Financial Markets, the Real Economy, and Self-fulfilling Uncertainties*

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Abstract

Uncertainty in both financial markets and the real economy rises sharply during recessions. We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production and aggregate economic activities. We argue that there exists mutual learning between financial markets and the real economy. Their joint information productions determine both the allocative efficiency in the real sector and the market efficiency in the financial sector. The mutual learning creates a strategic complementarity between information production in the financial sector and that in the real sector. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sectors produce little information in anticipation of the other producing little information. At the same time, aggregate output falls as the real allocative efficiency deteriorates. In the extension to the dynamic setting, our model characterizes a two-stage economic crisis: an adverse shock to fundamentals first triggers a mild recession along a unique equilibrium path, which then ignites a self-fulfilling uncertainty crisis and plunges the economy into a deep recession when capital stock of the economy deteriorates to a tipping point.

JEL classification: G01, G20, E2, E44

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1 Introduction

Uncertainty in both financial markets and the real economy rises sharply during recessions. The recent financial crisis of 2007-2009 presented one of the most striking episodes of such heightened uncertainty. The financial market uncertainty, measured by the VIX index, jumped by an astonishing 313% in the Great Recession. The increase in measured real uncertainty was equally astounding. For instance, the macroeconomic uncertainty measured in Jurado et al. (2015) almost doubled, and Bloom et al. (2012) report a 152% increase in the micro-level real uncertainty measured by the firm-level dispersion of output. What causes such sudden spikes in uncertainty? Why do financial uncertainty and real uncertainty move together? Why do they rise sharply in recessions? These challenging questions are of central importance for understanding the interaction between financial markets and the real economy. The purpose of this paper is to provide a theoretical framework to address these questions.

We develop a model of informational interdependence between financial markets and the real economy, linking uncertainty to information production (or acquisition) and aggregate economic activities. As the starting point of our theory, we argue that there exists *mutual learning* between financial markets and the real economy. Their joint information productions determine both the real production efficiency in the real sector and the price efficiency in the financial sector. As an example, oil producing companies scrutinize oil future prices when making their production decisions, while the financial market studies the financial reports from these producing companies to learn information when trading on oil futures. This mutual learning creates a strategic complementarity between information production in the financial sector and that in the real sector. A self-fulfilling surge in financial uncertainty and real uncertainty can naturally arise when both sides produce little information in anticipation of the other producing little information. At the same time, aggregate output falls as the real production efficiency deteriorates.

We formalize the idea in an extended Grossman-Stiglitz (1980) model. Our key innovation is that we introduce a real sector along the lines of Dixit-Stiglitz (1980) — in our framework, firms have to make investment decisions under imperfect information about *two* dimensions of uncertainty: their idiosyncratic productivity and demand shocks. We start with one firm and one financial market in our baseline partial equilibrium model for a given aggregate output. To reduce uncertainty, the firm can learn about its idiosyncratic productivity shock by incurring a cost, but it has to infer its demand shock from the information provided in the financial market where speculators (or traders) have a comparative advantage in acquiring information about the demand shock. In this context, the financial price is jointly determined by the firm's information production and thereby its disclosure and the demand information produced by financial market speculators. To understand strategic complementarity between these two sources of information, first suppose that the firm makes more accurate information disclosure about its productivity shock. The reduced uncertainty about the productivity shock attracts more informed traders and induces more aggressive trading. Hence, the amount of information produced on the demand shock in the financial market increases. Conversely, suppose that the financial market becomes more informative about the demand shock for some reason. The reduced uncertainty regarding the demand enables the firm to make better investment decisions and hence achieve a higher profit for every realized supply shock. This implies that the stake is higher for the firm to acquire information about its productivity shock. Hence, the firm has stronger incentives to acquire information about its productivity shock.

As the marginal benefit of acquiring information for the firm depends on aggregate output (besides financial price informativeness), the nature of equilibrium also depends crucially on the level of aggregate output. When the aggregate output is sufficiently high, the firm's incentive to acquire information is already strong enough and acquiring information is a dominant strategy for the firm. The resulting equilibrium is hence unique in which the firm produces and discloses more precise information and the financial market generates a more informative price signal. As a result, both financial and real uncertainty are low. At the other extreme, when the aggregate output is sufficiently low, not acquiring information is a dominant strategy for the firm. Anticipating this, speculators in the financial market also have little incentive to acquire information about the firm's demand shock. The equilibrium is hence also unique. However, when the aggregate output is in the intermediate range, the economy has two self-fulfilling equilibria. The information produced by the firm and the information generated by the financial market in one equilibrium (the "good" equilibrium) are much more precise than those in the other equilibrium (the "bad" equilibrium). Consequently, a surge in uncertainty can suddenly strike as a self-fulfilling equilibrium phenomenon.

We then extend the baseline model to a macroeconomic general-equilibrium framework with aggregate production to endogenize the aggregate output. The final consumption good is produced with a continuum of intermediate capital goods as the input according to a Dixit-Stiglitz production function. Each intermediate capital good is produced by one firm located on an island in the spirit of Lucas (1972). When information signals on some islands become noisier, the real investment decisions on those islands become less efficient and consequently the aggregate output declines. This causes the aggregate demand faced by other islands to drop. Thus, incentives to acquire information in the real sector on those other islands are also reduced, which decreases information acquisition in their financial sectors as well. The aggregate output hence declines further, which in turn affects those islands experiencing the original shock. In short, the complementarity in goods production due to the Dixit-Stiglitz demand externality across islands generates further complementarity in information production across islands. As a result, the complementarity forces for equilibrium multiplicity are strengthened in the general equilibrium. Similar to the partial equilibrium model, the economy may feature multiple (two) equilibria. In particular, in general equilibrium, a self-fulfilling rise of uncertainty is accompanied by the reduction in investment efficiency and the fall in aggregate output.

We derive four key implications of our macroeconomic model. First, an adverse shock originating in either the real sector or the financial sector that impairs their ability to conduct information acquisition can have a large impact on the aggregate economy due to the compound feedback loops. In fact, both aggregate investment and the endogenous aggregate TFP are decreasing in information precision. Hence, a small shock in information acquisition cost can have a large impact on all three quantities (aggregate investment, endogenous aggregate TFP, and aggregate output) in the same direction, in particular when it triggers a self-fulfilling "bad" equilibrium.[1] Second, our model endogenizes together the three variables — financial uncertainty, real uncertainty, and aggregate economic activities — and shows countercyclical uncertainty as observed in the data. Third, our model provides an information contagion channel, where a shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. Fourth, our model with a production economy sheds light on several puzzling empirical facts on asset price comovement. More information about idiosyncratic shocks results in individual asset prices being more responsive to idiosyncratic shocks and relatively less responsive to the aggregate shock. Hence, a lower degree of comovement of asset prices as well as a higher efficiency of resource allocation is expected to be accompanied by a higher GDP.

Finally, we extend the static model into an OLG dynamic framework, deriving several additional economic insights. The OLG model provides a dynamic equilibrium setting to study the process of saving and capital accumulation. The equilibrium, therefore, is dynamically linked across periods through savings. The nature of the equilibrium in each period is path-dependent, depending not only on the realization of the productivity shock in the current period but also on the capital accumulation in past periods. We show that our dynamic model possibly has two steady-state equilibria, meaning that the economy can exhibit self-fulfilling uncertainty traps. Interestingly, the transitional dynamics of our OLG model characterizes a two-stage economic crisis. An adverse shock to fundamentals first triggers a downward spiral of information production and economic activities. The initial impact is relatively small, but it makes way for a perfect storm of self-fulfilling uncertainties as the aggregate economy declines further. The resulting drop in output and the increase in measured uncertainties are huge. More specifically, we show that a medium-sized shock to the economy initially generates a mild recession along a unique equilibrium path. But as capital accumulation deteriorates the economy will eventually reach a tipping point, where multiple equilibria start to emerge. Then a self-fulfilling uncertainty crisis can suddenly throw the economy into a new "bad" equilibrium path. Output and capital fall dramatically and uncertainties spike.

[1] Brunnermeier (2009) discusses various other amplification mechanisms of financial markets.

In contrast, a small shock always generates a unique equilibrium path without the second stage.

These implications of our OLG model are also qualitatively consistent with some observed patterns of the recent financial crisis. The crisis originated in a relatively small mortgage sector, which started to decline in 2006. A mild economy-wide slowdown came later in the fourth quarter of 2007. The recession did not look particularly severe until the third quarter of 2008,[2] when full financial panic broke out after the collapse of Lehman Brothers and aggregate output fell sharply. It is now widely believed that a deterioration in fundamentals and a loss of confidence together drive this type of two-stage crisis.[3]

Empirical studies support our model's key implication on information quality across business cycles. Jiang, Habib and Gong (2015) show evidence that management forecast errors, measured by the difference between forecasted earnings per share (EPS) and actual EPS, increase during economic recessions. Loh and Stulz (2017) document that forecast errors of financial analysts are significantly higher during recessions/crises than good times.

Related literature. A burgeoning literature in finance studies the informational feedback effects of financial markets (see Bond, Edmans and Goldstein (2012) for an extensive survey of this literature). This literature argues that firm managers on the real side of the economy learn from financial prices. Among others,[4] Goldstein, Ozdenoren and Yuan (2013) and Sockin and Xiong (2015) develop clean model frameworks showing how prices in the secondary financial market can aggregate the dispersed information of speculators and guide firm managers or investors to make better real investment decisions. The learning in this literature is one way — the real sector learns from financial markets. On the other hand, the accounting literature emphasizes the opposite direction of learning — arguing that firm managers typically know more than financial market participants — and focuses on studying how firm managers (i.e., insiders) disclose information to the capital market, based on which financial speculators trade and security prices are formed. Our paper advances these two bodies of literature by introducing and studying *mutual (two-way) learning* between the real sector and financial markets. The two-way learning mechanism sheds light on important questions, such as how a financial price is formed, where the information comes from, and how the sources of information interact.

The finance literature pioneered by Grossman and Stiglitz (1980) and Verrecchia (1982) studies

[2] The economic growth rates in the first three quarters of 2008 were -0.5%, 4%, and 0.8%.

[3] The Federal Open Market Committee minutes repeatedly emphasize uncertainty as a key factor driving the 2001 and 2007-2009 recessions (see, e.g., Bloom et al. (2012)). On many occasions, the Chairman Ben Bernanke highlighted the loss of confidence as an important factor for the Great Recession. For example, at the stamp lecture on January 13, 2009, he said "Heightened systemic risks, falling asset values, and tightening credit have in turn taken a heavy toll on business and consumer confidence and precipitated a sharp slowing in global economic activity."

[4] For theoretical work, see, e.g., Fishman and Hagerty (1992), Leland (1992), Dow and Gorton (1997), Subrahmanyam and Titman (1999, 2013), Hirshleifer, Subrahmanyam, and Titman (2006), Foucault and Gehrig (2008), Goldstein and Guembel (2008), Ozdenoren and Yuan (2008), Bond, Goldstein, and Prescott (2010), Kurlat and Veldkamp (2015), Huang and Zeng (2016), Sockin (2016), and Foucault and Frésard (2016).

information production (or acquisition) in financial markets. The recent work of Goldstein and Yang (2015) analyzes a model where two different groups of financial traders are informed of different fundamentals of a security. They show that trading as well as information acquisition by these two groups of financial traders exhibit strategic complementarities. Ganguli and Yang (2009) study a model where traders can obtain private information about the supply of a stock in addition to that about its payoff. They show complementarity in information acquisition and the existence of multiple equilibria. Our model introduces the real sector and aggregate production into a Grossman-Stiglitz-type model; information acquisition in our model takes place both in the real sector and in financial markets. Adding to this literature, our paper shows that complementarity in information production exists between the real sector and the financial sector. Our model provides a micro-foundation for the two-factor structure of the asset payoff in Goldstein and Yang (2015). Our paper also adds to the recent literature by showing that the informational interplay between the real sector and the financial sector can give important macroeconomic implications.[5]

A large literature in macroeconomics documents robust evidence of countercyclical uncertainty — both real uncertainty and financial uncertainty increase during recessions. Real uncertainty is often proxied by firm-level dispersion in earnings, productivity and output, and the volatility of aggregate output forecast error, while financial uncertainty is often measured by financial market volatility and the VIX index (Bloom (2009), Bloom et al. (2012), Jurado et al. (2015)). An ongoing heated debate in this literature concerns the question of causality, i.e., whether uncertainty is a cause of or merely a response to recessions and where it comes from (see, e.g., Bachmann and Bayer (2013, 2014)). Interestingly, a recent paper by Ludvigson et al. (2017) empirically identifies that sharply higher real uncertainty in recessions is most often an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty in financial markets is a likely source of the fluctuations. Our paper contributes to the debate by providing a theoretical framework that is able to address the three variables — real uncertainty, financial uncertainty, and aggregate economic activities — simultaneously and show how they are related.[6]

Bacchetta, Tille and Wincoop (2012) also study the self-fulfilling nature of uncertainty. They construct an interesting endowment economy in which agents have mean-variance preferences. If the agents believe pure sunspots matter for asset prices, then the perceived risk of future prices will increase. As a result, the current asset prices will indeed be affected. The uncertainty is self-fulfilling because there also exists another equilibrium in which the asset prices are certain

[5] Goldstein and Yang (2017a) analyze a model where there are multiple dimensions of uncertainty and market prices convey information to real decision-makers. They focus on studying the effect of disclosing public information on real efficiency. They also study the information acquisition of financial speculators, but not that of the real sector.

[6] Benhabib, Liu and Wang (2016b) study endogenous information acquisition of firms that links real uncertainty and economic activities. However, there is no financial market in the model there and that paper does not touch upon financial uncertainty. Veldkamp (2005) and Van Nieuwerburgh and Veldkamp (2006) study learning asymmetries in business cycles, without involving financial markets.

and hence bear zero risk. Fajgelbaum, Schaal and Taschereau-Dumouchel (2016) propose a theory of self-reinforcing episodes of high uncertainty and low activity, through the mechanism of the "wait-and-see" effect together with agents learning from the actions of others. In contrast to these contributions, self-fulfilling uncertainty in our model comes from the information interdependence between financial markets and the real economy. This information interplay also allows us to study the impact of uncertainties on real economic activities.

Our model is related to a small body of the macroeconomics literature that studies how financial markets affect business cycle fluctuations through information channels.[7] Angeletos, Lorenzoni and Pavan (2010) build a two-stage feedback model where financial markets in the second stage learn from the volume of asset selling of entrepreneurs in the first stage, which generates strategic complementarity in investment that amplifies non-fundamental shocks. Benhabib, Liu and Wang (2016a) present a self-fulfilling business cycle model, where financial market sentiments affect the price of capital, which signals the fundamentals of the economy to the real side and consequently leads to real output that confirms the sentiments. David, Hopenhayn and Venkateswaran (2016) conduct a quantitative study that links imperfect information and resource misallocation, where firms learn from both internal sources and stock market prices about *one* dimension of fundamental uncertainty. The information is exogenous in their paper, and they conclude that firms turn primarily to internal sources for information, rather than to financial markets.[8] Compared with the aforementioned studies, ours shows that the amount of information in the economy is endogenous, and that there is feedback between the level of economic activity and the amount of information, amplified through the mutual learning between firms and financial markets.

Our model's cross-sectional implication on asset pricing comovements is closely related to and motivated by the study of Peng and Xiong (2006), who show that limited investor attention leads to category-learning behavior, i.e., investors tend to process more market- and sector-factor information than firm-specific information. As a consequence, return correlations between firms can be higher than their fundamental correlations and sectors with a higher average return correlation across firms will exhibit less informative stock prices. Unlike us, Peng and Xiong (2006) study pricing comovements in an exchange economy and hence they do not link the pricing comovements to fluctuations in the business cycle and investment efficiency as we do in our paper.

Finally, equilibrium multiplicity due to strategic complementarity in information production in our model also adds to the large literature on coordination failures and multiple equilibria. Multiple equilibria exist in these models because each individual agent would like to do what others do. So

[7] Among others, Reis (2006), Angeletos and Pavan (2007), Hellwig and Veldkamp (2009), Vives (2016), Colombo, Femminis and Pavan (2014) and Mäkinen and Ohl (2015) study information acquisition and efficiency.

[8] Because David, Hopenhayn and Venkateswaran (2016) consider neither endogenous information acquisition nor the feedback on information production between the real side and financial markets, incorporating our mechanism in their quantitative study may bring new results; see our numerical calibration in Section 6.

if there is enough heterogeneity to reduce their incentives to act like others, multiple equilibria can disappear. For example, Ball and Romer (1991) show that if the fixed menu cost is heterogeneous, equilibrium multiplicity is still possible but depends crucially on the shape of the distribution function of the fixed menu cost. Morris and Shin (1998) show that heterogeneity in information can lead to a unique equilibrium in an otherwise multiple-equilibria model of currency attacks. Schaal and Taschereau-Dumouchel (2015) apply the global games approach developed by Morris and Shin (1998) to remove multiple equilibria in a model in which firms coordinate on capacity utilization. Interestingly, we find that heterogeneity across firms in productivity or the information acquisition cost can actually increase, rather than reduce, the possibility of multiple equilibria in our model. This result is in sharp contrast to the types of coordination failure in other models.

The paper is organized as follows. In Section 2, we present the simple baseline model. In Section 3, we extend our baseline model to study endogenous information. In Section 4, we further extend the model to a macroeconomic framework. In Section 5, we extend the static model to an OLG dynamic framework. Section 6 provides numerical illustrations of our model. Section 7 concludes.

2 The Baseline Model

In this section, we present a simple baseline model with one firm, one financial market, and with exogenous information. The firm faces two uncertainties: demand shock and supply (or productivity) shock. The firm has some information about the supply shock while the financial market has some information about the demand shock. We show that there exists two-way learning between the firm and the financial market.

2.1 Setup

There are two types of agents: firm j and a group of financial market traders (speculators). There are two types of goods: an intermediate capital good and a final good. The price of the final good is normalized as the numeraire, $P \equiv 1$.

Intermediate Goods Firm Firm j is an intermediate goods firm. It produces the intermediate capital good Y_j using the final good as input according to the production function

$$Y_j = Z A_j K_j^{\eta} \quad \text{for } 0 < \eta < 1, \tag{1}$$

where Z is the common productivity shock to the whole economy (regarded as a constant in the baseline model), A_j is firm j's productivity, and K_j is the investment input of the final good.[9] The

[9] The label "final" here only means that this good is a *different* good from the intermediate good. In our static model, the input K_j (the final good) comes from endowment, which will be clear in Section 4.1. In the extended

input K_j fully depreciates after production. We will show that firm j borrows the investment input at interest rate $R_f \equiv 1$.

The market demand function of the intermediate capital good Y_j is assumed to be

$$Y_j = \left(\frac{1}{P_j}\right)^{\theta} \epsilon_j Y, \tag{2}$$

where P_j is the price of the capital good j (in terms of the final good), and ϵ_j measures the idiosyncratic *demand shock* to good j. Moreover, in the baseline model Y is an exogenous constant, which corresponds to the aggregate output (real GDP) (denote $y \equiv \log Y$), whereas parameter θ measures the price elasticity of demand.

Financial Market and Traders (Speculators) A financial market exists, where speculators trade a financial asset (a derivative) contingent on the firm's asset value or firm value (also its total income):[10]

$$V_j = Y_j P_j. \tag{3}$$

Specifically, we assume that the payoff of the financial derivative contract takes the form of

$$v_j = \log V_j,$$

where dv_j/dt also corresponds to the growth of the firm value.[11] Denote by q_j the *market trading price* of the financial derivative contract. That is, the long position of one unit of the financial asset (derivative) incurs an initial outlay of q_j and entitles to having the risky payoff v_j later.

The utility function of speculators is assumed to be

$$U(W^i) = -\exp\left(-\gamma W^i\right),$$

where W^i is the end-of-period wealth for speculator $i \in [0, 1]$, and γ is the risk-aversion (CARA) coefficient. The initial wealth for a speculator is assumed to be W_0 and the risk-free (gross) interest rate is $R_f \equiv 1$. This means that if a speculator takes a position of m units of the financial asset, his end-of-period wealth would be

$$W^i = (W_0 - mq_j) R_f + mv_j = W_0 + m(v_j - q_j).$$

dynamic model, the input K_j in the current period is the savings after consumption (i.e., the final consumption good) from the last period.

[10] For our one-period static model, in the balance sheet of the firm at the end of the period, the asset value of the firm is its total income; the debt value is the investment cost; and the equity value is the net profit. Hence, assuming a log-normal distribution of V_j is consistent with the literature where the asset value, not the equity value, is assumed to follow a geometric Brownian motion (see, e.g., Merton (1973) and He and Xiong (2012)).

[11] The log form is for tractability. Goldstein and Yang (2017a) study the financial asset's payoff being nonlinear with a log-normal distribution. They show numerically that being more informative about one factor motivates speculators to acquire more information about the other factor. In Appendix B, we also examine alternative specifications.

The assumption that speculators trade a derivative contract contingent on the firm's asset value, V_j, is made for tractability. This is along the line of the assumption in the literature that a firm's asset value or sales revenue follows a geometric Brownian motion (see, e.g., Merton (1973) and He and Xiong (2012)). The financial derivative can also be contingent on the firm's product price, P_j (that is, v_j takes the form of $v_j = \log P_j$). In the latter case, the financial market can be interpreted as a *commodity financial futures market* that specializes in trading *financial futures* regarding the intermediate capital good Y_j, in the spirit of Sockin and Xiong (2015). Assuming that the underlying asset of the derivative is either V_j or P_j is to ensure that the payoff of the underlying asset follows a log-normal distribution and thus to achieve tractability. This parallels the modeling device that assumes a *specific* function form of noisy trading (or asset supply) as in Goldstein, Ozdenoren and Yuan (2013), Sockin and Xiong (2015), and Goldstein and Yang (2017a).

The net aggregate supply of the financial asset (i.e., derivative) is assumed to be 0. The demand of noise/liquidity traders in the financial market is n_j, where n_j follows distribution $n_j \sim N(0, \tau_n^{-1})$.

Uncertainties and Information The firm faces two uncertainties: productivity (or supply) shock A_j and demand shock ϵ_j. Their prior distributions are $\log A_j \equiv a_j \sim \mathcal{N}(-\frac{1}{2}\tau_a^{-1}, \tau_a^{-1})$ and $\log \epsilon_j \equiv \varepsilon_j \sim \mathcal{N}(-\frac{1}{2}\tau_\varepsilon^{-1}, \tau_\varepsilon^{-1})$. And a_j and ε_j are independent. The common productivity shock Z is public information (denote $z = \log Z$).

In the baseline model, we assume that the firm and the financial market have some *exogenous* (imperfect) information about a_j and ε_j, respectively. Specifically, the firm possesses or is endowed with a noisy signal about its own productivity:

$$s_j = a_j + e_j,$$

where $e_j \sim N(0, \tau_s^{-1})$. Firm j will disclose its signal s_j to the financial market.[12] For simplicity, we assume that the firm has no private information about the demand shock, ε_j.[13]

In the financial market, as in Grossman and Stiglitz (1980), there is a continuum of traders with unit mass. The traders are of two types: informed and uninformed. An informed trader i has a noisy private signal

$$x_j^i = \varepsilon_j + \varrho_j^i,$$

where $\varrho_j^i \sim N(0, \tau_x^{-1})$ and ϱ_j^i is independent across different informed traders i. An uninformed trader has no private signal regarding ε_j. The proportion of informed traders is λ, which is exogenous in the baseline model.

[12] As will become clear later, a firm has incentives to disclose its information to the financial market because the disclosure can "attract" more information from the financial market, which can guide the firm to make better investment decisions. Goldstein and Yang (2017a) show that firms always have incentives to disclose their information orthogonal to traders (see also Bond and Goldstein (2015)).

[13] We will relax this assumption in Section 3.3 to allow the firm to have information on the demand shock as well.

Timeline The sequence of events (within the one period) in the baseline model is as follows:

T_1: Firm j discloses its signal s_j to the financial market.

T_2: Financial market trading takes place, and financial price q_j is realized.

T_3: Firm j makes its investment decision, K_j, based on information $\{s_j, q_j\}$.

T_4: The income or asset value, V_j, is realized. The payoff of the financial contract is delivered.

The timeline setting allows us to capture the two-way informational learning and production in a simple way. As the standard accounting literature studies, financial market participants learn from a firm's disclosure in trading securities and conducting price discovery. In other words, the firm's information acquisition often occurs before financial trading and financial price formation. On the other hand, the financial price, which is forward-looking, incorporates some information that the firm manager does not have and thus can learn in making real investment decisions, as a large finance literature on the feedback effects studies.

2.2 Equilibrium

The equilibrium consists of a financial market equilibrium at T_2 and the firm's investment decision at T_3. We conduct analysis by backward induction.

Firm j's Investment Decision at T_3 Firm j maximizes its *expected* profit:

$$K_j \equiv K(s_j, q_j) = \arg\max_{K_j} \mathbb{E}\left[P_j Y_j - R_f K_j | s_j, q_j\right] \tag{4}$$

with constraints (2) and (1). Here $\mathbb{E}(\cdot|s_j, q_j)$ is the conditional expectation operator over a_j and ε_j. It is well understood that a firm's objective is not always well defined in an asymmetric-information environment. For simplicity and convenience, we assume that firms maximize their expected profit. Alternatively, we can assume that firms are run by risk-neutral entrepreneurs.

Financial Market Trading at T_2 In the financial market, the information set of informed speculators is $\left\{s_j, q_j, x_j^i\right\}$ while that of uninformed speculators is $\{s_j, q_j\}$.

An informed speculator chooses his risky asset holdings, m^{Ii}, to maximize his utility:

$$m^{Ii}(s_j, q_j, x_j^i) = \arg\max_{m^{Ii}} \mathbb{E}\left[U(W^{Ii}) | s_j, q_j, x_j^i\right], \tag{5}$$

where $W^{Ii} = (W_0 - c) + m^{Ii}(v_j - q_j)$ and c denotes a constant expense, to be explained later. An uninformed speculator chooses his risky asset holdings, m^{Ui}, to maximize his utility:

$$m^{Ui}(s_j, q_j) = \arg\max_{m^{Ui}} \mathbb{E}\left[U(W^{Ui}) | s_j, q_j\right], \tag{6}$$

where $W^{Ui} = W_0 + m^{Ui}(v_j - q_j)$. In (5) and (6), $\mathbb{E}(\cdot)$ is the expectation operator over v_j

The equilibrium of our baseline model is formally defined as follows.

Definition 1 *An equilibrium consists of the financial price function* $q_j = q(s_j, \varepsilon_j, n_j)$ *and the firm's investment decision function* $K_j = K(s_j, q_j)$*, such that*

1. *Price* $q(s_j, \varepsilon_j, n_j)$ *clears the financial market at* T_2*:*

$$\lambda \int_0^1 m^{Ii} di + (1-\lambda) \int_0^1 m^{Ui} di + n_j = 0, \tag{7}$$

where, for given $K_j = K(s_j, q_j)$*,* m^{Ii} *and* m^{Ui} *solve (5) and (6), respectively.*

2. *Given price* $q(s_j, \varepsilon_j, n_j)$*, investment decision* $K(s_j, q_j)$ *solves the firm's problem (4).*

The equilibrium defined in Definition 1 highlights the two-way feedback (i.e., a fixed-point problem) between the financial market and the real economy. On the one hand, the financial price at T_2 should *reflect* the (forward-looking) investment decision at T_3 (and thereby the financial asset's fundamentals at T_4). On the other hand, the financial price at T_2 *influences and guides* the investment decision on the real side of the economy at T_3.

2.3 Characterization of Equilibrium

First, we characterize the financial market equilibrium. We conjecture that $\log K_j \equiv k_j = k(s_j, q_j)$ is a linear function in Definition 1. Plugging (2) and (1) into (3) yields

$$v_j = \frac{1}{\theta}\varepsilon_j + \left(1 - \frac{1}{\theta}\right)(z + a_j) + \eta \left(1 - \frac{1}{\theta}\right) k_j + \frac{1}{\theta} y, \tag{8}$$

which depends on ε_j, a_j and k_j. However, speculators are certain about k_j but not ε_j and a_j, because k_j is a function of signals s_j and q_j and thus speculators *perfectly foresee* the investment decision of the firm.

In solving (5), we find $m^{Ii} = \frac{\mathbb{E}[v_j|s_j,q_j,x_j^i]-q_j}{\gamma Var[v_j|s_j,q_j,x_j^i]}$. Similarly, (6) gives $m^{Ui} = \frac{\mathbb{E}[v_j|s_j,q_j]-q_j}{\gamma Var[v_j|s_j,q_j]}$. We also conjecture a linear price function:

$$q_j = \beta_0 + \beta_1(\varepsilon_j + \beta_2 s_j + \beta_3 n_j), \tag{9}$$

where β_0, β_1, β_2 and β_3 are coefficients. When combined with s_j, price q_j can be converted into another piece of public information about ε_j:

$$\tilde{q}_j(q_j, s_j) = \frac{q_j - \beta_0 - \beta_1\beta_2 s_j}{\beta_1} = \varepsilon_j + \beta_3 n_j \equiv \varepsilon_j + \varrho_j^q, \tag{10}$$

where $\varrho_j^q \sim N(0, \tau_q^{-1})$ with $\tau_q^{-1} = \beta_3^2 \tau_n^{-1}$. Information set $\{s_j, q_j\}$ is a one-to-one mapping to $\{s_j, \tilde{q}_j\}$.

Plugging (8) and (9) into the expressions of m^{Ii} and m^{Ui}, together with (7), yields the financial market equilibrium. We have Lemma 1.

Lemma 1 *In the equilibrium of the financial market, for a given λ, τ_q is an increasing function of τ_s, i.e., $\frac{\partial \tau_q}{\partial \tau_s} > 0$.*

Proof. *See Appendix.* ■

Lemma 1 states that when the precision of the firm's disclosed information about a_j increases, the informativeness of the financial price about ε_j also increases. The intuition is as follows. The total uncertainty over v_j is the sum of uncertainties over a_j and ε_j. When uncertainty over a_j decreases under a higher precision τ_s, informed traders have incentives to trade more aggressively, which overwhelms the trading of noise/liquidity traders, thus increasing the informativeness of the financial price.

Lemma 1 shows that the financial price comes partially from information disclosure in the real sector and partially from price discovery in the financial market. These two sources of information interact.

Next, we move to characterize firm j's investment decision at T_3. The first-order condition of (4) implies

$$k_j = k(s_j, \tilde{q}_j) = \phi_0' + \Theta(1 - \frac{1}{\theta}) \frac{\tau_s}{\tau_a + \tau_s} s_j + \frac{\Theta}{\theta} \frac{\tau_q}{\tau_\varepsilon + \tau_q} \tilde{q}_j, \tag{11}$$

where $\Theta = -\frac{1}{\eta(1-\frac{1}{\theta})-1} \in (1, \theta)$ and the constant coefficient ϕ_0' is provided in Appendix. Because $\tilde{q}_j$ is a linear function of s_j and q_j by (10), (11) implies that k_j is also a linear function of s_j and q_j, which confirms the earlier conjecture.

Lemma 2 *The firm's investment decision at T_3, $K(s_j, q_j)$, is given by (11) (together with (10)).*

Proof. *See Appendix.* ■

The realized profit for firm j at T_4 is $\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j) = P_j(\varepsilon_j, Y_j) Y_j(a_j, K_j) - K_j(s_j, \tilde{q}_j)$. Hence, the expected profit perceived at the stage of investment at T_3 is $\mathbb{E}[\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j)|s_j, \tilde{q}_j]$. Exploiting the law of iterated expectations, we find the ex ante expected profit of firm j perceived at T_0:

$$\begin{aligned}\Pi(\tau_s, \tau_q; Y, Z) &= \mathbb{E}\mathbb{E}[\pi(a_j, \varepsilon_j, s_j, \tilde{q}_j)|s_j, \tilde{q}_j] \\ &= \left[1 - \eta\left(1 - \frac{1}{\theta}\right)\right]\left[\eta\left(1 - \frac{1}{\theta}\right)\right]^{\Theta-1} \cdot \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)^{\Theta} \mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}} \epsilon_j^{\frac{1}{\theta}} | s_j, \tilde{q}_j\right)\right]^{\Theta}\right),\end{aligned}$$

where

$$\mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right)\right]^{\Theta}\right)=\exp\left\{\begin{array}{c}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\\ -\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\end{array}\right\}, \tag{12}$$

by noting that the outer $\mathbb{E}(\cdot)$ is the unconditional expectation operator over s_j and $\tilde{q}_j$.

It is easy to show that

$$\frac{\partial\Pi}{\partial\tau_s}>0 \qquad \text{and} \qquad \frac{\partial\Pi}{\partial\tau_q}>0. \tag{13}$$

The intuition behind the above comparative statics is easy to understand. When the firm has a more precise signal about a_j or ε_j, it makes a better investment decision because its investment can be more closely aligned with the realized productivity or demand shock. Moreover,

$$\frac{\partial^2\Pi}{\partial\tau_s\partial\tau_q}>0, \tag{14}$$

which means that the precisions of signals a_j and ε_j are *complementary* in affecting the firm's ex ante profit. In other words, the informational value of knowing about one shock increases with the information precision of the other shock. The intuition is as follows. When τ_q is higher, the firm has more accurate information about its demand. This enables the firm to make a better investment decision that is more closely aligned with the true demand shock and increases its profit multiplicatively for every realized productivity shock. Therefore, the incremental profit of knowing the realization of the productivity shock (versus not) is also higher for a higher τ_q. This means that the firm has *a higher stake* to acquire information about a_j when τ_q is higher. Similarly, we have

$$\frac{\partial^2\Pi}{\partial\tau_s\partial Y}>0 \qquad \text{and} \qquad \frac{\partial^2\Pi}{\partial\tau_s\partial Z}>0.$$

A key insight of the baseline model is that the learning between the financial market and the real economy occurs both ways. The financial market learns information from a firm's disclosure in trading, and conversely, the firm learns information from the financial price in making its real (investment) decision.

3 The Model with Endogenous Information

In this section, we study the model with acquisition of endogenous information. The purpose is to understand the information acquisition of the firm and that of the financial market, and how they interact.

3.1 Setup

We add T_0 to the timeline. At T_0, after the common productivity shock Z is realized (which becomes public information), the firm and the financial market *simultaneously* make their information acquisition decisions.

By paying an information acquisition cost $b > 0$ (in terms of the final good), the firm receives a signal $s_j = a_j + e_j$ with $e_j \sim N(0, \bar{\tau}_s^{-1})$; otherwise, it receives a less precise signal $s_j = a_j + e_j$ with $e_j \sim N(0, \underline{\tau}_s^{-1})$, where $\bar{\tau}_s > \underline{\tau}_s$. And $\underline{\tau}_s = 0$ corresponds to the extreme case where the firm receives a useless signal. In short, $\tau_s \in \{\underline{\tau}_s, \bar{\tau}_s\}$. In addition, in the spirit of the classic moral hazard problem (concerning hidden actions), we assume that a firm's choice of information precision τ_s is private information (i.e., *unobservable* to outsiders including financial market participants).

In the financial market, a trader can choose to be informed or uninformed. By paying an information acquisition cost $c > 0$ (in terms of the final good), a trader receives a private signal $x_j^i = \varepsilon_j + \varrho_j^i$ with $\varrho_j^i \sim N(0, \tau_x^{-1})$, as specified in the baseline model; otherwise, it receives no signal (or equivalently a useless signal). The proportion of informed speculators, λ, is endogenous.

Our assumption that the firm and financial markets have *comparative advantages* in acquiring information about different types of uncertainties is realistic. On the one hand, the firm has advantage over outsiders including financial analysts in obtaining information about its own productivity shock. On the other hand, financial analysts in major investment banks specializing in different regional or sectoral submarkets could, on aggregate, be better informed about the demand for the firm's product than the firm itself. Furthermore, in Section 3.3, we will relax the assumption to allow the firm to obtain information on the demand shock as well.

3.2 Equilibrium

Information Acquisition Decision of Speculators Proportion λ is determined such that an uninformed speculator and an informed one have the same expected utility:

$$\frac{EV(W^{Ii})}{EV(W^{Ui})} = 1, \tag{15}$$

where $EV(W^i) \equiv \mathbb{E}\left[U(W^i)|s_j, q_j\right]$. We have the following result.

Proposition 1 *In the equilibrium of the financial market with endogenous λ, τ_q is a function of τ_s and c, written as $\tau_q = \tau_q(\tau_s; c)$. We have the comparative statics $\frac{\partial \tau_q}{\partial \tau_s} > 0$ and $\frac{\partial \tau_q}{\partial c} < 0$.*

Proof. *See Appendix.* ■

Proposition 1 states that with taking the endogenous λ into account, the informativeness of the financial price about ε_j increases as the precision of the firm's information about a_j increases.

The intuition behind the comparative statics is as follows. There are two driving forces under the comparative statics $\frac{\partial \tau_q}{\partial \tau_s} > 0$. First, as in the earlier discussion of Lemma 1, when uncertainty over a_j decreases, informed traders trade more aggressively, increasing the informativeness of the financial price. Second, in the spirit of Grossman and Stiglitz (1980), (15) implies $e^{\gamma c} = \sqrt{\frac{Var[v_j|s_j,q_j]}{Var[v_j|s_j,q_j,x_j^i]}}$; that is,

$$e^{\gamma c} = \sqrt{\frac{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right] + Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j)}{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right] + Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j,x_j^i)}}, \tag{16}$$

the right-hand side (RHS) of which is the *gain* in information advantage for an informed speculator over an uninformed one. When one dimension of uncertainty (the term $Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]$) is reduced, the information advantage on the other dimension of uncertainty (the term $Var(\frac{1}{\theta}\varepsilon_j|\cdot)$) becomes more valuable. For example, in the extreme case when $Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right]$ is very large, being informed has little advantage over being uninformed. Hence, when the precision of signal s_j increases and thus $Var\left(a_j|s_j\right)$ decreases, an uninformed speculator has incentives to *switch* to being informed by paying a cost c.[14] When more speculators acquire information, price informativeness also improves. As for the comparative statics $\frac{\partial \tau_q}{\partial c} < 0$, a lower c induces more traders to become informed, causing price informativeness to improve.

Information Acquisition Decision of the Firm Considering that profit function $\Pi\left(\tau_s, \tau_q; Y, Z\right)$ given in (12) has the properties of $\frac{\partial \Pi}{\partial \tau_s} > 0$ and $\frac{\partial^2 \Pi}{\partial \tau_s \partial \tau_q} > 0$ shown in (13) and (14), we can obtain firm j's optimal information acquisition decision at T_0:

$$\tau_s = \begin{cases} \bar{\tau}_s & \text{if } \tau_q \geq \hat{\tau}_q(Y,Z,b) \\ \underline{\tau}_s & \text{otherwise} \end{cases}, \tag{17}$$

where threshold $\hat{\tau}_q \equiv \hat{\tau}_q(Y,Z,b)$ is defined as the unique root to the equation

$$\Pi\left(\tau_s = \bar{\tau}_s; \hat{\tau}_q, Y, Z\right) - \Pi\left(\tau_s = \underline{\tau}_s; \hat{\tau}_q, Y, Z\right) = b. \tag{18}$$

It is easy to show that $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial Y} < 0$, $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial Z} < 0$, and $\frac{\partial \hat{\tau}_q(Y,Z,b)}{\partial b} > 0$. In other words, the firm's information acquisition decision given by (17) is a *step* function, written as $\tau_s\left(\tau_q; Y, Z, b\right)$.

Proposition 2 *The optimal information acquisition decision of the firm at T_0, $\tau_s\left(\tau_q; Y, Z, b\right)$, is given by (17).*

Proof. *See Appendix.* ■

Proposition 2 states that if and only if the firm *expects* the financial efficiency τ_q to exceed the threshold value $\hat{\tau}_q(Y,Z,b)$ would it choose a high precision $\tau_s = \bar{\tau}_s$. This is because precisions

[14] Of course, there is a third force, which is the classic free-rider problem in Grossman and Stiglitz (1980). A more informative price reduces the incentive for a trader to acquire information.

of signals a_j and ε_j are *complementary* in affecting the firm's ex ante profit. Intuitively, when the uncertainty over ε_j is reduced, information about a_j becomes more valuable in maximizing the firm's expected profit. The equilibrium τ_s also depends on Y and Z; that is, when Y or Z is higher, the marginal benefit of increasing the signal precision τ_s is also higher, and so the firm is more likely to acquire information about a_j.

Proposition 2 is a novel result of our model. Earlier work in the literature such as Goldstein and Yang (2015) has shown information production complementarity *within* the financial market. Our paper shows information production complementarity *between* the real side of the economy and the financial side.

Full Equilibrium With both Proposition 1 and Proposition 2, we are now able to characterize the full equilibrium. Proposition 1 gives the reaction function $\tau_q(\tau_s; c)$ while Proposition 2 gives the reaction function $\tau_s\left(\tau_q; Y, Z, b\right)$. Let

$$\tau_q^* \equiv \tau_q(\tau_s = \underline{\tau}_s; c)$$

and

$$\tau_q^{**} \equiv \tau_q(\tau_s = \bar{\tau}_s; c);$$

clearly $\tau_q^{**} > \tau_q^*$ by Proposition 1. Proposition 3 follows.

Proposition 3 *The rational expectations equilibrium, characterized by the pair (τ_s, τ_q) for given (Y, Z, b, c), solves the system of equations $\tau_q(\tau_s; c)$ (given in Proposition 1) and $\tau_s\left(\tau_q; Y, Z, b\right)$ (given in Proposition 2). There are three possible equilibrium cases:*

i) Case 1: $\hat{\tau}_q > \tau_q^{**}$ The equilibrium is unique: $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^*\right)$;

ii) Case 2: $\hat{\tau}_q \in \left[\tau_q^*, \tau_q^{**}\right]$ There are multiple (two) equilibria: $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^*\right)$ or $\left(\bar{\tau}_s, \tau_q^{**}\right)$;

iii) Case 3: $\hat{\tau}_q < \tau_q^*$ The equilibrium is unique: $(\tau_s, \tau_q) = \left(\bar{\tau}_s, \tau_q^{**}\right)$,

where threshold $\hat{\tau}_q = \hat{\tau}_q(Y, Z, b)$ is given by (18).

Proof. *See Appendix.* ■

The intuition behind Proposition 3 is as follows. The *two-way* feedback in information production between the financial sector and the real sector can generate a unique equilibrium or multiple equilibria, depending on parameters Y, Z, b and c. For illustration, let us change Y while keeping Z, b and c constant. Recall that when aggregate output Y is sufficiently low (high), the firm's incentive to acquire information is already weak (strong) enough. Specifically, if Y is so low (and hence threshold $\hat{\tau}_q(Y, Z, b)$ is so high) such that not acquiring information is a *dominant strategy* for the firm (regardless of whether financial price informativeness $\tau_q = \tau_q^*$ or τ_q^{**}), then a unique equilibrium exists in which the real side does not acquire information and the financial efficiency is also at a lower level. This is case 1. Conversely, if aggregate output Y is so high (and hence threshold $\hat{\tau}_q(Y, Z, b)$ is so low) such that acquiring information is a *dominant strategy* for the firm, then a unique equilibrium exists in which the real side acquires information and the financial efficiency is also at a higher level. This is case 3. Between these two extreme cases, there are multiple *self-fulfilling* equilibria, which is case 2.

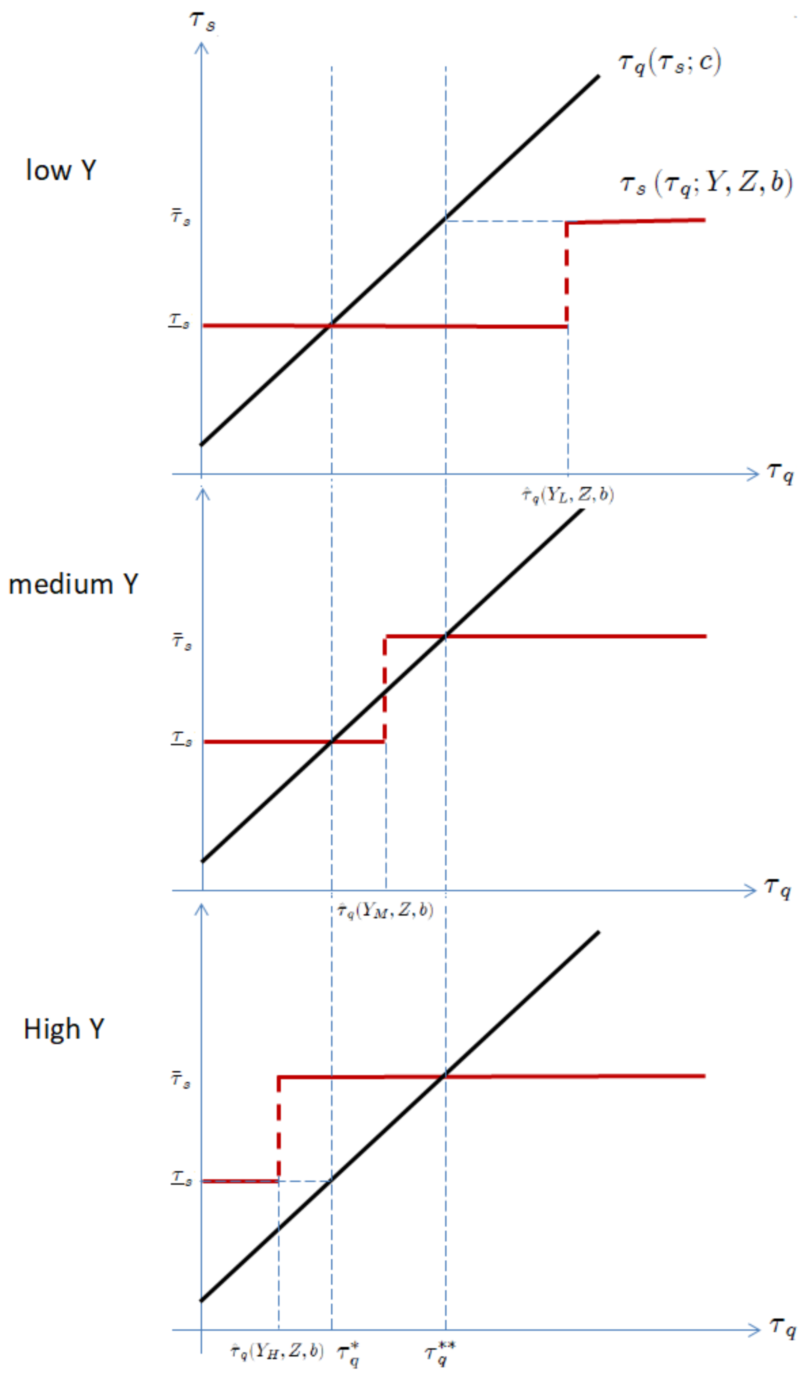

Figure 1: Three cases of equilibrium with $Y_L < Y_M < Y_H$ (Case 1: top; Case 2: middle; Case 3: bottom)

Figure 1 (with three panels) illustrates the three equilibrium cases, corresponding to different levels of aggregate output (i.e., $Y_L < Y_M < Y_H$). The two curves in each panel are the two reaction functions (i.e., $\tau_q(\tau_s; c)$ and $\tau_s(\tau_q; Y, Z, b)$) and their intersection(s) represent the equilibrium.[15] It is easy to see that when Y is kept constant, a change in b or c or Z (where a change in c corresponds to a horizontal shift of curve $\tau_q(\tau_s; c)$ in Figure 1) also leads to different cases of equilibrium.

3.3 Discussions

Before closing this section, we discuss two simplified assumptions of our model. First, for tractability and to obtain closed-form solutions, we have assumed the binary choice of information acquisition of the firm. We can instead assume that the firm's information acquisition is a continuous choice, and our model results would not change qualitatively. Second, for simplicity, in Section 2 we have assumed that the firm has no private information about the demand shock ε_j. We can relax this assumption, and our model results would be robust. To save space, we relegate the details to Appendix C.

4 The Macroeconomic Model

In this section, we extend the model to a macroeconomic framework. The extended model provides a macroeconomic background for the baseline model and endogenizes various exogenous specifications and variables of the baseline model. In particular, the aggregate output (i.e., real GDP), Y, is endogenized, which gives a number of novel implications.

4.1 Setup

Final goods firms The final (consumption) good is produced with a continuum of capital goods as input according to a Dixit-Stiglitz production function

$$Y = \left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{19}$$

where $j \in [0, 1]$, $\theta > 1$ is the elasticity of substitution between intermediate capital goods, and ϵ_j measures the *demand shock* to intermediate good j.

The representative competitive final goods firm maximizes its profits:

$$\max \; P \left[\int \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int P_j Y_j dj, \tag{20}$$

[15] Typically, $\tau_q(\tau_s; c)$ is a curve and not a straight line. But it is easy to show that when $\tau_x = +\infty$, it is a straight line.

where P_j is the price of intermediate good j. The price of the final good, P, is normalized as the numeraire price, i.e., $P \equiv 1$. The first-order condition of (20) with respect to Y_j gives the demand schedule for good j:

$$Y_j = \left(\frac{1}{P_j}\right)^{\theta} \epsilon_j Y,$$

which endogenizes the demand function of (2).

Intermediate Goods Firms There is a continuum of intermediate (capital) goods firms with a unit measure, indexed by j. The setup for a typical firm, firm j, is presented in Section 2.1. We may think of each intermediate good as being produced by one firm that is located on an island in the spirit of Lucas (1972). Both a_j and ε_j are i.i.d. across firms (or islands).

Financial markets In the financial market(s), there is a continuum of financial assets. Financial asset (derivative) j is contingent on intermediate goods firm j's asset value, $V_j = Y_j P_j$. The payoff of financial derivative contract j is $v_j = \log V_j$. The demand from noise/liquidity traders for financial asset j is $n_j \sim N(0, \tau_n^{-1})$, and n_j is independent across financial assets. It might be the case that each island has one financial market with financial asset j being traded in the financial market on island j or the case that there is only one financial market (exchange) where all the financial assets are traded. In the current framework of the aggregate economy, we may interpret noise trading as: 1) foreign capital flow, or 2) liquidity trading by some investors who must trade (for exogenous reasons such as balancing portfolios, endowment shocks, and so on).[16]

The setup for information acquisition for a firm and on a financial asset is the same as that in Section 3.

Investors The economy consists of a continuum of $[0, 1] \times [0, 1]$ investors. Each investor is endowed with W_0 units of the final good at T_0.[17] Each investor has three identities: capital supplier (i.e., lender), firm owner (i.e., shareholder), and financial market trader. The economy is decentralized, analogous to the Robinson Crusoe economy. The decisions of an investor made under different identities are independent.

We assume that W_0 is sufficiently high and a storage technology exists, so that in equilibrium $R_f = 1$.[18] An investor maximizes utility

$$U(C^i) = -\exp\left(-\gamma C^i\right) \tag{21}$$

[16] David, Hopenhayn and Venkateswaran (2016) also use the general equilibrium framework with noise traders.

[17] Clearly now, by calling the "final" good, we do not only mean the good produced with the intermediate inputs under Dixit-Stiglitz aggregation but also the endowment which can be used for either consumption or capital. The point is that the endowment good and the good produced under Dixit-Stiglitz aggregation are the *same* good.

[18] More specifically, W_0 is greater than the aggregate investment given in (25) later.

with constraint

$$C^i = W_0 + (\Pi - \chi) + D^i, \tag{22}$$

where C^i is the end-of-period wealth at T_4 for investor i. The term Π is the aggregate profit of firms; that is, $\Pi = \int (P_j Y_j - R_f K_j)\, dj$, which corresponds to (12) by the law of large numbers. The term $\Pi - \chi$ is the aggregate net profit of the firms distributed to an investor (as an owner of firms), where $\chi \in \{0, b\}$ is the aggregate information acquisition cost to the firms.[19] The term D^i is his payoff in financial market trading. If he is an informed trader in financial market j, then $D^i = -c + m^{Ii}(v_j - q_j)$; if he is an uninformed trader, then $D^i = m^{Ui}(v_j - q_j)$. Notice that for simplicity and expositional clarity, we have assumed here that an investor can trade only in one financial market (asset). In Section 4.4.2, we will provide a robustness analysis and show that our model insight is intact if investors are allowed to access all financial assets and hold a portfolio.

It is easy to show that the trading (asset holding) and information acquisition decision of an investor (as a financial market trader) is exactly the same as that in Sections 2 and 3.

4.2 Equilibrium

Within each island, the equilibrium is given by Propositions 1-3. Now we study the equilibrium of the aggregate economy, endogenizing Y. We consider the symmetric equilibrium, in which all intermediate goods firms have the same level of information precision.

To find the symmetric equilibrium, we proceed in two steps. First, suppose that the equilibrium information precision on the representative island (or equivalently all islands $j^- \neq j$) is given by (τ_s, τ_q), and work out the aggregate output Y. Second, given this Y, characterize the partial equilibrium on island j as studied in Section 3.2.

We take the first step and find that the aggregate output is given by

$$Y = ZAK^\eta, \tag{23}$$

where $A = A(\tau_s, \tau_q)$ is the *endogenous* aggregate TFP and $K = K(\tau_s, \tau_q; Z)$ is the aggregate investment in the economy, with

$$A = A(\tau_s, \tau_q) = \left\{ \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right] \right\}^{\frac{\theta}{\theta-1} - \eta} \tag{24}$$

and

$$K = K(\tau_s, \tau_q; Z) = \left[\eta \left(1 - \frac{1}{\theta} \right) Z \cdot A(\tau_s, \tau_q) \right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}}, \tag{25}$$

[19] We will consider the symmetric equilibrium, in which either all firms or none of them acquires information.

and the term $\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]$ is calculated in (12). The aggregate output can also be expressed as

$$Y = Y(A; Z) = \left[\eta\left(1 - \frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \left[Z \cdot A(\tau_s, \tau_q)\right]^{\frac{\theta\Theta}{\theta - \eta\Theta}}. \tag{26}$$

Equation (23) implies that despite heterogeneity among firms caused by idiosyncratic productivity shocks and demand shocks, our economy works as if there existed a representative firm with productivity A and aggregate investment K. Proposition 4 follows.

Proposition 4 *Both the endogenous aggregate TFP, A, and the aggregate investment, K, are increasing in τ_s and τ_q (given by (24) and (25), respectively). Hence, the aggregate output Y is increasing in τ_s and τ_q (given by (23)).*

Proof. *See Appendix.* ■

Proposition 4 highlights two effects of information frictions. First, given K, the endogenous aggregate TFP, A, measuring the efficiency of resource allocation, has the properties of $\frac{\partial A}{\partial \tau_s} > 0$ and $\frac{\partial A}{\partial \tau_q} > 0$. Efficient allocation requires more resources to be allocated to firms with higher realized A_j and ϵ_j. In other words, efficient investment K_j should be more aligned with realized A_j and ϵ_j. So, more precise information about A_j and ϵ_j achieved through information acquisition helps improve allocative efficiency. Second, higher uncertainty also leads to a lower level of aggregate investment, that is, $\frac{\partial K}{\partial \tau_s} > 0$ and $\frac{\partial K}{\partial \tau_q} > 0$.

Next, we take the second step. That is, given Y derived in the first step, characterize the partial equilibrium on island j. Denote by $\left(\tau_s^j, \tau_q^j\right)$ the equilibrium on island j. By the partial equilibrium result shown in Section 3.2, we have $\tau_s^j = \tau_s\left(\tau_q^j; Y, Z, b\right)$ and $\tau_q^j = \tau_q(\tau_s^j; c)$, where $Y = Y(\tau_s, \tau_q, Z)$ is given by (23).

By the symmetric equilibrium, we have $\left(\tau_s^j, \tau_q^j\right) = (\tau_s, \tau_q)$. Thus, the full equilibrium of the aggregate economy is given by the following joint equations:

$$\tau_s = \tau_s(\tau_q; Y, Z, b) \quad \text{(A firm's optimal information choice)} \tag{27}$$

$$\tau_q = \tau_q(\tau_s; c) \quad \text{(Financial market equilibrium)} \tag{28}$$

$$Y = Y(\tau_s, \tau_q, Z) = Z \cdot A(\tau_s, \tau_q)\left[K(\tau_s, \tau_q; Z)\right]^{\eta}, \quad \text{(Aggregate economy equilibrium)} \tag{29}$$

where (27), (28) and (29) are given by Proposition 2, Proposition 1 and Proposition 4, respectively.

Proposition 5 *The general equilibrium of the aggregate economy, characterized by triplet (τ_s, τ_q, Y), solves the system of equations (27)-(29) for given (b, c, Z). The general equilibrium has three cases:*

i) When $Z < \underline{Z}$*, the equilibrium is unique:* $(\tau_s, \tau_q, Y) = \left(\underline{\tau}_s, \tau_q^*, Y\left(\underline{\tau}_s, \tau_q^*, Z\right)\right)$*;*

ii) When $\underline{Z} \leq Z \leq \bar{Z}$*, there are multiple (two) equilibria:* $(\tau_s, \tau_q, Y) = \left(\underline{\tau}_s, \tau_q^*, Y\left(\underline{\tau}_s, \tau_q^*, Z\right)\right)$ *or* $\left(\bar{\tau}_s, \tau_q^{**}, Y\left(\bar{\tau}_s, \tau_q^{**}, Z\right)\right)$*;*

iii) When $Z > \bar{Z}$*, the equilibrium is unique:* $(\tau_s, \tau_q, Y) = \left(\bar{\tau}_s, \tau_q^{**}, Y\left(\bar{\tau}_s, \tau_q^{**}, Z\right)\right)$*,*

where threshold $\underline{Z} \equiv \underline{Z}(b, c)$ *is the unique root to the equation*

$$\Pi\left(\tau_s = \bar{\tau}_s; \tau_q^{**}, Y\left(\bar{A}, \underline{Z}\right), \underline{Z}\right) - \Pi\left(\tau_s = \underline{\tau}_s; \tau_q^{**}, Y\left(\bar{A}, \underline{Z}\right), \underline{Z}\right) = b$$

and threshold $\bar{Z} \equiv \bar{Z}(b, c)$ *is the unique root to the equation*

$$\Pi\left(\tau_s = \bar{\tau}_s; \tau_q^*, Y\left(\underline{A}, \bar{Z}\right), \bar{Z}\right) - \Pi\left(\tau_s = \underline{\tau}_s; \tau_q^*, Y\left(\underline{A}, \bar{Z}\right), \bar{Z}\right) = b,$$

with $\bar{A} = A\left(\bar{\tau}_s, \tau_q^{**}\right)$*,* $\underline{A} = A\left(\underline{\tau}_s, \tau_q^*\right)$*, and* $Y(A, Z)$ *being defined in (26).*

Proof. *See Appendix.* ■

The intuition behind equilibrium multiplicity in Proposition 5 is very similar to that behind equilibrium multiplicity in Proposition 3 — the root cause is *strategic complementarity* in information production. In the partial equilibrium, strategic complementarity in information production exists *within an island* (between the financial sector and the real sector), which can generate multiple equilibria. In the full equilibrium, strategic complementarity in information production exists *within an island* and *between islands*, and hence multiple equilibria become even more likely. In Proposition 3, the threshold for the existence of multiple equilibrium depends on Y or Z (or, more precisely, $Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}$ seen in (18) and (12)). The only difference now is that Y itself is endogenous and is a function of τ_s, τ_q and Z. Hence, the endogenous Y adds a further reinforcing channel, with the result that the general equilibrium becomes more sensitive to the change in Z than the partial equilibrium and multiple equilibria become more likely.[20]

To illustrate Proposition 5, Figure 2 depicts Y as a function of Z with c and b fixed (recall equation (26)). In Figure 2, when Z is low enough such that $Z \in (-\infty, \underline{Z})$, there is a unique "bad" equilibrium; when Z is high enough such that $Z \in \left(\bar{Z}, +\infty\right)$, there is a unique "good" equilibrium. When $Z \in \left[\underline{Z}, \bar{Z}\right]$, there are two *self-fulfilling* equilibria. Similarly, when Z is kept constant, a change in b or c also leads to different cases of equilibrium (where a change in b or c shifts the thresholds $\underline{Z}$ and $\bar{Z}$ in Figure 2).

[20] In Appendix D.2, we will also show that the results of Proposition 5 do not change qualitatively under the setup of continuous information acquisition of firms.

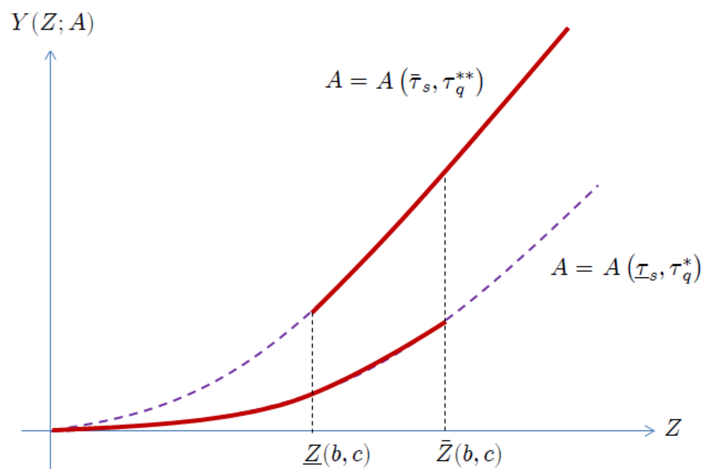

Figure 2: Aggregate output Y in general equilibrium

4.3 Implications

Now we discuss four key implications of the general equilibrium given in Proposition 5.

4.3.1 (Implication 1) Amplification Effects

A small adverse shock (i.e., a small increase in c or b, or a small decrease in Z) can have a large impact on the aggregate economy (aggregate output Y) due to the compound feedback loops of information amplification. For illustration, Figure 3 depicts the amplification effects when an adverse shock to c hits the economy (while b and Z stay the same), where each arrow represents an economic force given in equations (27)-(29). Detailed numerical illustrations of the comparative statics with respect to c, b, and Z will be provided in Section 6. Our *information channel* of amplification contrasts with the *financing channel* in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012), where an adverse shock originating in either the real sector or the financial sector can also lead to a large drop in the aggregate output.

In particular, the amplification in our model can arise from the presence of multiple equilibria (i.e., discontinuity). That is, a small aggregate shock or pure self-fulfilling beliefs in the absence of any aggregate shock can trigger the equilibrium to switch from one regime to the other, generating a very large drop in the aggregate-level output and investment. As illustrated in Figure 2, a small change in Z around $Z = \bar{Z}$ (i.e., a slight decrease in Z from above $\bar{Z}$ to below $\bar{Z}$) can trigger the equilibrium to switch from "good" to "bad", resulting in a large drop in Y. This implies that a positive shock and a negative shock to Z potentially have asymmetric effects on equilibrium output. Suppose that initially Z is (slightly) above $\bar{Z}$. When Z increases, the equilibrium output increases steadily. However, when Z declines, the equilibrium output may exhibit a sudden large decline if the economy falls to the bad equilibrium. It also implies that a small shock and a big shock to Z

can have dramatically different implications. While a small decline in Z leads to a steady decrease in output, a big decline in Z may trigger a self-fulfilling crisis. Conducting comparative statics with respect to c and b instead of Z shows similar patterns (see Section 6).

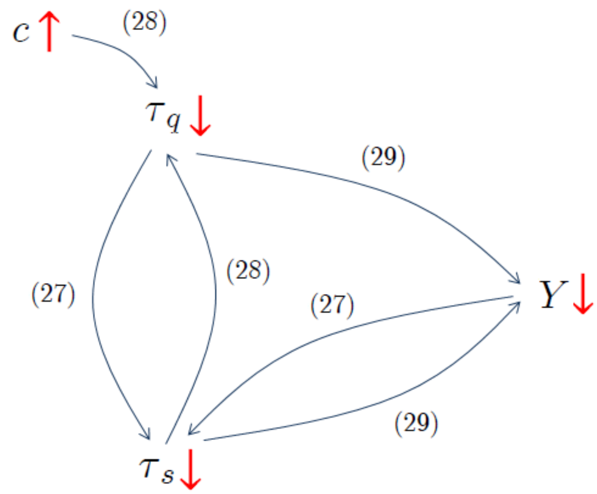

Figure 3: Information amplification

4.3.2 (Implication 2) Real Uncertainty and Financial Uncertainty

Our model endogenizes together the three variables — financial uncertainty, real uncertainty, and aggregate economic activities — and show how they are related. The residual financial uncertainty (or equivalently the financial market efficiency defined in Brunnermeier (2005) and Goldstein and Yang (2015)) is given by[21]

$$SD\left(\varepsilon_j | s_j, q_j\right) = \sqrt{\frac{1}{\tau_\varepsilon + \tau_q}}, \tag{30}$$

and the residual real uncertainty (or the forecast error) faced by a firm is given by

$$SD\left(a_j | s_j\right) = \sqrt{\frac{1}{\tau_a + \tau_s}}. \tag{31}$$

We have shown that an adverse shock in c or b or Z leads to a decrease in τ_s and τ_q together with a decrease in Y, which means that a rise in both real uncertainty and financial uncertainty is accompanied by a fall in aggregate GDP Y. In other words, uncertainty in both financial markets and the real economy rises during recessions.[22]

[21] Equivalently, we can define financial uncertainty as $SD[v_j|s_j, q_j] = \sqrt{\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j, q_j\right] + \left(1 - \frac{1}{\theta}\right)^2 Var\left[a_j|s_j\right]} = \sqrt{\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon + \tau_q} + \left(1 - \frac{1}{\theta}\right)^2 \frac{1}{\tau_a + \tau_s}}$.

[22] The evidence of countercyclical uncertainty in the macroeconomics literature is in line with the findings in the finance literature that the volatility of stock returns is higher in bad times than in good times.

4.3.3 (Implication 3) Contagion and Spillover

Our model implies information contagion and spillover. An adverse shock that directly affects only a small fraction of islands can generate a global recession on all islands through the endogenous information mechanism. Figure 4 illustrates the effects, where an adverse shock to c on some islands has a negative spillover effect on all other islands. Of course, an adverse shock to b has a similar effect. This implication of our model is consistent with a large amount of anecdotal evidence that idiosyncratic firm-level shocks can be the origin of aggregate fluctuations (i.e., microfoundation for aggregate shocks; see Gabaix (2011)).

Formally, we conduct a simple extension of our main model to allow for heterogeneity in b or c across islands. Let the information acquisition cost be $b = b^H$ for a fraction of islands, $j \in [0, \omega]$, and $b = b^L$ for the remaining fraction of islands, where $b^H > b^L$ and $0 \leq \omega \leq 1$. Suppose that in equilibrium the information precision is given by $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^*\right)$ for the first fraction of islands and by $(\tau_s, \tau_q) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ for the remaining fraction of islands. Then, the aggregate endogenous TFP is given by

$$A = \left[\omega \underline{A}^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} + (1-\omega)\, \bar{A}^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}\right]^{\frac{\theta}{\theta-1}-\eta} \tag{32}$$

and the aggregate output Y is given by (26), where $\underline{A} = A\left(\underline{\tau}_s, \tau_q^*\right)$ and $\bar{A} = A\left(\bar{\tau}_s, \tau_q^{**}\right)$.

Now we are able to formalize the contagion effect. For a given Z and c, define b^{**} such that $Z = \underline{Z}(b^{**}, c)$, where function $\underline{Z}$ is given in Proposition 5. Suppose that $b = b^L$ in the economy initially for all islands, where b^L is slightly lower than b^{**}. By Proposition 5, the economy initially has two equilibria, where one equilibrium is that all islands are with the "good" equilibrium — the information precision for all islands is $(\tau_s, \tau_q) = \left(\bar{\tau}_s, \tau_q^{**}\right)$. Consider now the case where a small fraction of islands suffer a shock in the sense that their information acquisition cost increases slightly to $b = b^H$ (which is slightly above b^{**}). Then, the islands suffering the shock inevitably falls into the "bad" equilibrium with $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^*\right)$. This decreases Y by (32). Because Y is reduced, all other islands are affected and can also inevitably fall into the "bad" equilibrium (by Proposition 3). That is, the unique "bad" equilibrium for the whole economy can be the outcome. In other words, the market efficiency, the real allocative efficiency, and the expected output in value (i.e., the real GDP $P_j Y_j$) on all islands will all go down. Again, a numerical illustration of this result will be provided in Section 6.

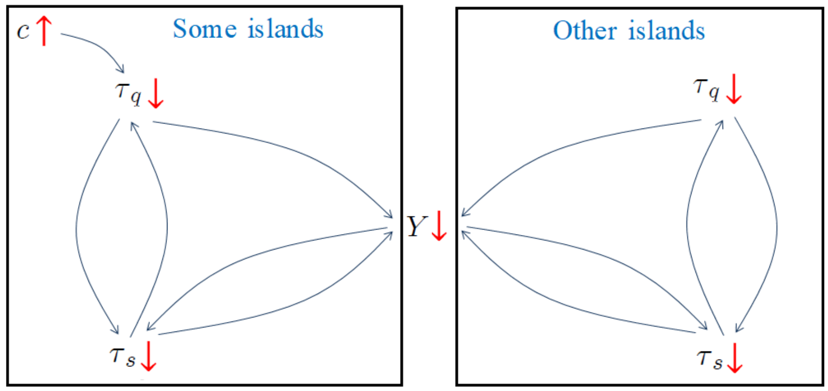

Figure 4: Information contagion

4.3.4 (Implication 4) Cross-sectional Implications

Our general-equilibrium model with production and a continuum of assets is particularly useful for studying asset price comovements. Empirical studies have established several interesting but puzzling patterns of comovements, summarized below.

i) A negative relation exists between the degree of comovement of stock prices in a sector and the informativeness of the stock prices (e.g., Durnew et al. (2003)). Stock prices move together more in poor economies than in rich economies (e.g., Morck, Yeung and Yu (2000)).

ii) There is a decline in comovement of asset prices across sectors over time (e.g., Campbell et al. (2001)).

iii) Correlations between U.S. stocks and the aggregate U.S. market are much higher for downside moves than for upside moves (e.g., Ang and Chen (2002)).

iv) Industries with larger firm-specific variation in stock returns have higher economic efficiency of corporate investment (e.g., Durnew, Morck and Yeung (2004)).

Now we show how our model can help shed light on these intriguing empirical regularities. The intuition is easy to understand. In our model, an individual asset price q_j is effectively driven by three components: aggregate productivity Z, the firm's signal $s_j = a_j + e_j$, and informed traders' signals $x_j^i = \varepsilon_j + \varrho_j^i$. The aggregate price index is instead driven by the aggregate productivity Z alone by the law of large numbers. When the signals s_j and x_j^i become more precise, the individual price q_j becomes more responsive to idiosyncratic shocks a_j and ε_j and relatively less responsive to the aggregate shock Z; therefore, the correlation between an individual asset price and the aggregate price index becomes lower. At the same time, the efficiency of resource allocation, characterized by the endogenous TFP A, becomes higher and the aggregate output increases. In short, a lower degree of comovement of asset prices as well as a higher efficiency of resource allocation is expected to be accompanied by a higher aggregate output.

Formally, for simplicity and to sharply deliver the message, we study two extreme cases of the model. Consider two countries which differ only in information acquisition costs. We assume that country I has extremely high information acquisition costs, say, $b^I = c^I = \infty$, whereas country II has information acquisition costs that are close to zero, namely, $b^{II} \to 0$ and $c^{II} \to 0$. Moreover, if agents acquire information, their signals are perfectly informative, namely $\tau_s = \tau_x = \infty$; otherwise they obtain useless signals. We also assume a reasonable parameter condition $\tau_n > \left(\frac{\gamma}{\Theta}\right)^2 \vartheta$, where $\vartheta = \left(1 - \frac{1}{\theta}\right)^2 \tau_a^{-1} + \left(\frac{1}{\theta}\right)^2 \tau_\varepsilon^{-1}$. Then, the asset prices in country I and country II, respectively denoted by q_j^I and q_j^{II}, are given by

$$\begin{aligned} q_j^I &= q_0^I + \frac{\Theta^2}{\theta - \eta\Theta} \log \underline{A} + \frac{\theta\Theta}{\theta - \eta\Theta} z + \gamma \left[\left(1 - \frac{1}{\theta}\right)^2 \tau_a^{-1} + \left(\frac{1}{\theta}\right)^2 \tau_\varepsilon^{-1} \right] n_j \\ q_j^{II} &= q_0^{II} + \frac{\Theta^2}{\theta - \eta\Theta} \log \bar{A} + \frac{\theta\Theta}{\theta - \eta\Theta} z + \Theta \left[\left(1 - \frac{1}{\theta}\right) a_j + \frac{1}{\theta} \varepsilon_j \right], \end{aligned}$$

where q_0^I and q_0^{II} are two constants which do not depend on z. Define $q^I = \int q_j^I dj$ and $q^{II} = \int q_j^{II} dj$. We have $corr(q_j^I, q^I) = \frac{\frac{\theta\Theta}{\theta - \eta\Theta}\sqrt{\tau_z^{-1}}}{\sqrt{\left(\frac{\theta\Theta}{\theta - \eta\Theta}\right)^2 \tau_z^{-1} + \gamma^2 \vartheta^2 \tau_n^{-1}}}$ and $corr(q_j^{II}, q^{II}) = \frac{\frac{\theta\Theta}{\theta - \eta\Theta}\sqrt{\tau_z^{-1}}}{\sqrt{\left(\frac{\theta\Theta}{\theta - \eta\Theta}\right)^2 \tau_z^{-1} + \Theta^2 \vartheta}}$, where τ_z^{-1} is the variance of aggregate shock z. So $corr(q_j^I, q^I) > corr(q_j^{II}, q^{II})$. For a given realization Z, the endogenous aggregate TFP of the two countries satisfies $A^I < A^{II}$ and the aggregate output satisfies $Y^I < Y^{II}$.

Similarly, we can consider one country with two types of islands. The setup for the two types of islands is the same as that for the two types of countries above. Assume that the fraction of the first type of islands is ω. Then, the asset prices on type-I islands and type-II islands are, respectively, given by

$$q_j^I = q_0^I + \frac{\Theta^2}{\theta - \eta\Theta} \log A + \frac{\theta\Theta}{\theta - \eta\Theta} z + \gamma \left[\left(1 - \frac{1}{\theta}\right)^2 \tau_a^{-1} + \left(\frac{1}{\theta}\right)^2 \tau_\varepsilon^{-1} \right] n_j$$

and

$$q_j^{II} = q_0^{II} + \frac{\Theta^2}{\theta - \eta\Theta} \log A + \frac{\theta\Theta}{\theta - \eta\Theta} z + \Theta \left[\left(1 - \frac{1}{\theta}\right) a_j + \frac{1}{\theta} \varepsilon_j \right],$$

where the endogens TFP A of the country is given by (32), and its aggregate output Y is given by (26). When ω increases, A decreases and, therefore, asset prices on all islands and aggregate output Y fall together. Define the aggregate price index as $q = \omega \int q_j^I dj + (1 - \omega) \int q_j^{II} dj$. The average correlation is computed as $\int corr(q_j, q) dj = \omega \frac{\frac{\theta\Theta}{\theta - \eta\Theta}\sqrt{\tau_z^{-1}}}{\sqrt{\left(\frac{\theta\Theta}{\theta - \eta\Theta}\right)^2 \tau_z^{-1} + \gamma^2 \vartheta^2 \tau_n^{-1}}} + (1 - \omega) \frac{\frac{\theta\Theta}{\theta - \eta\Theta}\sqrt{\tau_z^{-1}}}{\sqrt{\left(\frac{\theta\Theta}{\theta - \eta\Theta}\right)^2 \tau_z^{-1} + \Theta^2 \vartheta}}$, which is increasing in ω. It is also true that $corr(q_j^I, \int q_j^I dj) > corr(q_j^{II}, \int q_j^{II} dj)$.

The above model results explain the four empirical patterns listed at the beginning of this subsection. First, sector I (type-I islands), relative to sector II, has a higher degree of asset price comovement by $corr(q_j^I, \int q_j^I dj) > corr(q_j^{II}, \int q_j^{II} dj)$ and also a lower degree of financial price informativeness by $\tau_q^* < \tau_q^{**}$. Second, the improvement of information technology may correspond to a decrease in ω over time for a country, and hence our model implies that comovement in asset prices decline over time. Third, economic downside moves may correspond to periods with a higher ω, so asset price comovement is higher in such periods. Fourth, country I with a lower GDP has a higher degree of asset price comovement than country II. Sector I, which has a higher degree of asset price comovement, has a lower investment efficiency by $\underline{A} < \bar{A}$.

The puzzling empirical facts on asset price comovement, which are difficult to explain with traditional asset pricing theory, have inspired numbers of theoretical studies. Notably, Peng and Xiong (2006) develop a novel model based on limited investor attention. A key element of their model is that investors tend to process more aggregate information than firm-specific information. This is similar to our model, where aggregate shock Z is observable by financial investors while firm-specific shocks a_j and ε_j are imperfect information. Peng and Xiong (2006) study a pure exchange economy and show that their model can explain facts i) and ii). Our model with a production economy complements their work. By linking asset price comovements to information production and investment efficiency in a production economy, we are able to explain the two additional facts iii) and iv). In Section 6, we will also offer a different mechanism for why information on aggregate shocks is more easily available to investors based on the insight of Hayek (1945). Basically, we argue that when investors with dispersed information are trading some "aggregate" production factors in the economy such as capital and labor, the realization of aggregate shocks can be revealed through information aggregation of prices, while such "aggregate" assets may hardly exist for firm-specific shocks. Similarly, if investors can observe all asset prices on all islands, the realization of an aggregate shock can also be revealed by aggregating all asset prices.

4.4 Model Extensions

In the section, we conduct two extensions on our macroeconomic model.

4.4.1 Heterogeneity and Multiplicity

In this subsection, we extend our macroeconomic model by considering heterogeneity of firms and studying its effect on equilibrium multiplicity. Interestingly, we find that heterogeneity can actually increase, rather than reduce, the possibility of multiple equilibria in our model. This result is in sharp contrast to the types of coordination failure in other models, which have shown that heterogeneity can make multiple equilibria disappear.[23]

Formally, we consider the case where there is heterogeneity in b_j (the fixed cost of a firm's information acquisition). In Appendix D.1, we will also study heterogeneity in the observable part of firm-specific productivity and the results are similar. Let b_j be drawn from a continuous distribution with cumulative density function (c.d.f.) as $G(\cdot)$ in support $[0, \infty)$. This implies that in equilibrium a fraction of islands will have less precise information (i.e., $(\tau_s, \tau_q) = (\underline{\tau}_s, \tau_q^*)$) and the other fraction of islands will have more precise information (i.e., $(\tau_s, \tau_q) = (\bar{\tau}_s, \tau_q^{**})$). Let us denote the first fraction by ω and then the endogenous TFP is given by (32). Thus, the aggregate output Y is function of A. In the same spirit of Proposition 3, we find that there are three types of islands in equilibrium. Lemma 3 follows.

Lemma 3 *Suppose that ω is given and thus so are A and Y for a realized Z. In equilibrium, for islands with $b_j < b^*$, the equilibrium outcome is unique with $(\tau_s, \tau_q) = (\bar{\tau}_s, \tau_q^{**})$; for islands with $b_j > b^{**}$, the outcome is also unique with $(\tau_s, \tau_q) = (\underline{\tau}_s, \tau_q^*)$; and for islands with $b^* \leq b_j \leq b^{**}$, there are multiple equilibria with $(\tau_s, \tau_q) = (\underline{\tau}_s, \tau_q^*)$ or $(\bar{\tau}_s, \tau_q^{**})$, where b^* and b^{**} are determined by*

$$\frac{1}{\theta}\left(\frac{A(\bar{\tau}_s, \tau_q^*)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} Y - b^* = \frac{1}{\theta}\left(\frac{A(\underline{\tau}_s, \tau_q^*)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} Y \tag{33}$$

and

$$\frac{1}{\theta}\left(\frac{A(\bar{\tau}_s, \tau_q^{**})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} Y - b^{**} = \frac{1}{\theta}\left(\frac{A(\underline{\tau}_s, \tau_q^{**})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} Y, \tag{34}$$

respectively.

Proof. *See Appendix.* ■

The intuition behind Lemma 3 is very similar to that behind Proposition 3. For a given Y and Z, when b_j is sufficiently low, the *dominant strategy* for firm j is to acquire information (i.e., $\tau_s = \bar{\tau}_s$) even if the financial market is less informative (i.e., $\tau_q = \tau_q^*$). A similar argument applies to other ranges of b_j.

Lemma 3 essentially shows how the two thresholds b^* and b^{**} are determined for a given ω. An equilibrium means a fixed-point problem between (b^*, b^{**}) and ω. Proposition 6 follows.

Proposition 6 *When there is heterogeneity in $b_j \in [0,\infty)$ across firms, for a realized Z, there are always multiple equilibria in which ω, the endogenous TFP, and aggregate output are driven by a sunspot variable $0 \leq s \leq 1$, such that*

$$\omega = sG(b^*) + (1-s)G(b^{**}),$$

[23] See the discussions on Ball and Romer (1991), Morris and Shin (1998), and Schaal and Taschereau-Dumouchel (2015) in the literature review of the paper.

where b^* and b^{**} are respectively determined by (33) and (34). The endogenous TFP is given by (32) and the aggregate output is given by $Y = Y(A; Z)$ according to the formula in (26).

Proof. *See Appendix.* ■

Proposition 6 illustrates that heterogeneity in b_j increases the likelihood of multiple equilibria. In fact, Proposition 5 corresponds to the case of no heterogeneity in b_j, in which multiple equilibria occur only under an intermediate level of realized Z and feature two symmetric equilibria (i.e., $\omega = 0$ or $\omega = 1$). In contrast, Proposition 6 shows that for any realized Z there are multiple equilibria and for a given Z the multiple equilibria feature a continuum of ω driven by sunspots.

The reason for this intriguing result is that there are two layers of coordination problems in our model. Given other islands' information decisions, within an island, there exists a coordination problem between the firm and the financial market on that island. The second coordination problem occurs across islands due to the Dixit-Stiglitz demand externality. Heterogeneity in the information acquisition cost reduces the incentive of the firm on a particular island to coordinate with firms on other islands. So information production across islands will be less synchronized. However, enough heterogeneity in the information acquisition cost naturally divides islands into three types as shown in Proposition 3. As a result, two equilibria will always exist on some islands — islands on which the information acquisition cost b_j falls into a range such that the partial equilibrium in Proposition 3 has multiple (two) equilibria. Since the total fraction of islands with the "good" equilibrium is indeterminate, the aggregate economy hence features a continuum of equilibria.

4.4.2 Investors Holding a Portfolio

In Section 4.1, we assumed that an investor can trade only in one financial market (asset). Now we show that our model insight is intact if investors can hold a portfolio with access to financial assets on all islands.

For expositional clarity, we slightly modify the setup of the macroeconomic model in Section 4.1 by letting there be $j = 1, 2, 3, ...J$ discrete islands.[24] The Dixit-Stiglitz production function in (19) is alternatively assumed as

$$Y = J^{-\frac{1}{\theta-1}} \left[\sum_{j=1}^{J} \epsilon_j^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

[24] We can use the original setup with a continuum of islands, and the model result in this subsection will still be the same. But in that case, we need to specify that the dj units of type-j firms together have the total noise trading n_j. In other words, each island in the general equilibrium model should be the counterpart of the one island in the partial equilibrium model in terms of the financial asset supply (noise trading). For conceptual clarity, we instead use the setting of a discrete number of islands in this subsection.

where the normalization follows the standard macroeconomic literature such as Jaimovich and Floetotto (2008). The demand schedule for good j is then given by $Y_j = \frac{1}{J}\left(\frac{1}{P_j}\right)^{\theta}\epsilon_j Y$. Hence, as before, $v_j = \frac{1}{\theta}\varepsilon_j + \left(1-\frac{1}{\theta}\right)(z+a_j) + \eta\left(1-\frac{1}{\theta}\right)k_j + \frac{1}{\theta}y - \frac{1}{\theta}\log J$.

There is still a continuum of investors with unit mass. Investor i maximizes his utility of (21) with constraint (22), in which the payoff D^i from financial market trading becomes

$$D^i = \sum_{j=1}^{J} m_j^i(v_j - q_j) - \sum_{j=1}^{J} c_j^i,$$

where m_j^i is his position on asset j and c_j is an indicator function such that $c_j^i = c$ if he acquires information about ε_j and $c_j^i = 0$ otherwise. Hence, the utility maximization problem can be transformed into

$$\max -\mathbb{E}\left[\prod_{j=1}^{J} \exp\left(-\gamma\left[m_j^i(v_j - q_j) - c_j^i\right]\right)\right]. \tag{35}$$

Because v_j and q_j are independent across islands, the investor's decision on all islands together, $\left\{m_j^i\right\}$, is the same as the decision on each island *separately*. The first-order condition of (35) implies

$$m_j^i = \begin{cases} \frac{\mathbb{E}[v_j|s_j,q_j,x_j^i]-q_j}{\gamma Var[v_j|s_j,q_j,x_j^i]} \equiv m_j^{Ii} & \text{if } c_j^i = c \\ \frac{\mathbb{E}[v_j|s_j,q_j]-q_j}{\gamma Var[v_j|s_j,q_j]} \equiv m_j^{Ui} & \text{if } c_j^i = 0 \end{cases},$$

which is exactly the same as the trading (asset holding) decision of a speculator in Section 2. It is also easy to show that the results of information acquisition decisions for speculators in Section 3 apply here. In Appendix (the proof in Section 4.4.2), we will also show that the results for firms' information acquisition and the aggregate economy equilibrium do not change qualitatively.

5 The Dynamic Model

In this section, we extend the static model to an OLG framework. The OLG model provides a dynamic equilibrium setting to study the process of saving and capital accumulation. The exogenous endowment W_0 in the static model is now endogenized. The equilibrium, therefore, is dynamically linked across periods through savings — the nature of equilibrium in the next period endogenously depends on not only the aggregate productivity shock Z in that period but also the aggregate output in the current period.

We derive three additional economic insights. First, we show that the dynamic model possibly has two steady-state equilibria, which means that the exhibition of self-fulfilling uncertainty traps holds true in the dynamic setting. Second, when we study the transitional dynamics under a permanent negative shock, the dynamic model characterizes a two-stage economic crisis. Third, in the dynamic model, the aggregate shock Z is endogenously revealed through information aggregation of prices, rather than assumed constant and publicly observable as in the static model.

5.1 Setup

Agents In each period, there are three types of agents: investors (who were workers in the last period), entrepreneurs,[25] and workers. There is a continuum of $[0,1]\times[0,1]$ investors (workers) and a continuum of $[0,1]$ entrepreneurs. Each worker is endowed with one unit of time. A worker supplies labor for a wage when he is young, and saves up his wage as capital and becomes an investor when he is old. An investor earns income on his capital and then consumes. Each entrepreneur is a monopoly producer for an intermediate good on one island; he earns a profit and then consumes.

Production Production of an intermediate good needs the inputs of capital and labor subject to information frictions as in the baseline model. Specifically, the production function of intermediate good j is

$$Y_{jt} = Z_t A_{jt} \left(K_{jt}^{\eta} N_{jt}^{1-\eta} \right), \tag{36}$$

where K_{jt} is the input of capital, which fully depreciates after production, N_{jt} is the input of labor, and $\eta \in (0,1)$. There is a final good production sector as in the static model, with the production function

$$Y_t = \left[\int \epsilon_{jt}^{\frac{1}{\theta}} Y_{jt}^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}, \tag{37}$$

where $j \in [0,1]$, $\theta > 1$ is the elasticity of substitution between intermediate capital goods, and ϵ_{jt} measures the demand shock to intermediate good j. Denote $a_{jt} \equiv \log A_{jt}$ and $\varepsilon_{jt} \equiv \log \epsilon_{jt}$. The price of the final good is normalized as the numeraire price in each period t.

Timeline In each period t, there are five stages.

Stage 1: The old generation of workers becomes investors who possess capital which is carried over from the last period. A new generation of workers and a new generation of entrepreneurs are born.

Stage 2: Entrepreneurs and investors simultaneously make their information acquisition decisions as in Section 3. Entrepreneur j acquires information about productivity shock a_{jt} and investors acquire information about demand shocks $\{\varepsilon_{jt}\}$.

Stage 3: As in the timeline in the baseline model of Section 2, an entrepreneur first discloses his signal to the financial market, and then financial market trading takes place, and then

[25] For simplicity, we assume that entrepreneurs survive only for one period and in each period a new generation of entrepreneurs is born. Alternatively and equivalently, we can assume that entrepreneurs survive for two periods and in each period there are two overlapping generations of entrepreneurs; in that case, entrepreneurs sit idle in the first half of their life (young) and are active only in the second half of their life (old).

entrepreneurs make their investment decisions.

Stage 4: Production output is realized. Output is divided among workers (wages), investors (capital returns), and entrepreneurs (profits). The payoffs of financial contracts are delivered. Investors and entrepreneurs consume and then die.

Stage 5: Workers receive private signals about the aggregate productivity shock Z_{t+1} in the next period which is realized but is not public information. Workers trade their capital (wages) and bonds among themselves. The capital price and the return on bonds, R_{ft+1}, are realized. Workers then proceed to the next period.

5.2 Equilibrium

At stage 5 of period $t-1$, workers invest their wage income in capital and bonds based on their dispersed information about Z_t. A typical worker i faces the following budget constraint:

$$1 \cdot K_t^i + (1/R_{ft}) \cdot B_t^i = W_{t-1},$$

where W_{t-1} is his wage income, and K_t^i is the number of units of capital and B_t^i is the number of units of bonds that he invests in. One unit of consumption good at this stage can be transformed into one unit of capital and hence the price of capital is one. One unit of capital allows its owner to obtain the rental return R_t in the next period t. The bond is traded at the discounted price $1/R_{ft}$ and hence the return on the bond is R_{ft}; in other words, R_{ft} is the intertemporal interest rate between $t-1$ and t. Each worker receives a noisy private signal about Z_t. Since there is no aggregate noise trading, Z_t is revealed through the bond price $1/R_{ft}$ as in Vives (2014) and Benhabib, Liu and Wang (2016a). It must also be true that $R_{ft} = R_t$ in equilibrium. Since the net bond supply is zero, the aggregate capital in the next period t is $K_t = W_{t-1} \times 1 = W_{t-1}$.

In period t, investor i, who was a worker in the last period, starts off with K_t^i units of capital and B_t^i units of bonds. At stage 3, investors trade risky assets with payoff $v_{jt} = \log(Y_{jt}P_{jt})$ at prices q_{jt}. Hence, when trading risky assets, an investor's problem is

$$\max_{\varkappa_t^i, m_{jt}^i} \mathbb{E}\left(\exp\left[-\gamma\left(C_t^i - \varkappa_t^i c\right)\right] | \mathcal{I}_t^i\right)$$
$$\text{s.t.} \qquad C_t^i = R_t K_t^i + (v_{jt} - q_{jt}) m_{jt}^i + B_t^i$$

where C_t^i is investor i's consumption, K_t^i is his capital purchased in the previous period , B_t^i is his total purchase of bonds, m_{jt}^i is his position on risky asset j, and c is the information acquisition cost as in the static model. And $\varkappa_t^i$ is an indicator function, which equals 1 if the investor acquires information and 0 if not. Because Z_t is revealed by $1/R_{ft}$ in the last period $t-1$, the information set

$\mathcal{I}_t^i$ is $\left\{s_{jt}, q_{jt}, x_j^i, Z_t\right\}$ if acquiring information and $\{s_{jt}, q_{jt}, Z_t\}$ if not, where s_{jt} is entrepreneur j's signal and disclosure about a_{jt} and x_j^i is investor i's signal about ε_{jt}. For simplicity and tractability, we assume in the dynamic model that information acquisition costs (b and c) are direct utility costs to agents.

At the production stage, entrepreneur j's problem is to solve

$$\max_{K_{jt},N_{jt}} \mathbb{E}\left[P_{jt}Y_{jt} - W_t N_{jt} - R_t K_{jt} | \mathcal{I}_{jt}\right], \tag{38}$$

where $P_{jt} = \left(\frac{\epsilon_{jt}Y_t}{Y_{jt}}\right)^{\frac{1}{\theta}}$, Y_{jt} is given by (36), and W_t is the wage. The information set is $\mathcal{I}_{jt} = \{s_{jt}, q_{jt}, W_t, R_t, Z_t\}$. The first-order conditions of (38) imply

$$\begin{array}{r} \eta(1-\frac{1}{\theta})N_{jt}^{(1-\eta)(1-\frac{1}{\theta})}K_{jt}^{\eta(1-\frac{1}{\theta})-1}Z_t^{1-\frac{1}{\theta}}Y_t^{\frac{1}{\theta}}\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right] = R_t, \\ (1-\eta)(1-\frac{1}{\theta})N_{jt}^{(1-\eta)(1-\frac{1}{\theta})-1}K_{jt}^{\eta(1-\frac{1}{\theta})}Z_t^{1-\frac{1}{\theta}}Y_t^{\frac{1}{\theta}}\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right] = W_t, \end{array}, \tag{39}$$

which means $\frac{R_tK_{jt}}{W_tN_{jt}} = \frac{\eta}{1-\eta}$. Hence, in aggregate, $R_tK_t = (1-\frac{1}{\theta})\eta Y_t$ and $W_tN_t = (1-\frac{1}{\theta})(1-\eta)Y_t$, where $\int_0^1 K_{jt}dj = K_t$ and $\int_0^1 N_{jt}dj = N_t = 1$.

By (39), we also have the following resource allocation across firms (entrepreneurs):

$$\begin{array}{l} K_{jt} = \dfrac{\left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^{\theta}}{\displaystyle\int_0^1\left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^{\theta}dj}K_t \\ N_{jt} = \dfrac{\left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^{\theta}}{\displaystyle\int_0^1\left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^{\theta}dj}N_t \end{array}. \tag{40}$$

This gives the aggregate production function

$$Y_t = Z_tA_t\left(K_t^{\eta}N_t^{1-\eta}\right),$$

where

$$\begin{aligned} A_t &= A(\tau_{st},\tau_{qt}) = \left[\mathbb{E}\left(\left[\mathbb{E}\left(A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right)\right]^{\theta}\right)\right]^{\frac{1}{\theta-1}} \\ &= \exp\left(\frac{1}{2}(\theta-2)\frac{1}{\tau_a} - \frac{1}{2}\frac{(\theta-1)^2}{\theta}\frac{1}{\tau_a+\tau_{st}} - \frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_{qt}}\right), \end{aligned}$$

and τ_{st} is the precision of signal s_{jt} and τ_{qt} is the precision of financial price signal q_{jt}. Since $K_{t+1} = W_t$, Lemma 4 follows.

Lemma 4 *Given τ_{et} and τ_{qt}, the dynamics of the economy is characterized by*

$$Y_t = Z_t A_t K_t^{\eta},$$
$$K_{t+1} = (1-\frac{1}{\theta})(1-\eta)Y_t,$$
$$A_t = \exp\left(\frac{1}{2}(\theta-2)\frac{1}{\tau_a} - \frac{1}{2}\frac{(\theta-1)^2}{\theta}\frac{1}{\tau_a+\tau_{st}} - \frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_{qt}}\right).$$

Proof. *See Appendix.* ■

To determine τ_{et} and τ_{qt}, we need to study the information acquisition problem of entrepreneurs and investors. Similar to (16), information acquisition of investors gives

$$e^{\gamma c} = \sqrt{\frac{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_{st}} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_{qt}}}{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_{st}} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_x+\tau_{qt}}}}.$$

Hence, similar to Proposition 1, τ_{qt} is a function of τ_{et} and c, with comparative statics $\frac{\partial \tau_{qt}}{\partial \tau_{st}} > 0$ and $\frac{\partial \tau_{qt}}{\partial c} < 0$. The information acquisition problem of entrepreneurs gives similar results as in Propositions 2 and 3. We still focus on symmetric equilibria, in which $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ or $\left(\underline{\tau}_s, \tau_q^{*}\right)$. Define $\xi \equiv \left[\frac{A(\bar{\tau}_s,\tau_{qt})}{A(\underline{\tau}_s,\tau_{qt})}\right]^{\theta-1} = \exp\left[\frac{1}{2}\frac{(\theta-1)^3}{\theta}\left(\frac{\bar{\tau}_s-\underline{\tau}_s}{(\tau_a+\underline{\tau}_s)(\tau_a+\bar{\tau}_s)}\right)\right] > 1$, $\bar{A} = A(\bar{\tau}_s, \tau_q^{**})$ and $\underline{A} = A(\underline{\tau}_s, \tau_q^{*})$. Then, we find two thresholds $\bar{K}$ and $\underline{K}$, which are functions of Z_t and respectively solve

$$\frac{1}{\theta}\xi Y\left(Z_t, \underline{A}, \bar{K}\right) - b = \frac{1}{\theta} Y\left(Z_t, \underline{A}, \bar{K}\right)$$

and

$$\frac{1}{\theta} Y\left(Z_t, \bar{A}, \underline{K}\right) - b = \frac{1}{\theta}\frac{1}{\xi} Y\left(Z_t, \bar{A}, \underline{K}\right),$$

where $Y(Z_t, A_t, K_t) = Z_t A_t K_t^{\eta}$. Proposition 7 follows.

Proposition 7 *For a given Z_t, there is a unique equilibrium $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ when $K_t > \bar{K}$; there is a unique equilibrium $(\tau_{st}, \tau_{qt}) = \left(\underline{\tau}_s, \tau_q^{*}\right)$ when $K_t < \underline{K}$; and there are multiple (two) equilibria with $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ or $\left(\underline{\tau}_s, \tau_q^{*}\right)$ when $\underline{K} \leq K_t \leq \bar{K}$, where $\bar{K}$ and $\underline{K}$ are two thresholds, decreasing functions of Z_t.*

Proof. *See Appendix.* ■

Proposition 7 implies that for the dynamic model, whether a unique equilibrium or multiple equilibria exist depends not only on the realization of shock Z_t in the current period but also on the capital stock ($K_t = W_{t-1}N_{t-1}$) in the last period. In other words, the nature of equilibrium is path-dependent. This is in contrast to the result in Proposition 5 for the static model, where the existence of a unique equilibrium or multiple equilibria depends only on the realization of shock Z.

Based on Lemma 4 and Proposition 7, we can find the law of motion for capital:

$$K_{t+1} = \begin{cases} (1-\frac{1}{\theta})(1-\eta)Z_t\bar{A}K_t^{\eta} & \text{if } K_t \geq \underline{K}(Z_t) \quad \text{(a)} \\ (1-\frac{1}{\theta})(1-\eta)Z_t\underline{A}K_t^{\eta} & \text{if } K_t \leq \bar{K}(Z_t) \quad \text{(b)} \end{cases}. \tag{41}$$

Proposition 8 ***(Steady States)*** *Suppose that $Z_t = Z$, a constant. If $Z > Z^{**}$ or $Z < Z^{*}$, there is a unique steady-state equilibrium; if $Z^{*} \leq Z \leq Z^{**}$, there are two steady-state equilibria, where the two thresholds are $Z^{*} = \left[\frac{\left(\frac{b\theta}{\xi-1}\right)^{1-\eta}\xi^{1-\eta}}{\left[(1-\frac{1}{\theta})(1-\eta)\right]^{\eta}}\right]\frac{1}{\bar{A}}$ and $Z^{**} = \left[\frac{\left(\frac{b\theta}{\xi-1}\right)^{1-\eta}}{\left[(1-\frac{1}{\theta})(1-\eta)\right]^{\eta}}\right]\frac{1}{\underline{A}}$.*

Proof. *See Appendix.* ■

Proposition 8 highlights self-fulfilling uncertainty traps in the dynamic economy. Figure 5 illustrates Proposition 8. Two remarks are in order. First, even if we impose an equilibrium selection to choose the "good" equilibrium in each period (in which case the jump point in Figure 5 is unique at $K_t = \underline{K}$), there are still possibly multiple steady-state equilibria. Second, while Figure 5 indicates the existence of two possible equilibrium paths starting in the range $(\underline{K}, \bar{K})$, nothing prevents the equilibrium from switching between the two branches of capital accumulation in that K_t range. The coordination problem for agents, whether to acquire high or low information (i.e., $(\tau_{st}, \tau_{qt}) = (\bar{\tau}_s, \tau_q^{**})$ or $(\underline{\tau}_s, \tau_q^{*})$), is independent across periods since a new generation of agents replaces the old generation each period. Equilibrium switches between periods can be driven by a stochastic sunspot or sentiment process such as a Markov chain, or animal spirits.[26]

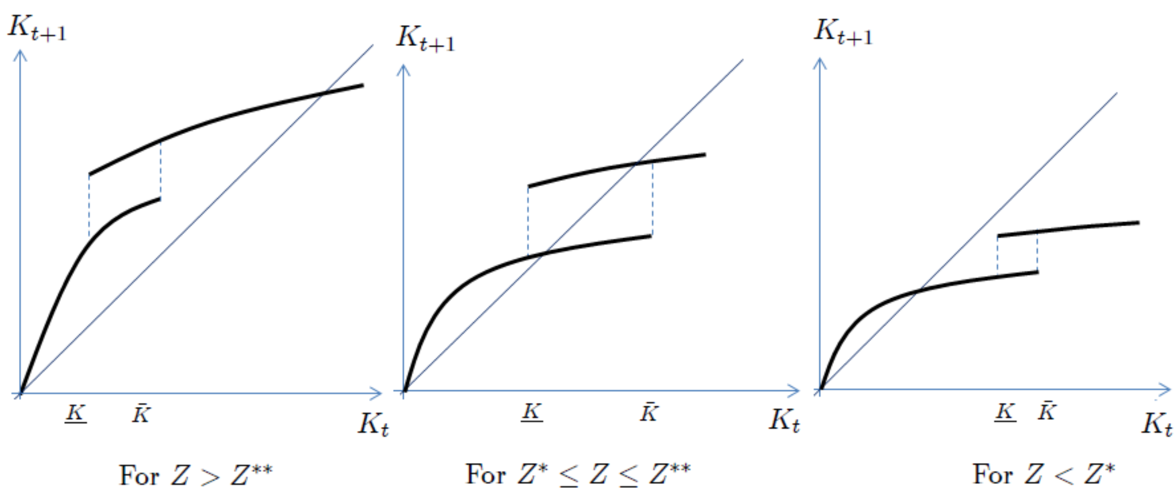

Figure 5: Two possible steady-state equilibria

[26] See, e.g., Benhabib, Dong and Wang (2018).

Next, we examine the transitional dynamics. Suppose that initially $Z_t = Z$, which is constant and a bit higher than Z^{**}, and suddenly at $t = t_0$ a permanent negative shock hits Z_t. Proposition 9 shows the transitional dynamics under a permanent negative shock.

Proposition 9 ***(Transitional Dynamics)*** *Suppose that initially $Z_t = Z > Z^{**}$ and the economy is in the steady state, and at $t = t_0$ a negative shock hits Z_t resulting in $Z_t = Z^{'} < Z$. If the shock is small enough such that $Z^{'} > Z^{**}$, the transitional dynamics of K_t are given by (41(a)) for $t > t_0$; if the shock is medium-sized such that $Z^{*} < Z^{'} < Z^{**}$, the transitional dynamics of K_t can be (41(a)) for $t_0 < t < t_1$ and (41(b)) for $t \geq t_1$, where t_1 is the time point of equilibrium switching.*

Proof. *See Appendix.* ■

Figure 6 illustrates Proposition 9. When the shock is small, the dynamic economy still has a unique "good" steady-state equilibrium. However, when the shock is medium-sized, it triggers a regime change: from the existence of a unique "good" steady-state equilibrium to the existence of two steady-state equilibria. The latter case characterizes a two-stage economic crisis. The negative shock itself does not cause a big decline in economy activities at the beginning. After the shock, the capital accumulation is initially along the *unique* path (i.e., the upper branch toward the new "good" steady-state equilibrium in Figure 6) and the capital stock is declining over time but the recession is mild. However, once the capital stock K_t has declined to a certain point such that $K_t \leq \bar{K}\left(Z^{'}\right)$, the second path of equilibrium is opened.[27] A sunspot or sentiment can suddenly switch the equilibrium path to the lower branch, in which case a surge in real uncertainty and financial uncertainty accompanied by a big drop in output strikes. After that, the economy further declines and gradually converges to the new "bad" steady-state equilibrium. Numerical illustrations for the transitional dynamics will be provided in Section 6.

[27] It is easy to show that if $Z^{'} \in (Z^{*}, Z^{**})$ is not too high, the dynamics of K_t enter the region of $K_t \leq \bar{K}\left(Z^{'}\right)$ before the new "good" steady state is reached; that is, $K^{**}\left(Z^{'}\right) < \bar{K}\left(Z^{'}\right)$, where $K^{**}\left(Z^{'}\right)$ is the capital in the "good" steady state for $Z_t = Z^{'}$.

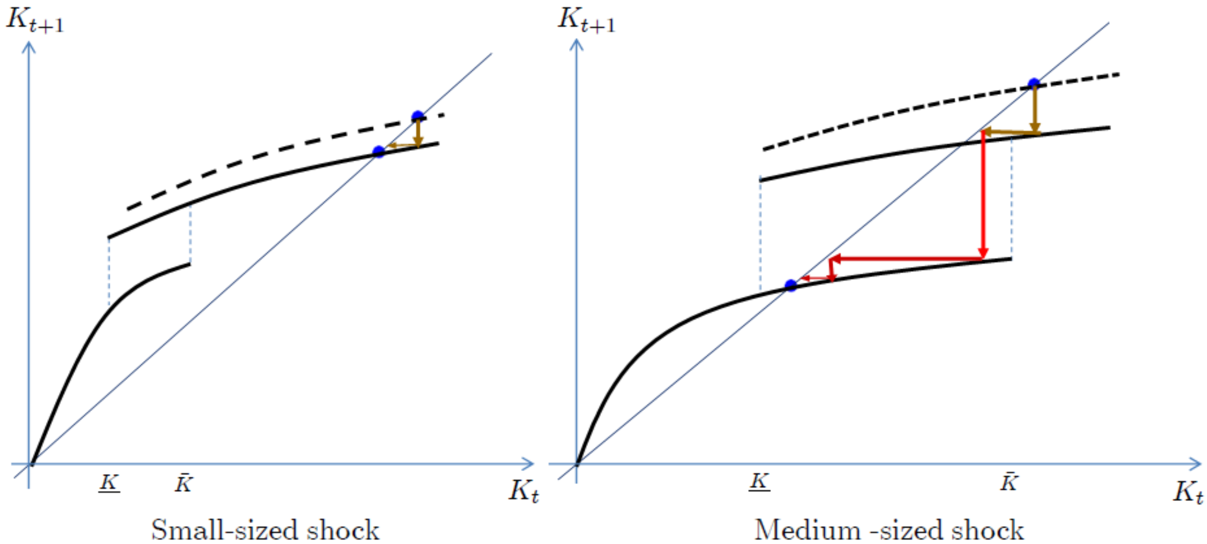

Figure 6: Transitional dynamics under a permanent negative shock on Z_t

6 Numerical Illustrations

Our analytic analysis in the previous sections has demonstrated that the information interplay between the real sector and the financial sector can have strong effects on the economy. Our model is too stylized to be calibrated with the data. We will therefore assign values to parameters in our model to conduct several numerical illustrations below.

Table 1 summarizes the parameter values chosen. We set the elasticity of substitution between intermediate goods, θ, to 6 as in David, Hopenhayn and Venkateswaran (2016). This implies that the gross markup is 15%. We set the degree of decreasing returns to scale of production η to 0.8, consistent with the recent estimates of Gopinath et al. (2016). We set the risk (CARA) coefficient γ to 8, considering that CARA can be calibrated as the relative risk aversion (RRA) divided by wealth and the wealth level of a typical investor in our model is GDP.[28] We borrow the unconditional residual uncertainty parameter from David et al. (2016), where $\tau_a = 4.9383$. We set $\bar{\tau}_s = 6.5746$, implying that a firm can reduce its residual uncertainty (standard deviation) on the productivity shock to 0.30 by paying the information acquisition cost. We set $\underline{\tau}_s = 3$, implying that the residual uncertainty in productivity equals 0.36 if a firm does not pay the information acquisition cost. We set $\tau_\varepsilon = 0.04$, implying that the contribution of demand shocks to firms' sales volatility is around a half of that of productivity shocks along the line of Foster, Haltiwanger and Syverson (2008). For simplicity, we set the precision of informed traders' signal τ_x to infinity, meaning that an informed trader can perfectly informed of the demand shock through his private signal. We set the common productivity $Z = 6.5$, the information acquisition cost for a firm $b = 0.03$, and the information

[28] We thank one referee for suggesting this way of calibration on γ.

acquisition cost for a financial trader $c = 0.105$ by considering that in equilibrium only a fraction of traders choose to acquire information. These parameter values lead to two self-fulfilling equilibria in our model.

Parameter	Description	Value
θ	Elasticity of substitution between intermediate goods	6
γ	Risk (CARA) coefficient	8
η	Degree of decreasing returns to scale of production	0.8
Z	Common productivity shock	6.5
τ_a	Precision of productivity shock prior	4.9383
τ_ε	Precision of demand shock prior	0.04
b	Information acquisition cost of the real side	0.03
c	Information acquisition cost of financial markets	0.105
$\underline{\tau}_s$	Low precision of signals of the real side	3
$\bar{\tau}_s$	High precision of signals of the real side	6.5746
τ_x	Precision of informed traders' signal	∞

Table 1: Parameter values

Table 2 summarizes key results of the two self-fulfilling equilibria in Section 4. First, both aggregate output and investment fall dramatically (by 58% and 58%, respectively) when the economy falls into the "bad" equilibrium. Second, information production from the financial sector and that from the real sector are both lower in the "bad" equilibrium than in the "good" equilibrium. The firms and financial traders face productivity shocks with a posterior standard deviation of $(\bar{\tau}_s + \tau_a)^{-\frac{1}{2}} = 0.2947$ in the "good" equilibrium and $(\underline{\tau}_s + \tau_a)^{-\frac{1}{2}} = 0.3549$ in the "bad" equilibrium. The financial price can reduce the posterior standard deviation of firm demand shocks to $\left(\tau_q^{**} + \tau_\varepsilon\right)^{-\frac{1}{2}} = 3.0789$ for the "good" equilibrium but only to $\left(\tau_q^{*} + \tau_\varepsilon\right)^{-\frac{1}{2}} = 3.7079$ for the "bad" equilibrium. These numbers imply a 20% increase in financial uncertainty and a 20% increase in real uncertainty from the "good" equilibrium to the "bad" equilibrium. Third, the resulting information production declines have important consequences for allocation efficiency. The endogenous TFP declines by about 16%. To understand the decline, we compute two alternative counterfactual endogenous TFP. We first compute $A(\bar{\tau}_s, \tau_q^*)$, the level of endogenous TFP when only the quality of the information provided by the financial market deteriorates while the quality of the information provided by firms stays at the level $\bar{\tau}_s$. We find that TFP would decline by about 13%. The other 3% decline in the endogenous TFP is due to the decline in firms' information production as indicated by $A(\underline{\tau}_s, \tau_q^{**})$, the level of endogenous TFP that the economy would obtain when only the quality of the information provided by firms deteriorates while the quality of the information provided by the financial market stays at the level τ_q^{**}.

	"Good" equilibrium	"Bad" equilibrium
τ_s	$\bar{\tau}_s = 6.5746$	$\underline{\tau}_s = 3$
τ_q	$\tau_q^{**} = 0.0655$	$\tau_q^{*} = 0.0327$
GDP (Y)	1.3562	0.5657
Aggregate investment (K)	0.9041	0.3771
Endogenous TFP (A)	$\bar{A} = 0.2262$	$\underline{A} = 0.1899$
TFP under changing τ_q only $(A(\bar{\tau}_s, \tau_q^{*}))$		0.1962
TFP under changing τ_s only $(A(\underline{\tau}_s, \tau_q^{**}))$		0.2189

Table 2: Numerical illustration for two self-fulfilling equilibria

As the nature of equilibrium depends crucially on the values of Z , b and c, our second exercise is hence to conduct complete comparative statics to understand their impact on equilibria in Section 4 in a quantitative sense. We fix all other parameter values (given in Table 1) but change one of Z, c and b each time and compute the equilibria accordingly. We report the results in Figure 7. Figure 7 has three columns, summarizing the comparative statics with respect to Z , b, and c. The first panel plots the equilibrium aggregate output, the second the posterior standard deviation of firms' productivity shocks (given in (31)), and the third the posterior standard deviation of firms' demand shocks inferred from financial prices (given in (30)).

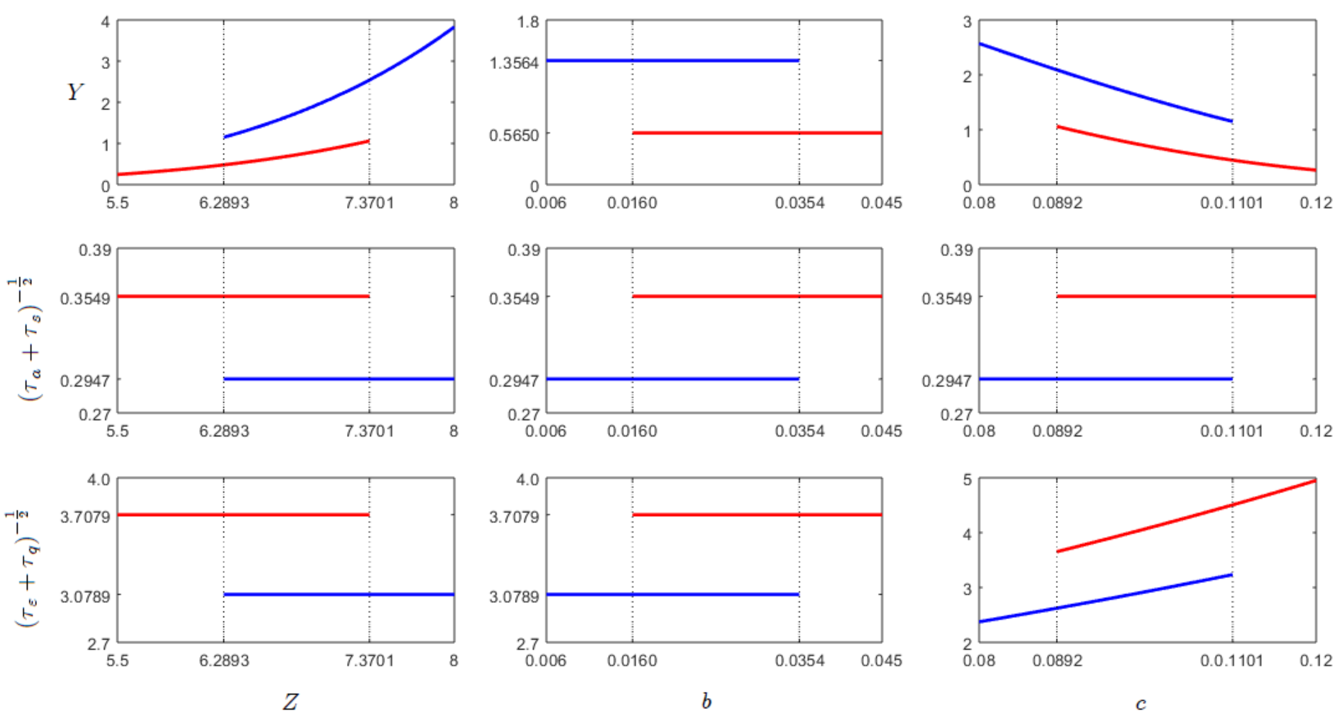

Figure 7: Comparative static analysis

The first column of Figure 7 shows that the equilibrium is unique if $Z > \bar{Z} = 7.3701$ or

$Z < \underline{Z} = 6.2893$. Suppose that the economy initially starts with aggregate common productivity $Z = 7.5205$. A small drop in Z by more than 2 percent would trigger a self-fulfilling crisis (also see Figure 2). In the second and third columns of Figure 7, the model generates two self-fulfilling equilibria when the information acquisition cost, c or b, is at the intermediate level. Due to the binary information choice of the firm, aggregate output is insensitive to the change in b under a given equilibrium. Similar to the effect of Z, if the initial level of b or c is close to their lower threshold for multiple equilibria, then a small shock (i.e., a small increase in b or c) can cause a sudden large decline in aggregate output.

Our next exercise is to show information contagion in our model. Again we assume that all parameters are initially as given in Table 1, except that we set $b = 0.0353$, slightly lower than the upper threshold of b to have multiple (two) equilibria. According to Figure 7, the economy initially has two equilibria. Now we assume that a small fraction $\kappa = 5\%$ of firms suffer a shock in the sense that their information acquisition cost b increases slightly to 0.0355, which means that a unique *"bad"* equilibrium takes hold in the economy of these islands by Figure 7. How about the other 95% of islands? The economy of the other 95% islands will inevitably fall into the bad equilibrium unless their acquisition cost b decreases below 0.0347.

Finally, we give numerical illustrations for our OLG model, particularly the results in Proposition 9. As production in the OLG model has inputs of both capital and labor, we set $\eta = 0.5$ as in Zhu (2012).[29] In order to quantitatively examine the effect of a small-sized shock versus a medium-sized shock on Z_t, we set $\underline{\tau}_s = 5.5$. The values of other parameters are as given in Table 1. Suppose that initially $Z_t = Z = 4.0961$. Based on Proposition 8, we find that the dynamic economy has a unique "good" steady-state equilibrium, in which $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$. Suppose that the economy initially stays in this steady state for the first three periods, $t = 1$ to 3. Assume that in period $t = 4$ suddenly there is a permanent shock on Z_t such that Z_t declines by 5% to $Z_t = Z' = 3.8913$. We examine what happens to the dynamic economy.

The left panels of Figure 8 show four phases of the dynamics. First, the shock has a direct impact. In period $t = 4$, the aggregate output Y_t immediately drops by 5%, which is exactly the size of the shock. The capital stock K_t declines by the same magnitude in the following period $t = 5$. Second, after the shock, the economy moves along a unique path, which is toward the new "good" steady state, until $t = 8$. The time between $t = 4$ to $t = 8$ corresponds a time of a mild recession, in which aggregate output Y_t and capital K_t gradually decline further by less than 5%. Third, in period $t = 9$, because the capital stock is already sufficiently low and enters the region of $K_t \leq \bar{K}(Z')$ as shown in Proposition 9, the second "bad" equilibrium path is opened, which gives room for equilibrium path switching. A sunspot or sentiment can suddenly switch

[29] Even if we set $\eta = 0.8$ as in earlier this section, the quantitative result here changes little.

the equilibrium path to the lower branch toward the "bad" steady state. Once that happens, the economy experiences a plunge with Y_t falling by roughly 9% in one shot at $t = 9$. At the same time, real uncertainty and financial uncertainty surge, as shown in the panels in the third and fourth rows of Figure 8. Fourth, after that, the economy moves along the path toward the "bad" steady state, and Y_t and K_t declines further by around 8%.

In contrast, if the shock is small, e.g., −2.5% as shown in the right panels of Figure 8, then only the first and second phases take place but not the third and fourth phases. After the shock, the economy moves along the unique path toward the new "good" steady state. The total drop in Y_t and K_t throughout the whole process is around 5%. There is no increase in real uncertainty or financial uncertainty in the entire process.

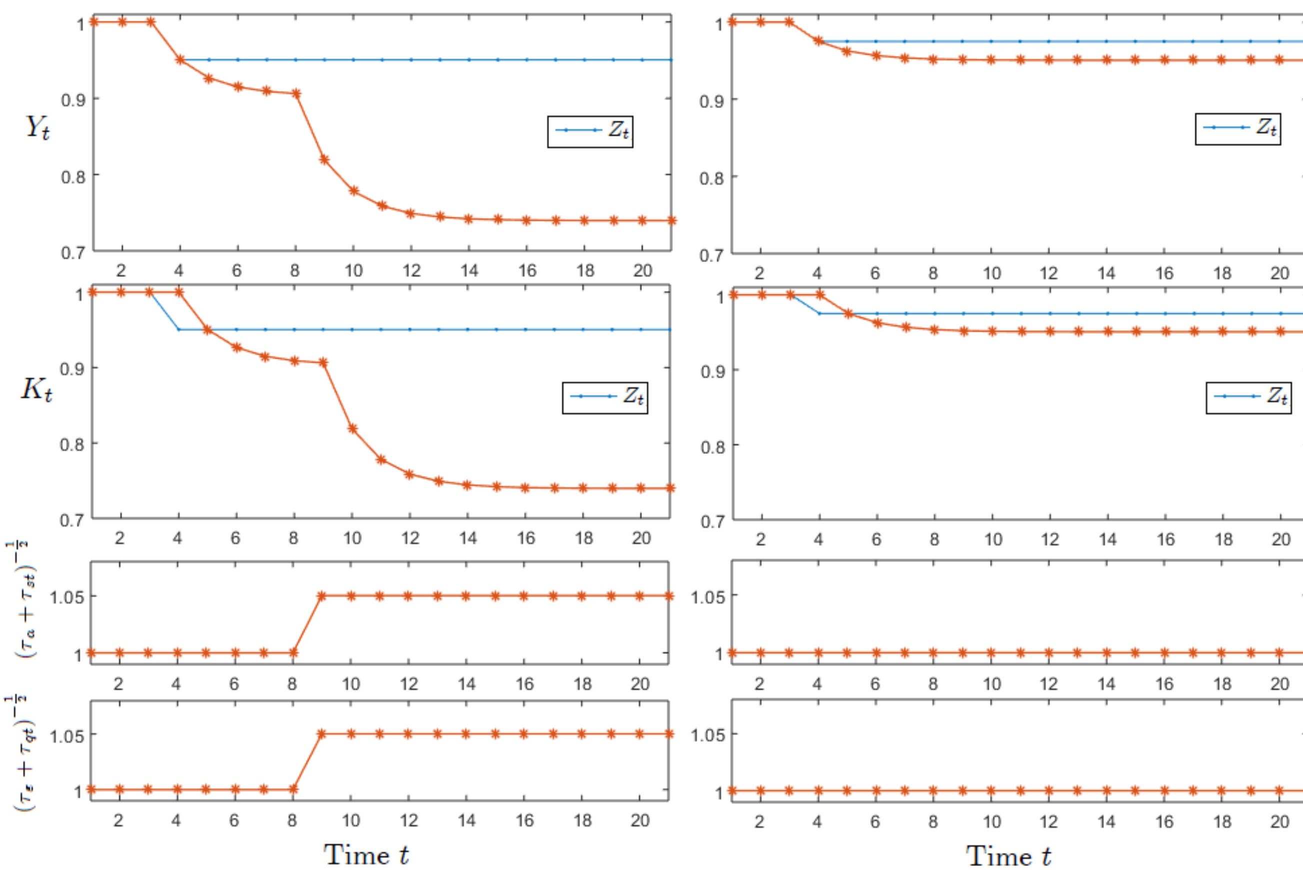

Figure 8: Transitional dynamics under a permanent shock on Z_t (Left: a medium-sized shock of size −5%; Right: a small-sized shock of size −2.5%) (Note: the initial value of each variable is normalized to 1)

7 Conclusion

We develop a model of informational interdependence between financial markets and the real economy. We endogenize financial and real uncertainty and show how they relate to aggregate economic activity. Information production in the real sector and that in the financial sector exhibit strategic complementarity. The key reason is that a financial price is a combination of firm disclosure and financial market price discovery. When a firm tries to maximize its monopoly profits in the real sector and speculators try to gain from arbitraging in financial markets, it is optimal for them to learn from each other. The mutual learning results in strategic complementarity in information production. In the general equilibrium, the amount of information available in the economy and the aggregate economic activity feed back into and reinforce each other. We derive a number of implications of our general-equilibrium macro model. In the extension to the dynamic OLG setting, our model shows self-fulfilling uncertainty traps and characterizes a two-stage economic crisis.

We have studied information production in a model with monopolistic competition with a constant markup. A vast IO literature has also studied information acquisition and disclosure under an oligopoly market structure (see, e.g., Vives (1984, 2008) and Yang (2018)). Examining how different market structures affect the two-way feedback between financial markets and the real economy in general equilibrium will be an interesting topic of future research, as it can shed some light on how informational frictions affect markups, a major driving force for business cycles (Rotemberg and Woodford (1999)).

Appendix

A Proofs

List of Main Notations of the Model:

A_j, a_j, τ_a	productivity shock of firm j; $\log A_j \equiv a_j \sim \mathcal{N}(-\frac{1}{2}\tau_a^{-1}, \tau_a^{-1})$
ϵ_j, ε_j, τ_ε	demand shock to intermediate good j; $\log \epsilon_j \equiv \varepsilon_j \sim \mathcal{N}(-\frac{1}{2}\tau_\varepsilon^{-1}, \tau_\varepsilon^{-1})$
η	degree of decreasing returns to scale of production
θ	elasticity of substitution between intermediate goods
Θ	$\Theta \equiv -\frac{1}{\eta\left(1-\frac{1}{\theta}\right)-1} \in (1, \theta)$
Z	aggregate productivity shock
Y, y	aggregate output of the final goods; $y = \log Y$
P	price of the final goods
Y_j	output of intermediate capital good j
K_j, k_j	investment capital input of firm j; $k_j = \log K_j$
P_j	price of intermediate good j
V_j, v_j	asset value or revenue of firm j; $v_j = \log V_j$
γ	risk aversion (CARA) coefficient of investors
n_j	demand of noise/liquidity traders in financial market j
s_j, e_j, τ_s, $\bar{\tau}_s$, $\underline{\tau}_s$	firm j's signal about a_j: $s_j = a_j + e_j$, where $e_j \sim N(0, \tau_s^{-1})$; $\tau_s \in \{\underline{\tau}_s, \bar{\tau}_s\}$
x_j^i, ϱ_j^i, τ_x	trader i's signal about ε_j: $x_j^i = \varepsilon_j + \varrho_j^i$, where $\varrho_j^i \sim N(0, \tau_x^{-1})$
λ	proportion of informed traders
q_j, $\tilde{q}_j$, ϱ_j^q, τ_q	trading price of v_j; $\tilde{q}_j = \frac{q_j - \beta_0 - \beta_1\beta_2 s_j}{\beta_1} = \varepsilon_j + \varrho_j^q$, where $\varrho_j^q \sim N(0, \tau_q^{-1})$
b, c	information acquisition cost for a firm and a trader, respectively
Π	ex ante expected profit of a firm
A	$A = A(\tau_s, \tau_q)$, endogenous aggregate TFP
K	$K = K(\tau_s, \tau_q; Z)$, aggregate investment in the economy
$\bar{A}$, $\underline{A}$	upper and lower endogenous TFP; $\bar{A} = A\left(\bar{\tau}_s, \tau_q^{**}\right)$ and $\underline{A} = A\left(\underline{\tau}_s, \tau_q^{*}\right)$
$\bar{Z}$, $\underline{Z}$	upper and lower thresholds of Z for multiple equilibria in general equilibrium
K_{jt}, K_t	capital input of firm j in the OLG model; $\int_0^1 K_{jt} dj = K_t$
N_{jt}, N_t	labor input of firm j in the OLG model; $\int_0^1 N_{jt} dj = N_t$
$\bar{K}$, $\underline{K}$	upper and lower thresholds of K_t for multiple equilibria for a given Z_t in OLG
Z^{**}, Z^{*}	upper and lower thresholds of Z_t for two steady-state equilibria in OLG
R_t	rental return of capital in period t in OLG
W_t	wage in period t in OLG
R_{ft}	bond return or intertemporal interest rate between $t-1$ and t in OLG

Proof of Lemma 1: For an informed trader,

$$\mathbb{E}[v_j|s_j,q_j,x_j^i]=\frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j,x_j^i\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[z+\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)+\frac{\tau_s}{\tau_a+\tau_s}s_j\right],$$

where $\mathbb{E}\left[\varepsilon_j|s_j,q_j,x_j^i\right]=\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_x+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\frac{\tau_x}{\tau_\varepsilon+\tau_x+\tau_q}x_j^i+\frac{\tau_q}{\tau_\varepsilon+\tau_x+\tau_q}\tilde{q}_j$ with $\tilde{q}_j(q_j,s_j)=\frac{q_j-\beta_0-\beta_1\beta_2 s_j}{\beta_1}$, and

$$Var[v_j|s_j,q_j,x_j^i]=\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j,q_j,x_j^i\right]+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s},$$

where $Var\left[\varepsilon_j|s_j,q_j,x_j^i\right]=\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}$.

For an uninformed trader,

$$\mathbb{E}[v_j|s_j,q_j]=\frac{1}{\theta}\mathbb{E}\left[\varepsilon_j|s_j,q_j\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[z+\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)+\frac{\tau_s}{\tau_a+\tau_s}s_j\right],$$

where $\mathbb{E}\left[\varepsilon_j|s_j,q_j\right]=\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j$, and

$$Var[v_j|s_j,q_j]=\left(\frac{1}{\theta}\right)^2 Var\left[\varepsilon_j|s_j,q_j\right]+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s},$$

where $Var\left[\varepsilon_j|s_j,q_j\right]=\frac{1}{\tau_\varepsilon+\tau_q}$.

Therefore, the market clearing condition, (7), implies

$$\begin{aligned}0 \;=\;& n_j+\lambda\frac{\frac{1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_x+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)\\+\frac{\tau_x}{\tau_\varepsilon+\tau_x+\tau_q}\varepsilon_j+\frac{\tau_q}{\tau_\varepsilon+\tau_x+\tau_q}\tilde{q}_j\end{array}\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\begin{array}{c}\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)\\+\frac{\tau_s}{\tau_a+\tau_s}s_j+z\end{array}\right]-q_j}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}\right]}\\&+(1-\lambda)\frac{\frac{1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)\\+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j\end{array}\right]+\eta\left(1-\frac{1}{\theta}\right)k\left(s_j,q_j\right)+\frac{1}{\theta}y+\left(1-\frac{1}{\theta}\right)\left[\begin{array}{c}\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)\\+\frac{\tau_s}{\tau_a+\tau_s}s_j+z\end{array}\right]-q_j}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}\right]}.\end{aligned} \tag{A.1}$$

It is straightforward to see that (A.1) can be transformed to

$$f\left(s_j,q_j,y,z\right)+\lambda\frac{\frac{1}{\theta}\frac{\tau_x}{\tau_\varepsilon+\tau_x+\tau_q}}{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}\right]}\varepsilon_j+n_j=0,$$

where $f\left(s_j,q_j,y,z\right)$ is a linear function of s_j, q_j, y and z. Hence,

$$\beta_3=\frac{\gamma\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}+\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}\right]}{\lambda\frac{1}{\theta}\frac{\tau_x}{\tau_\varepsilon+\tau_x+\tau_q}},$$

which, by substituting $\tau_q^{-1} = \beta_3^2 \tau_n^{-1}$, implies

$$\frac{\lambda}{\theta}\left[(\tau_a + \tau_s)\tau_x\right]\beta_3^3 - \gamma\left[\left(\frac{1}{\theta}\right)^2(\tau_a+\tau_s) + \left(1-\frac{1}{\theta}\right)^2(\tau_\varepsilon + \tau_x)\right]\beta_3^2 - \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n = 0. \quad \text{(A.2)}$$

(A.2) clearly has a unique positive solution with respect to β_3. In fact, if we write the LHS of (A.2) as function $\Lambda(\beta_3)$, it is easy to show that equation $\Lambda(\beta_3) + \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n = 0$ has a unique positive solution. Hence, equation $\Lambda(\beta_3) = 0$ has a unique positive solution, around which $\frac{\partial\Lambda}{\partial\beta_3} > 0$. We also prove that the unique positive solution of β_3 is decreasing in λ. In fact, $\frac{\partial\Lambda}{\partial\beta_3} > 0$ and $\frac{\partial\Lambda}{\partial\lambda} = \frac{1}{\theta}\left[(\tau_a+\tau_s)\tau_x\right]\beta_3^3 > 0$, so $\frac{d\beta_3}{d\lambda} = -\frac{\frac{\partial\Lambda}{\partial\lambda}}{\frac{\partial\Lambda}{\partial\beta_3}} < 0$. Also, the unique positive solution of β_3 is decreasing in τ_s. In fact,

$$\frac{\partial\Lambda}{\partial\tau_s} = \frac{\lambda}{\theta}\tau_x\beta_3^3 - \gamma\left(\frac{1}{\theta}\right)^2\beta_3^2 = \frac{\gamma\left(1-\frac{1}{\theta}\right)^2(\tau_\varepsilon+\tau_x)\beta_3^2 + \gamma\left(1-\frac{1}{\theta}\right)^2\tau_n}{\tau_a+\tau_s} > 0,$$

where the second equality is due to (A.2), so $\frac{d\beta_3}{d\tau_e} = -\frac{\frac{\partial\Lambda}{\partial\tau_s}}{\frac{\partial\Lambda}{\partial\beta_3}} < 0$ (or $\frac{d\tau_q}{d\tau_e} > 0$).

Proof of Lemma 2: The first-order condition of (4) implies

$$K_j = K(s_j,\tilde{q}_j) = \left[\eta\left(1-\frac{1}{\theta}\right)Y^{\frac{1}{\theta}}Z^{1-\frac{1}{\theta}}\right]^{\Theta}\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right)^{\Theta}, \quad \text{(A.3)}$$

where $\Theta = -\frac{1}{\eta\left(1-\frac{1}{\theta}\right)-1} \in (1,\theta)$. Combining (2), (1) and (A.3), we have

$$\begin{aligned}\pi(a_j,\varepsilon_j,s_j,\tilde{q}_j) &= P_j(\varepsilon_j,Y_j)Y_j(a_j,K_j) - K_j(s_j,\tilde{q}_j)\\ &= \left\{\begin{array}{l}\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1}\left(Y^{\frac{1}{\theta}}Z^{1-\frac{1}{\theta}}\right)^{\Theta}\left[\left(\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}\right)\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right)^{\Theta\eta\left(1-\frac{1}{\theta}\right)}\right]\\ -\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta}\left(Y^{\frac{1}{\theta}}Z^{1-\frac{1}{\theta}}\right)^{\Theta}\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right)^{\Theta}\end{array}\right\}.\end{aligned}$$

Hence,

$$\mathbb{E}\left[\pi(a_j,\varepsilon_j,s_j,\tilde{q}_j)|s_j,\tilde{q}_j\right] = \left[1-\eta\left(1-\frac{1}{\theta}\right)\right]\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1}\cdot\left(Y^{\frac{1}{\theta}}Z^{1-\frac{1}{\theta}}\right)^{\Theta}\left[\mathbb{E}\left(\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right)\right]^{\Theta}.$$

We have

$$\begin{aligned}\log\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}}A_j^{1-\frac{1}{\theta}}|s_j,\tilde{q}_j\right] &= \frac{1}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right) + \frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j\right] + \left(1-\frac{1}{\theta}\right)\left[\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right) + \frac{\tau_s}{\tau_a+\tau_s}s_j\right]\\ &\quad +\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q} + \frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}.\end{aligned}$$

So

$$\begin{aligned}\phi_0' &= \Theta \log\left[\eta\left(1-\frac{1}{\theta}\right)\right] + \frac{\Theta}{\theta}y + \left(1-\frac{1}{\theta}\right)\Theta\log Z + \frac{\Theta}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)\right] \\ &+\Theta(1-\frac{1}{\theta})\left[\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)\right] + \frac{1}{2}\Theta\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q} + \frac{1}{2}\Theta\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}.\end{aligned}$$

In addition,

$$\begin{aligned}&\mathbb{E}\exp\left\{\frac{\Theta}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\tilde{q}_j\right]+\Theta(1-\frac{1}{\theta})\left[\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)+\frac{\tau_s}{\tau_a+\tau_s}s_j\right]\right\} \\ =\ &\mathbb{E}\exp\left\{\begin{array}{l}\frac{\Theta}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_q}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\frac{\tau_q}{\tau_\varepsilon+\tau_q}\left(\varepsilon_j+\varrho_j^q\right)\right] \\ +\Theta(1-\frac{1}{\theta})\left[\frac{\tau_a}{\tau_a+\tau_s}\left(-\frac{1}{2}\tau_a^{-1}\right)+\frac{\tau_s}{\tau_a+\tau_s}\left(a_j+e_j\right)\right]\end{array}\right\} \\ =\ &\exp\left\{\begin{array}{l}\left[\frac{\Theta}{\theta}\left(-\frac{1}{2}\tau_\varepsilon^{-1}\right)+\frac{1}{2}\left(\frac{\Theta}{\theta}\right)^2\left(\frac{\tau_q}{\tau_\varepsilon+\tau_q}\right)^2\left(\frac{1}{\tau_\varepsilon}+\frac{1}{\tau_q}\right)\right] \\ +\left[\Theta(1-\frac{1}{\theta})\left(-\frac{1}{2}\tau_a^{-1}\right)+\frac{1}{2}\left[\Theta(1-\frac{1}{\theta})\right]^2\left(\frac{\tau_s}{\tau_a+\tau_s}\right)^2\left(\frac{1}{\tau_a}+\frac{1}{\tau_s}\right)\right]\end{array}\right\} \\ =\ &\exp\left\{\begin{array}{l}\left[\frac{\Theta}{\theta}\left(-\frac{1}{2}\frac{1}{\tau_\varepsilon}\right)+\frac{1}{2}\left(\frac{\Theta}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon}-\frac{1}{\tau_\varepsilon+\tau_q}\right)\right] \\ +\left[\Theta(1-\frac{1}{\theta})\left(-\frac{1}{2}\frac{1}{\tau_a}\right)+\frac{1}{2}\left(\Theta(1-\frac{1}{\theta})\right)^2\left(\frac{1}{\tau_a}-\frac{1}{\tau_a+\tau_s}\right)\right]\end{array}\right\}.\end{aligned}$$

Thus,

$$\mathbb{E}\left(\left[\mathbb{E}\left(A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right)\right]^\Theta\right) = \exp\left\{\begin{array}{l}\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2-\Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a}+\frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2-\Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon} \\ -\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\end{array}\right\}.$$

It is easy to show that

$$\frac{\partial\Pi}{\partial\tau_s}>0 \qquad \text{and} \qquad \frac{\partial\Pi}{\partial\tau_q}>0,$$

by noting that

$$sgn\left(\frac{\partial\Pi}{\partial\tau_s}\right) = sgn\left\{\begin{array}{l}\exp\left\{-\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\right\} \\ \cdot\Theta\left(\Theta-1\right)\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{(\tau_a+\tau_s)^2}\end{array}\right\}.$$

Moreover,

$$\frac{\partial^2\Pi}{\partial\tau_s\partial\tau_q}>0$$

by noting that

$$sgn\left(\frac{\partial^2\Pi}{\partial\tau_s\partial\tau_q}\right) = sgn\left\{\begin{array}{l}\exp\left\{-\Theta\left(\Theta-1\right)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]\right\} \\ \cdot\left[\Theta\left(\Theta-1\right)\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{(\tau_\varepsilon+\tau_q)^2}\right]\left[\Theta\left(\Theta-1\right)\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{(\tau_a+\tau_s)^2}\right]\end{array}\right\}. \quad \text{(A.4)}$$

Finally, it is easy to show that $\frac{\partial^2\Pi}{\partial\tau_s\partial Y}>0$ and $\frac{\partial^2\Pi}{\partial\tau_s\partial Z}>0$.

Proof of Proposition 1: The proof is quite similar to that in Grossman and Stiglitz (1980). By the definition $EV(W^i) \equiv \mathbb{E}\left[U(W^i)|s_j, q_j\right]$, we have $\frac{EV(W^{Ii})}{EV(W^{Ui})} = e^{\gamma c}\sqrt{\frac{Var[v_j|s_j,q_j,x_j^i]}{Var[v_j|s_j,q_j]}}$. Thus,

$$\frac{EV(W^{Ii})}{EV(W^{Ui})} = 1 \Longleftrightarrow \sqrt{\frac{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right] + Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j)}{Var\left[\left(1-\frac{1}{\theta}\right)a_j|s_j\right] + Var(\frac{1}{\theta}\varepsilon_j|s_j,q_j,x_j^i)}} = e^{\gamma c}$$

$$\Longleftrightarrow \sqrt{\frac{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}}} = e^{\gamma c}. \tag{A.5}$$

Let $F\left(\tau_q;\tau_s\right) = \frac{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}+\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}}$, which implies

$$\frac{\partial F}{\partial \tau_s} = \frac{-\left(1-\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_a+\tau_s}\right)^2\left[\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q} - \left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}\right]}{\left[\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]^2} < 0$$

and

$$\frac{\partial F}{\partial \tau_q} = \frac{\left\{\begin{array}{c}\left(\frac{1}{\theta}\right)^2\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s}\left[\left(\frac{1}{\tau_\varepsilon+\tau_q}\right)^2 - \left(\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}\right)^2\right] \\ +\left(\frac{1}{\theta}\right)^4\left(\frac{1}{\tau_\varepsilon+\tau_q}\right)\left(\frac{1}{\tau_\varepsilon+\tau_x+\tau_q}\right)\left(\frac{1}{\tau_\varepsilon+\tau_q} - \frac{1}{\tau_\varepsilon+\tau_x+\tau_q}\right)\end{array}\right\}}{\left[\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right]^2} > 0.$$

Because $F\left(\tau_q;\tau_s\right) = e^{-2\gamma c}$, by the implicit function theorem, we have $\frac{\partial \tau_q}{\partial \tau_s} = -\frac{\partial F/\partial \tau_s}{\partial F/\partial \tau_q} > 0$. Also, by $\frac{\partial F}{\partial \tau_q} > 0$, we have $\frac{\partial \tau_q}{\partial c} < 0$.

Proof of Proposition 2: Because $\frac{\partial \Pi}{\partial \tau_s} > 0$, we have $\Pi\left(\tau_s = \bar{\tau}_s;\tau_q,Y,Z\right) - \Pi\left(\tau_s = \underline{\tau}_s;\tau_q,Y,Z\right) > 0$ for a given τ_q, Y and Z. Because $\frac{\partial^2\Pi}{\partial\tau_s\partial\tau_q} > 0$, there exists a unique $\hat{\tau}_q$ such that

$$\Pi\left(\tau_s = \bar{\tau}_s;\hat{\tau}_q,Y,Z\right) - \Pi\left(\tau_s = \underline{\tau}_s;\hat{\tau}_q,Y,Z\right) = b. \tag{A.6}$$

Denote the LHS of (A.6) by function $\Gamma(\hat{\tau}_q,Y,Z)$ for a given $\bar{\tau}_s$ and $\underline{\tau}_s$. Because $\frac{\partial^2\Pi}{\partial\tau_s\partial\tau_q} > 0$, $\frac{\partial^2\Pi}{\partial\tau_s\partial Y} > 0$, and $\frac{\partial^2\Pi}{\partial\tau_s\partial Z} > 0$, we have that $\frac{\partial\Gamma}{\partial\hat{\tau}_q} > 0$, $\frac{\partial\Gamma}{\partial Y} > 0$, and $\frac{\partial\Gamma}{\partial Z} > 0$. Therefore, by the implicit function theorem, we have that $\frac{\partial\hat{\tau}_q(Y,Z,b)}{\partial Y} = -\frac{\frac{\partial\Gamma}{\partial Y}}{\frac{\partial\Gamma}{\partial\hat{\tau}_q}} < 0$, $\frac{\partial\hat{\tau}_q(Y,Z,b)}{\partial Z} = -\frac{\frac{\partial\Gamma}{\partial Z}}{\frac{\partial\Gamma}{\partial\hat{\tau}_q}} < 0$, and $\frac{\partial\hat{\tau}_q(Y,Z,b)}{\partial b} > 0$.

Proof of Proposition 3: By (17), the condition that guarantees $(\tau_s,\tau_q) = \left(\underline{\tau}_s,\tau_q^*\right)$ is one equilibrium is $\tau_q^* \leq \hat{\tau}_q(Y,Z,b)$. Similarly, the condition that guarantees $(\tau_s,\tau_q) = \left(\bar{\tau}_s,\tau_q^{**}\right)$ is one equilibrium is $\tau_q^{**} \geq \hat{\tau}_q(Y,Z,b)$. Considering that $\hat{\tau}_q(Y,Z,b)$ can be of the three cases: $\hat{\tau}_q > \tau_q^{**}$, $\hat{\tau}_q \in \left[\tau_q^*,\tau_q^{**}\right]$, and $\hat{\tau}_q < \tau_q^*$, it is straightforward to obtain Proposition 3.

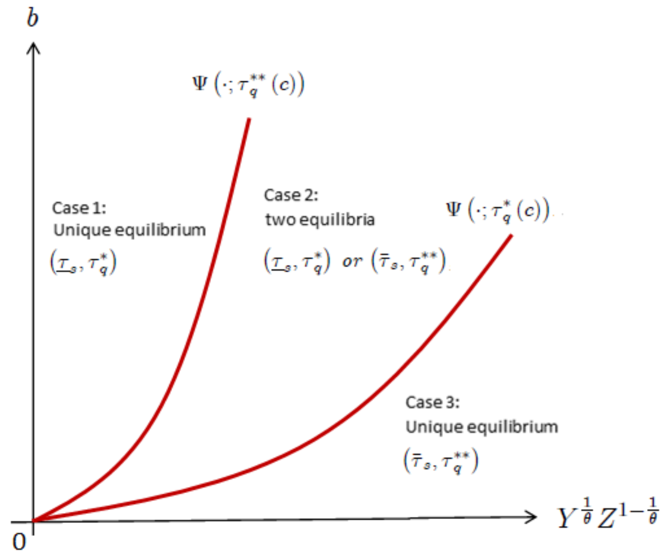

Figure A.1: Parameter regions of (Y, Z, b, c) for different cases of equilibrium

Figure A.1 graphically illustrates the full comparative statics — the parameter regions of (Y, Z, b, c) for a particular type of equilibrium to prevail, where the threshold curve $\Psi(\cdot; \tau_q)$, given by (18) with $\hat{\tau}_q$ being replaced by τ_q, represents b as a function of $Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}$ parameterized by τ_q. For a given c, a combination $\left(b, Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)$ determines which type of equilibrium will prevail. When c increases, the two threshold curves rotate clockwise and hence the parameter region of $\left(b, Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)$ in which case 3 of equilibrium prevails shrinks while that in which case 1 prevails expands.

Proof of Proposition 4: Substituting (A.3) and (1) into (19) yields

$$
\begin{aligned}
Y &= \left[\int \epsilon_j^{\frac{1}{\theta}} \left(Z A_j \left\{ \left[\eta \left(1 - \frac{1}{\theta}\right) Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}} \right]^{\Theta} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right\}^{\eta} \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \\
&= Z \left[\eta \left(1 - \frac{1}{\theta}\right) Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}} \right]^{\Theta\eta} \left[\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta\eta\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \\
&= Z^{\Theta} \left[\eta \left(1 - \frac{1}{\theta}\right) Y^{\frac{1}{\theta}} \right]^{\Theta\eta} \left[\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta-1} dj \right]^{\frac{\theta}{\theta-1}}, \qquad \text{(A.7)}
\end{aligned}
$$

where the last equality follows based on $\Theta = -\frac{1}{\eta\left(1-\frac{1}{\theta}\right)-1}$.

Exploiting the law of iterated expectations, we have

$$
\int \epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta-1} dj = \mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j \right] \right)^{\Theta} \right].
$$

Hence, (A.7) becomes

$$Y = Z^{\Theta \frac{\theta}{\theta - \Theta\eta}} \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta\eta\frac{\theta}{\theta-\Theta\eta}} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}}. \tag{A.8}$$

Similarly, the aggregate investment in the economy is given by

$$\begin{aligned} K &= \int K_j dj = \left[\eta\left(1-\frac{1}{\theta}\right) Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right]^{\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\} \\ &= Z^{\left(1-\frac{1}{\theta}\right)\Theta + \Theta\frac{\theta}{\theta-\Theta\eta}\frac{\Theta}{\theta}} \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta\eta\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+1}. \end{aligned}$$

where the second equality is obtained by substituting the expression of Y into (A.8). Because $\Theta\frac{\theta}{\theta-\Theta\eta} - \left[\left(1-\frac{1}{\theta}\right)\Theta + \Theta\frac{\theta}{\theta-\Theta\eta}\frac{\Theta}{\theta}\right]\eta = 1$, and $\Theta\eta\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta} + \Theta = \frac{\theta}{\theta-\Theta\eta}\Theta$ (that is, $\Theta\eta\frac{\theta}{\theta-\Theta\eta} = \left(\Theta\eta\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+\Theta\right)\eta$) and $\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta} - \left(\frac{\theta}{\theta-1}\frac{\theta}{\theta-\Theta\eta}\cdot\frac{\Theta}{\theta}+1\right)\eta = \frac{\theta}{\theta-1}-\eta$, it follows

$$Y = ZAK^{\eta},$$

where $A = A(\tau_s, \tau_q)$ is given by

$$\begin{aligned} A(\tau_s,\tau_q) &= \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\}^{\frac{\theta}{\theta-1}-\eta} \\ &= \exp\left\{\left(\frac{\theta}{\theta-1}-\eta\right)\left(\begin{array}{l} \Theta\left(1-\frac{1}{\theta}\right)\left(-\frac{1}{2}\frac{1}{\tau_a}\right) + \frac{1}{2}\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2\frac{1}{\tau_a} + \Theta\frac{1}{\theta}\left(-\frac{1}{2}\frac{1}{\tau_\varepsilon}\right) + \frac{1}{2}\left(\Theta\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon} \\ -\Theta(\Theta-1)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\frac{1}{\tau_a+\tau_s} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_q}\right] \end{array}\right)\right\}. \end{aligned}$$

Because $0<\eta<1$ and thus $\frac{\theta}{\theta-1}-\eta>0$, $A(\tau_s,\tau_q)$ is increasing in τ_s and τ_q.

Also, we can express K in terms of A. In fact,

$$\begin{aligned} K &= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} Z^{\left(1-\frac{1}{\theta}\right)\Theta} \left\{\mathbb{E}\left[\left(\mathbb{E}\left[\epsilon_j^{\frac{1}{\theta}} A_j^{1-\frac{1}{\theta}} | s_j, \tilde{q}_j\right]\right)^{\Theta}\right]\right\} \cdot Y^{\frac{\Theta}{\theta}} \\ &= \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} Z^{\left(1-\frac{1}{\theta}\right)\Theta} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} \left(ZAK^{\eta}\right)^{\frac{\Theta}{\theta}} = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} Z^{\left(1-\frac{1}{\theta}\right)\Theta+\frac{\Theta}{\theta}} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}} K^{\eta\frac{\Theta}{\theta}}. \end{aligned}$$

This implies

$$\begin{aligned} K^{1-\eta\frac{\Theta}{\theta}} = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta} Z^{\Theta} A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}} &\Longleftrightarrow K = \left[\eta\left(1-\frac{1}{\theta}\right)Z\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}} A^{\frac{\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta}}{1-\eta\frac{\Theta}{\theta}}} \\ &\Longleftrightarrow K = \left[\eta\left(1-\frac{1}{\theta}\right)Z\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}} A^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}}, \end{aligned}$$

where the last equality is obtained by $\Theta = -\frac{1}{\eta\frac{\theta-1}{\theta}-1} = -\frac{\frac{\theta}{\theta-1}}{\eta-\frac{\theta}{\theta-1}}$ and thus $\frac{1}{\frac{\theta}{\theta-1}-\eta}+\frac{\Theta}{\theta} = \Theta\frac{\theta-1}{\theta}+\frac{\Theta}{\theta} = \Theta$. Because $0 < \eta < 1$ and $\Theta \in (1, \theta)$, K is increasing in A and thus is increasing in τ_s and τ_q.

Finally, we have

$$Y = ZAK^{\eta} = ZA\left[\eta\left(1-\frac{1}{\theta}\right)Z\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} A^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} = \left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta} \cdot (ZA)^{\frac{\theta\Theta}{\theta-\eta\Theta}},$$

by noting $\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}\eta + 1 = \frac{\theta\Theta}{\theta-\eta\Theta}$.

Proof of Proposition 5: Suppose all other islands have the equilibrium $(\tau_s, \tau_q) = \left(\bar{\tau}_s, \tau_q^{**}\right)$. Then, by Propositions 2 and 4, for a given b and c, the condition of Z that guarantees $\left(\tau_s^j, \tau_q^j\right) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ is also one equilibrium on island j is $Z \geq \underline{Z}$, where $\underline{Z}$ satisfies

$$\Pi\left(\tau_s = \bar{\tau}_s; \tau_q^{**}, Y\left(\bar{A}, \underline{Z}\right), \underline{Z}\right) - \Pi\left(\tau_s = \underline{\tau}_s; \tau_q^{**}, Y\left(\bar{A}, \underline{Z}\right), \underline{Z}\right) = b.$$

Similarly, suppose all other islands have the equilibrium $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^{*}\right)$. Then, the condition of Z that guarantees $\left(\tau_s^j, \tau_q^j\right) = \left(\underline{\tau}_s, \tau_q^{*}\right)$ is also one equilibrium on island j is $Z \leq \bar{Z}$, where $\bar{Z}$ satisfies

$$\Pi\left(\tau_s = \bar{\tau}_s; \tau_q^{*}, Y\left(\underline{A}, \bar{Z}\right), \bar{Z}\right) - \Pi\left(\tau_s = \underline{\tau}_s; \tau_q^{*}, Y\left(\underline{A}, \bar{Z}\right), \bar{Z}\right) = b.$$

Considering that for a given b and c, Z can be one of the three cases: $Z < \underline{Z}$, $\underline{Z} \leq Z \leq \bar{Z}$, and $Z > \bar{Z}$, it is straightforward to obtain Proposition 5.

Proof of Lemma 3: By (12) and (24), the ex ante expected profit for an intermediate-goods firm choosing $\bar{\tau}_s$ for the given $\tau_q = \tau_q^*$, relative to the ex ante expected profit for an "average" intermediate-goods firm, is scaled by $\left(\frac{A(\bar{\tau}_s,\tau_q^*)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}$. Similarly, the ex ante expected profit for an intermediate-goods firm choosing $\underline{\tau}_s$ for the given $\tau_q = \tau_q^*$, relative to the ex ante expected profit for an "average" intermediate-goods firm, is scaled by $\left(\frac{A(\underline{\tau}_s,\tau_q^*)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}$. It is easy to show that the ex ante expected profit for an "average" intermediate-goods firm in the economy is $\frac{1}{\theta}Y$. Then, we can define a threshold b^*, given by (33). This implies that when $b_j \geq b^*$, $(\tau_s, \tau_q) = \left(\underline{\tau}_s, \tau_q^*\right)$ is one equilibrium for island j. By a similar argument, when $b_j \leq b^{**}$, $(\tau_s, \tau_q) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ is one equilibrium for island j. Given b^* and b^{**}, we can divide all islands into three types: $b_j < b$, $b^* \leq b_j \leq b^{**}$, and $b_j > b^{**}$. Therefore, Lemma 3 is obtained.

Proof of Proposition 6: Suppose an equilibrium is already found in which ω satisfies $G(b^*) < \omega < G(b^{**})$. Then, a slight increase in ω to ω^+ must be another equilibrium because $G(b^{*+}) < \omega^+ < G(b^{**+})$. For the corner case, if an equilibrium satisfies $G(b^*) = \omega < G(b^{**})$, it is easy to show that ω^+ or ω^- must be another equilibrium because either $G(b^{*+}) < \omega^+ < G(b^{**+})$ or $G(b^{*-}) < \omega^- < G(b^{**-})$ must hold. A similar argument applies to the corner case of $G(b^*) < \omega = G(b^{**})$.

Proof in Section 4.4.2: Given a firm's information $\{s_j, \tilde{q}_j\}$, we can find its investment decision. Then, the aggregate output can be calculated as

$$Y = ZK^\eta \frac{\left[\frac{1}{J}\sum_{j=1}^J \left(A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^{\Theta-1}\right)\right]^{\frac{\theta}{\theta-1}}}{\left[\frac{1}{J}\sum_{j=1}^J\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^{\Theta}\right]^\eta},$$

where $K = \sum_{j=1}^J K_j$. We focus on symmetric equilibrium in which the information precision on all islands is the same. When J is large enough, the law of large numbers implies

$$Y = ZAK^\eta$$

where A is again given by (24).

Now we consider firms' information acquisition problem. Again, by the law of large numbers, we find the ex ante expected profit of firm j perceived at T_0:

$$\frac{\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^\Theta}{\sum_{j=1}^J\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^\Theta}\frac{1}{\theta}Y = \frac{\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^\Theta}{\frac{1}{J}\sum_{j=1}^J\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^\Theta}\frac{1}{\theta}\frac{1}{J}Y$$

Notice that an individual firm's profit shrinks as J increases. Hence, we change the information acquisition cost to $\frac{1}{J}b$. By the law of large numbers, $\frac{1}{J}\sum_{j=1}^J\left\{\mathbb{E}\left[A_j^{1-\frac{1}{\theta}}\epsilon_j^{\frac{1}{\theta}}|s_j,\tilde{q}_j\right]\right\}^\Theta = A^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}$. Hence, for a given Y, Z and τ_q, a firm acquires information if and only if

$$\frac{1}{\theta}\frac{1}{J}Y\cdot\left(\frac{A(\bar{\tau}_s,\tau_q)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}} - \frac{1}{J}b \geq \frac{1}{\theta}\frac{1}{J}Y\cdot\left(\frac{A(\underline{\tau}_s,\tau_q)}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}.$$

This is the same condition as (18).

Proof of Lemma 4: Plugging $\frac{R_tK_{jt}}{W_tN_{jt}} = \frac{\eta}{1-\eta}$ into (39), we have $K_{jt} \propto \left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^\theta$ and $N_{jt} \propto \left\{\mathbb{E}\left[A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right]\right\}^\theta$. Hence, (40) is obtained. Substituting (36) and (40) into (37), we have the aggregate production function in Lemma 4, where

$$A_t = A(\tau_{st}, \tau_{qt}) = \left[\mathbb{E}\left(\left[\mathbb{E}\left(A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right)\right]^{\theta}\right)\right]^{\frac{1}{\theta-1}}$$
$$= \exp\left(\frac{1}{2}(\theta-2)\frac{1}{\tau_a} - \frac{1}{2}\frac{(\theta-1)^2}{\theta}\frac{1}{\tau_a+\tau_{st}} - \frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_{qt}}\right).$$

Proof of Proposition 7: Similar to (12), an entrepreneur's ex ante expected profit is proportional to $\mathbb{E}\left(\left[\mathbb{E}\left(A_{jt}^{1-\frac{1}{\theta}}\epsilon_{jt}^{\frac{1}{\theta}}|\mathcal{I}_{jt}\right)\right]^{\theta}\right)$ or $(A(\tau_{st}, \tau_{qt}))^{\theta-1}$. Suppose that all other islands have equilibrium $(\tau_{st}, \tau_{qt}) = \left(\underline{\tau}_s, \tau_q^*\right)$ and the financial market equilibrium on island j is $\tau_{qt} = \tau_q^*$. Then, the expected profit for the entrepreneur on island j to choose $\tau_{st} = \underline{\tau}_s$ is $\frac{1}{\theta}Y\left(Z_t, \underline{A}, K_t\right)$ and the expected profit to choose $\tau_{st} = \bar{\tau}_s$ is $\frac{1}{\theta}\xi Y\left(Z_t, \underline{A}, \bar{K}\right)$. Thus, the condition of K_t that guarantees the entrepreneur on island j chooses $\tau_{st} = \underline{\tau}_s$ is $K_t < \bar{K}$, where $\bar{K}$ satisfies

$$\frac{1}{\theta}\xi Y\left(Z_t, \underline{A}, \bar{K}\right) - b = \frac{1}{\theta}Y\left(Z_t, \underline{A}, \bar{K}\right).$$

That is, when $K_t < \bar{K}$, $(\tau_{st}, \tau_{qt}) = \left(\underline{\tau}_s, \tau_q^*\right)$ is one equilibrium for all islands. Similarly, suppose that all other islands have equilibrium $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ and the financial market equilibrium on island j is $\tau_{qt} = \tau_q^{**}$. Then, the condition of K_t that guarantees the entrepreneur in island j chooses $\tau_{st} = \bar{\tau}_s$ is $K_t > \underline{K}$, where $\underline{K}$ satisfies

$$\frac{1}{\theta}Y\left(Z_t, \bar{A}, \underline{K}\right) - b = \frac{1}{\theta}\frac{1}{\xi}Y\left(Z_t, \bar{A}, \underline{K}\right).$$

That is, when $K_t > \underline{K}$, $(\tau_{st}, \tau_{qt}) = \left(\bar{\tau}_s, \tau_q^{**}\right)$ is one equilibrium for all islands. It is easy to show that both $\bar{K}$ and $\underline{K}$ are decreasing functions of Z_t. Considering that K_t can be one of the three cases: $K_t > \bar{K}$, $\underline{K} \leq K_t \leq \bar{K}$, and $K_t < \underline{K}$, it is straightforward to obtain Proposition 7.

Proof of Proposition 8: By (41), we can find the steady-state K, which is given by

$$K = \begin{cases} \left[(1-\frac{1}{\theta})(1-\eta)Z\bar{A}\right]^{\frac{1}{1-\eta}} & \text{if } K \geq \underline{K}(Z) \\ \left[(1-\frac{1}{\theta})(1-\eta)Z\underline{A}\right]^{\frac{1}{1-\eta}} & \text{if } K \leq \bar{K}(Z) \end{cases}.$$

We can also find $\bar{K} = \left(\frac{b\theta}{\xi-1}\right)^{\frac{1}{\eta}}(Z\underline{A})^{-\frac{1}{\eta}}$ and $\underline{K} = \left(\frac{b\theta}{1-\frac{1}{\xi}}\right)^{\frac{1}{\eta}}\left(Z\bar{A}\right)^{-\frac{1}{\eta}}$. Clearly, $\bar{K} > \underline{K}$ when $\xi\underline{A} < \bar{A}$. It follows that

$$\left[(1-\frac{1}{\theta})(1-\eta)Z\underline{A}\right]^{\frac{1}{1-\eta}} \leq \left(\frac{b\theta}{\xi-1}\right)^{\frac{1}{\eta}}(Z\underline{A})^{-\frac{1}{\eta}} \Rightarrow Z \leq Z^{**}$$

and

$$\left[(1-\frac{1}{\theta})(1-\eta)Z\bar{A}\ \right]^{\frac{1}{1-\eta}} \geq \left(\frac{b\theta}{1-\frac{1}{\xi}}\right)^{\frac{1}{\eta}} (Z\bar{A})^{-\frac{1}{\eta}} \Rightarrow Z \geq Z^{*},$$

where $Z^{*} = \left[\frac{\left(\frac{b\theta}{\xi-1}\right)^{1-\eta}\xi^{1-\eta}}{\left[(1-\frac{1}{\theta})(1-\eta)\ \right]^{\eta}}\right]\frac{1}{\bar{A}}$ and $Z^{**} = \left[\frac{\left(\frac{b\theta}{\xi-1}\right)^{1-\eta}}{\left[(1-\frac{1}{\theta})(1-\eta)\ \right]^{\eta}}\right]\frac{1}{\underline{A}}$. Clearly, $Z^{*} < Z^{**}$ when $\xi^{1-\eta}\underline{A} < \bar{A}$. We have three cases of Z. If $Z > Z^{**}$, there is a unique steady-state equilibrium, in which $K^{**} = \left[(1-\frac{1}{\theta})(1-\eta)Z\bar{A}\ \right]^{\frac{1}{1-\eta}}$. When $Z < Z^{*}$, there is a unique steady-state equilibrium, in which $K^{*} = \left[(1-\frac{1}{\theta})(1-\eta)Z\underline{A}\right]^{\frac{1}{1-\eta}}$. If $Z^{*} \leq Z \leq Z^{**}$, there are two steady-state equilibria, in which both K above are possible.

Proof of Proposition 9: Because initially $Z_t = Z > Z^{**}$, the economy has a unique "good" steady-state equilibrium and $K^{**}(Z) > \bar{K}(Z)$, where $K^{**}(\cdot)$ is defined in the proof of Proposition 8. Immediately after the shock the capital is still in $K^{**}(Z)$ but Z_t becomes $Z_t = Z^{'} < Z$. If $Z^{'} > Z^{**}$, there is still a unique "good" steady-state equilibrium and the transitional path is unique. Now we examine the case where $Z^{*} < Z^{'} < Z^{**}$. On the one hand, if $Z^{'} \in (Z^{*}, Z^{**})$ is not too low, the capital immediately after the shock satisfies the condition that $K_t = K^{**}(Z) > \bar{K}\left(Z^{'}\right)$, by noting that the condition that $K^{**}(Z) > \bar{K}\left(Z^{'}\right)$ is certainly true if $Z^{'}$ is sufficiently close to Z^{**}. This means that immediately after the shock, the transitional path is unique. On the other hand, if $Z^{'} \in (Z^{*}, Z^{**})$ is not too high, the condition that $K^{**}\left(Z^{'}\right) < \bar{K}\left(Z^{'}\right)$ is true by noting that this condition is certainly true if $Z^{'}$ is sufficiently close to Z^{*}. This means that the dynamics of K_t enter the region of $K_t \leq \bar{K}\left(Z^{'}\right)$ earlier than reaching the new "good" steady state $K^{**}\left(Z^{'}\right)$ and thus the transitional path has two after the capital declines to $K_t \leq \bar{K}\left(Z^{'}\right)$.

B Alternative Specification of the Financial Asset's Payoff

In the main model, for tractability we have assumed that the payoff of the financial contract takes the log form. Now we show that the model result of the two-way complementarity in information acquisition is robust to some alternative model specifications.

Specifically, we consider that the demand curve of the real product is linear and the firm manager (entrepreneur) with a CARA preference maximizes the exponential value of firm profit. Veldkamp and Wolfers (2007) use the setting of the exponential utility of firm managers, and the assumption of a linear demand curve of the real product is also extensively studied in the literature (see, e.g., Vives (2008)).

Consider a firm whose product has a spot price p given by

$$p = 1 + a + \varepsilon - \theta Q$$

where Q is the supply of the product of the firm, a can be interpreted as the quality of the product and ε may reflect consumers' taste, and $\theta \geq 0$ captures the status of the aggregate economy and/or the market power of the firm. This demand function can be derived from consumers' choice problem, where consumers are assumed to have utility function $u(Q) = (1 + a + \varepsilon)Q - \frac{\theta}{2}Q^2$ and they maximize $u(Q) - pQ$.

The firm manager (entrepreneur) maximizes his CARA utility:

$$\max_{Q} \mathbb{E}\left[-e^{-\zeta(pQ-Q-\chi)}|\mathcal{I}_F\right], \tag{B.1}$$

where $pQ - Q$ is the firm's profit, ζ is the risk-aversion (CARA) coefficient, $\chi \in \{0, b\}$ is the information acquisition cost of the firm, and $\mathcal{I}_F = \{s, q\}$ is the information set, with s being the entrepreneur's signal about a and q being the financial price signal about ε as in the main model. If the firm pays the cost b, the precision of signal s is $\tau_s = \bar{\tau}_s$; otherwise $\tau_s = \underline{\tau}_s$. (B.1) is equivalent to maximizing

$$\mathbb{E}\left[a + \varepsilon|\mathcal{I}_F\right] Q - \theta Q^2 - \frac{\zeta}{2} Q^2 Var(a + \varepsilon|\mathcal{I}_F),$$

which implies

$$Q = \frac{\mathbb{E}\left[a + \varepsilon|\mathcal{I}_F\right]}{2\theta + \zeta Var(a + \varepsilon|\mathcal{I}_F)}.$$

In the spirit of Grossman and Stiglitz (1980), the firm's indifference condition of acquiring information or not is

$$e^{\zeta b} = \sqrt{\frac{2\theta + \zeta\left(\frac{1}{\tau_a + \underline{\tau}_s} + \frac{1}{\tau_\varepsilon + \hat{\tau}_q}\right)}{2\theta + \zeta\left(\frac{1}{\tau_a + \bar{\tau}_s} + \frac{1}{\tau_\varepsilon + \hat{\tau}_q}\right)}},$$

where $\hat{\tau}_q$ is the threshold of the financial price informativeness as in the main model. That is, if and only if $\tau_q \geq \hat{\tau}_q$, the firm acquires information (i.e., $\tau_s = \bar{\tau}_s$). In other words, τ_s is an increasing function of τ_q.

A financial market exists, where speculators trade a financial asset (future) contingent on p. That is, the long position of one unit of the financial asset (derivative) incurs an initial outlay of q and entitles to having the risky payoff p later. Note that different from the main model, here the payoff of the financial contract no longer takes the log form. Because p is a linear function of a and ε, the trading game is the standard REE model. We can find that the other way of information production complementarity — τ_q is an increasing function of τ_s — holds true.

In short, the result of the two-way complementarity in information production between the firm and the financial market holds for this alternative model setup.

C Discussions in Section 3.3

C. 1 Continuous Information Acquisition of the Firm

For tractability and closed-form solutions, we have assumed the binary choice of information acquisition of the firm. Now we assume that the firm's information acquisition is a continuous choice and show that our model results do not change qualitatively. Specifically, to obtain signal s_j with the precision level τ_s, the information acquisition cost function is $h(\tau_s, b)$, where $\frac{\partial h}{\partial \tau_s} > 0$, $\frac{\partial^2 h}{\partial \tau_s^2} > 0$, $\frac{\partial^2 h}{\partial \tau_s \partial b} > 0$, and $b > 0$ is a parameter. Then firm j's optimal information acquisition decision at T_0 is given by

$$\underset{\tau_s}{Max} \quad \Pi\left(\tau_s, \tau_q; Y, Z\right) - h(\tau_s, b), \tag{C.1}$$

the FOC of which is

$$\frac{\partial \Pi\left(\tau_s, \tau_q; Y, Z\right)}{\partial \tau_s} = \frac{\partial h(\tau_s, b)}{\partial \tau_s}. \tag{C.2}$$

Equation (C.2) gives a *continuous* response function of the firm: $\tau_s\left(\tau_q; Y, Z, b\right)$. Cost function $h(\tau_s, b)$ is assumed in such a way that the second-order condition for the maximization of (C.1) is satisfied, that is, $\frac{\partial^2 \Pi(\tau_s, \tau_q; Y, Z)}{\partial \tau_s^2} < \frac{\partial^2 h(\tau_s, b)}{\partial \tau_s^2}$. It is easy to show properties regarding $\tau_s\left(\tau_q; Y, Z, b\right)$; namely, $\frac{\partial \tau_s}{\partial \tau_q} > 0$, $\frac{\partial \tau_s}{\partial Y} > 0$, $\frac{\partial \tau_s}{\partial Z} > 0$, and $\frac{\partial \tau_s}{\partial b} < 0$.

We characterize the condition under which the curve $\tau_s\left(\tau_q; Y, Z, b\right)$ in (τ_q, τ_s)-space exhibits a faster increase in the middle and a slower increase on the two sides. We need to make sure that $\frac{\partial^2 \Pi}{\partial \tau_s \partial \tau_q}$ is increasing in τ_q for some region of τ_q (i.e., $\frac{\partial \frac{\partial^2 \Pi}{\partial \tau_s \partial \tau_q}}{\partial \tau_q} > 0$). By (A.4), we can find that $\frac{\partial \frac{\partial^2 \Pi}{\partial \tau_s \partial \tau_q}}{\partial \tau_q} > 0$ iff $4\left(\tau_\varepsilon + \tau_q\right) < \Theta\left(\Theta - 1\right)\left(\frac{1}{\theta}\right)^2$.

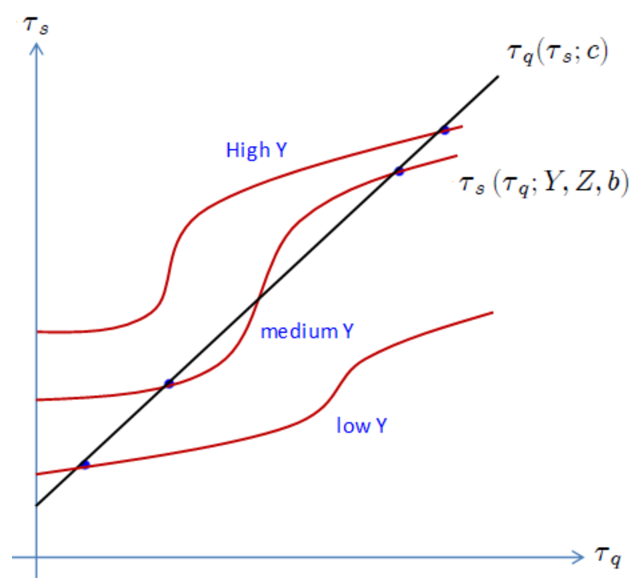

Figure C.1: Equilibrium under continuous choice of information acquisition of firms

The two reaction functions (curves) $\tau_q(\tau_s; c)$ and $\tau_s(\tau_q; Y, Z, b)$ give a unique equilibrium or two (stable) equilibria for different levels of Y or Z as in Proposition 3. Similar to Figure 2 (for the binary choice of information acquisition), Figure C.1 illustrates different cases of equilibrium under the continuous choice of information acquisition.[30]

C. 2 Signals about the Demand Shock of the Firm

For simplicity, in Section 2 we have assumed that the firm has no private information about the demand shock ε_j. Now we relax this assumption. Specifically, we assume that the firm also exogenously possesses a noisy signal on the demand shock:

$$x_j = \varepsilon_j + \rho_j, \tag{C.3}$$

where $\rho_j \sim N(0, \frac{1}{\tau_\rho})$. It discloses this piece of information to the financial market.

In this case, the financial market equilibrium gives (15), that is,

$$\sqrt{\frac{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}}{\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_s} + \left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_x+\tau_\rho+\tau_q}}} = e^{\gamma c}. \tag{C.4}$$

Based on (C.4), the key result of $\tau_q = \tau_q(\tau_s; c)$ with $\frac{\partial \tau_q}{\partial \tau_s} > 0$ in Proposition 1 still holds. Intuitively, the additional piece of disclosed information on ϵ_j is public information for all financial market speculators. This is equivalent to these speculators having more precise information about the prior distribution of ϵ_j, and nothing else is affected.

Firm j's information set becomes $\{s_j, x_j, q_j\}$. The ex ante expected profit of firm j perceived at T_0 in (12) becomes

$$\Pi(\tau_s, \tau_q, \tau_\rho; Y, Z) = \pi_0 \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)^\Theta \exp\left\{-\Theta(\Theta-1)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_s} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right]\right\}, \tag{C.5}$$

where π_0 is a constant term.[31] It is easy to show the properties of $\frac{\partial \Pi}{\partial \tau_s} > 0$, $\frac{\partial \Pi}{\partial \tau_q} > 0$ and

$$\begin{aligned} \frac{\partial^2 \Pi}{\partial \tau_s \partial \tau_q} &= \pi_0 \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}}\right)^\Theta \left\{ \begin{array}{c} \exp\left\{-\Theta(\Theta-1)\left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_s} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right]\right\} \\ \left[\Theta(\Theta-1)\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_a+\tau_s}\right)^2\right]\left[\Theta(\Theta-1)\frac{1}{2}\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2\right] \end{array} \right\} \\ &> 0. \end{aligned} \tag{C.6}$$

[30] For a medium level of Y, the two curves have three intersections. The middle intersection corresponds to an unstable equilibrium while the other two correspond to stable equilibria.

[31] Concretely, $\pi_0 = \left[1-\eta\left(1-\frac{1}{\theta}\right)\right]\left[\eta\left(1-\frac{1}{\theta}\right)\right]^{\Theta-1} \exp\left\{\frac{1}{2}\left\{\left[\Theta\left(1-\frac{1}{\theta}\right)\right]^2 - \Theta\left(1-\frac{1}{\theta}\right)\right\}\frac{1}{\tau_a} + \frac{1}{2}\left[\left(\Theta\frac{1}{\theta}\right)^2 - \Theta\frac{1}{\theta}\right]\frac{1}{\tau_\varepsilon}\right\}$.

These are the key properties that are necessary for Proposition 2 to still hold. So Proposition 3 also holds.[32]

Furthermore, we can let the firm *endogenously* choose to acquire signal x_j. Specifically, it is assumed that by paying an information acquisition cost $\hat{b} > 0$ (in terms of the final good), the firm receives a signal given in (C.3). It is easy to show that when $\hat{b}$ is sufficiently low, the firm endogenously chooses to acquire signal x_j.

Finally, it is interesting to note that

$$\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} = \pi_0 \left(Y^{\frac{1}{\theta}} Z^{1-\frac{1}{\theta}} \right)^{\Theta} \left\{ \begin{array}{l} \exp\left\{ -\Theta(\Theta-1) \left[\frac{1}{2}\left(1-\frac{1}{\theta}\right)^2 \frac{1}{\tau_a+\tau_s} + \frac{1}{2}\left(\frac{1}{\theta}\right)^2 \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q} \right] \right\} \\ \left[\Theta(\Theta-1)\frac{1}{2}\left(\frac{1}{\theta}\right)^2 \left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2 \right] \cdot \Theta(\Theta-1)\left(\frac{1}{\theta}\right)^2 \left(\frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q}\right)^2 \\ \left(\frac{1}{2} - \frac{1}{\tau_\varepsilon+\tau_\rho+\tau_q} \right) \end{array} \right\}.$$

When $\tau_\varepsilon + \tau_\rho + \tau_q > 2$, it follows that $\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} > 0$, which means that the firm's incentive to acquire information about the demand shock increases with financial efficiency (a strategic complementarity). When $\tau_\varepsilon + \tau_\rho + \tau_q < 2$, it follows that $\frac{\partial^2 \Pi}{\partial \tau_\rho \partial \tau_q} < 0$ (a strategic substitutability). The above results demonstrate that even if the firm and the financial market acquire information on the same shock, their information acquisitions can exhibit a strategic complementarity (for the parameter region of $\tau_\varepsilon + \tau_\rho + \tau_q > 2$). The intuition behind the strategic complementarity is similar to the explanation for (14). When τ_q is higher, the firm's profit is scaled up. So the firm has a higher stake to improve τ_ρ when τ_q is higher. Of course, there is another standard substitution effect. When the complementarity effect dominates the substitution effect (the condition $\tau_\varepsilon + \tau_\rho + \tau_q > 2$), the overall effect is a strategic complementarity.

D Extension in Section 4

D. 1 Extension in Section 4.4.1: Heterogeneity in Productively

Let us assume that

$$Y_j = Z\left(\exp \Delta_j\right) A_j K_j^{\eta},$$

where $\Delta_j \sim \mathcal{N}(-\frac{1}{2}\tau_\Delta^{-1}, \tau_\Delta^{-1})$, meaning the observable part of firm-specific productivity, and Z and Δ_j are public information. As in Section 4.4.1, it is easy to find

$$Y = Z\left(\exp \bar{\Delta}\right) A K^{\eta}$$

[32] According to Goldstein and Yang (2017a,b), the firm may optimally choose not to disclose its information on the demand shock. In this case, the results of our main model will certainly hold true.

where $\exp\bar{\Delta} = \left(\mathbb{E}\left[\exp\left[\Delta_j\left(\left(1-\frac{1}{\theta}\right)\Theta\right)\right]\right]\right)^{\frac{\theta}{\theta-1}-\eta}$, and A is given by (24) , and $K = \begin{bmatrix} \eta\left(1-\frac{1}{\theta}\right)Z \\ \cdot\left(\exp\bar{\Delta}\right)A \end{bmatrix}^{\frac{\Theta}{1-\eta\frac{\Theta}{\theta}}}$.

As in Section 4.4.1, denote by ω the fraction of islands that will have more precise information (i.e., $(\tau_s,\tau_q) = \left(\bar{\tau}_s,\tau_q^{**}\right)$). For a given ω, there are two thresholds Δ^* and Δ^{**}, which are respectively determined by

$$\frac{1}{\theta}\left(\frac{\exp\Delta^*}{\exp\bar{\Delta}}\frac{A(\bar{\tau}_s,\tau_q^{**})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}Y - b = \frac{1}{\theta}\left(\frac{\exp\Delta^*}{\exp\bar{\Delta}}\frac{A(\underline{\tau}_s,\tau_q^{**})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}Y$$

and

$$\frac{1}{\theta}\left(\frac{\exp\Delta^{**}}{\exp\bar{\Delta}}\frac{A(\bar{\tau}_s,\tau_q^{*})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}Y - b = \frac{1}{\theta}\left(\frac{\exp\Delta^{**}}{\exp\bar{\Delta}}\frac{A(\underline{\tau}_s,\tau_q^{*})}{A}\right)^{\frac{1}{\frac{\theta}{\theta-1}-\eta}}Y.$$

In equilibrium, for island with $\Delta_j > \Delta^{**}$, the equilibrium outcome is unique with $(\tau_s,\tau_q) = \left(\bar{\tau}_s,\tau_q^{**}\right)$; for islands with $\Delta_j < \Delta^*$, the outcome is also unique with $(\tau_s,\tau_q) = \left(\underline{\tau}_s,\tau_q^{*}\right)$; and for islands with $\Delta^* \le \Delta_j \le \Delta^{**}$, there are multiple equilibria with $(\tau_s,\tau_q) = \left(\underline{\tau}_s,\tau_q^{*}\right)$ or $\left(\bar{\tau}_s,\tau_q^{**}\right)$. An equilibrium means a fixed-point problem between (Δ^*,Δ^{**}) and ω. Therefore, similar to Proposition 6, we have the following result: When there is heterogeneity in Δ_j across firms, there are always multiple equilibria in which ω is not determined and only needs to satisfy

$$\Phi\left(\tau_\Delta^{1/2}\left[\Delta^* + \frac{1}{2}\tau_\Delta^{-1}\right]\right) \le \omega \le \Phi\left(\tau_\Delta^{1/2}\left[\Delta^{**} + \frac{1}{2}\tau_\Delta^{-1}\right]\right),$$

where $\Phi(\cdot)$ stands for the c.d.f. of the standard normal.

D. 2 Continuous Information Acquisition of Firms in General Equilibrium

We continue to use the setup of continuous information acquisition of firms in Appendix C.1 and show that the equilibrium of the full model is similar to that in Proposition 5. We still consider the symmetric equilibrium, in which all intermediate goods firms have the same level of information precision. We use Figure D.1 for illustration.

As shown in Appendix C.1, the response function of a firm, $\tau_s(\tau_q;Y,Z,b)$, has the properties of $\frac{\partial\tau_s}{\partial\tau_q} > 0$, $\frac{\partial\tau_s}{\partial Y} > 0$ and $\frac{\partial\tau_s}{\partial Z} > 0$. Then, when Y or Z increases, the curve of $\tau_s(\tau_q;Y,Z,b)$ in Figure D.1 shifts upward. We can find a unique tangent point H. Denote the coordinates of point H by $(\tau_q,\tau_s) = \left(\tau_q^H,\tau_s^H\right)$. Then, we can find the unique corresponding $\underline{Z}$ that satisfies $\tau_s^H = \tau_s\left(\tau_q^H;Y\left(A^H,\underline{Z}\right),\underline{Z},b\right)$, where $A^H = A\left(\tau_s^H,\tau_q^H\right)$ and functions $A(\tau_s,\tau_q)$ and $Y(A,Z)$ are respectively defined in (24) and (26). Similarly, we can find a unique tangent point L. Denote the coordinates of point L by $(\tau_q,\tau_s) = \left(\tau_q^L,\tau_s^L\right)$. Then, we can find the unique corresponding $\bar{Z}$ that satisfies $\tau_s^L = \tau_s\left(\tau_q^L;Y\left(A^L,\bar{Z}\right),\bar{Z},b\right)$, where $A^L = A\left(\tau_s^L,\tau_q^L\right)$. Therefore, when $Z < \underline{Z}$ or $Z > \bar{Z}$, there is a unique equilibrium of (τ_s,τ_q,Y); when $\underline{Z} \le Z \le \bar{Z}$, there are multiple equilibria.

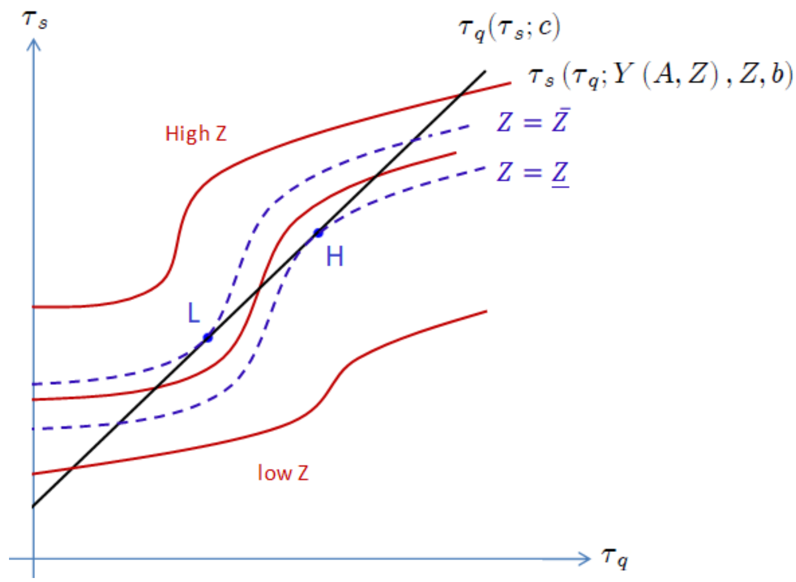

Figure D.1: General equilibrium under continuous information acquisition of firms

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