1 A flexible–price model

\[ \text{Max } U = \int_{0}^{\infty} e^{-rt} u(c, m^{np}) \, dt \quad (1) \]

**Assumption 1 (1)** \( u(\cdot, \cdot) \) is strictly increasing and strictly concave, and \( c \) and \( m^{np} \) are normal goods.

**Assumption 2** \( y(m^p) \) is positive, strictly increasing, strictly concave, \( \lim_{m^p \to 0} y'(m^p) = \infty \), and \( \lim_{m^p \to \infty} y'(m^p) = 0 \).

**Assumption 2’** \( y(m^p) \) is a positive constant.
\[ a \equiv (M^{np} + M^p + B)/P \]  

\[ \dot{a} = (R - \pi)a - R(m^{np} + m^p) + y(m^p) - c - \tau. \]  

\[ \lim_{t \to \infty} e^{-\int_0^t [R(s) - \pi(s)] ds} a(t) \geq 0 \]  

Taking as given \( a(0) \) and the time paths of \( \tau, R, \) and \( \pi \). The optimality conditions associated with the household’s problem are

\[ u_c(c, m^{np}) = \lambda \]  

\[ m^p [y'(m^p) - R] = 0 \]  

\[ \frac{u_m(c, m^{np})}{u_c(c, m^{np})} = R \]  

\[ \lambda (r + \pi - R) = \dot{\lambda} \]  

\[ \lim_{t \to \infty} e^{-\int_0^t [R(s) - \pi(s)] ds} a(t) = 0 \]
Assumption together with equation (6) and \( R > 0 \) implies

\[ m^p = m^p(R), \]

with \( m^{p'} \equiv \frac{dm^p}{dR} < 0 \). Alternatively, equation (6), \( R > 0 \), and assumption 2' imply that \( m^p = m^{p'} = 0 \). Using equation (7) and assumption, \( m^{np} \) can be expressed as a function of consumption and the nominal interest rate,

\[ m^{np} = m^{np}(c, R), \]

that is increasing in \( c \) and decreasing in \( R \).
1.0 The government

\[ R = \rho(\pi), \]  
\[ \rho(\cdot) \text{ is continuous, non-decreasing, and strictly positive and there exists at least one } \pi^* > -r \text{ such that } \rho(\pi^*) = r + \pi^*. \]  
Following Leeper (1991), we will refer to the monetary policy as active if \( \rho'(\pi^*) > 1 \) and as passive if \( \rho'(\pi^*) < 1 \).

The sequential budget constraint of the government is given by
\[ \dot{B} = RB - \dot{M}^{np} - \dot{M}^p - P\tau, \]
which can be written as
\[ \dot{a} = (R - \pi)a - R(m^{np} + m^p) - \tau. \]  
The nominal value of initial government liabilities, \( A(0) \), is predetermined:
\[ a(0) = \frac{A(0)}{P(0)}. \]
We classify fiscal policies into two categories: Ricardian fiscal policies and non-Ricardian. Ricardian fiscal policies are those that ensure that the present discounted value of total government liabilities converges to zero—that is, equation (9) is satisfied—under all possible, equilibrium or off-equilibrium, paths of endogenous variables such as the price level, the money supply, inflation, or the nominal interest rate.

Throughout the paper we will restrict attention to one particular price-neutral fiscal policy that takes the form
\[ \tau = Ra - R(m^{np} + m^p) \] (15)

We will also analyze a particular non-Ricardian policy consisting of an exogenous path for lump-sum taxes
\[ \tau = \bar{\tau}. \] (16)
Equilibrium

In equilibrium the goods market clear
\[ c = y(m^p). \] (17)

Using equations (10)–(12) and (17) to replace \( m^p, m^{np}, R, \) and \( c \) in equation (5),
\[ u_c(c, m^{np}) = \lambda = \lambda(\pi) \] (18)

\[ \lambda'(\pi) = \rho'[u_{cc}y'm^{pl} + u_{cm}(m^{np}_c y'm^{pl} + m^{np}_R)] \] (19)

where \( m^{np}_c \) and \( m^{np}_R \) denote the partial derivatives of \( m^{np} \) with respect to \( c \) and \( R \), respectively. (10)–(12), and (17), equations (8), (9), (13), and (15) can be rewritten as
\[ \dot{\pi} = \frac{\lambda(\pi)[r + \pi - \rho(\pi)]}{\lambda'(\pi)} \] (20)

\[ \dot{a} = [\rho(\pi) - \pi]a - \rho(\pi)[m^{np}y(m^p(\rho(\pi)), \rho(\pi)) + m^p(\rho(\pi))] - \tau \] (21)

\[ \lim_{t \to \infty} e^{-\int_0^t [\rho(\pi) - \pi(s)] ds} a(t) = 0 \] (22)

\[ \tau = \rho(\pi) [a - [m^{np}y(m^p(\rho(\pi))), \rho(\pi)) + m^p(\rho(\pi))]] \] (23)
or

\[ \tau = \tilde{\tau}. \]  \hspace{1cm} (24)
Definition 1 (Perfect-foresight equilibrium in the flexible-price economy) In the flexible-price economy, a perfect-foresight equilibrium is a set of sequences $\{\pi, a, \tau\}$ and an initial price level $P(0) > 0$ satisfying (14), (20)–(22) and either if fiscal policy is non-Ricardian $\tau = \bar{\tau}$ or $\tau = R a - R(m^{np} + m^p)$ if fiscal policy is Ricardian, given $A(0) > 0$.

Given a sequence for $\pi$, equations (10)–(12), (17), and (18) uniquely determine the equilibrium sequences of $\{c, m^{np}, m^p, \lambda, R\}$ independently of whether the equilibrium price level is unique.

Definition 2 (Real and Nominal Indeterminacy) The equilibrium displays real indeterminacy if there exists an infinite number of equilibrium sequences $\{\pi\}$. The equilibrium exhibits nominal indeterminacy if for any equilibrium sequence $\{\pi\}$, there exists an infinite number of initial price levels $P(0) > 0$ consistent with a perfect-foresight equilibrium.
In what follows, we restrict the analysis to equilibria in which the inflation rate remains bounded in a neighborhood around a steady–state value, \( \pi^* \), which is defined as a constant value of \( \pi \) that solves (20), that is, a solution to \( r + \pi = \rho(\pi) \). By assumption \( \pi^* \) exists and is greater than \(-r\). Note that \( \pi^* \) may not be unique. In particular, if there exists a steady state \( \pi^* \) with \( \rho'(\pi^*) > 1 \), then since \( \rho(\cdot) \) is assumed to be continuous and strictly positive there must also exist a steady state with \( \rho' < 1 \).

It follows from equation (20) that if the sign of \( \lambda'(\pi^*) \) is the opposite of the sign of \( 1 - \rho'(\pi^*) \), any initial inflation rate near the steady state \( \pi^* \) will give rise to an inflation trajectory that converges to \( \pi^* \). If, on the other hand, \( \lambda'(\pi^*) \) and \( 1 - \rho'(\pi^*) \) are of the same sign, the only sequence of inflation rates consistent with equation (20) that remains in the neighborhood of \( \pi^* \) is the steady state \( \pi^* \).
Note that the case $\rho'(\pi) = 0$ for all $\pi$ corresponds to a pure interest rate peg. In this case it follows from equation (19) that $\lambda$ is constant, and therefore the inflation rate is also constant, which implies that under a pure interest rate peg the economy exhibits real determinacy.
Under a Ricardian fiscal policy the set of equilibrium conditions includes equation (23). Given a sequence \( \{\pi\} \) and an initial price level \( P(0) > 0 \), equations (21) and (23) can be used to construct a pair of sequences \( \{a, \tau\} \). Because the fiscal policy is Ricardian, the transversality condition (22) is always satisfied. If instead the fiscal authority follows the non–Ricardian fiscal policy given in (16), combining (14), (21), and (22) yields

\[
\frac{A(0)}{P(0)} = \int_0^\infty e^{-\int_0^t \rho(\pi)-\pi} ds \\
\{\rho(\pi) \left[ m^{np}(y(m^p(\rho(\pi))), \rho(\pi)) + m^p(\rho(\pi)) \right] + \tilde{\tau}\} ds
\]

which given \( A(0) > 0 \) and a sequence for \( \pi \) uniquely determines the initial price level \( P(0) \).
The above analysis demonstrates that for the class of monetary–fiscal regimes considered nominal determinacy depends only on fiscal policy and not on monetary policy — a result that has been emphasized in the recent literature on the fiscal determination of the price level and that we summarize in the following proposition:

**Proposition 1** If fiscal policy is Ricardian, the equilibrium exhibits nominal indeterminacy. Under the non-Ricardian fiscal policy given by (16), the equilibrium displays nominal determinacy.
**Proposition 2** Suppose preferences are separable in consumption and money ($u_{cm} = 0$) and money is productive (assumption 2 holds), then if monetary policy is active ($\rho'(\pi^*) > 1$), the equilibrium displays real indeterminacy, whereas if monetary policy is passive ($\rho'(\pi^*) < 1$), there exists a unique perfect–foresight equilibrium in which the real allocation converges to the steady state.

**Proposition 3** Suppose that money is not productive (assumption 2' holds) and consumption and money are substitutes ($u_{cm} < 0$). Then, if monetary policy is active ($\rho'(\pi^*) > 1$), the real allocation is indeterminate, and if monetary policy is passive ($\rho'(\pi^*) < 1$) there exists a unique perfect–foresight equilibrium in which the real allocation converges to the steady state.
1. Real Indeterminacy in the Flexible-Price Model

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>Non-productive money ($y' = 0$)</th>
<th>Productive money ($y' &gt; 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive ($\rho'(\pi^*) &lt; 1$)</td>
<td>$u_{cm} &gt; 0$</td>
<td>$u_{cm} &lt; 0$</td>
</tr>
<tr>
<td>Active ($\rho'(\pi^*) &gt; 1$)</td>
<td>$D$</td>
<td>$I$</td>
</tr>
</tbody>
</table>

Note: The notation is: D, determinate; I, indeterminate; A, ambiguous. (Under A the real allocation may be determinate or indeterminate depending on specific parameter values.)

**Proposition 4** Suppose that money is not productive (assumption 2' holds) and consumption and money are complements ($u_{cm} > 0$). Then, if monetary policy is passive ($\rho'(\pi^*) < 1$), the real allocation is indeterminate, and if monetary policy is active ($\rho'(\pi^*) > 1$) there exists a unique perfect-foresight equilibrium in which the real allocation converges to the steady state.
Combining the case of non-productive money (assumption 2') with preferences that are separable in consumption and real balances ($u_{cm} = 0$) results in the continuous time version of the economy analyzed in Leeper (1991). In this case equation (5) implies that $\lambda$ is constant. It then follows that $\pi$, $R$ and $m^{np}$ are also constant, and the only equilibrium real allocation is the steady state. This result differs from that obtained by Leeper who finds that under passive monetary policy the inflation rate is indeterminate. The difference stems from the fact that in Leeper’s discrete–time model the nominal interest rate in period $t$ is assumed to be a function of the change in the price level between periods $t - 1$ and $t$, whereas in the continuous time model analyzed here, the inflation rate is the right hand side derivative of the price level, so its discrete–time counterpart is better approximated by the change in the price level between periods $t$ and $t + 1$. 

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In fact, it is straightforward to show that if in Leeper’s discrete–time model, with $u_{cm} = 0$ and an endowment economy, the feedback rule is assumed to be forward looking — that is, $R_t = \rho(P_{t+1}/P_t)$ — the equilibrium displays real determinacy.

$$\rho_{cm}(\hat{m}_t - \hat{m}_{t+1}) = \hat{R}_t - \hat{\pi}_{t+1}$$

$$\hat{m}_t = -\epsilon_m \hat{R}_t$$

$$\hat{R}_t = \epsilon_p \hat{\pi}_{t+1}$$

$$\hat{\pi}_{t+2} = \left[1 + \frac{\epsilon_p - 1}{\epsilon_p \rho_{cm} \epsilon_m R}\right] \hat{\pi}_{t+1}.$$  

In an economy with $u_{cm} = 0$ and productive money,

$$\rho_{cc}(\hat{y}_{t+1} - \hat{y}_t) = \epsilon_p \hat{\pi}_{t+1} - \hat{\pi}_{t+1}$$

$$\hat{y}_t = -\epsilon_y R \epsilon_p \hat{\pi}_{t+1}$$

$$\hat{\pi}_{t+2} = \left[1 + \frac{1 - \epsilon_p}{\epsilon_p \rho_{cc} \epsilon_y R \epsilon_p}\right] \hat{\pi}_{t+1}.$$
2 A sticky–price model

Specifically, we assume that there exists a continuum of household–firm units indexed by \( j \), each of which produces a differentiated good \( Y^j \) and faces a demand function

\[ Y^d d\left(\frac{P^j}{P}\right), \]

where \( Y^d \) denotes the level of aggregate demand, \( P^j \) the price firm \( j \) charges for its output, and \( P \) the aggregate price level. Such a demand function can be derived by assuming that households have preferences over a composite good that is produced from differentiated intermediate goods via a Dixit-Stiglitz production function. The function \( d(\cdot) \) is assumed to satisfy \( d(1) = 1 \) and \( d'(1) < -1 \). The restriction imposed on \( d'(1) \) is necessary for the individual firm’s problem to be well defined in a symmetric equilibrium. The production of good \( j \) is assumed to take real money balances, \( m^{pj} \), as the only input

\[ Y^j = y(m^{pj}) \]

where \( y(\cdot) \) satisfies assumption .

16
The household’s lifetime utility function is assumed to be of the form

\[
U^j = \int_0^\infty e^{-rt} \left[ u(c^j, m^{npj}) - \frac{\gamma}{2} \left( \frac{\dot{P}^j}{P^j} - \pi^* \right)^2 \right] dt
\]

(26)

\( \pi^* > -r \) denotes the steady-state inflation rate. The household’s instant budget constraint and no-Ponzi-game restriction are

\[
\dot{a}^j = (R - \pi)a^j - R(m^{npj} + m^{pj}) + \frac{P^j}{P}y(m^{pj}) - c^j - \tau
\]

(27)

\[
\lim_{t \to \infty} e^{-\int_0^t [R(s) - \pi(s)] ds} a^j(t) \geq 0
\]

(28)

In addition, firms are subject to the constraint that given the price they charge, their sales are demand-determined

\[
y(m^{pj}) = Y^d \left( \frac{P^j}{P} \right)
\]

(29)

The household chooses sequences for \( c^j, m^{npj}, m^{pj}, P^j \geq 0 \) and \( a^j \) so as to maximize (26) subject to (27)–(29) taking as given \( a^j(0), P^j(0) \), and the time paths of \( \tau, R, Y^d, \) and \( P \).
The demand for real balances for non-production purposes can be expressed as

$$m^{npj} = m^{np}(c^j, R)$$  

(30)

which by assumption is increasing in $c^j$ and decreasing in $R$. 

18
Equilibrium

In a symmetric equilibrium all household–firm units choose identical sequences for consumption, asset holdings, and prices. Thus, $c^j = c$, $m^{pj} = m^p$, $m^{npj} = m^{np}$, $a^j = a$, $P^j = P$, $\lambda^j = \lambda$, $\mu^j = \mu$, and $\pi^j = \pi$. In addition,

$$u_c(y(m^p), m^{np}(y(m^p), \rho(\pi))) = \lambda.$$ (31)

$$m^p = m^p(\lambda, \pi); \quad m^p_{\lambda} < 0, m^p_{\pi} u_{cm} < 0$$ (32)

Let $\eta \equiv d'(1) < -1$ denote the equilibrium price elasticity of the demand function faced by the individual firm.

$$\dot{\lambda} = \lambda [r + \pi - \rho(\pi)]$$ (33)

$$\gamma \dot{\pi} = \gamma r(\pi - \pi^*) - y(m^p)\lambda \left[1 + \eta \left(1 - \frac{\rho(\pi)}{y'(m^p)}\right)\right]$$ (34)

$$\dot{a} = [\rho(\pi) - \pi]a - \rho(\pi) [m^{np}(y(m^p), \rho(\pi)) + m^p] - \tau$$

$$0 = \lim_{t \to \infty} e^{-\int_0^t [\rho(\pi) - \pi] ds} a(t)$$ (35)

$$\tau = -\rho(\pi) [m^{np}(y(m^p), \rho(\pi)) + m^p] + Ra$$ (36)
or

\[ \tau = \bar{\tau} \]
Definition 3  (Perfect–foresight equilibrium in the sticky–price economy) In the sticky–price economy, a perfect–foresight equilibrium is a set of sequences \( \{ \lambda, \pi, \tau, a \} \) satisfying (33)–(35) and either \( \tau = \bar{\tau} \) if the fiscal regime is non–Ricardian or \( \tau = R a - R (m^{np} + m^p) \). if the fiscal regime is Ricardian, given \( a(0) \).

Given the equilibrium sequences \( \{ \lambda, \pi, \tau, a \} \), the corresponding equilibrium sequences \( \{ c, m^{np}, m^p, R \} \) are uniquely determined by (12), (17), (30), and (32).
Ricardian fiscal policy

\[
\begin{pmatrix}
\dot{\lambda} \\
\dot{\pi}
\end{pmatrix} = A \begin{pmatrix}
\lambda - \lambda^* \\
\pi - \pi^*
\end{pmatrix}
\]

(37)

\[
A = \begin{bmatrix}
0 & u_c(1 - \rho') \\
A_{21} & A_{22}
\end{bmatrix}
\]

\[
A_{21} = -\frac{u_c c^* \eta R^* y'' m^p_\lambda}{\gamma y''^2} > 0
\]

\[
A_{22} = r + \frac{u_c c^* \eta}{\gamma} \left[ \frac{\rho'}{y'} - \frac{R^*}{y''^2} y'' m^p_\pi \right]
\]

**Proposition 5** If fiscal policy is Ricardian and monetary policy is passive ($\rho' (\pi^*) < 1$), then there exists a continuum of perfect-foresight equilibria in which $\pi$ and $\lambda$ converge asymptotically to the steady state $(\pi^*, \lambda^*)$.

**Proposition 6** If fiscal policy is Ricardian and monetary policy is active ($\rho' (\pi^*) > 1$), then, if $A_{22} > 0 (< 0)$, there exists a unique (a continuum of) perfect-foresight equilibria in which $\pi$ and $\lambda$ converge to the steady state $(\pi^*, \lambda^*)$. 

22
Consider a utility function that is separable logarithmic in consumption, so that \( u_c c^* = 1 \). In this case, the trace of \( A \) is given by

\[
\text{trace} (A) = r + \frac{(1 + \eta) \rho'}{\gamma R^*} \tag{38}
\]

Let \( \bar{\rho}' \equiv -\frac{r R^* \gamma}{1 + \eta} \) denote the value of \( \rho' \) at which the trace vanishes. Clearly, \( \bar{\rho}' \) may be greater or less than one. If \( \bar{\rho}' \leq 1 \), then the equilibrium is indeterminate for any active monetary policy. We highlight this result in the following corollary.

**Corollary 7** Suppose fiscal policy is Ricardian and preferences are log–linear in consumption and real balances. If \( \bar{\rho}' \equiv -\frac{r R^* \gamma}{1 + \eta} \) is less than or equal to one, then there exists a continuum of perfect–foresight equilibria in which \( \pi \) and \( \lambda \) converge to the steady state \( (\pi^*, \lambda^*) \) for any active monetary policy.
Periodic perfect–foresight equilibria

Proposition 8  Consider an economy with preferences given by $u(c, m^{np}) = (1 - s)c^{1-s} + V(m^{np})$, technology given by $y(m^p) = (m^p)^{\alpha}$, and monetary policy given by a smooth interest–rate feedback rule, $\rho(\pi) > 0$, which for $\pi > \bar{\pi}$ takes the form $\rho(\pi) = R^* + a(\pi - R^* + r)$ where $s > 0$, $0 < \alpha < 1$, $R^* - r > \bar{\pi}$, $R^* - r - R^*/a$, $a > 0$, and $R^* > 0$. Let fiscal policy be Ricardian and let the parameter configuration satisfy $\tilde{a} \equiv \frac{-r\alpha\gamma}{\eta} \left( \frac{\eta}{1+\eta} \frac{R^*}{\alpha} \right)^{1-\alpha s} > 1$ and $1 < s < 1/\alpha$. Then there exists an infinite number of active monetary policies satisfying $a < \tilde{a}$ for each of which the perfect foresight equilibrium is indeterminate and $\pi$ and $\lambda$ converge asymptotically to a deterministic cycle.
2.0.0 Non-Ricardian Fiscal Policy

Suppose now that the government follows the non-Ricardian fiscal policy described in equation (16), that is, a fiscal policy whereby the time path of real lump–sum taxes is exogenous. Using

\[ \dot{a} = [\rho(\pi) - \pi]a \]

(39)

\[ -\rho(\pi) \left[ m^{np}(y(m^p(\lambda, \pi)), \rho(\pi)) + m^p(\lambda, \pi) \right] - \bar{\tau}. \]

(34), and (39), which can be written as

\[
\begin{pmatrix}
\dot{\lambda} \\
\dot{\pi} \\
\dot{a}
\end{pmatrix} =
\begin{bmatrix}
A & 0 \\
\epsilon & r
\end{bmatrix}
\begin{pmatrix}
\lambda - \lambda^* \\
\pi - \pi^* \\
a - a^*
\end{pmatrix}
\]

(40)

where \( A \) is defined in (37) and \( \epsilon \) is a one by two vector whose elements are the steady–state derivatives of \( R(m^{np} + m^p) \) with respect to \( \lambda \) and \( \pi \).

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Proposition 9  If fiscal policy is non–Ricardian and monetary policy is passive ($\rho' (\pi^*) < 1$), then there exists a unique perfect–foresight equilibrium in which $\{\lambda, \pi\}$ converge asymptotically to the steady state $(\pi^*, \lambda^*)$.

Proposition 10  If fiscal policy is non–Ricardian and monetary policy is active ($\rho' (\pi^*) > 1$), then if $A_{22} > 0 (< 0)$, there exists no (a continuum of) perfect–foresight equilibria in which $\{\lambda, \pi\}$ converge asymptotically to the steady state $(\pi^*, \lambda^*)$.

In the case that monetary policy is active and both eigenvalues of $A$ are positive, there may exist bounded equilibria that converge to a stable cycle around the steady state. Note that for the system (33), (34), and (39) the dynamics of $\{\lambda, \pi\}$ are independent of $a$, and thus the analysis of periodic equilibria of the previous section still applies. For example, in the special case introduced earlier if cycles exist, any initial condition for $(\lambda, \pi)$ in the
neighborhood of the steady state will converge to a cycle. To assure that $a$ does not explode, however, we must restrict ourselves to a one dimensional manifold in $\{\lambda, \pi\}$. This follows because while cycles restricted to the $\{\lambda, \pi\}$ plane are attracting, in the three dimensional space the cycle in $\{\lambda, \pi, a\}$ will have only a two dimensional stable manifold: initial values of $\lambda$ and $\pi$ will have to be chosen to assure that the triple $\{\lambda, \pi, a\}$ converges to the cycle and $a$ remains bounded.
2. Real indeterminacy in the Sticky-Price Model

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>Fiscal Policy</th>
<th>Ricardian</th>
<th>Non-Ricardian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive ($\rho'(\pi^*) &lt; 1$)</td>
<td>I</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Active ($\rho'(\pi^*) &gt; 1$)</td>
<td>I</td>
<td>I</td>
<td></td>
</tr>
<tr>
<td>$A_{22} &lt; 0$</td>
<td>I or D</td>
<td>I or NE</td>
<td></td>
</tr>
<tr>
<td>$A_{22} &gt; 0$</td>
<td>I or D</td>
<td>I or NE</td>
<td></td>
</tr>
</tbody>
</table>

Note: The notation is D, determinate; I, indeterminate; NE, no perfect-foresight equilibrium exists.
3.1 Backward- and forward-looking feedback rules

3.1.0 Flexible-price model

We now analyze a generalization of the interest-rate feedback rule in which the nominal interest rate depends not only on current but also on past or future rates of inflation. Consider first the following backward-looking feedback rule

\[ R = \rho (q \pi + (1 - q) \pi^p); \quad \rho' > 0; \ q \in [0, 1] \]

(41)

where \( \pi^p \) is a weighted average of past rates of inflation and is defined as

\[ \pi^p = b \int_{-\infty}^{t} \pi e^{b(s-t)} ds; \quad b > 0 \]

(42)

Differentiating this expression with respect to time yields

\[ \dot{\pi}^p = b (\pi - \pi^p) \]

(43)

The rest of the equilibrium conditions are identical to those obtained earlier for a flexible
price system. In particular, we have that

$$\lambda'(R)\dot{R} = \lambda(R)[r + \pi - R]$$  \hspace{1cm} (44)

where

$$\lambda'(R) = [u_{cc} y' m^{pl} + u_{cm} (m_{c}^{np} y' m^{pl} + m^{np}_{R})]$$  \hspace{1cm} (45)

Using equation (41) to eliminate $\pi$ from (43) and (44) and linearizing around the steady state results in the following system of linear differential equations

$$\begin{bmatrix} \dot{R} \\ \dot{\pi}^p \end{bmatrix} = \begin{bmatrix} \frac{\lambda}{\lambda'} \left( \frac{1}{\rho' q_1} - 1 \right) & -\frac{\lambda}{\lambda'} \frac{(1-q_1)}{q_1} \\ b \frac{1}{\rho' q_1} & -\frac{b}{q_1} \end{bmatrix} \begin{bmatrix} R - R^* \\ \pi^p - \pi^* \end{bmatrix}$$

Let $J$ denote the Jacobian matrix of this system. Because $R$ is a jump variable and $\pi^p$ is predetermined, the real allocation is locally unique if the real parts of the eigenvalues of $J$ have opposite signs, or, equivalently, if the determinant of $J$ is negative. On the other hand, the real allocation is locally indeterminate if both eigenvalues have negative real parts, that is, if the determinant of $J$ is positive and its trace is negative. The
determinant and trace of $J$ are given by

$$det(J) = \frac{\lambda}{\lambda'} \frac{b}{\rho'q_1} (\rho' - 1)$$

$$trace(J) = \frac{\lambda}{\lambda'} \left( \frac{1}{\rho'q_1} - 1 \right) - \frac{b}{q_1}$$
Now consider the two polar cases of money entering only through preferences \((y' = 0)\) and money entering only through production \((u_{cm} = m^p_R = 0)\). If money enters only through preferences and money and consumption are Edgeworth complements \((u_{cm} > 0)\), then equation (45) implies that \(\lambda'\) is negative. It follows directly from the above two expressions that the conditions governing the local determinacy of \(R\) are identical to those obtained under a purely contemporaneous feedback rule. Namely, the equilibrium is unique under active monetary policy \((\rho' > 1)\) and is indeterminate under passive monetary policy \((\rho' < 1)\).

When money enters only through production or only through preferences with consumption and money being Edgeworth substitutes, \(\lambda'\) is positive. Thus, the equilibrium is always locally determinate under passive monetary policy, as was the case under purely contemporaneous feedback rules.
However, contrary to the case of purely contemporaneous feedback rules, if monetary policy is active, then equilibria in which $R$ converges to its steady state may not exist. To see this, note that in this case the determinant of $J$ is positive, so that the real parts of the roots of $J$ have the same sign as the trace of $J$. However, the trace of $J$ can have either sign. If the trace is positive, then no equilibrium exists. If it is negative, the equilibrium is indeterminate. For large enough values of $\rho'$ the trace of $J$ becomes negative. Thus, highly active monetary policy induces indeterminacy. Furthermore, the larger the emphasis the feedback rule places on contemporaneous inflation ($q$ close to one) or the lower the weight it assigns to inflation rates observed in the distant past ($b$ large), the smaller is the minimum value of $\rho'$ beyond which the equilibrium becomes indeterminate. In the limit, as $q$ approaches unity or $b$ approaches infinity, the equilibrium becomes indeterminate un-
der every active monetary policy, which is the result obtained under purely contemporaneous feedback rules. On the other hand, as the monetary policy becomes purely backward looking \((q \to 0)\), no equilibrium in which \(R\) converges to its steady state exists under active monetary policy.