Financial Markets, the Real Economy, and Self-fulfilling Uncertainties

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Abstract

Uncertainty in both financial markets and the real economy rises sharply during recessions. We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production (acquisition) and aggregate economic activities to explain this intriguing empirical fact. We argue that there exists *mutual learning* between financial markets and the real economy. Their joint information productions determine both the real production efficiency in the real sector and the price efficiency in the financial sector. The mutual learning makes information production in the financial sector and that in the real sector a strategic complementarity. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sectors produce little information in anticipation of the other sector to do so. At the same time, aggregate output falls as the real production efficiency deteriorates. Our model has other implications on aggregate economic activities.

JEL classification: G01, G20, E2, E44

Keywords: Information production, Strategic complementarity, Real economy, Two-way learning, Economic uncertainty

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1 Introduction

Uncertainty in both financial markets and the real economy rises sharply during recessions. The recent financial crisis of 2007-2009 presented one of the most striking episodes of such heightened uncertainty. The financial market uncertainty, measured by the VIX index, jumped by an astonishing 313% in the Great Recession. The increase in measured real uncertainty was equally impressive. For instance, the macroeconomic uncertainty measured in Jurado et al. (2015) almost doubled, and Bloom et al. (2012) reports a 152% increase in the micro-level real uncertainty measured by the firm-level dispersion of output. What causes such sudden spikes in uncertainty? Why do financial uncertainty and real uncertainty move together? Why do they rise sharply in recessions? These challenging questions are of central importance for understanding the interaction between financial markets and the real economy. Despite a large and flourishing literature on uncertainty since the pioneering work of Bloom (2009), these questions are still largely unanswered. The purpose of this paper is to provide a theoretical framework to address these questions.

We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production (or acquisition) and aggregate economic activities. As the starting point of our theory, we argue that there exists *mutual learning* between financial markets and the real economy. Their joint information productions determine both the real production efficiency in the real sector and the price efficiency in the financial sector. As an example, oil producing companies scrutinize oil future prices when they make their production decisions, while the financial market studies the financial reports from these producing companies to learn information when trading on oil futures. This mutual learning makes information production in the financial sector and that in the real sector a strategic complementarity. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sides produce little information in anticipation that the other side will do so. At the same time, aggregate output falls as the real production efficiency deteriorates.

We formalize the idea in an extended Grossman-Stiglitz (1980) model. Our key innovation is that we introduce a real sector along the line of Dixit-Stiglitz (1980) — in our framework, firms have to make investment decisions under imperfect information about *two* dimensions of uncertainty: their idiosyncratic productivity and demand shocks. We start with one firm and one financial market in our baseline partial equilibrium model for a given aggregate output. To reduce uncertainty, the firm can learn about its idiosyncratic productivity shock by incurring a cost, but it has to infer its demand shock from the information provided in the financial market where speculators (or traders) have a comparative advantage in acquiring information about the demand shock. In this context, the financial price is jointly determined by the firm's information production and thereby its disclosure and the demand information produced by financial market speculators. To understand strategic complementarity in these two sources of information, first consider that the firm makes more accurate information disclosure about its productivity shock. The reduced uncertainty about the productivity shock attracts more informed traders and induces more aggressively trading. Hence, information production on the demand shock in the financial market increases. Conversely, suppose that the financial market becomes more informative about the demand shock for some reason. The reduced uncertainty regarding the demand implies that the stake is higher for better investment that is more closely aligned with the true realized productivity shock. Hence, the firm has stronger incentives to acquire precise information about its productivity shock.

As the marginal benefit of acquiring information for the firm depends on aggregate output (besides financial price informativeness), the nature of equilibrium also crucially depends on the level of aggregate output. When the aggregate output is sufficiently high, the resulting equilibrium is unique in which the firm produces and discloses more precise information and the financial market generates a more informative price signal. As a result, both financial and real uncertainty are low. When the aggregate output is sufficiently low, not acquiring information is a dominant strategy for the firm. Anticipating this, speculators in the financial market also have little incentive to acquire information about the firm's demand shock. The equilibrium is hence also unique. However, when the aggregate output is in the intermediate range, the economy has two self-fulfilling equilibria. The information produced by the firm and the information generated by financial market in one equilibrium (the "good" equilibrium) are much more precise than those in the other equilibrium (the "bad" equilibrium). Consequently, a sudden outburst of uncertainty can arise as a self-fulfilling equilibrium phenomenon.

We then extend the baseline model to a macroeconomic general-equilibrium framework with aggregate production to endogenize the aggregate output. The final consumption good is produced with the inputs of a continuum of intermediate capital goods according to a Dixit-Stiglitz production function. Each intermediate capital good is produced by one firm located on an island in the spirit of Lucas (1972). We show that complementarity in information production exists between the financial sector and the real sector within an island and between islands. The complementarity between two islands arises from strategic complementarity in production between firms. When information signals on some islands become noisier, the real investment decisions on those islands become less efficient and consequently the aggregate output declines. This causes the aggregate demand faced by other islands to drop. Thus, incentives to acquire information in the real sector on those other islands are also reduced, which decreases information acquisition in their financial sectors as well. The aggregate output hence declines further, which in turn affects those islands experiencing the original shock. Similar to the partial equilibrium model, the economy may feature two equilibria: a "good" equilibrium with more information production and higher real economic activities, and a “bad” equilibrium with the opposite features.

Although our model is too stylized to be confronted with the actual data, it does explain, qualitatively, several prominent features of macroeconomic fluctuations. First, our quantitative exercise shows that with reasonable parameter values our model can generate a very large drop in the aggregate-level output and investment in the absence of any aggregate shock. Second, the measured uncertainty in both financial markets and the real economy has a sizable spike in recessions. The residual uncertainty faced by firms and by financial investors increases substantially when the economy shifts from the “good” to the “bad” equilibrium. Third, our model implies that financial panics seemingly start with some gradual deterioration in economic fundamentals. When the deterioration reaches a tipping point, uncertainty surges and the real economic activities collapse.[1]

Our model has additional implications for aggregate economic activities. First, our model highlights how information frictions can be an important reason for resource misallocation. Efficient allocation requires more resources to be allocated to firms with higher realized productivity and stronger demand shocks. Higher residual uncertainty under less information leads to a higher degree of resource misallocation. In our model, the endogenous aggregate total factor productivity (TFP), which maps the degree of resource misallocation, is an increasing function of information precision. As information precision is procyclical, our model is consistent with an established empirical fact that aggregate productivity is procyclical. Second, a small shock to the financial sector that impairs its ability to perform price discovery can have a large impact on the aggregate economy due to the compound feedback loops. In fact, both aggregate investment and the endogenous aggregate TFP are decreasing in information precision. Hence, a small shock to the financial sector can have a large impact on all three quantities (aggregate investment, endogenous aggregate TFP, and aggregate output) in the same direction. Third, our model provides an information contagion channel, where a shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. This is consistent with some evidence that idiosyncratic firm-level shocks can be the origin of aggregate fluctuations (see Gabaix (2011)).

Related literature. A burgeoning literature in finance studies the informational feedback effects of financial markets (see Bond, Edmans and Goldstein (2012) for an extensive survey of this literature). This literature argues that firm managers on the real side of the economy learn from financial prices. Among others,[2] Goldstein, Ozdenoren and Yuan (2013) and Sockin and Xiong

[1] The Delinquency Rate on Single-Family Residential Mortgages had begun a steady climb in August 2006 before it led to the collapse of several large financial institutions and the subsequent financial crisis in 2008-2010.

[2] For theoretical work, see, e.g., Fishman and Hagerty (1992), Leland (1992), Dow and Gorton (1997), Subrahmanyam and Titman (1999, 2013), Hirshleifer, Subrahmanyam, and Titman (2006), Foucault and Gehrig (2008), Goldstein and Guembel (2008), Ozdenoren and Yuan (2008), Bond, Goldstein, and Prescott (2010), Kurlat and Veldkamp (2015), Huang and Zeng (2016), Sockin (2016), Foucault and Frésard (2016), and Dessaint et al. (2017).

(2015) develop clean model frameworks showing how prices in the secondary financial market can aggregate dispersed information of speculators and guide firm managers or investors to make better real investment decisions. The learning in this literature is one way — the real sector learns from financial markets. On the other hand, the accounting literature emphasizes the opposite direction of learning — arguing that firm managers typically know more than financial market participants — and focuses on studying how firm managers (i.e., insiders) disclose information to the capital market, based on which financial speculators trade and security prices are formed. Our paper advances these two bodies of literature by introducing and studying *mutual (two-way) learning* between the real sector and financial markets. The two-way learning mechanism sheds light on important questions, such as how a financial price is formed, where the information comes from, and how the sources of information interact.

The finance literature pioneered by Grossman and Stiglitz (1980) and Verrecchia (1982) studies information production (or acquisition) in financial markets. The recent work of Goldstein and Yang (2015) analyzes a model where two different groups of financial traders are informed of different fundamentals of a security. They show that trading as well as information acquisition by these two groups of financial traders exhibit strategic complementarities. Ganguli and Yang (2009) study a model where traders can obtain private information about the supply of a stock in addition to that about its payoff. They show complementarity in information acquisition and the existence of multiple equilibria. Our model introduces the real sector and aggregate production into a Grossman-Stiglitz-type model; information acquisition in our model takes place both in the real sector and in financial markets. Adding to this literature, our paper shows that complementarity in information production exists between the real sector and the financial sector, which has important macroeconomic implications.[3]

A large literature in macroeconomics documents robust evidence of countercyclical uncertainty — both real uncertainty and financial uncertainty increase during recessions. Real uncertainty is often proxied by firm-level dispersion in earnings, productivity and output, and the volatility of aggregate output forecast error, while financial uncertainty is often measured by financial market volatility and the VIX index (Bloom (2009), Bloom et al. (2012), Jurado et al. (2015)). An ongoing heated debate in this literature concerns the question of causality, i.e., whether uncertainty is a cause or merely a response to recessions and where uncertainty comes from (see, e.g., Bachmann and Bayer (2013, 2014)). Interestingly, a recent paper by Ludvigson et al. (2017) empirically identifies that sharply higher real uncertainty in recessions is most often an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is

[3] Goldstein and Yang (2016) analyze a model where there are multiple dimensions of uncertainty and market prices convey information to real decision-makers. They focus on studying the effect of disclosing public information on real efficiency. There is no information production (acquisition) in their model, while studying the interaction of information production in the real sector and that in the financial sector is a key emphasis of our model.

a likely source of the fluctuations. Our paper contributes to the debate by providing a theoretical framework that is able to address the three variables simultaneously — real uncertainty, financial uncertainty, and aggregate economic activities — and show how they are related.[4]

Bacchetta, Tille and Wincoop (2012) also study the self-fulfilling nature of uncertainty. They construct an interesting endowment economy in which agents have mean-variance preferences so that the equilibrium asset prices are negatively linked to the perceived risk of future prices. If the agents believe that pure sunspots matter for asset prices, then the perceived risk of future prices increases. As a result, the current asset prices will indeed be affected. The uncertainty is self-fulfilling because there also exists another equilibrium in which the asset prices are certain and hence bear zero risk. Fajgelbaum, Schaal and Taschereau-Dumouchel (2016) propose a theory of self-reinforcing episodes of high uncertainty and low activity, through the mechanism of the "wait-and-see" effect together with agents learning from the actions of others. In contrast to these contributions, self-fulfilling uncertainty in our model comes from the information interdependence between financial markets and the real economy. This information interplay also allows us to study the impact of uncertainties on real economic activities.

Finally, our model is related to a small body of macroeconomics literature that studies how financial markets affect business cycle fluctuations through information channels.[5] Angeletos, Lorenzoni and Pavan (2010) build a two-stage feedback model where financial markets in the second stage learn from the volume of asset selling of entrepreneurs in the first stage, which generates strategic complementarity in investment that amplifies non-fundamental shocks at that stage. Benhabib, Liu and Wang (2016a) present a self-fulfilling business cycle model, where financial market sentiments affect the price of capital, which signals the fundamentals of the economy to the real side and consequently leads to real output that confirms the sentiments. David, Hopenhayn and Venkateswaran (2016) conduct a quantitative study that links imperfect information and resource misallocation, where firms learn from both private sources and imperfectly informative stock market prices about *one* dimension of fundamental uncertainty. The information is exogenous in David, Hopenhayn and Venkateswaran, and they conclude that firms turn primarily to internal sources for information, rather than to financial markets.[6] Compared with the aforementioned studies, ours shows that the amount of information in the economy is endogenous, and that there is feedback between the level of economic activity and the amount of information, amplified through the mutual learning

[4] Benhabib, Liu and Wang (2016b) study endogenous information acquisition of firms that links real uncertainty and economic activities. However, there is no financial market in the model there and that paper does not touch upon financial uncertainty. Veldkamp (2005) and Nieuwerburgh and Veldkamp (2006) study learning asymmetries in business cycles, without involving financial markets.

[5] Among others, Reis (2006), Angeletos and Pavan (2007), Hellwig and Veldkamp (2009), Vives (2016), Colombo, Femminis and Pavan (2014) and Mäkinen and Ohl (2015) study information acquisition and efficiency.

[6] Because David, Hopenhayn and Venkateswaran (2016) consider neither endogenous information acquisition nor the feedback on information production between the real side and financial markets, incorporating our mechanism in their quantitative study may bring new results; see our numerical calibration in Section 5.

between firms and financial markets.

The paper is organized as follows. In Section 2, we present the simple baseline model. In Section 3, we extend our baseline model to study endogenous information. In Section 4, we further extend the model to a macroeconomic framework. Section 5 provides quantitative analysis of our model. Section 6 concludes.

2 The Baseline Model

In this section, we present a simple baseline model with one firm, one financial market, and with exogenous information. The firm faces two uncertainties: demand shocks and supply (or productivity) shocks. The firm has some information about the supply shocks while the financial market has some information about the demand shocks. We show that there exists two-way learning between the firm and the financial market.

2.1 Setup

There are two types of agents: firm j and a group of financial market traders (speculators). There are two types of goods: an intermediate capital good and a final consumption good. The price of the consumption good is normalized as the numeraire, $P \equiv 1$.

Intermediate Goods Firm Firm j is an intermediate goods firm. It produces the intermediate capital good Y_j using the input of the final consumption good according to the production function

$$Y_j = ZA_jK_j^{\eta} \quad \text{for } 0 < \eta < 1, \tag{1}$$

where Z is the common productivity shock to the whole economy (regarded as a constant in the baseline model), A_j is Firm j's productivity, and K_j is the investment input of the final consumption good.[7] We will show that firm j borrows the investment input at interest rate $R_f \equiv 1$.

The market demand function of the intermediate capital good Y_j is assumed to be

$$Y_j = \left(\frac{1}{P_j}\right)^{\theta} \epsilon_j Y, \tag{2}$$

where P_j is the price of the capital good j (in terms of the final consumption good), and ϵ_j measures the idiosyncratic *demand shock* to good j. Moreover, in the baseline model Y is an exogenous constant, which corresponds to the aggregate output (real GDP) (denote $y \equiv \log Y$), whereas parameter θ measures the price elasticity of demand.

[7] The input K_j will fully depreciate after production.

Financial Market and Traders (Speculators) A financial market exists, where speculators trade a financial asset (a derivative) contingent on the firm's asset value or firm value (also its total income):[8]

$$V_j = Y_j P_j. \tag{3}$$

Specifically, we assume that the payoff of the financial derivative contract takes the form of

$$v_j = \log V_j,$$

where v_j also corresponds to the growth of the firm value. Denote the *market trading price* of the financial derivative contract by q_j. That is, the long position of one unit of the financial asset (derivative) incurs an initial outlay of q_j and entitles to having the risky payoff v_j later.

The utility function of speculators is assumed to be

$$U(W^i) = -\exp\left(-\gamma W^i\right),$$

where W^i is the end-of-period wealth for speculator i, and γ is risk aversion (CARA) coefficient. The initial wealth for a speculator is assumed to be W_0 and the risk-free (gross) interest rate is $R_f \equiv 1$. This means that if a speculator takes a position of d units of the financial asset, his end-of-period wealth would be

$$W^i = (W_0 - dq_j) R_f + dv_j = W_0 + d(v_j - q_j).$$

The assumption that speculators trade a derivative contract contingent on the firm's asset value, V_j, is made for tractability. This is along the line of the assumption in the literature that a firm's asset value or sales revenue follows a geometric Brownian motion. The financial derivative can also be contingent on the firm's product price, P_j (that is, v_j takes the form of $v_j = \log P_j$). In the latter case, the financial market can be interpreted as a *commodity financial futures market* that specializes in trading *financial futures* regarding the intermediate capital good Y_j, in the spirit of Sockin and Xiong (2015). Assuming that the underlying asset of the derivative is either V_j or P_j is to ensure that the payoff of the underlying asset follows a log-normal distribution and thus to achieve tractability. This parallels the modeling device that assumes a *specific* function form of noisy trading (or asset supply) as in Goldstein, Ozdenoren and Yuan (2013), Sockin and Xiong (2015), and Goldstein and Yang (2016).

The net aggregate supply of the financial asset (i.e., derivative) is assumed to be 0. The demand

[8] In the balance sheet of the firm at the end of the period, the asset value of the firm is its total income; the debt value is the investment cost; and the equity value is the net profit. This is consistent with the literature where the asset value, not the equity value, is assumed to follow a geometric Brownian motion (a log-normal distribution).

of noise/liquidity traders in the financial market is n_j, where n_j follows distribution $n_j \sim N(0, \sigma_n^2)$.

Uncertainties and Information The firm faces two uncertainties: productivity (or supply) shock A_j and demand shock ϵ_j. Their prior distributions are $\log A_j \equiv a_j \sim \mathcal{N}(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$ and $\log \epsilon_j \equiv \varepsilon_j \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$ (denote $\tau_a = 1/\sigma_a^2$ and $\tau_\varepsilon = 1/\sigma_\varepsilon^2$). a_j and ε_j are independent. The common productivity shock Z is public information (denote $z = \log Z$).

In the baseline model, we assume that the firm and the financial market have some *exogenous* (imperfect) information about a_j and ε_j, respectively. Specifically, the firm possesses or is endowed with a noisy signal about its own productivity:

$$s_j = a_j + e_j,$$

where $e_j \sim N(0, \sigma_e^2)$ (denote the precision of the signal by $\tau_e \equiv \frac{1}{\sigma_e^2}$). Firm j will disclose its signal s_j to the financial market.[9] For simplicity, we assume that the firm has no private information about the demand shock, ε_j.

In the financial market, as in Grossman and Stiglitz (1980), there are two types of traders: informed and uninformed traders. An informed trader i has a noisy private signal

$$x_j^i = \varepsilon_j + \varrho_j^i,$$

where $\varrho_j^i \sim N(0, \sigma_\varrho^2)$ and ϱ_j^i is independent across informed traders (denote $\tau_\varrho \equiv \frac{1}{\sigma_\varrho^2}$). An uninformed trader has no private signal regarding ε_j. The proportion of informed traders is λ, which is exogenous in the baseline model. Note that for the extreme case of $\sigma_\varrho^2 = 0$, an informed trader's signal represents perfect information about ε_j.

Timeline The sequence of events in the baseline model is as follows:

T_1: Firm j discloses its signal s_j to the financial market.

T_2: Financial market trading takes place, and financial price q_j is realized.

T_3: Firm j makes its investment decision, K_j, based on information $\{s_j, q_j\}$.

T_4: The income or asset value, V_j, is realized. The payoff of the financial contract is delivered.

2.2 Equilibrium

The equilibrium consists of a financial market equilibrium at T_2 and the firm's investment decision at T_3. We conduct analysis by backward induction.

[9] As will become clear later, a firm has incentives to disclose its information to the financial market because the disclosure can "attract" more information from the financial market, which can guide the firm to make better investment decisions.

Firm j's Investment Decision at T_3 Firm j maximizes its *expected* profit:

$$K_j \equiv K(s_j, q_j) = \arg\max_{K_j} \mathbb{E}\left[P_j Y_j - R_f K_j | s_j, q_j\right] \tag{4}$$

with constraints (2) and (1). Here $\mathbb{E}(\cdot|s_j, q_j)$ is the conditional expectation operator over a_j and ε_j.

Financial Market Trading at T_2 In the financial market, the information set of informed speculators is $\left\{s_j, q_j, x_j^i\right\}$ while that of uninformed speculators is $\{s_j, q_j\}$.

An informed speculator chooses his risky asset holdings, d^{Ii}, to maximize his utility:

$$d^{Ii}(s_j, q_j, x_j^i) = \arg\max_{d^{Ii}} \mathbb{E}\left[U(W^{Ii})|s_j, q_j, x_j^i\right], \tag{5}$$

where $W^{Ii} = (W_0 - c) + d^{Ii}(v_j - q_j)$ and c denotes a constant expense, to be explained later. An uninformed speculator chooses his risky asset holdings, d^{Ui}, to maximize his utility:

$$d^{Ui}(s_j, q_j) = \arg\max_{d^{Ui}} \mathbb{E}\left[U(W^{Ui})|s_j, q_j\right], \tag{6}$$

where $W^{Ui} = W_0 + d^{Ui}(v_j - q_j)$. In (5) and (6), $\mathbb{E}(\cdot)$ is the expectation operator over v_j

The equilibrium of our baseline model is formally defined as follows.

Definition 1 *An equilibrium consists of the financial price function $q_j = q(s_j, \varepsilon_j, n_j)$ and the firm's investment decision function $K_j = K(s_j, q_j)$, such that*

1. *Price $q(s_j, \varepsilon_j, n_j)$ clears the financial market at T_2:*

$$\lambda \int d^{Ii} + (1-\lambda) \int d^{Ui} + n_j = 0, \tag{7}$$

where, for given $K_j = K(s_j, q_j)$, d^{Ii} and d^{Ui} solve (5) and (6), respectively.

2. *Given price $q(s_j, \varepsilon_j, n_j)$, investment decision $K(s_j, q_j)$ solves the firm's problem (4).*

The equilibrium defined in Definition 1 highlights the two-way feedback (i.e., a fixed-point problem) between the financial market and the real economy. On the one hand, the financial price at T_2 should *reflect* the (forward-looking) investment decision at T_3 (and thereby the financial asset's fundamentals at T_4). On the other hand, the financial price at T_2 *influences and guides* the investment decision on the real side of the economy at T_3.

2.3 Characterization of Equilibrium

First, we characterize the financial market equilibrium. We conjecture that $\log K_j \equiv k_j = k(s_j, q_j)$ is a linear function in Definition 1. Plugging (2) and (1) into (3) yields

$$v_j = \frac{1}{\theta}\varepsilon_j + \left(1 - \frac{1}{\theta}\right)(z + a_j) + \eta\left(1 - \frac{1}{\theta}\right)k_j + \frac{1}{\theta}y, \tag{8}$$

which depends on ε_j, a_j and k_j. However, speculators are certain about k_j but not ε_j and a_j, because k_j is a function of signals s_j and q_j and thus speculators *perfectly foresee* the investment decision of the firm.

In solving (5), we find that

$$d^{Ii} = \frac{\mathbb{E}[v_j|s_j, q_j, x_j^i] - q_j}{\gamma Var[v_j|s_j, q_j, x_j^i]}. \tag{9}$$

Similarly, (6) gives

$$d^{Ui} = \frac{\mathbb{E}[v_j|s_j, q_j] - q_j}{\gamma Var[v_j|s_j, q_j]}. \tag{10}$$

We also conjecture a linear price function:

$$q_j = \beta_0 + \beta_1(\varepsilon_j + \beta_2 s_j + \beta_3 n_j), \tag{11}$$

where β_0, β_1, β_2 and β_3 are coefficients. Combined with s_j, price q_j can be converted into another piece of public information about ε_j:

$$\tilde{q}_j(q_j, s_j) = \frac{q_j - \beta_0 - \beta_1\beta_2 s_j}{\beta_1} = \varepsilon_j + \beta_3 n_j \equiv \varepsilon_j + \varrho_j^q, \tag{12}$$

where $\varrho_j^q \sim N(0, \sigma_q^2)$ with $\sigma_q^2 = \beta_3^2\sigma_n^2$ (written as $\tau_q \equiv \frac{1}{\sigma_q^2}$ representing the precision of the price signal). Information set $\{s_j, q_j\}$ is a one-to-one mapping into $\{s_j, \tilde{q}_j\}$. In particular, the informativeness of price q_j (together with signal s_j) about ε_j is fully captured by the term ϱ_j^q. In other words, given σ_n^2 (and s_j), β_3 measures *price informativeness* of q_j about ε_j.

Plugging (8) and (11) into (9) and (10), together with (7), yields the financial market equilibrium. We have Lemma 1.

Lemma 1 *In the equilibrium of the financial market, for a given λ, τ_q is an increasing function of τ_e, i.e., $\frac{\partial \tau_q}{\partial \tau_e} > 0$.*

Proof. *See Appendix.* ■

Lemma 1 states that when the precision of the firm's disclosed information about a_j increases, the informativeness of the financial price about ε_j also increases. The intuition is as follows. The total uncertainty over v_j is the sum of uncertainties over a_j and ε_j. When uncertainty over a_j decreases under a higher τ_e, informed traders have incentives to trade more aggressively, which overwhelms the trading of noise/liquidity traders, thus increasing the informativeness of the financial price.

Lemma 1 shows that the financial price comes partially from information disclosure in the real sector and partially from price discovery in the financial market. These two sources of information interact. This is a novel insight of our paper.

Illustration *We use the extreme case of $\sigma_\varrho^2 = 0$ (in which case informed traders are perfectly informed of ε_j) to provide explicit solutions to the financial market equilibrium. For an informed trader,*

$$\mathbb{E}[v_j|s_j,q_j,\varepsilon_j] = \frac{1}{\theta}\varepsilon_j + \eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right) + \frac{1}{\theta}y + \left(1-\frac{1}{\theta}\right)\left[z + \frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right) + \frac{\tau_e}{\tau_a+\tau_e}s_j\right]$$

and

$$Var[v_j|s_j,q_j,\varepsilon_j] = \left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}.$$

For an uninformed trader,

$$\mathbb{E}[v_j|s_j,q_j] = \frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j\right] + \eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right) + \frac{1}{\theta}y + \left(1-\frac{1}{\theta}\right)\left[z + \frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right) + \frac{\tau_e}{\tau_a+\tau_e}s_j\right]$$

and

$$Var[v_j|s_j,q_j] = \left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j,q_j\right] + \left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}.$$

Therefore, the market clearing condition, (7), implies

$$\begin{aligned} 0 \quad = \quad & n_j + \lambda\frac{\frac{1}{\theta}\varepsilon_j + \eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right) + \frac{1}{\theta}y + \left(1-\frac{1}{\theta}\right)\left[z + \frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right) + \frac{\tau_e}{\tau_a+\tau_e}s_j\right]}{\gamma\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}} \\ & + (1-\lambda)\frac{\frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j\right] + \eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right) + \frac{1}{\theta}y + \left(1-\frac{1}{\theta}\right)\left[z + \frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right) + \frac{\tau_e}{\tau_a+\tau_e}s_j\right]}{\gamma\left[\left(\frac{1}{\theta}\right)^2 Var\left(\varepsilon_j|s_j,q_j\right) + \left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}\right]}. \end{aligned} \tag{13}$$

It is straightforward to see that (13) can be transformed to

$$f\left(s_j,q_j,y,z\right) + \lambda\frac{\frac{1}{\theta}}{\gamma\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}}\varepsilon_j + n_j = 0, \tag{14}$$

where $f(s_j, q_j, y, z)$ is a linear function of s_j, q_j and y. Hence,

$$\beta_3 = \frac{\gamma\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e}}{\lambda\frac{1}{\theta}} \qquad \text{or} \quad \tau_q = \frac{\lambda^2 \frac{1}{\theta^2}}{\gamma^2\left(1-\frac{1}{\theta}\right)^4} \frac{(\tau_a+\tau_e)^2}{\sigma_n^2}. \tag{15}$$

From (15), it is easy to see that τ_q is an increasing function of τ_e for a given λ. In fact, as the precision of the firm's disclosed information increases, informed speculators trade more aggressively (see (14)) and so the price becomes more informative.

Next, we move to characterize firm j's investment decision at T_3. As shown in (12), firm j's information set $\{s_j, q_j\}$ at T_3 is equivalent to the information set $\{s_j, \tilde{q}_j\}$. The first-order condition of (4) implies

$$K_j = K(s_j, \tilde{q}_j) = \left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right]^{\Theta} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}, \tag{16}$$

where $\Theta = -\frac{1}{\eta(1-\frac{1}{\theta})-1} \in (1, \theta)$. We find that

$$k_j = k(s_j, \tilde{q}_j) = \phi_0' + \Theta(1-\frac{1}{\theta})\frac{\tau_e}{\tau_a+\tau_e} s_j + \frac{\Theta}{\theta}\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j, \tag{17}$$

where the constant coefficient ϕ_0' is provided in Appendix. Because $\tilde{q}_j$ is a linear function of s_j and q_j by (12), (17) implies that k_j is also a linear function of s_j and q_j, which confirms the earlier conjecture.

Lemma 2 *The firm's investment decision at T_3, $K(s_j, q_j)$, is given by (17) (together with (12)).*

Proof. *See Appendix.* ■

The realized profit for firm j at T_4 is $\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j) = P_j(\varepsilon_j, Y_j) Y_j(a_j, K_j) - K_j(s_j, \tilde{q}_j)$. Hence, the expected profit perceived at the stage of investment at T_3 is

$$\mathbb{E}\left[\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j) | s_j, \tilde{q}_j\right] = \left[1-\eta\left(1-\frac{1}{\theta}\right)\right]\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1} \cdot \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)^{\Theta} \left[\mathbb{E}\left(\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right)\right]^{\Theta}.$$

Exploiting the law of iterated expectations, we find that the ex ante expected profit of firm j perceived at T_0 is

$$\begin{aligned} \Pi(\tau_e, \tau_q; Y, Z) &= \mathbb{E}\mathbb{E}\left[\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j) | s_j, \tilde{q}_j\right] \\ &= \left[1-\eta\left(1-\frac{1}{\theta}\right)\right]\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1} \cdot \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)^{\Theta} \mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}} \epsilon_j^{\frac{1}{\theta}} | s_j, \tilde{q}_j\right)\right]^{\Theta}\right), \end{aligned}$$

where

$$\mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right)\right]^{\Theta}\right)=\exp\left\{\begin{array}{c}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\\ -\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\end{array}\right\}, \tag{18}$$

by noting that the outer $\mathbb{E}(\cdot)$ is the unconditional expectation operator over s_j and $\tilde{q}_j$.

It is easy to show that

$$\frac{\partial\Pi}{\partial\tau_e}>0 \qquad \text{and} \qquad \frac{\partial\Pi}{\partial\tau_q}>0. \tag{19}$$

The intuition behind the above comparative statics is easy to understand. When the firm has a more precise signal about a_j or ε_j, it makes a better investment decision because its investment can be more closely aligned with the realized productivity or demand shock. Moreover,

$$\frac{\partial^2\Pi}{\partial\tau_e\partial\tau_q}>0, \tag{20}$$

which means that the precisions of signals a_j and ε_j are *complementary* in affecting the firm's ex ante profit.

A key insight of the baseline model is that the learning between the financial market and the real economy occurs both ways. The financial market learns information from a firm's disclosure in trading, and conversely, the firm learns information from the financial price in making its real (investment) decision.

3 The Model with Endogenous Information

In this section, we study the model with acquisition of endogenous information. The purpose is to understand the information acquisition of the firm and that of the financial market, and how they interact.

We add T_0 to the timeline. At T_0, after the common productivity shock Z is realized (which becomes public information), the firm and the financial market *simultaneously* make their information acquisition decisions.

Setup By paying an information acquisition cost $b>0$ (in terms of the final consumption good), the firm receives a signal $s_j=a_j+e_j$ with $e_j\sim N(0,\underline{\sigma}_e^2)$; otherwise, it receives a less precise signal $s_j=a_j+e_j$ with $e_j\sim N(0,\bar{\sigma}_e^2)$, where $\bar{\sigma}_e^2>\underline{\sigma}_e^2$. That $\bar{\sigma}_e^2=\infty$ corresponds to the extreme case where the firm receives a useless signal. In short, $\sigma_e^2\in\left\{\underline{\sigma}_e^2,\bar{\sigma}_e^2\right\}$. Also denote $\bar{\tau}_e\equiv 1/\underline{\sigma}_e^2$ and $\underline{\tau}_e\equiv 1/\bar{\sigma}_e^2$. In addition, in the spirit of the classic moral hazard problem (concerning hidden actions), we assume that a firm's choice of information precision, $\tau_e\in\{\bar{\tau}_e,\underline{\tau}_e\}$, is private information (i.e., *unobservable* by outsiders including financial market participants).

In the financial market, a trader can choose to be informed or uninformed. By paying an information acquisition cost $c > 0$ (in terms of the final consumption good), a trader receives a private signal $x_j^i = \varepsilon_j + \varrho_j^i$ with $\varrho_j^i \sim N(0, \sigma_\varrho^2)$, as specified in the baseline model; otherwise, it receives no signal (or equivalently a useless signal). The proportion of informed speculators, λ, is endogenous.

Our assumption that the firm and financial markets have *comparative advantages* in acquiring information on different uncertainties is realistic. For example, financial analysts in major investment banks specializing in different regional or sectoral submarkets can on aggregate be better informed about the demand for the firm's product than the firm itself. We can relax the assumption and alternatively assume that the firm can also obtain a noisy signal on the demand shock in addition to its signal on the productivity shock, under which our main results do not change (see Appendix B). In fact, as long as the firm learns information (about the demand shock) from the financial price, our result does not change qualitatively, no matter whether the firm itself also has some additional information about the demand shock.

Information Acquisition Decision of Speculators Proportion λ is determined such that an uninformed speculator and an informed one have the same ex ante utility:

$$\frac{EV(W^{Ii})}{EV(W^{Ui})} = 1, \tag{21}$$

where $EV(W^i) \equiv \mathbb{E}\left[U(W^i)|s_j, q_j\right]$. We have the following result.

Proposition 1 *In the equilibrium of the financial market with endogenous λ, τ_q is a function of τ_e and c, written as $\tau_q = \tau_q(\tau_e; c)$. We have the comparative statics $\frac{\partial \tau_q}{\partial \tau_e} > 0$ and $\frac{\partial \tau_q}{\partial c} < 0$.*

Proof. *See Appendix.* ■

Proposition 1 states that with taking endogenous λ into account, the informativeness of the financial price about ε_j increases as the precision of the firm's information about a_j increases. The intuition behind the comparative statics is as follows. There are two driving forces under the comparative statics $\frac{\partial \tau_q}{\partial \tau_e} > 0$. First, as in the earlier discussion of Lemma 1, when uncertainty over a_j decreases, informed traders trade more aggressively, increasing the informativeness of the financial price. Second, in the spirit of Grossman and Stiglitz (1980), (21) implies $e^{\gamma c} = \sqrt{\frac{Var[v_j|s_j,q_j]}{Var[v_j|s_j,q_j,x_j^i]}}$ (that is, $e^{\gamma c} = \sqrt{\frac{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]+Var\left(\frac{1}{\theta}\varepsilon_j|s_j,q_j\right)}{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]+Var\left(\frac{1}{\theta}\varepsilon_j|s_j,q_j,x_j^i\right)}}$), the RHS of which, written as function $G(\tau_q; \tau_e)$, is the *gain* in information advantage for an informed speculator over an uninformed one. When one dimension of uncertainty (the term $Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]$) is reduced, the information advantage on the other dimension of uncertainty (the term $Var(\frac{1}{\theta}\varepsilon_j|\cdot)$) becomes more useful. For example, in the extreme case when $Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]$ is very large, being informed has little advantage over being uninformed. Hence, when the precision of signal s_j increases and thus $Var\left(a_j|s_j\right)$ decreases, an uninformed speculator has incentives to *switch* to being informed by paying a cost c. When more speculators acquire information, price informativeness also improves. As for comparative statics $\frac{\partial \tau_q}{\partial c} < 0$, a lower c induces more traders to become informed causing price informativeness to improve.

Illustration *We continue to use the extreme case of* $\sigma_\varrho^2 = 0$ *(in which case informed traders are perfectly informed of* ε_j*) to provide explicit solutions to the financial market equilibrium. After taking endogenous* λ into account*, we have*

$$\sqrt{\frac{Var\left(a_j|s_j\right) + Var(\varepsilon_j|s_j, q_j)}{Var\left(a_j|s_j\right) + Var(\varepsilon_j|s_j, q_j, \varepsilon_j)}} = e^{\gamma c} \quad \Longrightarrow \quad \sqrt{\frac{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e}}} = e^{\gamma c}, \tag{22}$$

which implies that τ_q *is increasing in* τ_e *and decreasing in* c*. Recall that the LHS of equation (22) is written as function* $G\left(\tau_q; \tau_e\right)$*. Figure 1 illustrates the solution (denoted by* $\tau_q = \tau_q(\tau_e; c)$*) to equation (22).*

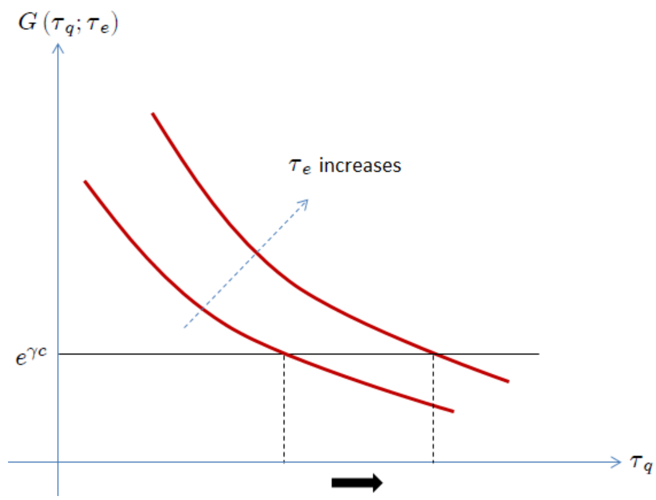

Figure 1: A graphical illustration of equilibrium τ_q as a function of τ_e and c

Information Acquisition Decision of the Firm Considering that profit function $\Pi\left(\tau_e, \tau_p; Y, Z\right)$ given in (18) has the properties of $\frac{\partial \Pi}{\partial \tau_e} > 0$ and $\frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0$ shown in (19) and (20), we can obtain firm j's optimal information acquisition decision at T_0:

$$\tau_e = \begin{cases} \bar{\tau}_e & \text{if } \tau_q \geq \hat{\tau}_q(Y, Z, b) \\ \underline{\tau}_e & \text{otherwise} \end{cases}, \tag{23}$$

where threshold $\hat{\tau}_q \equiv \hat{\tau}_q(Y, Z, b)$ is defined as the unique root to the equation

$$\Pi\left(\tau_e = \bar{\tau}_e; \hat{\tau}_q, Y, Z\right) - \Pi\left(\tau_e = \underline{\tau}_e; \hat{\tau}_q, Y, Z\right) = b. \tag{24}$$

It is easy to show that $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial Y} < 0$, $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial Z} < 0$, and $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial b} > 0$. We have the following result.

Proposition 2 *The optimal information acquisition decision of the firm at T_0, $\tau_e\left(\tau_q; Y, Z, b\right)$, is given by (23).*

Proof. *See Appendix.* ■

Proposition 2 states that if and only if the firm *expects* the financial efficiency τ_q to exceed the threshold value $\hat{\tau}_q(Y, Z, b)$, would it choose a high precision $\tau_e = \bar{\tau}_e$. This is because precisions of signals a_j and ε_j are *complementary* in affecting the firm's ex ante profit. Intuitively, when the uncertainty over ε_j is reduced, knowing more about a_j would better enable the firm to maximize its expected profit. The equilibrium τ_e also depends on Y and Z; that is, when Y or Z increases, the marginal benefit of increasing the signal precision also increases for the firm, and so the firm is more likely to acquire more precise information.

Proposition 2 is a novel result of our model. Earlier work in the literature such as Goldstein and Yang (2015) has shown information production complementarity *within* the financial market. Our paper shows information production complementarity *between* the real side of the economy and the financial side.

Full Equilibrium With both Proposition 1 and Proposition 2, we are now able to characterize the full equilibrium. Proposition 1 gives the reaction function $\tau_q(\tau_e; c)$ while Proposition 2 gives the reaction function $\tau_e\left(\tau_q; Y, Z, b\right)$. Let

$$\tau_q^* \equiv \tau_q(\tau_e = \underline{\tau}_e; c)$$

and

$$\tau_q^{**} \equiv \tau_q(\tau_e = \bar{\tau}_e; c);$$

clearly $\tau_q^{**} > \tau_q^*$ by Proposition 1. Proposition 3 follows.

Proposition 3 *The rational expectations equilibrium has three cases:*

i) Case 1: $\tau_q^* > \hat{\tau}_q$ There is a unique equilibrium: $(\tau_e, \tau_q) = \left(\bar{\tau}_e, \tau_q^{**}\right)$;

ii) Case 2: $\hat{\tau}_q \in \left[\tau_q^*, \tau_q^{**}\right]$ There are multiple (two) equilibria: $(\tau_e, \tau_q) = \left(\bar{\tau}_e, \tau_q^{**}\right)$ or $\left(\underline{\tau}_e, \tau_q^*\right)$;

iii) Case 3: $\tau_q^{**} < \hat{\tau}_q$ There is a unique equilibrium: $(\tau_e, \tau_q) = \left(\underline{\tau}_e, \tau_q^*\right)$,

where threshold $\hat{\tau}_q = \hat{\tau}_q(Y, Z, b)$ is given by (24).

Proof. *See Appendix.* ■

The intuition behind Proposition 3 is as follows. If aggregate output Y is so high (and hence the threshold $\hat{\tau}_q(Y, Z, b)$ is so low by (24)) such that acquiring information is a *dominant strategy* for the firm (regardless of financial price informativeness τ_q), then a unique equilibrium exists in which the real side acquires information and the financial efficiency is also in a higher level. This is case 1. Conversely, if aggregate output Y is so low (and hence the threshold $\hat{\tau}_q(Y, Z, b)$ is so high) such that not acquiring information is a *dominant strategy* for the firm, then a unique equilibrium exists in which the real side does not acquire information and the financial efficiency is also in a lower level. This is case 3. Between these two extreme cases, there are *self-fulfilling* multiple equilibria, which is case 2.

Figure 2 illustrates the three equilibrium cases, corresponding to different levels of aggregate output (i.e., $Y_H > Y_M > Y_L$). Recall that function $G(\tau_q; \tau_e)$ measures the gain in information advantage for an informed speculator over an uninformed one, whereas the utility cost of information acquisition for an informed speculator is $e^{\gamma c}$. Similarly, it is easy to see that when Y is kept constant, a change in b or c or Z (where a change in c corresponding to a vertical shift in the horizontal line $e^{\gamma c}$ in Figure 2) also leads to different cases of equilibrium.

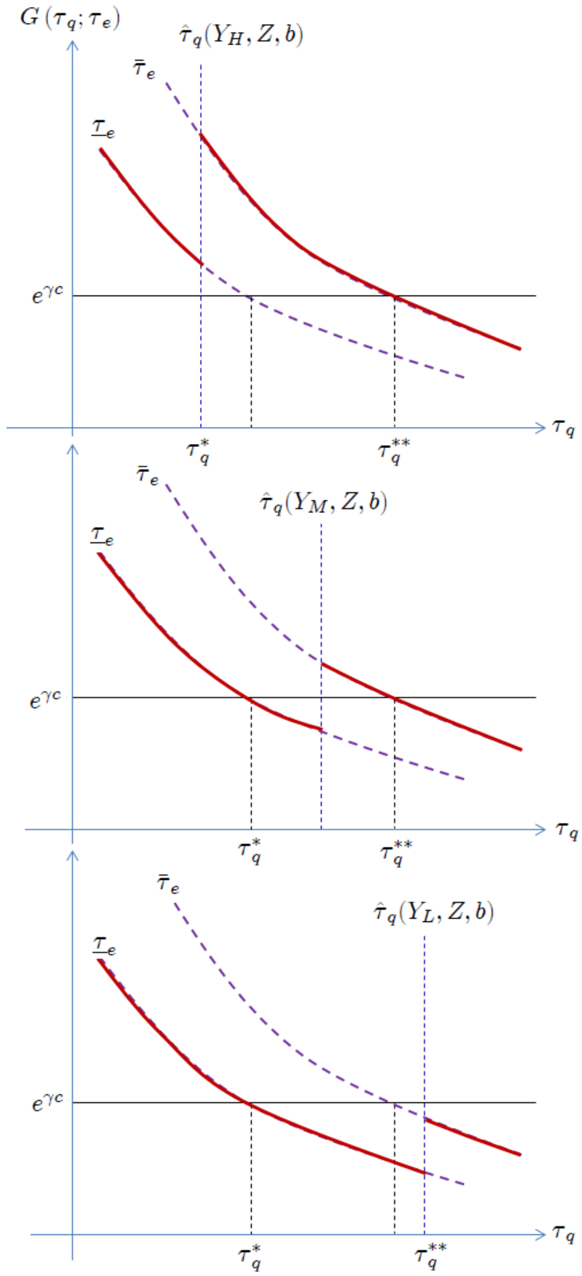

Figure 2: Three Cases of Equilibrium with $Y_H > Y_M > Y_L$ (Case 1: Top; Case 2: Middle; Case 3: Bottom)

4 The Macroeconomic Model

In this section, we extend the model to a macroeconomic framework. The extended model provides a macroeconomic background of the baseline model and endogenizes various exogenous specifications and variables of the baseline model. In particular, the aggregate output (i.e., real GDP), Y, is endogenized, which gives a number of new implications.

4.1 Setup

Final goods firms The final consumption good is produced with inputs of a continuum of capital goods according to a Dixit-Stiglitz production function

$$Y = \left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{25}$$

where $j \in [0, 1]$, $\theta > 1$ is the elasticity of substitution between intermediate capital goods, and ϵ_j measures the *demand shock* to intermediate good j.

The representative competitive final goods firm maximizes its profits:

$$\max \; P \left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int P_j Y_j dj, \tag{26}$$

where P_j is the price of intermediate good j. The price of the consumption good, P, is normalized as the numeraire price, i.e., $P \equiv 1$. The first-order condition of (26) with respect to Y_j gives

$$\left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} \epsilon_j^{\frac{1}{\theta}} Y_j^{-\frac{1}{\theta}} = P_j.$$

This implies the demand schedule for good j:

$$Y_j = \left(\frac{1}{P_j} \right)^{\theta} \epsilon_j Y,$$

which endogenizes the demand function of (2).

Intermediate Goods Firms There is a continuum of intermediate (capital) goods firms, indexed by j. The setup for a typical firm, firm j, is presented in Section 2.1. We may think of each intermediate good as being produced by one firm that is located on an island in the spirit of Lucas (1972). Both a_j and ε_j are i.i.d. across firms (or islands).

Financial markets Each island has a financial market, which trades the financial asset (derivative) contingent on the intermediate goods firm's asset value, $V_j = Y_j P_j$, on that island. The payoff of the financial derivative contract is $v_j = \log V_j$. The demand from noise/liquidity traders in financial market j is $n_j \sim N(0, \sigma_n^2)$, and n_j is independent across islands. In the current framework of the aggregate economy, we may interpret noise trading as: 1) foreign capital flow, or 2) liquidity trading by some investors who must trade (for exogenous reasons such as balancing portfolios, endowment shocks, and so on).

The setup for information acquisition on each island is the same as that in Section 3.

Investors The economy consists of a continuum of investors with a unit measure. Each investor is endowed with W_0 units of the final consumption good at T_0. Each investor has three identities: capital suppliers (i.e., lenders), firm owners (i.e., shareholders), and financial market traders. The economy is decentralized, analogous to the Robinson Crusoe economy. The decisions of an investor made under different identities are independent.

We assume that W_0 is sufficiently high and a storage technology exists, so that in equilibrium $R_f = 1$. An investor maximizes utility:

$$U(C^i) = -\exp\left(-\tau C^i\right) \tag{27}$$

with

$$C^i = W_0 + (\Pi - \chi) + D^i,$$

where C^i is the end-of-period wealth at T_4 for investor i. The term Π is the aggregate profit of firms, that is, $\Pi = \int (P_j Y_j - R_f K_j)\, dj$, the value of which is given in (18). The term $\Pi - \chi$ is the aggregate net profit of firms distributed to an investor (as an owner of firms), where $\chi \in \{0, b\}$ is the aggregate information acquisition cost to the firms.[10] The term D^i is investor i's position in financial market trading. We make a weak assumption that a trader can access trading in all financial markets. In a symmetric equilibrium, a trader is informed on λ fraction of islands and uninformed on the remaining $1-\lambda$ fraction of islands. In addition, all firms have λ proportion of informed traders and $1-\lambda$ proportion of uninformed traders, so $D^i = -\lambda c + \int_{j\in I_i} d_j^{Ii}(v_j - q_j)dj + \int_{j\in[0,1]\setminus I_i} d_j^{Ui}(v_j - q_j)dj$, where I_i is the set of islands on which trader i is informed, d_j^{Ii} is that trader's position on island j as an informed trader, and d_j^{Ui} is that trader's position on island j as an uninformed trader.

4.2 Equilibrium

Within each island, the equilibrium is given by Lemmas 1-2 and Propositions 1-3. Now we study the equilibrium of the aggregate economy, endogenizing Y. We consider the symmetric equilibrium, in which all intermediate goods firms have the same level of information precision.

Substituting (16) and (1) into (25) yields

$$Y = Z^{\Theta}\left[\eta\left(1-\frac{1}{\theta}\right)Y^{\frac{1}{\theta}}\right]^{\Theta\eta}\left[\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}}|s_j, \tilde{q}_j\right]\right)^{\Theta-1} dj\right]^{\frac{\theta}{\theta-1}}. \tag{28}$$

Exploiting the law of iterated expectations, we have

$$\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}}|s_j, \tilde{q}_j\right]\right)^{\Theta-1} dj = \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}}|s_j, \tilde{q}_j\right]\right)^{\Theta}\right].$$

[10] We will consider the symmetric equilibrium, in which either all firms or none of them acquires information.

Hence, (28) implies

$$Y = Z^{\Theta \frac{\theta}{\theta - \Theta\eta}} \left[\eta \left(1 - \frac{1}{\theta}\right)\right]^{\Theta\eta \frac{\theta}{\theta - \Theta\eta}} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}\frac{\theta}{\theta - \Theta\eta}}.$$

Similarly, the aggregate investment in the economy is given by

$$K = \int K_j dj = Z^{\left(1-\frac{1}{\theta}\right)\Theta} \left[\eta \left(1 - \frac{1}{\theta}\right) Y^{\frac{1}{\theta}}\right]^{\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}. \tag{29}$$

Therefore, we have

$$Y = ZAK^{\eta}, \tag{30}$$

where $A = A(\tau_e, \tau_q)$ is given by

$$\begin{aligned} A(\tau_e, \tau_q) &= \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1} - \eta} \\ &= \exp\left\{\left(\frac{\theta}{\theta - 1} - \eta\right)\left(\begin{array}{c} \Theta\left(1 - \frac{1}{\theta}\right)\left(-\frac{1}{2}\frac{1}{\tau_a}\right) + \frac{1}{2}\left[\Theta\left(1 - \frac{1}{\theta}\right)\right]^2 \frac{1}{\tau_a} + \Theta\frac{1}{\theta}\left(-\frac{1}{2}\frac{1}{\tau_\varepsilon}\right) + \frac{1}{2}\left(\Theta\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon} \\ -\Theta(\Theta - 1)\left[\frac{1}{2}\left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_e} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon + \tau_q}\right] \end{array}\right)\right\}, \end{aligned} \tag{31}$$

which is the *endogenous* aggregate TFP.

Equation (30) implies that despite heterogeneity among firms caused by idiosyncratic productivity shocks and demand shocks, our economy works as if there existed a representative firm with productivity A and aggregate investment K.

By (29) and (30), the aggregate investment can also be expressed as

$$K = K(\tau_e, \tau_q; Z) = \left[\eta\left(1 - \frac{1}{\theta}\right) Z \cdot A(\tau_e, \tau_q)\right]^{\frac{\Theta}{1 - \eta\frac{\Theta}{\theta}}}.$$

Because $0 < \eta < 1$ and $\Theta \in (1, \theta)$, K is increasing in A and thus is increasing in τ_e and τ_q. This way, the aggregate output can also be expressed as

$$Y = Y(\tau_e, \tau_q; Z) = \left[\eta\left(1 - \frac{1}{\theta}\right)\right]^{\frac{\Theta}{1 - \eta\frac{\Theta}{\theta}}\eta} \left[Z \cdot A(\tau_e, \tau_q)\right]^{\frac{\theta\Theta}{\theta - \eta\Theta}}. \tag{32}$$

Proposition 4 follows.

Proposition 4 *Both the endogenous aggregate TFP, A, and the aggregate investment, K, are increasing in τ_e and τ_q. Hence, the aggregate output Y is increasing in τ_e and τ_q.*

Proof. *See Appendix.* ■

Proposition 4 highlights two effects of information frictions. First, given K, the endogenous aggregate TFP, A, measuring the efficiency of resource allocation, has the properties of $\frac{\partial A}{\partial \tau_e} > 0$ and $\frac{\partial A}{\partial \tau_q} > 0$. Efficient allocation requires more resources to be allocated to firms with higher realized A_j and ϵ_j. In other words, efficient investment K_j should be more aligned with realized A_j and ϵ_j. So, more precise information about A_j and ϵ_j achieved through information acquisition helps improve allocative efficiency. Second, higher uncertainty also leads to a lower level of aggregate investment, that is, $\frac{\partial K}{\partial \tau_e} > 0$ and $\frac{\partial K}{\partial \tau_q} > 0$.

We parameterize our economy by (b, c, Z), and characterize the full equilibrium of the aggregate economy, which is given by the following joint equations:

$$\tau_e = \tau_e(\tau_q; Y, Z, b) \qquad \text{(A firm's optimal information choice)} \quad (33)$$

$$\tau_q = \tau_q(\tau_e; c) \qquad \text{(Financial market equilibrium)} \quad (34)$$

$$Y = Z \cdot A(\tau_e, \tau_q)\left[K(\tau_e, \tau_q; Z)\right]^{\eta}, \qquad \text{(Aggregate economy equilibrium)} \quad (35)$$

where (33), (34) and (35) are given by Proposition 2, Proposition 1 and Proposition 4, respectively.

Proposition 5 *The full equilibrium of the aggregate economy is characterized by triplet* (τ_e, τ_q, Y)*, which solves the system of equations (33)-(35) for given* (b, c, Z).

We discuss three important implications of Proposition 5. First, a small shock (i.e., a small increase in c or b, or a small decrease in Z) can have a large impact on the aggregate economy (aggregate output Y) due to the compound feedback loops of information amplification, as illustrated in Figures 3; detailed quantitative analysis of the comparative statics with respect to c, b, and Z will be provided in the next section. Our information channel of amplification contrasts with the financing channel in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012), where a negative shock originating in either the real sector or the financial sector can lead to a large drop in the aggregate output.

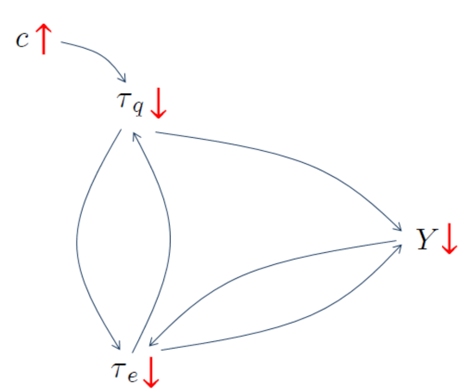

Figure 3: Information amplification

In particular, the amplification in our model can arise from the presence of multiple equilibria (i.e., discontinuity). That is, pure self-fulfilling beliefs in the absence of any aggregate shock or a small aggregate shock can trigger the equilibrium switching from one regime to the other, generating a very large drop in the aggregate-level output and investment. To illustrate the effect, we fix c and b, and express Y as a function of Z (recall equation (32)); see the formal expression of function $Y(Z;A)$ including the thresholds $\underline{Z}$ and $\bar{Z}$ in (A.7) in the appendix. Figure 4 depicts function $Y(Z;A)$.

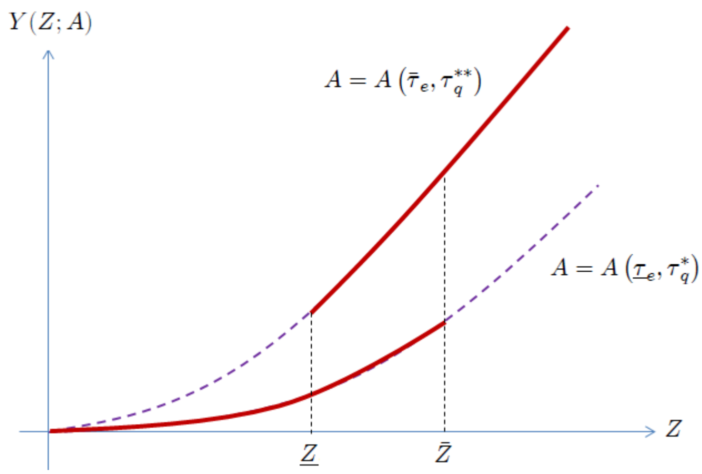

Figure 4: Aggregate output $Y(Z;A)$ in the presence of multiple (two) equilibria

Recall that A is the endogenous TFP (given in (31)). In Figure 4, when Z is low enough such that $Z \in (-\infty, \underline{Z})$, there is a unique "bad" equilibrium; when Z is high enough such that $Z \in (\bar{Z}, +\infty)$, there is a unique "good" equilibrium. When $Z \in [\underline{Z}, \bar{Z}]$, there are two *self-fulfilling* equilibria. Clearly, a small change in Z around $Z = \bar{Z}$ (i.e., a slight decrease in Z from above $\bar{Z}$ to below $\bar{Z}$) can trigger the equilibrium switching from "good" to "bad", resulting in a large drop in Y. Figure 4 shows several interesting implications. The first is that a small shock and a big shock to Z can have dramatically different implications. While a small decline in Z leads to a steady decrease in output, a big decline in Z may trigger a self-fulfilling crisis. The second implication is that a positive shock and a negative shock to Z potentially have asymmetric effects on equilibrium output. When Z increases, the equilibrium output increases steadily. However, when Z declines, the equilibrium output may exhibit a sudden large decline if the economy falls to the bad equilibrium. Conducting comparative statics with respect to c and b instead of Z shows similar patterns (see next section).

Second, our model implies information contagion and spillover, as illustrated in Figures 5. That is, a shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. This result is consistent with a large amount of anecdotal evidence that idiosyncratic firm-level shocks can be the origin of aggregate fluctuations (i.e., microfoundation for aggregate shocks; see Gabaix (2011)). Again, a quantitative analysis of this result will be provided in the next section.

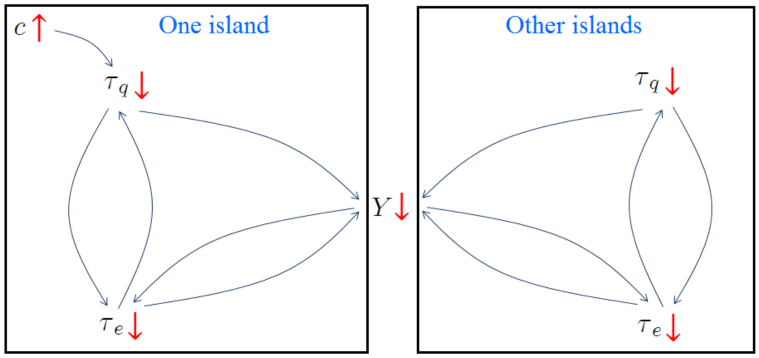

Figure 5: Information contagion

Third, our model endogenizes together the three variables — financial uncertainty, real uncertainty, and aggregate economic activities — and show how they are related. The residual financial uncertainty (or equivalently the financial market efficiency defined in Brunnermeier (2005) and Goldstein and Yang (2015)) is given by[11]

$$SD\left(\varepsilon_j | s_j, q_j\right) = \sqrt{\frac{1}{\tau_\varepsilon + \tau_q}}, \tag{36}$$

and the residual real uncertainty (or the forecast error) faced by a firm is given by

$$SD\left(a_j | s_j\right) = \sqrt{\frac{1}{\tau_a + \tau_e}}. \tag{37}$$

We have shown that an adverse shock in c or b or Z leads to a decrease in τ_e and τ_q together with a decrease in Y, which means that a rise in both real uncertainty and financial uncertainty is accompanied by a fall in aggregate GDP Y. In other words, uncertainty in both financial markets and the real economy rises during recessions.[12]

5 Quantitative Analysis

Our analytic analysis in the previous sections has demonstrated that the information interplay between the real sector and the financial sector can have strong effects on the economy. Our model is too stylized to be calibrated with the data. We will therefore assign values to parameters in our model to conduct several numerical illustrations below.

[11] Equivalently, we can define financial uncertainty as $SD[v_j|s_j, q_j] = \sqrt{\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j, q_j\right] + \left(1 - \frac{1}{\theta}\right)^2 Var\left[a_j|s_j\right]} = \sqrt{\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon + \tau_q} + \left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_e}}$.

[12] The evidence of countercyclical uncertainty in the macroeconomics literature is in line with the findings in the finance literature that the volatility of stock returns is higher in bad times than in good times (see, e.g., Schwert (1989a,b)).

Table 1 summarizes the parameter values chosen. We set the elasticity of substitution between intermediate goods θ to 6 as in David, Hopenhanya and Venkateswaran (2016). This implies that the gross markup is 15%. We set the degree of decreasing returns to scale of production η to 0.8, consistent with the recent estimates of Gopinath et al (2016). We set the risk (CARA) coefficient γ to 2 . This implies that a risky lottery drawn from a normal distribution with standard deviation of 0.2 $ will need an average payoff of 1.04 $ dollar to be equivalent to 1 $ for sure in financial speculations, which seems to be reasonable and in line with the risk premium between stocks and bonds. The commonly estimated standard deviation of firm-level physical productivity shocks σ_a is typically very large. However, it is difficult to imagine that firms would face such enormous uncertainty when making investment. We therefore borrow the unconditional residual uncertainty parameter from David, Hopenhanya and Venkateswaran (2016), where $\sigma_a = 0.45$ (or $\tau_a = 4.9383$). We set $\bar{\tau}_e = 6.5746$, implying that a firm can reduce its residual uncertainty on the productivity shock to 0.30 by paying the information acquisition cost. We set $\underline{\tau}_e = 1$, implying that the residual uncertainty in productivity equals 0.41 if a firm does not pay the information acquisition cost. Foster, Haltiwanger and Syverson (2008) show that demand shocks are more important than physical productivity shocks for firm turnover. We therefore set $\sigma_\varepsilon = 2.2502$ (or $\tau_\varepsilon = 0.1975$), implying that demand and productivity shocks contribute equally to firms' sales volatility. We set the precision of informed traders' signal τ_ϱ to 0.2922, meaning that an informed trader can reduce the conditional volatility of demand shock to 1.429 through his private signal. We set the common productivity $Z = 2$, information acquisition cost for a financial trader $c = 0.09$, and information acquisition cost for firm $b = 0.065$. These parameter values lead to two self-fulfilling equilibria in our model.

Parameter	Description	Value
τ_a	Precision of productivity shock prior	4.9383
τ_ε	Precision of demand shock prior	0.1975
τ_ϱ	Precision of informed traders' signal	0.2922
θ	Elasticity of substitution between intermediate goods	6
γ	Risk (CARA) coefficient	2
η	Degree of decreasing returns to scale of production	0.8
c	Information acquisition cost of financial markets	0.09
b	Information acquisition cost of the real side	0.065
Z	Common productivity shock	2
$\underline{\tau}_e$	Low precision of signals of the real side	1
$\bar{\tau}_e$	High precision of signals of the real side	6.5746

Table 1: Parameter values

Table 2 summarizes key results of the two self-fulfilling equilibria. First, both aggregate output and investment fall dramatically (by 42% and 42%, respectively) when the economy falls into the "bad" equilibrium . Second, information production from the financial sector and from the real sector is both lower in the"bad" equilibrium than in the "good" equilibrium. The firms and financial traders face productivity shocks with a posterior standard deviation of $(\bar{\tau}_e + \tau_a)^{-\frac{1}{2}} = 0.2947$ in the "good" equilibrium and $(\underline{\tau}_e + \tau_a)^{-\frac{1}{2}} = 0.4104$ in the "bad" equilibrium. The financial price can reduce the posterior standard deviation of firm demand shocks to $\left(\tau_q^{**} + \tau_\varepsilon\right)^{-\frac{1}{2}} = 1.8380$ for the "good" equilibrium but only to $\left(\tau_q^{*} + \tau_\varepsilon\right)^{-\frac{1}{2}} = 2.1567$ for the "bad" equilibrium. These numbers imply a 17% increase in financial uncertainty and a 33% increase in real uncertainty from the "bad" equilibrium to the "good" equilibrium. Third, the resulting information production declines have important consequences for allocation efficiency. The endogenous TFP declines by about 10%. To understand the decline, we compute two alternative counterfactual endogenous TFP. We first compute $A(\bar{\tau}_e, \tau_q^*)$, the level of endogenous TFP when only the quality of the information provided by the financial market deteriorates while the quality of the information provided by the firm stays at the level $\bar{\tau}_e$. We find that TFP would decline by about 4%. The other 6% decline in endogenous TFP is due to the decline in the firm's information production as indicated by $A(\underline{\tau}_e, \tau_q^{**})$, the level of endogenous TFP that the economy would obtain when only the quality of the information provided by the firm deteriorates while the quality of the information provided by the financial market stays at the level τ_q^{**}.

	"Good" Equilibrium	"Bad" equilibrium
τ_e	$\bar{\tau}_e = 6.5746$	$\underline{\tau}_e = 1$
τ_q	$\tau_q^{**} = 0.0985$	$\tau_q^{*} = 0.0175$
GDP (Y)	1.5109	0.8701
Aggregate Investment (K)	1.0073	0.5801
Endogenous TFP (A)	$\bar{A} = 0.7511$	$\underline{A} = 0.6726$[13]
TFP under changing τ_q^* only ($A(\bar{\tau}_e, \tau_q^*)$)		0.7199
TFP under changing $\underline{\tau}_e$ only ($A(\underline{\tau}_e, \tau_q^{**})$)		0.7017

Table 2: Numerical illustration for two self-fulfilling equilibria

As the nature of equilibrium crucially depends on the values of Z , c and b, our second exercise is hence to conduct *complete* comparative statics to understand their impact on equilibria in a quantitative sense. We fix all other parameter values (given in Table 1) but change one of Z, c and b each time and compute the equilibria accordingly. We report our results in Figure 6. Figure 6 has three columns, summarizing the comparative statics with respect to Z , c, and b in turn. The first panel plots the equilibrium aggregate output, the second panel plots the posterior standard deviation of firms' productivity shocks (given in (37)), and the third panel reports the posterior

[13] Note that $\bar{A} = A(\bar{\tau}_e, \tau_q^{**})$ and $\underline{A} = A(\underline{\tau}_e, \tau_q^{*})$; see (A.7).

standard deviation of firms' demand shocks inferred from the financial price (given in (36)).

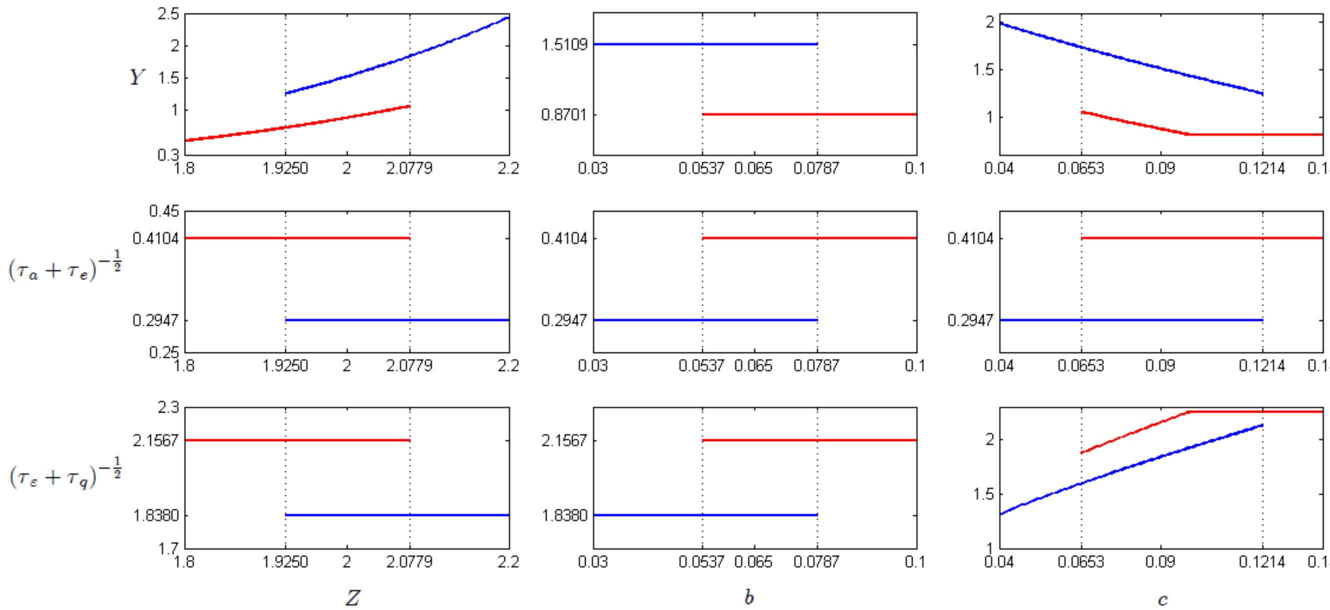

Figure 6: Comparative static analysis

The first column of Figure 6 shows that the equilibrium is unique if $Z > \bar{Z} = 2.0779$ or $Z < \underline{Z} = 1.9250$. Suppose the economy initially starts with aggregate common productivity $Z = 2.1203$. A small drop in Z by more than 2 percent would trigger a self-fulfilling crisis (also see Figure 4). In the second and third columns of Figure 6, the model generates two self-fulfilling equilibria when the information acquisition cost, c or b, is at the intermediate level.[14] Due to the binary information choice of the firm, aggregate output is insensitive to the change in b when the equilibrium is unique. Similar to the effect of Z, if the initial level of b or c is close to their lower threshold for multiple equilibria, then a small shock (i.e., a small increase in c or b) can cause a sudden large decline in aggregate output.

Our final exercise is to show information contagion in our model. Again we assume that all parameters are initially given in Table 1, except that we set $b = 0.0786$, slightly lower than the upper threshold of b to have multiple (two) equilibria. According to Figure 6, the economy initially has two equilibria. Now we assume that a small fraction $\kappa = 5\%$ of firms suffer a shock in the sense that their information acquisition cost b increases slightly to 0.0788, which means that a unique *"bad"* equilibrium takes over in the economy of these islands by Figure 6. How about the other 95% islands? The economy of the other 95% islands will unavoidably fall into the bad equilibrium unless their acquisition cost b decreases below 0.0778.

[14] In the third common (the comparative statics with regard to c), when c is higher than a certain level, no traders acquire information about the demand shocks and hence the financial price efficiency τ_q becomes a constant.

6 Conclusion

We develop a model of informational interdependence between financial markets and the real economy. We endogenize financial and real uncertainty and show how they relate to aggregate economic activity. Information production in the real sector and that in the financial sector exhibit strategic complementarity. The key reason is that a financial price is a combination of firm disclosure and financial market price discovery. When a firm tries to maximize its monopoly profits in the real sector and speculators try to gain from arbitraging in financial markets, it is optimal for them to learn from each other. The mutual learning results in strategic complementarity in information production. In the general equilibrium, the amount of information available in the economy and the aggregate economic activity feed back into each other and reinforce each other.

Besides the implications on economic uncertainty, our model has additional implications regarding macroeconomic activities, including the effect of financial market efficiency on resource misallocation, amplification and contagion in business cycle fluctuations, and self-fulfilling economic crises. Our model is essentially static and qualitative, which can clearly and cleanly deliver key mechanisms and implications of the model. Extending our static model to a dynamic economy and fully examining the quantitative importance of our mechanisms are left for future research.

Appendix

A Proofs

Proof of Lemma 1: For an informed trader,

$$\mathbb{E}[v_j|s_j,q_j,x_j^i]=\frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j,x_j^i\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right)+\frac{\tau_e}{\tau_a+\tau_e}s_j\right]$$

where $E\left[\varepsilon_j|s_j,q_j,x_j^i\right]=\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\varrho+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)+\frac{\tau_\varrho}{\tau_\varepsilon+\tau_\varrho+\tau_q}x_j^i+\frac{\tau_q}{\tau_\varepsilon+\tau_\varrho+\tau_q}\tilde{q}_j$ with $\tilde{q}_j(q_j,s_j)=\frac{q_j-\beta_0-\beta_1\beta_2 s_j}{\beta_1}$, and

$$Var[v_j|s_j,q_j,x_j^i]=\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j,q_j,x_j^i\right]+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}$$

where $Var\left[\varepsilon_j|s_j,q_j,x_j^i\right]=\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}$.

For an uninformed trader,

$$\mathbb{E}[v_j|s_j,q_j]=\frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right)+\frac{\tau_e}{\tau_a+\tau_e}s_j\right]$$

where $\mathbb{E}\left[\varepsilon_j|s_j,q_j\right]=\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j$, and

$$Var[v_j|s_j,q_j]=\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j,q_j\right]+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}$$

where $Var\left[\varepsilon_j|s_j,q_j\right]=\frac{1}{\tau_\varepsilon+\tau_q}$.

Therefore, the market clearing condition, (7), implies

$$\begin{aligned}0 &= n_j+\lambda\frac{\frac{1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_\varrho+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\+\frac{\tau_\varrho}{\tau_\varepsilon+\tau_\varrho+\tau_q}\varepsilon_j+\frac{\tau_q}{\tau_\varepsilon+\tau_\varrho+\tau_q}\tilde{q}_j\end{array}\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\begin{array}{c}\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right)\\+\frac{\tau_e}{\tau_a+\tau_e}s_j\end{array}\right]}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}\right]}\\ &\quad+(1-\lambda)\frac{\frac{1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j\end{array}\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right)+\frac{\tau_e}{\tau_a+\tau_e}s_j\right]}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}\right]}.\end{aligned} \tag{A.1}$$

It is straightforward to see that (A.1) can be transformed to

$$f\left(s_j,q_j,y\right)+\lambda\frac{\frac{1}{\theta}\frac{\tau_\varrho}{\tau_\varepsilon+\tau_\varrho+\tau_q}}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}\right]}\varepsilon_j+n_j=0,$$

where $f(s_j, q_j, y)$ is a linear function of s_j, q_j and y. Hence,

$$\beta_3 = \frac{\gamma\left[\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q} + \left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e}\right]}{\lambda\frac{1}{\theta}\frac{\tau_\varrho}{\tau_\varepsilon+\tau_\varrho+\tau_q}},$$

which, by substituting $\tau_q \equiv \frac{1}{\beta_3^2\sigma_n^2}$, implies

$$\frac{\lambda}{\theta}\left[(\tau_a+\tau_e)\tau_\varrho\right]\beta_3^3 - \gamma\left[\left(\frac{1}{\theta}\right)^2(\tau_a+\tau_e) + \left(1-\frac{1}{\theta}\right)^2(\tau_\varepsilon+\tau_\varrho)\right]\beta_3^2 - \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n = 0, \quad \text{(A.2)}$$

where $\tau_n = \frac{1}{\sigma_n^2}$.

(A.2) clearly has a unique positive solution with respect to β_3. In fact, if we write LHS of (A.2) as function $\Lambda(\beta_3)$, it is easy to show that equation $\Lambda(\beta_3) + \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n = 0$ has a unique positive solution. Hence, equation $\Lambda(\beta_3) = 0$ has a unique positive solution, around which $\frac{\partial\Lambda}{\partial\beta_3} > 0$. We also prove that the unique positive solution of β_3 is decreasing in λ. In fact, $\frac{\partial\Lambda}{\partial\beta_3} > 0$ and $\frac{\partial\Lambda}{\partial\lambda} = \frac{1}{\theta}\left[(\tau_a+\tau_e)\tau_\varrho\right]\beta_3^3 > 0$, so $\frac{d\beta_3}{d\lambda} = -\frac{\frac{\partial\Lambda}{\partial\lambda}}{\frac{\partial\Lambda}{\partial\beta_3}} < 0$. Also, the unique positive solution of β_3 is decreasing in τ_e. In fact,

$$\frac{\partial\Lambda}{\partial\tau_e} = \frac{\lambda}{\theta}\tau_\varrho\beta_3^3 - \gamma\left(\frac{1}{\theta}\right)^2\beta_3^2 = \frac{\gamma\left(1-\frac{1}{\theta}\right)^2(\tau_\varepsilon+\tau_\varrho)\beta_3^2 + \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n}{\tau_a+\tau_e} > 0,$$

where the second equality is due to (A.2), so $\frac{d\beta_3}{d\tau_e} = -\frac{\frac{\partial\Lambda}{\partial\tau_e}}{\frac{\partial\Lambda}{\partial\beta_3}} < 0$ (or $\frac{d\tau_q}{d\tau_e} > 0$).

Proof of Lemma 2: We have

$$\begin{aligned}\log\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right] &= \frac{1}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j\right] + (1-\frac{1}{\theta})\left[\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right) + \frac{\tau_e}{\tau_a+\tau_e}s_j\right] \\ &\quad +\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q} + \frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}.\end{aligned}$$

So

$$\begin{aligned}\phi_0' &= \Theta\log\left[\eta\left(1-\frac{1}{\theta}\right)\right] + \frac{\Theta}{\theta}y + \frac{\Theta}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\right] + \Theta(1-\frac{1}{\theta})\left[\frac{\tau_a}{\tau_a+\tau_e}\left(-\frac{1}{2}\sigma_a^2\right)\right] \\ &\quad +\frac{1}{2}\Theta\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q} + \frac{1}{2}\Theta\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}.\end{aligned}$$

In addition,

$$\mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right)\right]^{\Theta}\right)=\exp\left\{\begin{array}{c}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\\ -\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\end{array}\right\}.$$

It is easy to show that

$$\frac{\partial\Pi}{\partial\tau_e}>0 \qquad \text{and} \qquad \frac{\partial\Pi}{\partial\tau_q}>0,$$

by noting that

$$sgn\left(\frac{\partial\Pi}{\partial\tau_e}\right)=sgn\left\{\begin{array}{l}\exp\left\{\begin{array}{c}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\\ -\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\end{array}\right\}\\ \cdot\Theta\left(\Theta-1\right)\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{(\tau_a+\tau_e)^2}\end{array}\right\}.$$

Moreover,

$$\frac{\partial^2\Pi}{\partial\tau_e\partial\tau_q}>0 \text{ and } \frac{\partial^2\Pi}{\partial\tau_e\partial Y}>0$$

Proof of Proposition 1: The proof is quite similar to that in Grossman and Stiglitz (1980). By the definition $EV(W^i)\equiv\mathbb{E}\left[U(W^i)|s_j,q_j\right]$, we have

$$\frac{EV(W^{Ii})}{EV(W^{Ui})}=e^{\gamma c}\sqrt{\frac{Var[v_j|s_j,q_j,x_j^i]}{Var[v_j|s_j,q_j]}}.$$

Thus,

$$\begin{aligned}&\frac{EV(W^{Ii})}{EV(W^{Ui})}=1\\ \iff&\sqrt{\frac{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]+Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j)}{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]+Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j,x_j^i)}}=e^{\gamma c}\\ \iff&\sqrt{\frac{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}}}=e^{\gamma c}\end{aligned}\qquad\text{(A.3)}$$

Let $F\left(\tau_q;\tau_e\right)=\frac{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}}$, which implies

$$\begin{aligned}\frac{\partial F}{\partial\tau_e}&=\frac{-\left(1-\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_a+\tau_e}\right)^2\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}-\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}\right]}{\left[\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]^2}\\ &<0\end{aligned}$$

and

$$\frac{\partial F}{\partial \tau_q} = \frac{\left\{ \begin{array}{c} \left(\frac{1}{\theta}\right)^2 \left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e} \left[\left(\frac{1}{\tau_\varepsilon+\tau_q}\right)^2 - \left(\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}\right)^2 \right] \\ + \left(\frac{1}{\theta}\right)^4 \left(\frac{1}{\tau_\varepsilon+\tau_q}\right) \left(\frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}\right) \left(\frac{1}{\tau_\varepsilon+\tau_q} - \frac{1}{\tau_\varepsilon+\tau_\varrho+\tau_q}\right) \end{array} \right\}}{\left[\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_q} \right]^2} > 0.$$

Because $F(\tau_q; \tau_e) = e^{-2\gamma c}$, by differentiating both sides of the equation, we have

$$\frac{\partial \tau_q}{\partial \tau_e} = -\frac{\partial F / \partial \tau_e}{\partial F / \partial \tau_q} > 0.$$

Also, by $\frac{\partial F}{\partial \tau_q} > 0$, we have

$$\frac{\partial \tau_q}{\partial c} < 0.$$

Proof of Proposition 2: Because $\frac{\partial \Pi}{\partial \tau_e} > 0$, we have $\Pi(\tau_e = \bar{\tau}_e; \tau_q, Y) - \Pi(\tau_e = \underline{\tau}_e; \tau_q, Y) > 0$ for a given τ_q and Y. Because $\frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0$, there exisits a unique $\hat{\tau}_q$ such that

$$\Pi(\tau_e = \bar{\tau}_e; \hat{\tau}_q, Y) - \Pi(\tau_e = \underline{\tau}_e; \hat{\tau}_q, Y) = b. \tag{A.4}$$

Denote the LHS of (A.4) by function $\Gamma(\hat{\tau}_q, Y)$ for given a $\bar{\tau}_e$ and $\underline{\tau}_e$. Because $\frac{\partial^2 \Pi}{\partial \tau_e \partial \tau_q} > 0$ and $\frac{\partial^2 \Pi}{\partial \tau_e \partial Y} > 0$, we have that $\frac{\partial \Gamma}{\partial \hat{\tau}_q} > 0$ and $\frac{\partial \Gamma}{\partial Y} > 0$. Therefore,

$$\frac{\partial \hat{\tau}_q(Y, b)}{\partial Y} = -\frac{\frac{\partial \Gamma}{\partial Y}}{\frac{\partial \Gamma}{\partial \hat{\tau}_q}} < 0$$

and

$$\frac{\partial \hat{\tau}_q(Y, b)}{\partial b} > 0.$$

Proof of Proposition 3: Based on the results in Propositions 1 and 2, it is straightforward to obtain Proposition 3.

Proof of Proposition 4: Substituting (16) and (1) into (25)

$$
\begin{aligned}
Y &= \left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\
&= \left[\int \epsilon_j^{\frac{1}{\theta}} \left(A_j \left\{\left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}}\right]^{\Theta} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right\}^{\eta}\right)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\
&= \left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}}\right]^{\Theta\eta} \left[\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta\eta\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\
&= \left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}}\right]^{\Theta\eta} \left[\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta-1} dj\right]^{\frac{\theta}{\theta-1}}, \qquad \text{(A.5)}
\end{aligned}
$$

where the last equality follows based on $\Theta = -\frac{1}{\eta\left(1-\frac{1}{\theta}\right)-1}$.

Exploiting the law of iterated expectations, we have

$$
\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta-1} dj = \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right].
$$

Hence, (A.5) becomes

$$
Y = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta\eta\frac{\theta}{\theta-\Theta\eta}} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}}. \qquad \text{(A.6)}
$$

Similarly, the aggregate investment in the economy is given by

$$
\begin{aligned}
K &= \int K_j dj = \left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}}\right]^{\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\} \\
&= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta\eta\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+1}.
\end{aligned}
$$

where the second equality is obtained by substituting the expression of Y in (A.6). Because $\Theta\eta\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+\Theta = \frac{\theta}{\theta-\Theta\eta}\Theta$ and $\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta} - \left(\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+1\right)\eta = \frac{\theta}{\theta-1} - \eta$,

$$
Y = AK^{\eta},
$$

where $A = A(\tau_e, \tau_q)$ is given by

$$
\begin{aligned}
A(\tau_e, \tau_q) &= \left\{ \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right] \right\}^{\frac{\theta}{\theta-1}-\eta} \\
= \quad \exp & \left\{ \left(\frac{\theta}{\theta-1} - \eta \right) \left(\begin{array}{c} \Theta\left(1-\frac{1}{\theta}\right)\left(-\frac{1}{2}\frac{1}{\tau_a}\right) + \frac{1}{2}\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2 \frac{1}{\tau_a} + \Theta\frac{1}{\theta}\left(-\frac{1}{2}\frac{1}{\tau_\varepsilon}\right) + \frac{1}{2}\left(\Theta\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon} \\ -\Theta(\Theta-1)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_q}\right] \end{array} \right) \right\}.
\end{aligned}
$$

Because $0 < \eta < 1$ and thus $\frac{\theta}{\theta-1} - \eta > 0$, $A(\tau_e, \tau_q)$ is increasing in τ_e and τ_q.

Also, we can express K in terms of A:

$$
\begin{aligned}
K &= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} \left\{ \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right] \right\} \cdot Y^{\frac{\Theta}{\theta}} \\
&= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} \left\{ \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right] \right\} \cdot (AK^{\eta})^{\frac{\Theta}{\theta}} \\
&= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} (AK^{\eta})^{\frac{\Theta}{\theta}} = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}} K^{\eta\frac{\Theta}{\theta}} \\
\Longleftrightarrow \quad & K^{1-\eta\frac{\Theta}{\theta}} = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}} \\
\Longleftrightarrow \quad & K = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}} A^{\frac{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}}{1-\eta\frac{\Theta}{\theta}}} \\
\Longleftrightarrow \quad & K = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}} A^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}},
\end{aligned}
$$

where the last equality is obtained by $\Theta = -\frac{1}{\eta\frac{\theta-1}{\theta}-1} = -\frac{\frac{\theta}{\theta-1}}{\eta-\frac{\theta}{\theta-1}}$ and thus $\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta} = \Theta\frac{\theta-1}{\theta}+\frac{\Theta}{\theta} = \Theta$. Because $0 < \eta < 1$ and $\Theta \in (1, \theta)$, K is increasing in A and thus is increasing in τ_e and τ_q.

Some results in Section 4.2: Function $Y(Z; A)$ is given by

$$
Y(Z;A) = \left\{ \begin{array}{ll}
\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \bar{A}^{\frac{\theta\Theta}{\theta-\eta\Theta}} Z^{\frac{\theta\Theta}{\theta-\eta\Theta}} & \text{if } Z \in (\bar{Z}, +\infty) \\
\left\{ \begin{array}{l} \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \bar{A}^{\frac{\theta\Theta}{\theta-\eta\Theta}} Z^{\frac{\theta\Theta}{\theta-\eta\Theta}} \\ \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \underline{A}^{\frac{\theta\Theta}{\theta-\eta\Theta}} Z^{\frac{\theta\Theta}{\theta-\eta\Theta}} \end{array} \right. & \text{if } Z \in [\underline{Z}, \bar{Z}] \\
\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \underline{A}^{\frac{\theta\Theta}{\theta-\eta\Theta}} Z^{\frac{\theta\Theta}{\theta-\eta\Theta}} & \text{if } Z \in (-\infty, \underline{Z})
\end{array} \right. \tag{A.7}
$$

where

$$\Pi\left(\tau_e=\bar{\tau}_e;\tau_q^*,Y\left(\underline{A},\bar{Z}\right),\bar{Z}\right)-\Pi\left(\tau_e=\underline{\tau}_e;\tau_q^*,Y\left(\underline{A},\bar{Z}\right),\bar{Z}\right)=b;$$
$$\Pi\left(\tau_e=\bar{\tau}_e;\tau_q^{**},Y\left(\bar{A},\underline{Z}\right),\underline{Z}\right)-\Pi\left(\tau_e=\underline{\tau}_e;\tau_q^{**},Y\left(\bar{A},\underline{Z}\right),\underline{Z}\right)=b;$$
$$\bar{A}=A\left(\bar{\tau}_e,\tau_q^{**}\right);$$
$$\underline{A}=A\left(\underline{\tau}_e,\tau_q^{*}\right);$$
$$\tau_q^*\equiv\tau_q(\tau_e=\underline{\tau}_e;c);$$
$$\tau_q^{**}\equiv\tau_q(\tau_e=\bar{\tau}_e;c).$$

B Model extension in Section 3

We assume that the firm also receives a noisy signal on the demand shock:

$$x_j=\varepsilon_j+\rho,$$

where $\rho\sim N(0,\frac{1}{\tau_\rho})$. The firm may obtain or buy this piece of information from informed traders. Under this alternative setup, we prove that the main result in Proposition 2 does not change.

Firm j's information set becomes $\{s_j,x_j,q_j\}$. The ex ante expected profit of firm j perceived at T_0 in (18) becomes

$$\Pi\left(\tau_e,\tau_q,\tau_\rho;Y\right)=B\cdot Y^{\frac{\Theta}{\theta}}\exp\left\{-\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right]\right\},$$

where $B=\left[1-\eta\left(1-\frac{1}{\theta}\right)\right]\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1}\cdot\exp\left\{\begin{array}{c}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}\\+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\end{array}\right\}$. It is easy to show that

$$\begin{aligned}\frac{\partial^2\Pi}{\partial\tau_e\partial\tau_q} &= B\cdot Y^{\frac{\Theta}{\theta}}\left\{\begin{array}{c}\exp\left\{-\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_e}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right]\right\}\\ \left[\Theta\left(\Theta-1\right)\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_a+\tau_e}\right)^2\right]\left[\Theta\left(\Theta-1\right)\frac{1}{2}\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2\right]\end{array}\right\}\\ &> 0,\end{aligned}$$

which means that as the financial efficiency (τ_q) increases, the firm has stronger incentives to acquire information about the productivity shock a_j.

We also show that

$$\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} = B \cdot Y^{\frac{\Theta}{\theta}} \left\{ \begin{array}{l} \exp\left\{-\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_e} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right]\right\} \\ \left[\Theta\left(\Theta-1\right)\frac{1}{2}\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2\right] \cdot \Theta\left(\Theta-1\right)\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2 \\ \left(\frac{1}{2} - \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right) \end{array} \right\}.$$

So, when $\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q} < \frac{1}{2}$, it follows that $\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} > 0$, which means that the firm's incentive to acquire information about the demand shock increases with financial efficiency. When $\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q} > \frac{1}{2}$, it follows that $\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} < 0$, which means that the firm's incentive to acquire information about the demand shock decreases with financial efficiency (i.e., strategic substitutability). Therefore, if the firm incurs a cost in acquiring information about the demand shock or the productivity shock, it may be endogenously optimal for the firm to choose to acquire information about the latter.

References

Angeletos, George-Marios, Guido Lorenzoni and Alessandro Pavan (2010). Beauty Contests and Irrational Exuberance: A Neoclassical Approach, mimeo.

Angeletos, George-Marios and Alessandro Pavan (2007). Efficient Use of Information and Social Value of Information, Econometrica, 75, 4, 1103-1142.

Bacchetta, Philippe, Cédric Tille, Eric van Wincoop (2012). Self-Fulfilling Risk Panics, American Economic Review, 102(7), 3674-3700.

Bachmann, Rudiger and Christian Bayer (2013), Wait-and-See Business Cycles? Journal of Monetary Economics, 60(6), 704-719

Bachmann, Rudiger and Christian Bayer (2014), Investment Dispersion and the Business Cycle, American Economic Review, 104(4), 1392-1416.

Benhabib, Jess, Xuewen Liu and Pengfei Wang (2016a). Sentiments, Financial Markets, and Macroeconomic Fluctuations, Journal of Financial Economics, 120 (2), 420-443.

Benhabib, Jess, Xuewen Liu and Pengfei Wang (2016b). Endogenous Information Acquisition and Countercyclical Uncertainty, Journal of Economic Theory, 165, 601-642.

Bernanke, Ben and Mark Gertler (1989). Agency Costs, Net Worth, and Business Fluctuations, American Economic Review 79 (1989) 14-31.

Bloom, Nicholas (2009), The Impact of Uncertainty Shocks, Econometrica, 77(3), 623-685.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten and Stephen Terry (2012), Really Uncertain Business Cycles, mimeo.

Bond, Philip, Alex Edmans and Itay Goldstein (2012), The Real Effects of Financial Markets, The Annual Review of Financial Economics is 4, 339-60.

Bond, Philip, Itay Goldstein and Edward Simpson Prescott (2010). Market-Based Corrective Actions, Review of Financial Studies, 23, 781-820.

Brunnermeier, Markus (2005). Information leakage and market efficiency, Review of Financial Studies, 18 (2), 417-457.

Chen, Qi, Itay Goldstein and Wei Jiang (2007). Price Informativeness and Investment Sensitivity to Stock Price, Review of Financial Studies, 20, 619-650.

Colombo, Luca, Gianluca Femminis and Alessandro Pavan (2014). Information Acquisition and Welfare, Review of Economic Studies, 81, 1438-1483.

David, Joel, Hugo Hopenhaynz and Venky Venkateswaranx (2016). Information, Misallocation and Aggregate Productivity, Quarterly Journal of Economics, 131 (2): 943-1005.

Dessaint, Olivier, Thierry Foucault, Laurent Frésard, and Adrien Matray (2017). Ripple Effects of Noise on Corporate Investment, mimeo.

Diamond, Douglas and Robert Verrecchia (1981). Information aggregation in a noisy rational expectations economy, Journal of Financial Economics, 1981, 9(3), pp. 221-35.

Dow, James and Gary Gorton (1997). Stock Market Efficiency and Economic Efficiency: Is There a Connection? Journal of Finance, 52, 1087-1129.

Dow, James and Rohit Rahi (2003). Informed Trading, Investment, and Economic Welfare, Journal of Business, 76, 430-454.

Fajgelbaum, Pablo, Edouard Schaal and Mathieu Taschereau-Dumouchel (2016), Uncertainty Traps, Quarterly Journal of Economics, forthcoming.

Fishman, Michael and Kathleen Hagerty (1992). Insider Trading and the Efficiency of Stock Prices, RAND Journal of Economics, 23, 106-122.

Foster, Lucia, John Haltiwanger and Chad Syverson (2008). Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability? American Economic Review, 98(1), 394-425.

Foucault, Thierry and Laurent Frésard (2016). Corporate Strategy, conformism, and the stock market, mimeo.

Foucault, Thierry and Thomas Gehrig (2008). Stock Price Informativeness, Cross-Listings and Investment Decisions, Journal of Financial Economics, 88, 146-168.

Gabaix, Xavier (2011). The Granular Origins of Aggregate Fluctuations, Econometrica, 79(3), 733-772.

Ganguli, Jayant and Liyan Yang (2009). Complementarities, Multiplicity, and Supply Information, Journal of the European Economic Association, 7(1), 90-115.

Goldstein, Itay and Alexander Guembel (2008). Manipulation and the Allocational Role of Prices, Review of Economic Studies, 75, 133-164.

Goldstein, Itay, Emre Ozdenoren and Kathy Yuan (2013). Trading frenzies and their impact on real investment, Journal of Financial Economics, Elsevier, vol. 109(2), pages 566-582.

Goldstein, Itay and Liyan Yang (2015). Information Diversity and Market Efficiency Spirals, Journal of Finance, 70(4), 1723-1765.

Goldstein, Itay and Liyan Yang (2016). Good Disclosure, Bad Disclosure, mimeo.

Gopinath, Gita, Sebnem Kalemli-Ozcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez (2015). Capital Allocation and Productivity in South Europe, mimeo.

Grossman, Sanford and Joseph Stiglitz (1980). On the Impossibility of Informationally Efficient Markets, American Economic Review, 70, 3, 393-408.

Hellwig, Christian and Laura Veldkamp (2009). Knowing What Others Know: Coordination Motives in Information Acquisition, Review of Economic Studies 76 (1): 223–251.

Hirshleifer, David, Avanidhar Subrahmanyam, and Sheridan Titman (2006). Feedback and the success of irrational investors, Journal of Financial Economics 81, 311-338.

Hsieh, C. and P. Klenow (2009). Misallocation and Manufacturing TFP in China and India, Quarterly Journal of Economics 124: 1403-48.

Huang, Shiyang and Yao Zeng (2016). Investment Waves under Cross Learning, mimeo.

Jermann, Urban and Vincenzo Quadrini (2012). Macroeconomic Effects of Financial Shocks, American Economic Review, 102(1), 238-71.

Jurado, Kyle, Sydney Ludvigson and Serena Ng (2015). Measuring Uncertainty, American Economic Review, 105(3): 1177-1216.

Kiyotaki, Nobuhiro and John Moore (1997). Credit Cycles, Journal of Political Economy 105 (2): 211-48.

Kurlat, Pablo and Laura Veldkamp (2015). Shall We Regulate Financial Information?, Journal of Economic Theory, 158, 697-720.

Leland, Hayne (1992). Insider Trading: Should it be Prohibited? Journal of Political Economy, 100, 859-887.

Lucas, Robert E., Jr. (1972). Expectations and the Neutrality of Money. Journal of Economic Theory 4, 103-124.

Ludvigson, Sydney, Sai Ma and Serena Ng (2017). Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response? mimeo.

Luo, Yuanzhi (2005). Do Insiders Learn from Outsiders? Evidence from Mergers and Acquisitions, Journal of Finance, 60, 1951-1982.

Nieuwerburgh, Stijn Van and Laura Veldkamp (2006). Learning Asymmetries in Real Business Cycles, Journal of Monetary Economics, v.53(4), p.753-772.

Ozdenoren, Emre and Kathy Yuan (2008). Feedback Effects and Asset Prices, Journal of Finance, 63(4), 1939-1975.

Restuccia, D., and R. Rogerson (2008). Policy distortions and aggregate productivity with heterogeneous establishments, Review of Economic Dynamics 11, 707-720.

Schwert, William (1989a). Why does stock market volatility change over time? Journal of Finance, 44, 1115-1153.

Schwert, William (1989b). Business cycles, financial crises, and stock volatility, Carnegie-Rochester Conference Series on Public Policy, 31, 83-125.

Sockin, Michael (2016). Not so Great Expectations: A Model of Growth and Informational Frictions, mimeo.

Sockin, Michael and Wei Xiong (2015). Informational Frictions and Commodity Markets, Journal of Finance 70, 2063-2098.

Subrahmanyam, Avanidhar and Sheridan Titman (1999). The Going-Public Decision and the Development of Financial Markets, Journal of Finance, 54, 1045-1082.

Subrahmanyam, Avanidhar, Sheridan Titman (2013). Financial Market Shocks and the Macroeconomy, Review of Financial Studies Volume 26, Issue 11, Pp. 2687-2717.

Titman, Sheridan (2013). Financial Markets and Investment Externalities. Journal of Finance 68(4), 1307-1329.

Veldkamp, Laura (2005). Slow Boom, Sudden Crash, Journal of Economic Theory, 124, 230-257.

Verrecchia, Robert (1982). Information Acquisition in a Noisy Rational Expectations Economy, Econometrica 50, 1415-1430.

Vives, Xavier (2016). Endogenous Public Information and Welfare in Market Games, Review of Economic Studies, forthcoming.