25)

where $j \in [0, 1]$, $\theta > 1$ is the elasticity of substitution between intermediate capital goods, and $\epsilon_j$ measures the demand shock to intermediate good $j$.

The representative competitive final goods firm maximizes its profits:

$$\max \; P \left[ \int \epsilon_j^\frac{1}{\sigma} Y_j^\frac{\theta - 1}{\sigma} \, dj \right]^\frac{1}{\theta - 1} - \int P_j Y_j \, dj,$$

(26)

where $P_j$ is the price of intermediate good $j$. The price of the consumption good, $P$, is normalized as the numeraire price, i.e., $P = 1$. The first-order condition of (26) with respect to $Y_j$ gives

$$\left[ \int \epsilon_j^\frac{1}{\sigma} Y_j^\frac{\theta - 1}{\sigma} \, dj \right]^\frac{1}{\theta - 1} \epsilon_j^\frac{1}{\sigma} Y_j^\frac{1}{\sigma} = P_j.$$

This implies the demand schedule for good $j$:

$$Y_j = \left( \frac{1}{P_j} \right)^\theta \epsilon_j Y,$$

which endogenizes the demand function of (2).

Intermediate Goods Firms There is a continuum of intermediate (capital) goods firms, indexed by $j$. The setup for a typical firm, firm $j$, is presented in Section 2.1. We may think of each intermediate good as being produced by one firm that is located on an island in the spirit of Lucas (1972). Both $a_j$ and $\epsilon_j$ are i.i.d. across firms (or islands).

Financial markets Each island has a financial market, which trades the financial asset (derivative) contingent on the intermediate goods firm’s asset value, $V_j = Y_j P_j$, on that island. The payoff of the financial derivative contract is $v_j = \log V_j$. The demand from noise/liquidity traders in financial market $j$ is $n_j \sim N(0, \sigma_n^2)$, and $n_j$ is independent across islands. In the current framework of the aggregate economy, we may interpret noise trading as: 1) foreign capital flow, or 2) liquidity trading by some investors who must trade (for exogenous reasons such as balancing portfolios, endowment shocks, and so on).

The setup for information acquisition on each island is the same as that in Section 3.

Investors The economy consists of a continuum of investors with a unit measure. Each
investor is endowed with $W_0$ units of the final consumption good at $T_0$. Each investor has three identities: capital suppliers (i.e., lenders), firm owners (i.e., shareholders), and financial market traders. The economy is decentralized, analogous to the Robinson Crusoe economy. The decisions of an investor made under different identities are independent.

We assume that $W_0$ is sufficiently high and a storage technology exists, so that in equilibrium $R_f = 1$. An investor maximizes utility:

$$U(C^i) = -\exp(-\tau C^i)$$

with

$$C^i = W_0 + (\Pi - \chi) + D^i,$$

where $C^i$ is the end-of-period wealth at $T_4$ for investor $i$. The term $\Pi$ is the aggregate profit of firms, that is, $\Pi = \int (P_j Y_j - R_f K_j) \, dj$, the value of which is given in (18). The term $\Pi - \chi$ is the aggregate net profit of firms distributed to an investor (as an owner of firms), where $\chi \in \{0, b\}$ is the aggregate information acquisition cost to the firms. The term $D^i$ is investor $i$’s position in financial market trading. We make a weak assumption that a trader can access trading in all financial markets.

In a symmetric equilibrium, a trader is informed on $\lambda$ fraction of islands and uninformed on the remaining $1 - \lambda$ fraction of islands. In addition, all firms have $\lambda$ proportion of informed traders and $1 - \lambda$ proportion of uninformed traders, so $D^i = -\lambda c + \int_{j \in I_i} d_{ij}^i (v_j - q_j) \, dj + \int_{j \in [0,1] \setminus I_i} d_{ij}^{u,i} (v_j - q_j) \, dj$, where $I_i$ is the set of islands on which trader $i$ is informed, $d_{ij}^{I,i}$ is that trader’s position on island $j$ as an informed trader, and $d_{ij}^{u,i}$ is that trader’s position on island $j$ as an uninformed trader.

### 4.2 Equilibrium

Within each island, the equilibrium is given by Lemmas 1-2 and Propositions 1-3. Now we study the equilibrium of the aggregate economy, endogenizing $Y$. We consider the symmetric equilibrium, in which all intermediate goods firms have the same level of information precision.

Substituting (16) and (1) into (25) yields

$$Y = Z^\Theta \left[ \eta \left( 1 - \frac{1}{\theta} \right) Y_s^{\frac{1}{\theta}} \right]^{\Theta \eta} \left[ \int e_j^{\frac{1}{2} A_j^{1-\frac{1}{\theta}}} \left( \mathbb{E} \left[ e_j^{\frac{1}{2} A_j^{1-\frac{1}{\theta}} | s_j, q_j \right] \right)^{\Theta - 1} \, dj \right]^{\frac{\theta}{\theta - 1}}.$$  

(28)

Exploiting the law of iterated expectations, we have

$$\int e_j^{\frac{1}{2} A_j^{1-\frac{1}{\theta}}} \left( \mathbb{E} \left[ e_j^{\frac{1}{2} A_j^{1-\frac{1}{\theta}} | s_j, q_j \right] \right)^{\Theta - 1} \, dj = \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{2} A_j^{1-\frac{1}{\theta}} | s_j, q_j \right] \right)^{\Theta} \right].$$

10 We will consider the symmetric equilibrium, in which either all firms or none of them acquires information.
Hence, (28) implies

\[ Y = Z^\Theta \eta^\frac{\theta}{\sigma - \theta \eta} \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\theta \eta \frac{\theta}{\sigma - \theta \eta}} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ \epsilon_j A_j^{1 - \frac{\theta}{\sigma}} | s_j, \tilde{q}_j \right] \right)^\theta \right] \right\}^{\frac{\theta}{\sigma - \theta \eta}}. \]

Similarly, the aggregate investment in the economy is given by

\[ K = \int K_j d\tilde{q}_j = Z^{(1 - \frac{\theta}{\sigma}) \Theta} \left[ \eta \left( 1 - \frac{1}{\theta} \right) Y^\frac{\theta}{\sigma} \right]^{\Theta} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ \epsilon_j A_j^{1 - \frac{\theta}{\sigma}} | s_j, \tilde{q}_j \right] \right)^\theta \right] \right\}^{\frac{\theta}{\sigma - \theta \eta}}. \] (29)

Therefore, we have

\[ Y = ZAK^\eta, \] (30)

where \( A = A(\tau_e, \tau_q) \) is given by

\[
A(\tau_e, \tau_q) = \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ \epsilon_j A_j^{1 - \frac{\theta}{\sigma}} | s_j, \tilde{q}_j \right] \right)^\theta \right] \right\}^{\frac{\theta}{\sigma - \theta \eta}} \\
\quad = \exp \left\{ \frac{\theta}{\eta} \left( \frac{\theta}{\eta} - 1 \right) \left( \Theta - 1 \right) \left[ \frac{1}{2} (1 - \frac{1}{\theta})^2 \tau_e + \frac{1}{2} (1 - \frac{1}{\theta})^2 \tau_a + \frac{1}{2} (\Theta^\frac{1}{\theta})^2 \frac{1}{\tau_e} \right] \right. \\
\quad \quad \quad \quad \quad \quad \left. - \Theta (\Theta - 1) \left[ \frac{1}{2} (1 - \frac{1}{\theta})^2 \frac{1}{\tau_a + \tau_e} + \frac{1}{2} (\frac{1}{\theta})^2 \frac{1}{\tau_a + \tau_q} \right] \right\}. \] (31)

which is the endogenous aggregate TFP.

Equation (30) implies that despite heterogeneity among firms caused by idiosyncratic productivity shocks and demand shocks, our economy works as if there existed a representative firm with productivity \( A \) and aggregate investment \( K \).

By (29) and (30), the aggregate investment can also be expressed as

\[ K = K(\tau_e, \tau_q; Z) = \left[ \eta \left( 1 - \frac{1}{\theta} \right) Z \cdot A(\tau_e, \tau_q) \right]^{\frac{\theta}{1 - \eta \frac{\theta}{\sigma}}}. \]

Because \( 0 < \eta < 1 \) and \( \Theta \in (1, \theta) \), \( K \) is increasing in \( A \) and thus is increasing in \( \tau_e \) and \( \tau_q \). This way, the aggregate output can also be expressed as

\[ Y = Y(\tau_e, \tau_q; Z) = \left[ \eta \left( 1 - \frac{1}{\theta} \right) Z \cdot A(\tau_e, \tau_q) \right]^{\frac{\theta}{1 - \eta \frac{\theta}{\sigma}}}. \] (32)

Proposition 4 follows.

**Proposition 4** Both the endogenous aggregate TFP, \( A \), and the aggregate investment, \( K \), are increasing in \( \tau_e \) and \( \tau_q \). Hence, the aggregate output \( Y \) is increasing in \( \tau_e \) and \( \tau_q \).

**Proof.** See Appendix. \( \blacksquare \)
Proposition 4 highlights two effects of information frictions. First, given $K$, the endogenous aggregate TFP, $A$, measuring the efficiency of resource allocation, has the properties of $\frac{\partial A}{\partial \epsilon} > 0$ and $\frac{\partial A}{\partial q} > 0$. Efficient allocation requires more resources to be allocated to firms with higher realized $A_j$ and $\epsilon_j$. In other words, efficient investment $K_j$ should be more aligned with realized $A_j$ and $\epsilon_j$. So, more precise information about $A_j$ and $\epsilon_j$ achieved through information acquisition helps improve allocative efficiency. Second, higher uncertainty also leads to a lower level of aggregate investment, that is, $\frac{\partial K}{\partial \epsilon} > 0$ and $\frac{\partial K}{\partial q} > 0$.

We parameterize our economy by $(b, c, Z)$, and characterize the full equilibrium of the aggregate economy, which is given by the following joint equations:

\begin{align*}
\tau_e &= \tau_e(\tau_q; Y, Z, b) \quad \text{(A firm’s optimal information choice)} \quad (33) \\
\tau_q &= \tau_q(\tau_e; c) \quad \text{(Financial market equilibrium)} \quad (34) \\
Y &= Z \cdot A(\tau_e, \tau_q) [K(\tau_e, \tau_q; Z)]^\eta, \quad \text{(Aggregate economy equilibrium)} \quad (35)
\end{align*}

where (33), (34) and (35) are given by Proposition 2, Proposition 1 and Proposition 4, respectively.

**Proposition 5** The full equilibrium of the aggregate economy is characterized by triplet $(\tau_e, \tau_q, Y)$, which solves the system of equations (33)-(35) for given $(b, c, Z)$.

We discuss three important implications of Proposition 5. First, a small shock (i.e., a small increase in $c$ or $b$, or a small decrease in $Z$) can have a large impact on the aggregate economy (aggregate output $Y$) due to the compound feedback loops of information amplification, as illustrated in Figures 3; detailed quantitative analysis of the comparative statics with respect to $c$, $b$, and $Z$ will be provided in the next section. Our information channel of amplification contrasts with the financing channel in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012), where a negative shock originating in either the real sector or the financial sector can lead to a large drop in the aggregate output.

![Figure 3: Information amplification](image-url)
In particular, the amplification in our model can arise from the presence of multiple equilibria (i.e., discontinuity). That is, pure self-fulfilling beliefs in the absence of any aggregate shock or a small aggregate shock can trigger the equilibrium switching from one regime to the other, generating a very large drop in the aggregate-level output and investment. To illustrate the effect, we fix $c$ and $b$, and express $Y$ as a function of $Z$ (recall equation (32)); see the formal expression of function $Y(Z; A)$ including the thresholds $Z$ and $\bar{Z}$ in (A.7) in the appendix. Figure 4 depicts function $Y(Z; A)$.

Recall that $A$ is the endogenous TFP (given in (31)). In Figure 4, when $Z$ is low enough such that $Z < Z_2(1, \bar{Z})$, there is a unique “bad” equilibrium; when $Z$ is high enough such that $Z > \bar{Z}$, there is a unique “good” equilibrium. When $Z_2(Z, \bar{Z})$, there are two self-fulfilling equilibria. Clearly, a small change in $Z$ around $Z = \bar{Z}$ (i.e., a slight decrease in $Z$ from above $\bar{Z}$ to below $\bar{Z}$) can trigger the equilibrium switching from “good” to “bad”, resulting in a large drop in $Y$. Figure 4 shows several interesting implications. The first is that a small shock and a big shock to $Z$ can have dramatically different implications. While a small decline in $Z$ leads to a steady decrease in output, a big decline in $Z$ may trigger a self-fulfilling crisis. The second implication is that a positive shock and a negative shock to $Z$ potentially have asymmetric effects on equilibrium output. When $Z$ increases, the equilibrium output increases steadily. However, when $Z$ declines, the equilibrium output may exhibit a sudden large decline if the economy falls to the bad equilibrium. Conducting comparative statics with respect to $c$ and $b$ instead of $Z$ shows similar patterns (see next section).

Second, our model implies information contagion and spillover, as illustrated in Figures 5. That is, a shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. This result is consistent with a large amount of anecdotal evidence that idiosyncratic firm-level shocks can be the origin of aggregate fluctuations (i.e., microfoundation for aggregate shocks; see Gabaix (2011)). Again, a quantitative
analysis of this result will be provided in the next section.

Third, our model endogenizes together the three variables — financial uncertainty, real uncertainty, and aggregate economic activities — and show how they are related. The residual financial uncertainty (or equivalently the financial market efficiency defined in Brunnermeier (2005) and Goldstein and Yang (2015)) is given by

$$SD(\varepsilon_j|s_j, q_j) = \sqrt{\frac{1}{\tau_x + \tau_q}},$$

and the residual real uncertainty (or the forecast error) faced by a firm is given by

$$SD(a_j|s_j) = \sqrt{\frac{1}{\tau_a + \tau_e}}.$$  

We have shown that an adverse shock in $c$ or $b$ or $Z$ leads to a decrease in $\tau_e$ and $\tau_q$ together with a decrease in $Y$, which means that a rise in both real uncertainty and financial uncertainty is accompanied by a fall in aggregate GDP $Y$. In other words, uncertainty in both financial markets and the real economy rises during recessions.

5 Quantitative Analysis

Our analytic analysis in the previous sections has demonstrated that the information interplay between the real sector and the financial sector can have strong effects on the economy. Our model is too stylized to be calibrated with the data. We will therefore assign values to parameters in our model to conduct several numerical illustrations below.

---

11 Equivalently, we can define financial uncertainty as $SD(\varepsilon_j|s_j, q_j) = \sqrt{(\frac{1}{\tau_x})^2 Var[\varepsilon_j|s_j, q_j] + (1 - \frac{1}{\tau})^2 Var[a_j|s_j] = \sqrt{\frac{1}{\tau_x + \tau_q} + (1 - \frac{1}{\tau})^2 \frac{1}{\tau_a + \tau_e}}}$.  
12 The evidence of countercyclical uncertainty in the macroeconomics literature is in line with the findings in the finance literature that the volatility of stock returns is higher in bad times than in good times (see, e.g., Schwert (1989a,b)).
Table 1 summarizes the parameter values chosen. We set the elasticity of substitution between intermediate goods $\theta$ to 6 as in David, Hopenhanya and Venkateswaran (2016). This implies that the gross markup is 15%. We set the degree of decreasing returns to scale of production $\eta$ to 0.8, consistent with the recent estimates of Gopinath et al (2016). We set the risk (CARA) coefficient $\gamma$ to 2. This implies that a risky lottery drawn from a normal distribution with standard deviation of 0.2 $\$ will need an average payoff of 1.04 $\$ dollar to be equivalent to 1 $\$ for sure in financial speculations, which seems to be reasonable and in line with the risk premium between stocks and bonds. The commonly estimated standard deviation of firm-level physical productivity shocks $\sigma_a$ is typically very large. However, it is difficult to imagine that firms would face such enormous uncertainty when making investment. We therefore borrow the unconditional residual uncertainty parameter from David, Hopenhanya and Venkateswaran (2016), where $\sigma_a = 0.45$ (or $\tau_a = 4.9383$). We set $\tau_e = 6.5746$, implying that a firm can reduce its residual uncertainty on the productivity shock to 0.30 by paying the information acquisition cost. We set $\tau_a = 1$, implying that the residual uncertainty in productivity equals 0.41 if a firm does not pay the information acquisition cost. Foster, Haltiwanger and Syverson (2008) show that demand shocks are more important than physical productivity shocks for firm turnover. We therefore set $\sigma_e = 2.2502$ (or $\tau_e = 0.1975$), implying that demand and productivity shocks contribute equally to firms’ sales volatility. We set the precision of informed traders’ signal $\tau_{\theta}$ to 0.2922, meaning that an informed trader can reduce the conditional volatility of demand shock to 1.429 through his private signal. We set the common productivity $Z = 2$, information acquisition cost for a financial trader $c = 0.09$, and information acquisition cost for firm $b = 0.065$. These parameter values lead to two self-fulfilling equilibria in our model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_a$</td>
<td>Precision of productivity shock prior</td>
<td>4.9383</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Precision of demand shock prior</td>
<td>0.1975</td>
</tr>
<tr>
<td>$\tau_{\theta}$</td>
<td>Precision of informed traders’ signal</td>
<td>0.2922</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between intermediate goods</td>
<td>6</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk (CARA) coefficient</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Degree of decreasing returns to scale of production</td>
<td>0.8</td>
</tr>
<tr>
<td>$c$</td>
<td>Information acquisition cost of financial markets</td>
<td>0.09</td>
</tr>
<tr>
<td>$b$</td>
<td>Information acquisition cost of the real side</td>
<td>0.065</td>
</tr>
<tr>
<td>$Z$</td>
<td>Common productivity shock</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Low precision of signals of the real side</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{\tau}_e$</td>
<td>High precision of signals of the real side</td>
<td>6.5746</td>
</tr>
</tbody>
</table>

**Table 1: Parameter values**

Table 2 summarizes key results of the two self-fulfilling equilibria. First, both aggregate output
and investment fall dramatically (by 42% and 42%, respectively) when the economy falls into the “bad” equilibrium. Second, information production from the financial sector and from the real sector is both lower in the “bad” equilibrium than in the “good” equilibrium. The firms and financial traders face productivity shocks with a posterior standard deviation of $(\tau_e + \tau_a)^{-\frac{1}{2}} = 0.2947$ in the “good” equilibrium and $(\tau_e + \tau_a)^{-\frac{1}{2}} = 0.4104$ in the “bad” equilibrium. The financial price can reduce the posterior standard deviation of firm demand shocks to $(\tau_q^{**} + \tau_e)^{-\frac{1}{2}} = 1.8380$ for the “good” equilibrium but only to $(\tau_q^* + \tau_e)^{-\frac{1}{2}} = 2.1567$ for the “bad” equilibrium. These numbers imply a 17% increase in financial uncertainty and a 33% increase in real uncertainty from the “bad” equilibrium to the “good” equilibrium. Third, the resulting information production declines have important consequences for allocation efficiency. The endogenous TFP declines by about 10%.

To understand the decline, we compute two alternative counterfactual endogenous TFP. We first compute $A(\bar{\tau}, \tau_q^*)$, the level of endogenous TFP when only the quality of the information provided by the financial market deteriorates while the quality of the information provided by the firm stays at the level $\bar{\tau}$. We find that TFP would decline by about 4%. The other 6% decline in endogenous TFP is due to the decline in the firm’s information production as indicated by $A(\bar{\tau}, \tau_q^{**})$, the level of endogenous TFP that the economy would obtain when only the quality of the information provided by the firm deteriorates while the quality of the information provided by the financial market stays at the level $\tau_q^{**}$.

<table>
<thead>
<tr>
<th></th>
<th>“Good” Equilibrium</th>
<th>“Bad” equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_e$</td>
<td>$\bar{\tau}_e = 6.5746$</td>
<td>$\bar{\tau}_e = 1$</td>
</tr>
<tr>
<td>$\tau_q$</td>
<td>$\tau_q^{**} = 0.0985$</td>
<td>$\tau_q^* = 0.0175$</td>
</tr>
<tr>
<td>GDP (Y)</td>
<td>1.5109</td>
<td>0.8701</td>
</tr>
<tr>
<td>Aggregate Investment (K)</td>
<td>1.0073</td>
<td>0.5801</td>
</tr>
<tr>
<td>Endogenous TFP (A)</td>
<td>$\bar{A} = 0.7511$</td>
<td>$\bar{A} = 0.6726$</td>
</tr>
<tr>
<td>TFP under changing $\tau_q$ only ($A(\bar{\tau}, \tau_q^*)$)</td>
<td>0.7199</td>
<td>0.7017</td>
</tr>
<tr>
<td>TFP under changing $\bar{\tau}$ only ($A(\bar{\tau}, \tau_q^{**})$)</td>
<td>0.7199</td>
<td>0.7017</td>
</tr>
</tbody>
</table>

Table 2: Numerical illustration for two self-fulfilling equilibria

As the nature of equilibrium crucially depends on the values of $Z$, $c$ and $b$, our second exercise is hence to conduct complete comparative statics to understand their impact on equilibria in a quantitative sense. We fix all other parameter values (given in Table 1) but change one of $Z$, $c$ and $b$ each time and compute the equilibria accordingly. We report our results in Figure 6. Figure 6 has three columns, summarizing the comparative statics with respect to $Z$, $c$, and $b$ in turn. The first panel plots the equilibrium aggregate output, the second panel plots the posterior standard deviation of firms’ productivity shocks (given in (37)), and the third panel reports the posterior

\[ \bar{A} = A(\bar{\tau}, \tau_q^{**}) \] and \[ \bar{A} = A(\bar{\tau}, \tau_q^*); \text{ see } (A.7). \]
standard deviation of firms’ demand shocks inferred from the financial price (given in (36)).

Figure 6: Comparative static analysis

The first column of Figure 6 shows that the equilibrium is unique if \( Z > \bar{Z} = 2.0779 \) or \( Z < \bar{Z} = 1.9250 \). Suppose the economy initially starts with aggregate common productivity \( Z = 2.1203 \). A small drop in \( Z \) by more than 2 percent would trigger a self-fulfilling crisis (also see Figure 4). In the second and third columns of Figure 6, the model generates two self-fulfilling equilibria when the information acquisition cost, \( c \) or \( b \), is at the intermediate level.\(^{14}\) Due to the binary information choice of the firm, aggregate output is insensitive to the change in \( b \) when the equilibrium is unique. Similar to the effect of \( Z \), if the initial level of \( b \) or \( c \) is close to their lower threshold for multiple equilibria, then a small shock (i.e., a small increase in \( c \) or \( b \)) can cause a sudden large decline in aggregate output.

Our final exercise is to show information contagion in our model. Again we assume that all parameters are initially given in Table 1, except that we set \( b = 0.0786 \), slightly lower than the upper threshold of \( b \) to have multiple (two) equilibria. According to Figure 6, the economy initially has two equilibria. Now we assume that a small fraction \( \kappa = 5\% \) of firms suffer a shock in the sense that their information acquisition cost \( b \) increases slightly to 0.0788, which means that a unique “bad” equilibrium takes over in the economy of these islands by Figure 6. How about the other 95% islands? The economy of the other 95% islands will unavoidably fall into the bad equilibrium unless their acquisition cost \( b \) decreases below 0.0778.

\(^{14}\)In the third common (the comparative statics with regard to \( c \)), when \( c \) is higher than a certain level, no traders acquire information about the demand shocks and hence the financial price efficiency \( \tau_q \) becomes a constant.
6 Conclusion

We develop a model of informational interdependence between financial markets and the real economy. We endogenize financial and real uncertainty and show how they relate to aggregate economic activity. Information production in the real sector and that in the financial sector exhibit strategic complementarity. The key reason is that a financial price is a combination of firm disclosure and financial market price discovery. When a firm tries to maximize its monopoly profits in the real sector and speculators try to gain from arbitraging in financial markets, it is optimal for them to learn from each other. The mutual learning results in strategic complementarity in information production. In the general equilibrium, the amount of information available in the economy and the aggregate economic activity feed back into each other and reinforce each other.

Besides the implications on economic uncertainty, our model has additional implications regarding macroeconomic activities, including the effect of financial market efficiency on resource misallocation, amplification and contagion in business cycle fluctuations, and self-fulfilling economic crises. Our model is essentially static and qualitative, which can clearly and cleanly deliver key mechanisms and implications of the model. Extending our static model to a dynamic economy and fully examining the quantitative importance of our mechanisms are left for future research.
Proof of Lemma 1: For an informed trader,

\[
\mathbb{E}[v_j|s_j, q_j, x_j] = \frac{1}{\theta} \mathbb{E}[\varepsilon_j|s_j, q_j, x_j] + \eta \left( 1 - \frac{1}{\theta} \right) k(s_j, q_j) + \frac{1}{\theta} y + \left( 1 - \frac{1}{\theta} \right) \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_e}{\tau_a + \tau_e} \right]
\]

where \( E[\varepsilon_j|s_j, q_j, x_j] = \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_a}{\tau_a + \tau_e + \tau_q} x_j \), and \( q_j(s_j, q_j) = g_j - \beta_0 - \beta_1 s_j \), and

\[
\text{Var}[v_j|s_j, q_j, x_j] = \left( \frac{1}{\theta} \right)^2 \text{Var}[\varepsilon_j|s_j, q_j, x_j] + \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}
\]

where \( \text{Var}[\varepsilon_j|s_j, q_j, x_j] = \frac{1}{\tau_a + \tau_e + \tau_q} \).

For an uninformed trader,

\[
\mathbb{E}[v_j|s_j, q_j] = \frac{1}{\theta} \mathbb{E}[\varepsilon_j|s_j, q_j] + \frac{1}{\theta} y + \left( 1 - \frac{1}{\theta} \right) \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_e}{\tau_a + \tau_e} \right]
\]

where \( \mathbb{E}[\varepsilon_j|s_j, q_j] = \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_a}{\tau_a + \tau_e} \tilde{q}_j \), and

\[
\text{Var}[v_j|s_j, q_j] = \left( \frac{1}{\theta} \right)^2 \text{Var}[\varepsilon_j|s_j, q_j] + \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}
\]

where \( \text{Var}[\varepsilon_j|s_j, q_j] = \frac{1}{\tau_a + \tau_e} \).

Therefore, the market clearing condition, (7), implies

\[
0 = n_j + \lambda \frac{1}{\theta} \left[ \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \tilde{q}_j \right] + \eta \left( 1 - \frac{1}{\theta} \right) k(s_j, q_j) + \frac{1}{\theta} y + \left( 1 - \frac{1}{\theta} \right) \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_e}{\tau_a + \tau_e} \right]
\]

\[
+ \left( 1 - \lambda \right) \frac{1}{\theta} \left[ \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \tilde{q}_j \right] + \eta \left( 1 - \frac{1}{\theta} \right) k(s_j, q_j) + \frac{1}{\theta} y + \left( 1 - \frac{1}{\theta} \right) \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_a^2 \right) + \frac{\tau_e}{\tau_a + \tau_e} \right]
\]

\[
\gamma \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_q} + \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}
\]

\[
\gamma \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_q} + \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}
\]

(A.1)

It is straightforward to see that (A.1) can be transformed to

\[
f(s_j, q_j, y) + \lambda \frac{1}{\theta} \left[ \frac{\tau_a}{\tau_a + \tau_e + \tau_q} \right] \varepsilon_j + n_j = 0,
\]
where $f(s_j, q_j, y)$ is a linear function of $s_j, q_j$ and $y$. Hence,

$$\beta_3 = \gamma \left[ \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_q + \tau_e} + \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e} \right],$$

which, by substituting $\tau_q \equiv \frac{1}{\beta_3 \sigma_n^2}$, implies

$$\frac{\lambda}{\theta} \left[ (\tau_a + \tau_e) \tau_e \right] \beta_3^3 - \gamma \left[ \left( \frac{1}{\theta} \right)^2 (\tau_a + \tau_e) + \left( 1 - \frac{1}{\theta} \right)^2 (\tau_e + \tau_e) \right] \beta_3^2 - \gamma \left( 1 - \frac{1}{\theta} \right)^2 \tau_n = 0, \quad (A.2)$$

where $\tau_n = \frac{1}{\sigma_n^2}$.

(A.2) clearly has a unique positive solution with respect to $\beta_3$. In fact, if we write LHS of (A.2) as function $\Lambda(\beta_3)$, it is easy to show that equation $\Lambda(\beta_3) + \gamma (1 - \frac{1}{\theta})^2 \tau_n = 0$ has a unique positive solution. Hence, equation $\Lambda(\beta_3) = 0$ has a unique positive solution, around which $\frac{\partial \Lambda}{\partial \beta_3} > 0$. We also prove that the unique positive solution of $\beta_3$ is decreasing in $\lambda$. In fact, $\frac{\partial \Lambda}{\partial \lambda} = 0$ and $\frac{\partial \Lambda}{\partial \beta_3} = \frac{1}{\theta} \left[ (\tau_a + \tau_e) \tau_e \right] \beta_3^3 > 0$, so $\frac{d \beta_3}{d \lambda} = -\frac{\partial \Lambda}{\partial \beta_3} < 0$. Also, the unique positive solution of $\beta_3$ is decreasing in $\tau_e$. In fact,

$$\frac{\partial \Lambda}{\partial \tau_e} = \frac{\lambda}{\theta} \tau_e \beta_3^3 - \gamma \left( \frac{1}{\theta} \right)^2 \beta_3^2 = \frac{\gamma (1 - \frac{1}{\theta})^2 (\tau_e + \tau_q) \beta_3^2 + \gamma (1 - \frac{1}{\theta})^2 \tau_n}{\tau_a + \tau_e} > 0,$$

where the second equality is due to (A.2), so $\frac{d \beta_3}{d \tau_e} = -\frac{d \Lambda}{d \beta_3} < 0$ (or $\frac{d \tau_q}{d \tau_e} > 0$).

**Proof of Lemma 2:** We have

$$\log E \left[ e_j^2 A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] = \frac{1}{\theta} \left[ \frac{\tau_e}{\tau_a + \tau_q} \left( -\frac{1}{2} \sigma_{\tilde{q}}^2 \right) + \frac{\tau_q}{\tau_e + \tau_q} \bar{d}_j \right] + \left( 1 - \frac{1}{\theta} \right) \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_n^2 \right) + \frac{\tau_e}{\tau_a + \tau_e} s_j \right]
+ \frac{1}{2} \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_q} + \frac{1}{2} \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}.$$

So

$$\phi_0 = \Theta \log \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right] + \Theta \left[ \frac{\tau_e}{\tau_a + \tau_q} \left( -\frac{1}{2} \sigma_{\tilde{q}}^2 \right) \right] + \Theta \left[ \frac{\tau_a}{\tau_a + \tau_e} \left( -\frac{1}{2} \sigma_n^2 \right) \right]
+ \frac{1}{2} \Theta \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_q} + \frac{1}{2} \Theta \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e}.$$
In addition,

\[
\mathbb{E}\left( \mathbb{E} \left( A_j^{1-\frac{1}{\Theta}} \epsilon_j^{1/\Theta} | s_j, \tilde{q}_j \right) \right)^{\Theta^*} = \exp \left\{ \frac{1}{2} \left[ \left( \Theta \left( 1 - \frac{1}{\Theta} \right) \right)^2 - \Theta \left( 1 - \frac{1}{\Theta} \right) \right] \frac{1}{\tau_a} + \frac{1}{2} \left[ \left( \Theta \frac{1}{\Theta} \right)^2 - \Theta \frac{1}{\Theta} \right] \frac{1}{\tau_e} \right\}.
\]

It is easy to show that

\[
\frac{\partial \Pi}{\partial \tau_e} > 0 \quad \text{and} \quad \frac{\partial \Pi}{\partial \tau_q} > 0,
\]

by noting that

\[
\text{sgn} \left( \frac{\partial \Pi}{\partial \tau_e} \right) = \text{sgn} \left\{ \exp \left\{ \frac{1}{2} \left[ \left( \Theta \left( 1 - \frac{1}{\Theta} \right) \right)^2 - \Theta \left( 1 - \frac{1}{\Theta} \right) \right] \frac{1}{\tau_a} + \frac{1}{2} \left[ \left( \Theta \frac{1}{\Theta} \right)^2 - \Theta \frac{1}{\Theta} \right] \frac{1}{\tau_e} \right\} \right\}.
\]

Moreover,

\[
\frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0 \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial \tau_e \partial Y} > 0.
\]

**Proof of Proposition 1:** The proof is quite similar to that in Grossman and Stiglitz (1980). By the definition \( EV(W^i) \equiv \mathbb{E} [U(W^i)|s_j, q_j] \), we have

\[
\frac{EV(W^{i})}{EV(W^{U})} = e^{\gamma c} \sqrt{\frac{\text{Var}[v_j|s_j, q_j, x_j]}{\text{Var}[v_j|s_j, q_j]}},
\]

Thus,

\[
\frac{EV(W^{i})}{EV(W^{U})} = 1 \iff \frac{\text{Var} \left[ \left( 1 - \frac{1}{\Theta} \right) a_j | s_j \right]}{\text{Var} \left[ \left( 1 - \frac{1}{\Theta} \right) a_j | s_j \right] + \text{Var} \left( \frac{1}{\Theta} \epsilon_j | s_j, q_j \right)} = e^{\gamma c}
\]

\[
\iff \sqrt{\frac{(1 - \frac{1}{\Theta})^2 \frac{1}{\tau_a + \tau_e} + (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}}{(1 - \frac{1}{\Theta})^2 \frac{1}{\tau_a + \tau_e} + (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}}} = e^{\gamma c}
\]

(A.3)

Let \( F(\tau_q; \tau_e) = \frac{(1 - \frac{1}{\Theta})^2 \frac{1}{\tau_a + \tau_e} + (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}}{(1 - \frac{1}{\Theta})^2 \frac{1}{\tau_a + \tau_e} + (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}} \), which implies

\[
\frac{\partial F}{\partial \tau_e} = -\left( 1 - \frac{1}{\Theta} \right)^2 \left( \frac{1}{\tau_a + \tau_e} \right)^2 \left[ \frac{(\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q} - (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}}{(1 - \frac{1}{\Theta})^2 \frac{1}{\tau_a + \tau_e} + (\frac{1}{\Theta})^2 \frac{1}{\tau_e + \tau_q}} \right]^2 < 0.
\]

31
and

\[
\frac{\partial F}{\partial \tau_q} = \left\{ \frac{(1 - \frac{1}{\theta})^2}{\tau_a + \tau_e} \frac{1}{\tau_e + \tau_q} \left[ \left( \frac{1}{\tau_a + \tau_e} - \left( \frac{1}{\tau_a + \tau_e + \tau_q} \right)^2 \right) \right] + \frac{(1 - \frac{1}{\theta})^4}{\tau_a + \tau_e} \frac{1}{\tau_q + \tau_e + \tau_q} \right\} \frac{1}{\tau_a + \tau_e + \tau_q} > 0.
\]

Because \( F(\tau_e; \tau_q) = e^{-2\gamma c} \), by differentiating both sides of the equation, we have

\[
\frac{\partial \tau_q}{\partial \tau_e} = -\frac{\partial F / \partial \tau_e}{\partial F / \partial \tau_q} > 0.
\]

Also, by \( \frac{\partial F}{\partial \tau_q} > 0 \), we have

\[
\frac{\partial \tau_q}{\partial c} < 0.
\]

**Proof of Proposition 2:** Because \( \frac{\partial \Pi}{\partial \tau_e} > 0 \), we have \( \Pi(\tau_e = \bar{\tau}_e; \tau_q, Y) - \Pi(\tau_e = \bar{\tau}_e; \tau_q, Y) > 0 \) for a given \( \tau_q \) and \( Y \). Because \( \frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0 \), there exists a unique \( \tilde{\tau}_q \) such that

\[
\Pi(\tau_e = \bar{\tau}_e; \tilde{\tau}_q, Y) - \Pi(\tau_e = \bar{\tau}_e; \tilde{\tau}_q, Y) = b. \tag{A.4}
\]

Denote the LHS of (A.4) by function \( \Gamma(\tilde{\tau}_q, Y) \) for given a \( \bar{\tau}_e \) and \( \bar{\tau}_e \). Because \( \frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0 \) and \( \frac{\partial^2 \Pi}{\partial \tau_q \partial Y} > 0 \), we have that \( \frac{\partial \Gamma}{\partial \tau_q} > 0 \) and \( \frac{\partial \Gamma}{\partial Y} > 0 \). Therefore,

\[
\frac{\partial \tilde{\tau}_q(Y, b)}{\partial Y} = -\frac{\partial \Gamma}{\partial \tau_q} < 0
\]

and

\[
\frac{\partial \tilde{\tau}_q(Y, b)}{\partial b} > 0.
\]

**Proof of Proposition 3:** Based on the results in Propositions 1 and 2, it is straightforward to obtain Proposition 3.
Proof of Proposition 4: Substituting (16) and (1) into (25)

\[
Y = \left[ \int e_j^{\frac{1}{\theta}} Y_j^{\frac{\theta - 1}{\theta}} \, dj \right]^{\frac{\theta}{\theta - 1}}
\]

\[
= \left[ \int e_j^{\frac{1}{\theta}} \left( A_j \left\{ \eta \left( 1 - \frac{1}{\theta} \right) Y_j^{\frac{1}{\theta}} \right\} \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\frac{\eta}{\theta - 1}} \right) \, dj \right]^{\frac{\theta}{\theta - 1}}
\]

\[
= \left[ \eta \left( 1 - \frac{1}{\theta} \right) Y_j^{\frac{1}{\theta}} \right]^{\frac{\eta}{\theta - 1}} \left[ \int e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\frac{\eta}{\theta - 1}} \, dj \right]^{\frac{\theta}{\theta - 1}}, \quad (A.5)
\]

where the last equality follows based on \( \Theta = -\frac{1}{\eta (1 - \frac{1}{\theta}) - 1} \).

Exploiting the law of iterated expectations, we have

\[
\int e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\frac{\eta}{\theta - 1}} \, dj = \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\frac{\eta}{\theta - 1}} \right].
\]

Hence, (A.5) becomes

\[
Y = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\eta}{\theta - 1}} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\frac{\eta}{\theta - 1}} \right] \right\}^{\frac{\theta}{\theta - 1}} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1}. \quad (A.6)
\]

Similarly, the aggregate investment in the economy is given by

\[
K = \int K_j \, dj = \left[ \eta \left( 1 - \frac{1}{\theta} \right) Y_j^{\frac{1}{\theta}} \right]^{\Theta} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\theta}} A_j^{1 - \frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right] \right\}^{\frac{\theta}{\theta - 1}} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} + \Theta.
\]

where the second equality is obtained by substituting the expression of \( Y \) in (A.6). Because \( \Theta \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} + \Theta = \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} - \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \frac{\theta}{\theta - 1} \eta = \frac{\theta}{\theta - 1} - \eta \),

\[
Y = AK^\eta,
\]
where $A = A(\tau_e, \tau_q)$ is given by

$$A(\tau_e, \tau_q) = \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\tilde{eta}}} A_j^{1 - \frac{1}{\tilde{eta}}} | s_j, \tilde{q}_j \right] \right)^{\theta} \right] \right\}^{-\frac{\theta}{\theta - 1 - \eta}}$$

$$= \exp \left\{ \left( \frac{\theta}{\theta - 1} - \eta \right) \left( \Theta (1 - \frac{1}{\theta}) \left( -\frac{1}{2} \tau_e^{2} \right) + \frac{1}{2} \left[ \Theta (1 - \frac{1}{\theta}) \right]^{2} \frac{1}{\tau_a^{2}} + \Theta^{2} \left( -\frac{1}{2} \tau_e^{2} \right) + \frac{1}{2} \left( \Theta^{2} \right)^{2} \frac{1}{\tau_e^{2}} \right) \right\}.$$ 

Because $0 < \eta < 1$ and thus $\frac{\theta}{\theta - 1} - \eta > 0$, $A(\tau_e, \tau_q)$ is increasing in $\tau_e$ and $\tau_q$.

Also, we can express $K$ in terms of $A$:

$$K = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\Theta} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\tilde{eta}}} A_j^{1 - \frac{1}{\tilde{eta}}} | s_j, \tilde{q}_j \right] \right)^{\theta} \right] \right\} \cdot Y^{\frac{\Theta}{\theta}}$$

$$= \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\Theta} \left\{ \mathbb{E} \left[ \left( \mathbb{E} \left[ e_j^{\frac{1}{\tilde{eta}}} A_j^{1 - \frac{1}{\tilde{eta}}} | s_j, \tilde{q}_j \right] \right)^{\theta} \right] \right\} \cdot (AK)^{\frac{\Theta}{\theta}}$$

$$= \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\Theta} A^{\frac{1}{\theta - 1 - \eta}} (AK)^{\frac{\Theta}{\theta}} = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\Theta} A^{\frac{1}{\theta - 1 - \eta}} K^{\frac{\Theta}{\theta}}$$

$$\iff K^{1 - \frac{\Theta}{\theta}} = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\Theta} A^{\frac{1}{\theta - 1 - \eta} + \frac{\Theta}{\theta}}$$

$$\iff K = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\Theta}{\theta - 1 - \eta}} A^{\frac{1}{\theta - 1 - \eta} - \frac{\Theta}{\theta}}$$

$$\iff K = \left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\Theta}{\theta - 1 - \eta}} A^{\frac{1}{\theta - 1 - \eta} - \frac{\Theta}{\theta}},$$

where the last equality is obtained by $\Theta = -\frac{1}{\eta \frac{\theta}{\theta - 1} - 1} = -\frac{\theta}{\eta \frac{\theta}{\theta - 1}}$ and thus $\frac{1}{\theta - 1 - \eta} + \frac{\Theta}{\theta} = \Theta^{2} - \frac{1}{\theta} + \Theta = \Theta$.

Because $0 < \eta < 1$ and $\Theta \in (1, \theta)$, $K$ is increasing in $A$ and thus is increasing in $\tau_e$ and $\tau_q$.

**Some results in Section 4.2:** Function $Y(Z; A)$ is given by

$$Y(Z; A) = \begin{cases} 
\left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\Theta}{\theta - 1 - \eta}} A^{\frac{1}{\theta - 1 - \eta} - \frac{\Theta}{\theta}} Z^{\frac{\Theta}{\theta - 1 - \eta}} & \text{if } Z \in (\bar{Z}, +\infty) \\
\left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\Theta}{\theta - 1 - \eta}} A^{\frac{1}{\theta - 1 - \eta} - \frac{\Theta}{\theta}} Z^{\frac{\Theta}{\theta - 1 - \eta}} & \text{if } Z \in [Z, \bar{Z}] \\
\left[ \eta \left( 1 - \frac{1}{\theta} \right) \right]^{\frac{\Theta}{\theta - 1 - \eta}} A^{\frac{1}{\theta - 1 - \eta} - \frac{\Theta}{\theta}} Z^{\frac{\Theta}{\theta - 1 - \eta}} & \text{if } Z \in (-\infty, Z]
\end{cases}$$

(A.7)
where

\[ \Pi(\tau_e = \bar{\tau}_e; \tau_q^*, Y (\bar{A}, \bar{Z}), \bar{Z}) \neq \Pi(\tau_e = \bar{\tau}_e; \tau_q^*, Y (\bar{A}, \bar{Z}), \bar{Z}) = b; \]

\[ \Pi(\tau_e = \bar{\tau}_e; \tau_q^{**}, Y (\bar{A}, \bar{Z}), \bar{Z}) \neq \Pi(\tau_e = \bar{\tau}_e; \tau_q^{**}, Y (\bar{A}, \bar{Z}), \bar{Z}) = b; \]

\[ A = A(\bar{\tau}_e, \tau_q^*); \]

\[ A = A(\bar{\tau}_e, \tau_q^*); \]

\[ \tau_q^* = \tau_q(\tau_e = \bar{\tau}_e; c); \]

\[ \tau_q^{**} = \tau_q(\tau_e = \bar{\tau}_e; c). \]

### B Model extension in Section 3

We assume that the firm also receives a noisy signal on the demand shock:

\[ x_j = \varepsilon_j + \rho, \]

where \( \rho \sim N(0, \frac{1}{\tau_q^*}) \). The firm may obtain or buy this piece of information from informed traders. Under this alternative setup, we prove that the main result in Proposition 2 does not change.

Firm \( j \)'s information set becomes \( \{s_j, x_j, q_j\} \). The ex ante expected profit of firm \( j \) perceived at \( T_0 \) in (18) becomes

\[ \Pi(\tau_e, \tau_q, \tau_p; Y) = B \cdot Y^\Theta^* \exp \left\{ -\Theta(\Theta - 1) \left[ \frac{1}{2} \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e} + \frac{1}{2} \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_e + \tau_p + \tau_q} \right] \right\}, \]

where \( B = [1 - \eta(1 - \frac{1}{\theta})] [\eta(1 - \frac{1}{\theta})]^{\Theta-1} \cdot \exp \left\{ \frac{1}{2} \left[ \left( \Theta(1 - \frac{1}{\theta}) \right)^2 - \Theta \left( 1 - \frac{1}{\theta} \right) \right] \frac{1}{\tau_a} + \frac{1}{2} \left[ \left( \Theta \frac{1}{\theta} \right)^2 - \Theta \frac{1}{\theta} \right] \frac{1}{\tau_e} \right\}. \] It is easy to show that

\[ \frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} = B \cdot Y^\Theta^* \left\{ \exp \left\{ -\Theta(\Theta - 1) \left[ \frac{1}{2} \left( 1 - \frac{1}{\theta} \right)^2 \frac{1}{\tau_a + \tau_e} + \frac{1}{2} \left( \frac{1}{\theta} \right)^2 \frac{1}{\tau_e + \tau_p + \tau_q} \right] \right\} \right\} \]

\[ > 0, \]

which means that as the financial efficiency \( (\tau_q) \) increases, the firm has stronger incentives to acquire information about the productivity shock \( a_j \).
We also show that
\[
\frac{\partial^2 \Pi}{\partial \tau_q \partial \tau_p} = B \cdot Y \exp \left\{ -\Theta (\Theta - 1) \left[ \frac{1}{2} (1 - \frac{1}{\beta})^2 \frac{1}{\tau_a + \tau_e} + \frac{1}{2} (\frac{1}{\beta})^2 \frac{1}{\tau_e + \tau_p + \tau_q} \right] \right\} \cdot \Theta (\Theta - 1) \left( \frac{1}{\beta} \right)^2 \left( \frac{1}{\tau_e + \tau_p + \tau_q} \right)^2.
\]

So, when \( \frac{1}{\tau_e + \tau_p + \tau_q} < \frac{1}{2} \), it follows that \( \frac{\partial^2 \Pi}{\partial \tau_q \partial \tau_p} > 0 \), which means that the firm’s incentive to acquire information about the demand shock increases with financial efficiency. When \( \frac{1}{\tau_e + \tau_p + \tau_q} > \frac{1}{2} \), it follows that \( \frac{\partial^2 \Pi}{\partial \tau_q \partial \tau_p} < 0 \), which means that the firm’s incentive to acquire information about the demand shock decreases with financial efficiency (i.e., strategic substitutability). Therefore, if the firm incurs a cost in acquiring information about the demand shock or the productivity shock, it may be endogenously optimal for the firm to choose to acquire information about the latter.

References


Chen, Qi, Itay Goldstein and Wei Jiang (2007). Price Informativeness and Investment Sensi-


Dessaint, Olivier, Thierry Foucault, Laurent Frésard, and Adrien Matray (2017). Ripple Effects of Noise on Corporate Investment, mimeo.


Huang, Shiyang and Yao Zeng (2016). Investment Waves under Cross Learning, mimeo.


