Endogenous Information Acquisition and Countercyclical Uncertainty*

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Abstract

We introduce endogenous information acquisition in an otherwise standard business cycle model. In our framework, information is a productive input, which is essentially specialized labor, so information acquisition is linked to the labor market and thereby the macroeconomic condition. Our model demonstrates that strategic complementarity (substitutability) in optimal information acquisition coincides with strategic complementarity (substitutability) in production. Our paper shows that when firms acquire information optimally, information acquisition is endogenously procyclical and therefore economic uncertainty faced by the firms is countercyclical. Two-way feedback exists between economic uncertainty and macroeconomic activities, resulting in an amplification effect of TFP shocks and possibly generating multiple equilibria. Our basic model can also be extended to explain countercyclical aggregate volatility.

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1 Introduction

Recessions are often associated with increases in uncertainty, measured by either macro-level or micro-level volatility. In a recent influential paper, Bloom (2009) shows that measured uncertainty is countercyclical, suggesting that "uncertainty shocks" may be quantitatively important features of business cycles. Indeed, this finding has inspired a growing literature arguing that uncertainty shocks are an important driving force of business cycles in general and of the recent Great Recession in particular.[1] For example, Christiano, Motto and Rostagno (2014) find that the exogenous fluctuations in the volatility of cross-sectional uncertainty are the most important shock driving the business cycles in an estimated sticky price model augmented with the financial accelerator mechanism. Nevertheless, the direction of causality between economic uncertainty and macroeconomic activities is still an open question. For example, several recent empirical works (e.g., Bachmann and Bayer (2013, 2014), Bachmann, Elstner and Sims (2013), Bachmann, Elstner and Hristov (2015)) document evidence that uncertainty appears to be an outcome, rather than the cause, of recessions. Models in which "uncertainty fluctuations" are merely a symptom of business cycles have also been proposed in the literature (e.g., Bachmann and Moscarini (2012), Ilut, Kehrig and Schneider (2015)).

In this paper, we revisit the relationship between uncertainty and macroeconomic activities theoretically, in the context of endogenous information acquisition models. Rather than treating volatility shocks as exogenous, we provide a model that endogenizes them under equilibrium information acquisition. We show that several commonly-used measures of uncertainty in the literature, such as firm-level dispersion in productivity, firm residual idiosyncratic uncertainty, and the volatility of aggregate output forecast error, are endogenously countercyclical when firms opt to acquire information procyclically. Our model shows that the causality between uncertainty and economic activities can go both ways. A recession caused by primitive shocks such as TFP shocks increases uncertainty because of reduced information acquisition. The raised uncertainty in turn amplifies and reinforces the decline in output. Recession in the fear of high economic uncertainty hence can become a self-fulfilling prophecy if this two-way feedback is strong enough.

To demonstrate this idea, we incorporate endogenous information acquisition into an otherwise standard Dixit-Stiglitz monopolistic competition model, in which each intermediate goods firm (entrepreneur) employs labor to produce a differentiated good but faces uncertainty about its idiosyncratic demand shock. To reduce uncertainty, firms can acquire information about their idiosyncratic demand shocks before production by incurring some *labor costs*, so information acquisition is linked to the labor market and thereby the macroeconomic condition. Information acquisition

[1] The Federal Open Market Committee minutes repeatedly emphasize uncertainty as a key factor driving the 2001 and 2007-2009 recessions (see, e.g., Bloom et al. (2012)).

enables a firm to earn higher expected profits because with more precise information it can align its production more closely with the true demand for its product. A two-way feedback arises under endogenous information acquisition. On the one hand, uncertainty about idiosyncratic demand shocks distorts the allocation of resources (labor) across firms ex post. Reducing uncertainty through information acquisition improves resource allocation, thus increasing the endogenous aggregate TFP and boosting macroeconomic activities. On the other hand, the macroeconomic condition affects labor costs and firms' incentives to acquire information ex ante, which, in turn, determine firms' residual uncertainty or forecast error (on their idiosyncratic demand shocks) at the time of production.

To illustrate the two-way feedback, consider a recession caused initially by a negative TFP shock. The resulting gain from information acquisition decreases as the expected profit, which partially depends on the aggregate production, goes down. However, the labor cost of information acquisition also decreases as real wages (depending on the aggregate economy) and the cost of information acquisition also go down. We show that under reasonable specifications of preferences, the decrease in benefit outweighs the decrease in cost during the recession, leading to a decrease in information acquisition. As firms have less precise information, they face greater residual uncertainty, resulting in greater resource misallocation. This reinforces the negative impact of the initial drop in TFP on aggregate production, deepening the recession. Conversely, there is more information acquisition and the residual uncertainty faced by firms is lower during a boom. In short, information acquisition is endogenously procyclical and the residual idiosyncratic uncertainty (forecast error) faced by firms is countercyclical in equilibrium.

We further show that this two-way feedback, if strong enough, can generate multiple equilibria in which high uncertainty and low aggregate economic activities can become self-fulfilling. This happens when the effect of complementarity in production across intermediate goods firms is stronger than the general equilibrium effect on the real wage. In such a case, production of intermediate goods firms exhibits strategic complementarity. If other firms acquire information, this increases the overall efficiency of resource allocation and hence aggregate output, which in turn increases an individual firm's incentives to acquire information. Information acquisition therefore exhibits strategic complementarity, leading to multiple equilibria. If a firm expects that all other firms acquire information, it does so as well, resulting in an efficient equilibrium in which all firms acquire information. If instead a firm expects that no other firms acquire information, it has no incentive to do so either, resulting in an inefficient equilibrium in which no firm acquires information. The inefficient equilibrium is characterized as higher uncertainty and lower aggregate production than those associated with the efficient equilibrium.

We then extend our baseline model to show that countercyclical idiosyncratic uncertainty also manifests as countercyclical aggregate volatility and countercyclical aggregate uncertainty. We assume that besides the "private" signal about the idiosyncratic demand shock, each firm also receives a "public" signal, which is the sum of its idiosyncratic demand shock and the economy-wide common sentiment shock about aggregate demand in the spirit of Angeletos and La'O (2013a) and Benhabib, Wang and Wen (2015). When firms make their production decision, they then face not only the aforementioned idiosyncratic uncertainty but also the aggregate uncertainty of aggregate output coming from the time-varying common sentiment shock. As in the baseline model, a firm can acquire more information (i.e., making its "private" signal more precise) to reduce its uncertainty about its idiosyncratic demand shock. As before, when the economy is in a boom due to a positive TFP shock, firms have stronger incentives to acquire information. So the "public" signal becomes less important to firms' production decisions. This means that a firm's production is less responsive to its "public" signal and thus the common sentiment shock. This decreases the volatility of aggregate output and also decreases the uncertainty (forecast error) in aggregate output faced by each firm. At the same time, as in the baseline model, the idiosyncratic uncertainty faced by the firms also decreases. In short, a higher (lower) TFP leads to lower (higher) aggregate volatility and aggregate uncertainty.

Our model yields several predictions in line with the empirical data. First, we find evidence that the total employment for information acquisition-related occupations is strongly procyclical, lending direct support to the mechanism of procyclical information acquisition of our model.[2] Second, empirical studies support our model's implication of countercyclical idiosyncratic uncertainty as measured by the volatility of idiosyncratic forecast error.[3] Third, empirical evidence (Basu and Fernald (2001), David, Hopenhayn and Venkateswaran (2015)) also supports our model's prediction that reallocation helps explain procyclical aggregate endogenous productivity.[4] Finally, in addition to the prediction of countercyclical aggregate volatility consistent with the data, our extended model in Section 3 implies that aggregate output is more difficult to predict in recessions, which is consistent with the evidence in Jurado, Ludvigson and Ng (2014). The prediction of our extended model in Section 3 is also consistent with the notion that sentiment shocks are likely more important in recessions than in booms.[5]

Relation to the Literature. We add to the fast-growing literature that follows Bloom (2009) in studying the interaction between uncertainty and economic activities. Many recent papers have

[2] Our estimation is based on the Occupational Employment Statistics (OES) survey data (available from 2001 to 2015), where we use the occupation of "Market Research Analysts" as a proxy for the intensity of information acquisition. We find that the employment for this occupation is strongly procyclical. The conclusion is also confirmed when we use several other occupations, such as "Survey Researchers" and "Cost Estimators", as proxies. For longer-period data, we conduct an estimation based on the Current Employment Statistics, where we use the employment in the "Marketing Research and Public Polling" industry as the measure of employment in information acquisition. The data again show that information acquisition is procyclical.

[3] See the evidence in Bachmann, Elstner and Sims (2013), Bachmann, Elstner and Hristov (2015), Campbell et al. (2001), Gilchrist, Sim and Zakrajšek (2014), Kehrig (2011) and Berger and Varvar (2011).

[4] Information acquisition in David, Hopenhayn and Venkateswaran (2015) is exogenous, unlike in our paper.

[5] For example, Kurov (2010) and Lutz (2015) provide some indirect evidence for such a notion using investor sentiments and stock market data.

extended Bloom's partial equilibrium analysis to the general equilibrium. Since an increase in stochastic volatility typically generates a comovement problem between consumption and investment in a frictionless economy, various frictions have been explored. For example, Christiano, Motto and Rostagno (2014), Arellano, Bai and Kehoe (2012), and Gilchrist, Sim and Zakrajsek (2014) introduce financial frictions and show that financial frictions can propagate uncertainty shocks. Basu and Bundick (2012) study uncertainty shocks in a New Keynesian model. Bloom et al. (2012) introduce microeconomic rigidities, such as adjustment costs in capital and labor. In contrast to our model, these papers treat uncertainty shocks as exogenous and focus on their impact on the economy. A few theoretical papers model uncertainty fluctuations as the outcome of business cycles. Notably, Bachmann and Moscarini (2012) consider the endogenous uncertainty in cross-sectional measures of dispersion.[6] In their model, firms face uncertainty about the elasticity of their demand but can learn gradually from the volume of sales. They show that bad economic times are the best times to price-experiment. Hence information acquisition is countercyclical and generates countercyclical cross-sectional measures of dispersion. In our model, firms know their demand elasticity but need to learn the position of their demand curve, which gives rise to a simpler learning problem and allows us to study time-series aggregate volatility. Ilut, Kehrig and Schneider (2015) show that if hiring decisions respond more to bad news than to good news, both aggregate conditional volatility and the cross-sectional dispersion of employment growth are countercyclical. There is no endogenous information acquisition mechanism in their framework.

Our paper is related to the work of Mäkinen and Ohl (2015), who extend the literature on asymmetric learning and countercyclical uncertainty (see, e.g., Van Nieuwerburgh and Veldkamp (2006) and Veldkamp (2005)) to encompass costly acquisition. The authors demonstrate that firms' information demand exhibits countercyclicality, and show that the equilibrium price system moderates aggregate fluctuations by disincentivizing information acquisition. Information acquisition in their model is endogenous as in our paper. Our paper and theirs complement each other in that their paper models the learning about the economy's aggregate state and emphasizes the price system transmitting information, where information acquisition is a strategic substitute. Fajgelbaum, Schaal and Taschereau-Dumouchel (2014) propose a theory of self-reinforcing episodes of high uncertainty and low activity, through the mechanism of the "wait-and-see" effect together with agents learning from the actions of others. Straub and Ulbricht (2014) explore the joint propagation of uncertainty and financial distress, looking at a model where adverse shocks to the financial sector endogenously generate uncertainty about firm-specific fundamentals. The mechanisms studied in these papers are related to but different from our model's emphasis on the two-way feedback between micro-level uncertainty and macroeconomic activity.

[6] Cui (2012) and D'Erasmo and Moscoso-Boedo (2012) also study endogenous cross-sectional dispersion and use approaches different from ours.

Our paper is also related to the literature on information acquisition and welfare. Among others, Reis (2006), Angeletos and Pavan (2007), Hellwig and Veldkamp (2009), Vives (2013), Colombo, Femminis and Pavan (2014) and Mäkinen and Ohl (2015) study information acquisition and efficiency. In business-cycle models, Angeletos and La'O (2013b), Llosa and Venkateswaran (2013) and Mackowiak and Wiederholt (2013) contrast the equilibrium acquisition of information with the efficient acquisition of information. In our model, there is under-acquisition of information in the competitive equilibrium. When an individual firm acquires information, it has positive externality to consumer surplus and negative externality to other firms' profit. The overall effect is that the positive externality exceeds the negative externality, so too few individual firms acquire information in the competitive equilibrium compared with the second-best efficient equilibrium.

The rest of the paper is organized as follows. Section 2 presents the baseline model, highlighting the basic mechanism. Section 3 extends the baseline model to study aggregate volatility and aggregate uncertainty. Section 4 concludes.

2 The Baseline Model

2.1 Model Setup

The economy is populated by a large representative household comprising a continuum of identical workers and a continuum of entrepreneurs, with a unit measure of each. The household derives utility from leisure and from consumption of a composite final good produced with a continuum of differentiated intermediate goods. Workers supply labor to entrepreneurs in a competitive labor market. Entrepreneur j is the monopolist of differentiated intermediate good j. The demand for each intermediate good j is affected by an idiosyncratic demand shock ϵ_{jt} and by aggregate demand driven by an aggregate productivity shock A_t. At the beginning of each period, after observing the aggregate productivity shock A_t, entrepreneur j decides whether to acquire information regarding ϵ_{jt}, and then produces accordingly. At the end of each period, the workers and entrepreneurs pool their wage and profit income for the household.

The Representative Household The household maximizes its utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \psi N_t - \psi N_{et} \right], \tag{1}$$

where C_t is the consumption of the household, N_t is the total working hours of workers, and N_{et} is the total working hours entrepreneurs spend on information acquisition (to be specified later). Note that in our model *without capital*, a reasonable parameter specification is to set γ below 1. In fact, in our model, γ plays the role of the income elasticity of labor supply; empirically, labor supply is not very sensitive to income and wages are not very pro-cyclical.[7]

The budget constraint for the household is

$$P_t C_t \leq W_t N_t + \Pi_t, \tag{2}$$

where P_t is the price of the consumption good, W_t is the nominal wage, and Π_t denotes total profit income earned by entrepreneurs. For the representative household there is no transfer of resources across periods, so the infinite-horizon maximization problem becomes the repeated one-period maximization problem. Our model is essentially static. The first-order condition of the maximization problem of (1) yields

$$\psi C_t^{\gamma} = \frac{W_t}{P_t}. \tag{3}$$

When making its consumption decision (or labor supply decision) according to (3), the representative household sees the nominal wage W_t, and it forms (rational) expectations of the equilibrium aggregate price P_t and thus of the real wage $\frac{W_t}{P_t}$.[8]

The Final Goods Producer The consumption good is produced by a competitive final goods firm facing competitive factor markets according to the Dixit-Stiglitz aggregate production function:

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1. \tag{4}$$

The final good producer maximizes its profit:

$$\max_{y_{jt}} P_t \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} - \int_0^1 p_{jt} y_{jt} dj,$$

where p_{jt} is the price of intermediate good j. The first-order condition with respect to input y_{jt} implies

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} \epsilon_{jt} Y_t, \tag{5}$$

which shows that the demand for intermediate good j is affected by both idiosyncratic shock ϵ_{jt} and aggregate demand Y_t. We assume that ϵ_{jt} is independent across time and goods and $\log \epsilon_{jt} \equiv \varepsilon_{jt} \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$, so the mean of ϵ_{jt} is $\mathbb{E}\epsilon_{jt} = 1$; denote $\tau_\varepsilon = 1/\sigma_\varepsilon^2$. It is also easy to show that

$$P_t = \left[\int_0^1 \epsilon_{jt} p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}.$$

Intermediate Goods Producers Entrepreneurs are the producers of intermediate goods.

[7] We appreciate one referee's clarification on this point. See more explanations later.

[8] In our baseline model, there is no aggregate uncertainty, so P_t is deterministic and is perfectly foreseen under rational expectations.

Entrepreneur j is the monopolist of intermediate good j with production function

$$y_{jt} = A_t n_{jt}. \tag{6}$$

Entrepreneur j produces y_{jt} to maximize his profit under demand uncertainty driven by ϵ_{jt}.[9] To reduce the uncertainty before production, entrepreneur j can spend m working hours to acquire some information about ϵ_{jt} (for example, via a market survey). If he chooses to do so, he receives a signal given by

$$s_{jt} = \varepsilon_{jt} + e_{jt},$$

where $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$, so the precision of the signal is $\tau_e = 1/\sigma_e^2$. If the entrepreneur does not acquire information, he knows only the prior, unconditional distribution of ε_{jt}; equivalently, he receives a useless signal s_{jt} with $\sigma_e^2 = \infty$.

Entrepreneurs hire workers based on the nominal wage before the actual production and trades take place. Entrepreneurs of course have to form expectations of the equilibrium aggregate price P_t and hence the real wage when they make their hiring decision.

An informed entrepreneur j chooses y_{jt} to maximize his expected profit

$$y_{jt} = y(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[p_{jt} y_{jt} - W_t n_{jt} | s_{jt}\right] \tag{7}$$

with constraints (5) and (6).[10] Here $\mathbb{E}(\cdot|s_{jt})$ is the conditional expectation operator over ϵ_{jt}. Denote the realized profit for an informed entrepreneur by $\pi(\epsilon_{jt}, s_{jt}) = p_{jt}\left(\epsilon_{jt}, y_{jt}\right) y_{jt} - W_t n_{jt}\left(y_{jt}\right)$. Likewise, an uninformed entrepreneur j solves

$$\tilde{y}_{jt} = \arg\max_{\tilde{y}_{jt}} \mathbb{E}\left[p_{jt} \tilde{y}_{jt} - W_t n_{jt}\right] \tag{8}$$

with constraints $\tilde{y}_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} \epsilon_{jt} Y_t$ and $\tilde{y}_{jt} = A_t n_{jt}$. Here $\mathbb{E}$ is simply the unconditional expectation operator over ϵ_{jt}. Denote the realized profit for an uninformed entrepreneur by $\tilde{\pi}_t(\epsilon_{jt}) = p_{jt}\left(\epsilon_{jt}, \tilde{y}_{jt}\right) \tilde{y}_{jt} - W_t n_{jt}\left(\tilde{y}_{jt}\right)$.

Throughout the paper, we normalize the wage as the numeraire price:

$$W_t = 1.$$

Information Acquisition To acquire a signal s_{jt}, an entrepreneur needs to spend a fixed

[9] In Appendix B, we examine the price setting instead of the quantity setting, and our insight carries over.

[10] The profit in terms of utility units is the amount of profit multiplied by ψ. Maximizing profits is equivalent to maximizing the "shareholder's value".

amount of time m. The ex ante expected profit for a firm acquiring information is

$$\pi_t^I = \mathbb{E}_{\epsilon_{jt}, s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt})\right] = \mathbb{E}_{s_{jt}}\mathbb{E}_{\epsilon_{jt}|s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt})|s_{jt}\right].$$

Throughout the paper, $\mathbb{E}_x(\cdot)$ denotes the unconditional expectation operator over x and $\mathbb{E}_{x|y}(\cdot|y)$ denotes the conditional expectation operator over x conditional on y. The ex ante expected profit for an entrepreneur not acquiring information is

$$\pi_t^U = \mathbb{E}_{\epsilon_{jt}}\left[\tilde{\pi}_t(\epsilon_{jt})\right].$$

As entrepreneurs are identical ex ante, all of them will acquire information if $\pi_t^I - m > \pi_t^U$ and none will acquire information if $\pi_t^I - m < \pi_t^U$. If $\pi_t^I - m = \pi_t^U$, entrepreneurs are indifferent in acquiring information or not. Denote by λ_t the fraction of entrepreneurs who acquire information.[11] We must have

$$\begin{cases} \pi_t^I - m > \pi_t^U & \text{if } \lambda_t = 1 \\ \pi_t^I - m = \pi_t^U & \text{if } \lambda_t \in (0,1) \\ \pi_t^I - m < \pi_t^U & \text{if } \lambda_t = 0 \end{cases}. \tag{9}$$

Timeline We summarize the sequence of actions by consumers and firms, the information structure, and the rational expectations equilibria of our baseline model.

1. At the beginning of each period, after observing A_t, an entrepreneur makes his decision on whether to acquire a signal about ϵ_{jt}. Signal s_{jt} is obtained if he pays a constant cost m in terms of working hours; otherwise no signal (or a useless one) is obtained.

2. Based on signal s_{jt}, nominal wage $W_t \equiv 1$, and rational expectations of P_t (or Y_t), an informed entrepreneur decides how much labor n_{jt} to hire to produce his intermediate good. An uninformed entrepreneur chooses n_{jt} based on the prior of ϵ_{jt}.

3. Given the production of y_{jt} and $\tilde{y}_{jt}$, price p_{jt} adjusts to equate demand and supply according to equation (5).

4. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

Definition 1 formally defines the equilibrium of our baseline model.

[11] The setup of binary choice of information acquisition (i.e., acquiring or not) coupled with a fraction λ_t of firms acquiring information (in the spirit of Grossman and Stiglitz (1980)) in the baseline model is for obtaining explicit and analytical solutions. In Appendix D, we study continuous information acquisition.

Definition 1 *An REE is a sequence of aggregate allocations* $\{C(A_t), Y(A_t), N(A_t), \Pi(A_t), \lambda(A_t)\}$*, individual productions* $y_{jt} = y(A_t, s_{jt})$ *for informed entrepreneurs and* $y_{jt} = \tilde{y}(A_t)$ *for uninformed entrepreneurs, and prices* $\{P(A_t), p(s_{jt}, \epsilon_{jt})\}$*, such that for each realization of* A_t*, (i)* $C(A_t)$ *and* $N(A_t)$ *maximize households' utility given the equilibrium price* $P_t = P(A_t)$ *and aggregate profit* $\Pi(A_t)$*; (ii) equation (5) maximizes the final goods firm's profit given shocks* ϵ_{jt} *and equilibrium prices* $p(s_{jt}, \epsilon_{jt})$*; (iii) given* P_t *and signal* s_{jt}*,* $y(A_t, s_{jt})$ *maximizes the expected profit of an informed entrepreneur and* $\tilde{y}(A_t)$ *maximizes the expected profit of an uninformed entrepreneur; (iv) A* $\lambda(A_t)$ *fraction of entrepreneurs acquire information about their* ϵ_{jt}*, so* $\Pi(A_t) = \lambda_t \pi_t^I + (1-\lambda_t)\pi_t^U$*; and (v) all markets clear, namely,* $C(A_t) = Y(A_t)$ *and* $N_t = \int_0^1 \frac{y_{jt}}{A_t} dj$*.*

We now proceed to characterize the equilibrium.

2.2 Characterization of Equilibrium

First, we work out a firm's optimal production given its information acquisition decision. Next, we aggregate all firms' production to obtain the aggregate output Y_t as a function of λ_t and A_t. Then, we compare π_t^I and π_t^U to solve firms' information acquisition problem, which yields another function involving λ_t and Y_t. Finally, we use these two functions to determine Y_t and λ_t simultaneously as functions of A_t.

Equilibrium Y_t for a Given λ_t Substituting (5) and (6) into (7), we have

$$y_{jt}(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[\left(P_t \cdot y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{1}{A_t} y_{jt}\right) | s_{jt}\right] \tag{10}$$

for an informed entrepreneur. This yields

$$y_{jt} = y(A_t, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} Y_t \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt})\right]^{\theta}, \tag{11}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt})\right]^{\theta} = \exp\left[\frac{\tau_e}{\tau_\varepsilon + \tau_e} s_{jt} + \frac{1}{2}\frac{1-\theta}{\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right]$. Similarly, we obtain the production for an uninformed entrepreneur:

$$\tilde{y}_{jt} = \tilde{y}(A_t) = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} Y_t \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta}, \tag{12}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta} = \exp(\frac{1}{2}\frac{1-\theta}{\theta}\frac{1}{\tau_\varepsilon})$. Since a λ_t fraction of firms produce according to (11) and $1 - \lambda_t$ of them produce according to (12), the aggregate production defined in equation (4) becomes

$$Y_t = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} Y_t \left[\int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}\right]\right)^{\theta-1} dj + \int_{\lambda_t}^1 \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj\right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

The labor demand is simply given by $n_{jt} = y_{jt}/A_t$. Hence labor market clearing gives

$$N_t = \frac{1}{A_t}\left(1-\frac{1}{\theta}\right)^{\theta}(P_t A_t)^{\theta} Y_t \left[\int_0^{\lambda_t}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj + \int_{\lambda_t}^{1}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj\right]. \tag{14}$$

Exploiting the law of iterated expectations (see the proof of Proposition 1 in Appendix A), (13) can be transformed into

$$\frac{1}{P_t} = (1-\frac{1}{\theta})\,(A_t z_t) \tag{15}$$

while (13) and (14) together yield

$$Y_t = N_t\,(A_t z_t)\,, \tag{16}$$

where $z_t = z(\lambda_t)$ is the *endogenous* TFP given by

$$z(\lambda_t) = \left[\lambda_t \overline{z}^{\theta-1} + (1-\lambda_t)\,\underline{z}^{\theta-1}\right]^{\frac{1}{\theta-1}} \tag{17}$$

with $\overline{z} \equiv \exp\left(-\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right)$ and $\underline{z} \equiv \exp\left(-\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon}\right)$. It is easy to see that $z'(\lambda_t) > 0$, and $z(\lambda_t = 1) = \overline{z}$ and $z(\lambda_t = 0) = \underline{z}$. That is, if more firms acquire information, the aggregate production becomes more efficient. In fact, efficient allocation requires more resources to be allocated to firms with higher realized ϵ_{jt}, that is, efficient production should be contingent on realized ϵ_{jt}. So, more precise information about ϵ_{jt} achieved through information acquisition helps improve allocative efficiency.

Equations (15) and (16) are intuitive. (16) implies that despite heterogeneity among firms originating in idiosyncratic demand shocks, our economy works as if there existed a representative firm with productivity $A_t z(\lambda_t)$. (15) means that the real wage, $\frac{1}{P_t}$, is proportional to labor productivity $A_t z(\lambda_t)$, where the proportion $1-\frac{1}{\theta}$ is the share of labor cost in aggregate GDP (i.e., the average profit-to-revenue ratio in the economy is $\frac{1}{\theta}$).

In equilibrium, (3) becomes $P_t = \frac{1}{\psi}Y_t^{-\gamma}$, which together with (15) yields aggregate output as a function of λ_t and A_t:

$$Y_t = \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{A_t z(\lambda_t)}{\psi}\right)^{\frac{1}{\gamma}}. \tag{18}$$

This implies that the aggregate production increases with λ_t.

Equilibrium λ_t under Information Acquisition We now turn to firms' information acquisition problem to derive another relationship between λ_t and Y_t. Exploiting the law of iterated expectations (see the proof of Proposition 1 in Appendix A), we obtain the ex ante expected profit for an informed entrepreneur,

$$\pi_t^I = \frac{1}{\theta-1}\frac{1}{A_t}Y_t z_t^{-\theta}\overline{z}^{\theta-1},$$

and the expected profit for an uninformed entrepreneur,

$$\pi_t^U = \frac{1}{\theta - 1} \frac{1}{A_t} Y_t z_t^{-\theta} \underline{z}^{\theta - 1}.$$

In fact, by using (15), the following relationship holds:

$$\frac{1}{\theta} = \frac{\lambda_t \pi_t^I + (1 - \lambda_t) \pi_t^U}{Y_t P_t};$$

namely, the average profit-to-revenue ratio in the economy is $\frac{1}{\theta}$. It is easy to see that $\pi_t^I > \pi_t^U$. So informed entrepreneurs always enjoy a higher expected profit. In other words, information is valuable to firms. However, acquiring information is costly. If $\lambda_t \in (0, 1)$, (9) implies

$$\pi_t^I - \pi_t^U = \frac{1}{\theta - 1} \frac{1}{A_t} Y_t z_t^{-\theta} \left(\overline{z}^{\theta - 1} - \underline{z}^{\theta - 1} \right) = m \tag{19}$$

in equilibrium. Substituting $A_t = \frac{\theta}{\theta - 1} \frac{\psi Y_t^\gamma}{z_t}$ from (18) into (19), we obtain the second equilibrium relationship between Y_t and λ_t:

$$\frac{1}{\theta} \left(z(\lambda_t) \right)^{1 - \theta} Y_t^{1 - \gamma} \left(\overline{z}^{\theta - 1} - \underline{z}^{\theta - 1} \right) = \psi m. \tag{20}$$

Under $\gamma < 1$, equation (20) defines λ_t as an increasing function of Y_t. The LHS of equation (20) is the benefit of acquiring information in *utility* units, and the RHS is the utility loss from foregoing leisure. When Y_t increases, the benefit increases (under $\gamma < 1$), leading to stronger incentives to acquire information. Oppositely, under $\gamma > 1$, (20) defines λ_t as a decreasing function of Y_t.

Put slightly differently, we can rewrite (20) as

$$\frac{1}{\theta} Y_t \left(z(\lambda_t) \right)^{1 - \theta} \left(\overline{z}^{\theta - 1} - \underline{z}^{\theta - 1} \right) = \frac{m}{\frac{1}{\psi} Y_t^{-\gamma}}. \tag{21}$$

The LHS of (21) is the benefit of information acquisition in *consumption* units, and the RHS is the cost in terms of real wage (by noting $P_t = \frac{1}{\psi} Y_t^{-\gamma}$). Under $\gamma < 1$, the real wage does not increase as fast as aggregate output Y_t does. Hence, when Y_t goes up, the increase in benefit outruns the increase in cost, so an individual entrepreneur has incentives to switch from being uninformed to being informed by paying the cost. As λ_t goes up, the equilibrium (equation (21)) will be eventually restored. Under $\gamma > 1$, the opposite applies.

Full Equilibrium Equations (18) and (20) jointly determine Y_t and λ_t. Substituting the expression of Y_t in (18) into the LHS of (20) yields

$$\frac{1}{\theta} \left[\left(1 - \frac{1}{\theta} \right) \frac{1}{\psi} \right]^{\frac{1 - \gamma}{\gamma}} A_t^{\frac{1 - \gamma}{\gamma}} \left(z(\lambda_t) \right)^{\frac{1 - \theta\gamma}{\gamma}} \left(\overline{z}^{\theta - 1} - \underline{z}^{\theta - 1} \right) = \psi m. \tag{22}$$

When $\theta\gamma > 1$, holding A_t constant, the LHS of (22) is decreasing in λ_t; that is, when more other firms acquire information, the benefit of information acquisition for a particular individual firm is decreasing. In other words, when $\gamma \in \left(\frac{1}{\theta}, \infty\right)$, information acquisition of intermediate goods firms exhibits strategic substitutability; when $\gamma \in \left(0, \frac{1}{\theta}\right)$, it exhibits strategic complementarity. Lemma 1 follows.

Lemma 1 *Strategic complementarity (substitutability) in information acquisition coincides with strategic complementarity (substitutability) in production.*

Holding the exogenous TFP A_t constant, by (11) together with (3), y_{jt} is decreasing in Y_t iff $\theta\gamma > 1$. In fact, from (5) and (3) together with (11), an increase in aggregate output Y_t has two opposite effects on a particular individual firm in production: the demand curve shifts upward and the production cost (in terms of real wage) also goes up due to the general equilibrium effect. Under $1 - \theta\gamma > 0$, the first effect is stronger than the second effect, so production of intermediate goods firms exhibits strategic complementarity. We will show that when information acquisition exhibits strategic complementarity (i.e., $\gamma \in \left(0, \frac{1}{\theta}\right)$), equilibrium multiplicity can arise.

Based on the above analysis, we can partition γ into three regions: $\gamma \in \left(0, \frac{1}{\theta}\right)$, $\gamma \in \left(\frac{1}{\theta}, 1\right)$, and $\gamma \in (1, \infty)$. We discuss these three cases of γ in order.

i) Case of $\gamma \in \left(\frac{1}{\theta}, 1\right)$

We mainly focus on this case because of the empirical relevance of the parameters.[12] In this case, equation (20) defines λ_t as an increasing function of Y_t and information acquisition exhibits strategic substitutability. Figure 1 gives a diagrammatic analysis of the full equilibrium for this case.

[12] In our model, the real wage is given by $W_t = \psi Y_t^{\gamma}$. So γ measures the elasticity of the real wage to output changes. This equation also implies that γ is the ratio of the volatility of the real wage (in log) to the volatility of output (in log). The US data show that γ is below 1. For example, Stock and Watson (1999) report that the volatilities of output and the real wage are 1.66 and 0.64, respectively, which implies $\gamma = 0.39$. Kydland (1995) and Cheron and Langot (2004) have similar findings. Also, the observed procyclical movement in labor requires $\gamma < 1$ in our model without capital. The new Keynesian models routinely assume that $\gamma < 1$, so there are strategic complementarities in price setting by firms that generate a persistent real effect of monetary shocks (see the related lengthy discussion in the book of Woodford (2003)).

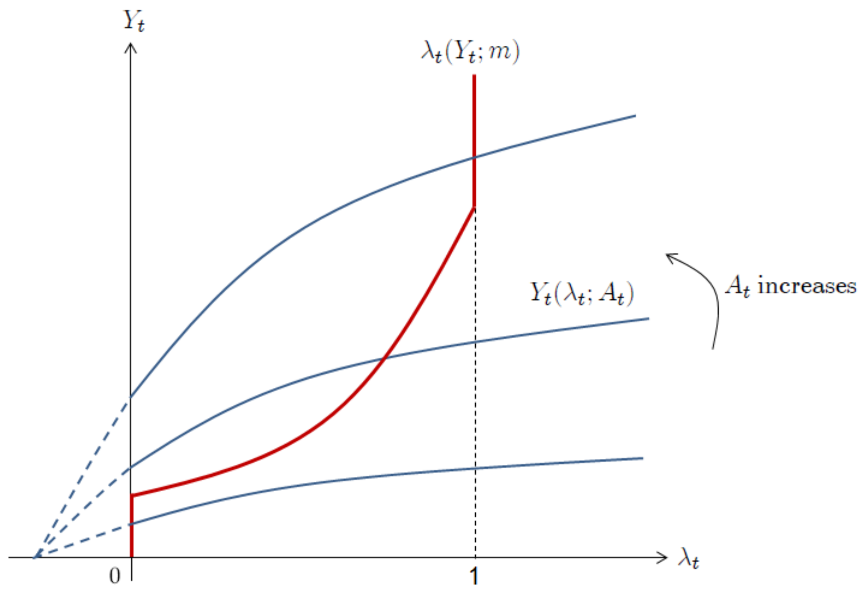

Figure 1: Equilibrium with Endogenous Information Acquisition for the Case of $\gamma \in \left(\frac{1}{\theta}, 1\right)$

In Figure 1, $Y_t(\lambda_t; A_t)$ is given by equation (18) while $\lambda_t(Y_t; m)$ is given by (20).[13] The vertical lines of $\lambda_t = 0$ and $\lambda_t = 1$ correspond to corner solutions in (9). When output Y_t is below a threshold, no entrepreneurs will acquire information. In other words, $\lambda_t(Y_t; m)$ becomes a vertical line at $\lambda_t = 0$. Likewise, if output Y_t is very large, all entrepreneur acquires information, in which case $\lambda_t(Y_t; m)$ becomes a vertical line again at $\lambda_t = 1$. When output is in an intermediate range, an increase in output enhances the incentive to acquire information (under $\gamma < 1$), so $\lambda_t(Y_t; m)$ is upward sloping. By (18), $Y_t(\lambda_t; A_t)$ is also upward sloping. With the help of this graph, it is easy to see how the equilibrium, corresponding to the intersection between two curves, evolves as A_t changes. Note that under $\gamma > \frac{1}{\theta}$ (strategic substitutability in information acquisition), the slope of the curve given by (18) is smaller than that given by (20) at their interior intersection (i.e., $\frac{1}{\theta-1}\frac{1}{\gamma} < \frac{1}{1-\gamma}$), so the equilibrium is always unique. When A_t increases, the unique equilibrium shifts toward the upper-right corner, which means that both Y_t and λ_t increase (weakly).

To obtain a complete characterization of Y_t and λ_t, we first calculate the endogenous TFP z_t by solving equation (22) *with considering corner solutions in (9)*. Then, we can calculate Y_t and λ_t. See the complete characterizations in Appendix A.[14] Once we have λ_t and Y_t, it is straightforward

[13] Concretely, the function given by (18) can be written as $Y_t = c_0 \cdot A_t^{\frac{1}{\gamma}} (c_1 + c_2\lambda_t)^{\frac{1}{\theta-1}\frac{1}{\gamma}}$, while the function given by (20) is $Y_t = c_3 \cdot (c_1 + c_2\lambda_t)^{\frac{1}{1-\gamma}}$, where $c_0 = \left(1 - \frac{1}{\theta}\right)^{\frac{1}{\gamma}} \left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}}$, $c_1 = \underline{z}^{\theta-1}$, $c_2 = \overline{z}^{\theta-1} - \underline{z}^{\theta-1}$ and $c_3 = \left(\frac{\theta\psi m}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}\right)^{\frac{1}{1-\gamma}}$.

[14] For the special case of $\gamma = \frac{1}{\theta}$, there is a continuum of equilibria (corresponding to $\lambda_t \in [0, 1]$ together with (18)) when A_t is such that $c_0 \cdot A_t^{\frac{1}{\gamma}} = c_3$; there is a unique equilibrium (corresponding to $\lambda_t = 1$) when A_t is higher, and there is a unique equilibrium (corresponding to $\lambda_t = 0$) when A_t is lower. For the special case of $\gamma = 1$, there is a unique equilibrium determined by equations (18) and (20); $\lambda_t(Y_t; m)$ becomes a vertical line in Figure 1.

to obtain the rest of the variables. Based on (16) and (18), we have the aggregate labor:

$$N_t = \left(1 - \frac{1}{\theta}\right)\left(\frac{1}{\psi}\right) Y_t^{1-\gamma}. \tag{23}$$

We apply equation (15) to obtain the aggregate price P_t, equations (11) and (12) to obtain firms' production y_{jt}, and equation (5) to obtain price p_{jt}. Proposition 1 summarizes the equilibrium.

Proposition 1 *If $\gamma \in \left(\frac{1}{\theta}, 1\right)$, the equilibrium with endogenous information acquisition is unique; $\log Y_t(A_t; m)$ is continuous and increasing in A_t, and $\lambda_t(A_t; m)$ is continuous and increasing in A_t.*

Proof. *See Appendix A.* ■

Under endogenous information acquisition, there is an amplification effect, as shown in Figure A-1 in the appendix. An initial higher TFP shock increases aggregate output, and a higher aggregate output increases incentives for firms to acquire information, which increases aggregate output further (through the endogenous TFP $z(\lambda_t)$), and so on. That is, there is an upward spiral (multiplier effect).

ii) Case of $\gamma \in \left(0, \frac{1}{\theta}\right)$

In this case, information acquisition exhibits strategic complementarity and multiple equilibria are possible. Figure 2 presents the equilibrium.

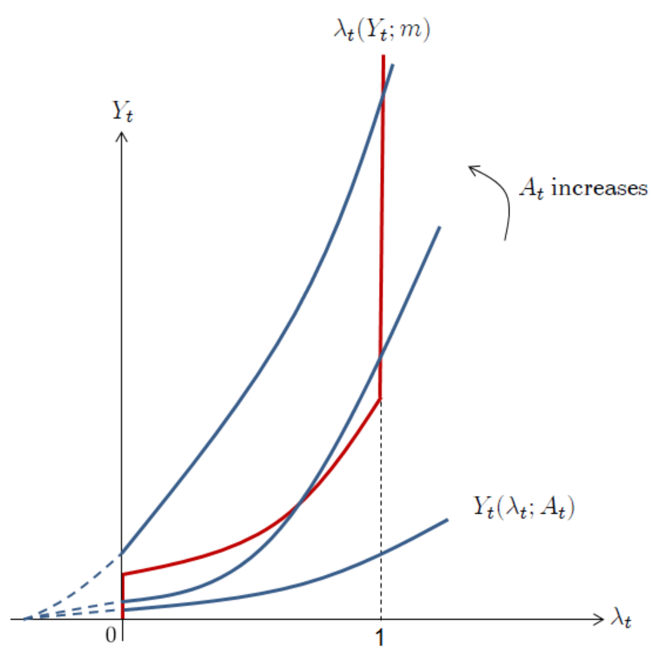

Figure 2: Equilibrium with Endogenous Information Acquisition for the Case of $\gamma \in \left(0, \frac{1}{\theta}\right)$)

In Figure 2, when A_t is sufficiently high or sufficiently low, there is a unique equilibrium (corresponding to the corner solution in (9)). When A_t is in the intermediate range, there are multiple (three) equilibria, corresponding to $\lambda_t = 0$, $\lambda_t = 1$ and the interior solution of λ_t to equation (22).[15] Note that under $\gamma < \frac{1}{\theta}$ (strategic complementarity in information acquisition), the slope of the curve given by (18) is greater than that given by (20) at their interior intersection (i.e., $\frac{1}{\theta-1}\frac{1}{\gamma} > \frac{1}{1-\gamma}$). We provide the concrete characterization of Y_t and λ_t in the appendix (see the proof of Proposition 1).

We also prove that in the case of multiple (three) equilibria, the equilibrium of $\lambda_t = 1$ is the most efficient and the equilibrium of $\lambda_t = 0$ is the least efficient. In the least efficient inefficient equilibrium, firms acquire less information and face higher uncertainty, and the aggregate output is lower.

iii) Case of $\gamma \in (1, \infty)$

For completeness, we also consider the case of $\gamma \in (1, \infty)$. In this case, information acquisition exhibits strategic substitutability and the equilibrium is always unique. Nevertheless, equation (20) defines λ_t as a decreasing function of Y_t. Figure 3 presents the equilibrium in this case. It is clear from the figure that an increase in A_t will lead to less information acquisition (i.e., a lower λ_t) in equilibrium; see Benhabib, Liu and Wang (2015) for further details.

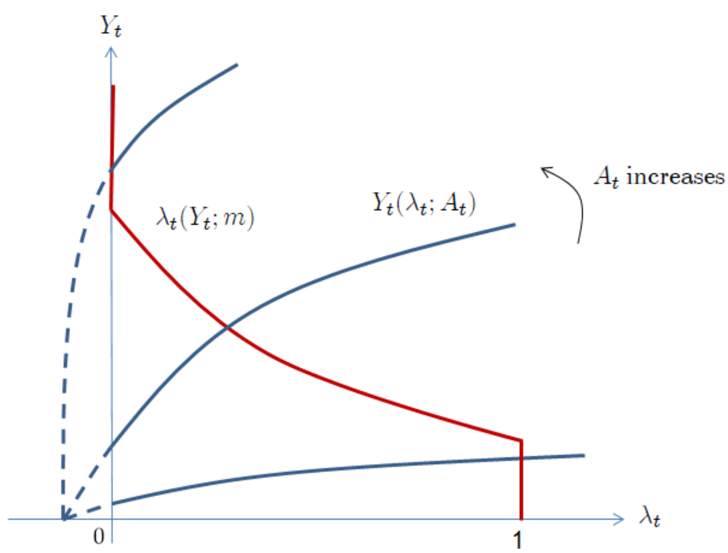

Figure 3: Equilibrium with Endogenous Information Acquisition for the Case of $\gamma \in (1, \infty)$)

Before closing this subsection, we would like to point out that shocks on ψ have effects similar to shocks on A_t (under even less restrictive parameter values). In fact, we can verify that as long as $\gamma > \frac{1}{\theta}$, there is a unique equilibrium, in which both Y_t and λ_t are decreasing in ψ; there are multiple (three) equilibria under $\gamma < \frac{1}{\theta}$.

[15] The middle equilibrium, corresponding to the interior solution of λ_t, is unstable to small deviations from this equilibrium.

2.3 Implications for Idiosyncratic Uncertainty and Firm-level Dispersion

Having obtained the equilibrium Y_t and λ_t, we now discuss the implications of the baseline model.

Idiosyncratic Uncertainty In equilibrium, the residual idiosyncratic uncertainty (forecast error) faced by an informed firm is

$$SD\left(\varepsilon_{jt}|s_{jt}\right) = \sqrt{\frac{1}{\tau_e + \tau_\varepsilon}}$$

which is lower than $\sqrt{\frac{1}{\tau_\varepsilon}}$, the residual idiosyncratic uncertainty faced by an uninformed firm. Considering that a higher λ_t is accompanied by a higher Y_t in equilibrium, Corollary 1 follows immediately.

Corollary 1 *In the economy of the baseline model, information acquisition is endogenously procyclical and the residual idiosyncratic uncertainty faced by firms is countercyclical (under $\gamma \in \left(\frac{1}{\theta}, 1\right)$).*

Corollary 1 is a key result of our paper. It highlights that information acquisition in our model is endogenous and procyclical, which has implications for countercyclical uncertainty faced by firms.

Firm-level Dispersion The empirical literature often uses firm-level dispersion as a proxy for economic uncertainty. We examine two measures of firm-level dispersion. First, we calculate the standard deviation of production (in log) at the firm level for a given A_t and m. As $\log y_{jt} = \log n_{jt} + \log A_t$, employment at the firm level has the same standard deviation as production. We obtain

$$SD\left(\log y_{jt}|A_t\right) = \begin{cases} 0 & \text{if } \log A_t < \log \underline{A} \\ \left[\lambda_t\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\right) + \frac{\lambda_t-\lambda_t^2}{4\theta^2}\frac{1}{\tau_\varepsilon}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^2\right]^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ \left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log A_t > \log \overline{A} \end{cases}, \quad (24)$$

where we calculate volatility as $SD(x) = \sqrt{\mathbb{E}^2(x) - [\mathbb{E}(x)]^2}$. The second line in (24) is increasing in $\lambda_t \in [0,1]$ under parameter condition $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon} \le 4\theta^2$.[16]

Second, we calculate the standard deviation of revenue-based TFP (or productivity) in log at the firm level, defined as $SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]\right)$. As $\log y_{jt} = \log n_{jt} + \log A_t$, the standard deviation

[16] The requirement of a parameter condition is due to the *mixed* distribution of $\log y_{jt}$ in our baseline model: a fraction $1-\lambda_t$ of firms form a mass-point for the distribution of production, while the remaining fraction λ_t is non-atomic.

of $\log p_{jt}$ at the firm level is the same as that of $\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]$. We obtain

$$SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]|A_t\right)=\begin{cases}\frac{1}{\theta}\sigma_\varepsilon & \text{if } \log A_t<\log\underline{A}\\ \left\{\begin{array}{l}\lambda_t\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_e}\right)+\left(1-\lambda_t\right)\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon}\\ +\lambda_t\left(1-\lambda_t\right)\left[\frac{1}{2\theta^2}\frac{1}{\tau_\varepsilon+\tau_e}-\frac{1}{2\theta^2}\frac{1}{\tau_\varepsilon}\right]^2\end{array}\right\}^{\frac{1}{2}} & \text{if } \log\underline{A}\le\log A_t\le\log\overline{A}\\ \frac{1}{\theta}\left(\frac{1}{\tau_\varepsilon+\tau_e}\right)^{\frac{1}{2}} & \text{if } \log A_t>\log\overline{A}\end{cases}. \tag{25}$$

The second line in (25) is decreasing in $\lambda_t\in[0,1]$ under parameter condition $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\le 4\theta^2$.

Corollary 2 *Suppose $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\le 4\theta^2$. In the economy of the baseline model, the firm-level dispersion in production or employment is procyclical, while the firm-level dispersion in productivity or sale price is countercyclical (under $\gamma\in\left(\frac{1}{\theta},1\right)$).*

Proof. *See Appendix A.* ■

The intuition behind Corollary 2 is the following. In a boom with a higher aggregate output, a larger fraction of firms acquire information and thus their production is more responsive to their true demand shocks, which leads to a higher dispersion of production and a lower dispersion of productivity across firms. We can verify that the dispersion of sales across firms is procyclical. Note that more precise information about idiosyncratic shocks ϵ_{jt} leads to firms' sale prices being more similar; in the extreme case where firms have perfect information about idiosyncratic shocks (i.e., $\tau_e=\infty$), for example, their sale prices would be the same (i.e., they would achieve the same optimal markup).

Corollaries 1 and 2 together might give a theoretical clarification for two concepts: economic uncertainty faced by the firms and firm-level dispersion as a measure of uncertainty. In our model, firms face a decrease in economic uncertainty in a boom, which is also the case in the recent work of Fajgelbaum, Schaal and Taschereau-Dumouchel (2014) and Straub and Ulbricht (2015). However, the uncertainty measured by firm-level dispersion (in production) can increase in a boom. In other words, firm-level dispersion as a proxy for uncertainty does not necessarily covary with uncertainty.[17]

2.4 Implications for Total Factor Productivity (TFP)

We now study the implications of our model for aggregate TFP. Since the endogenous TFP $z(\lambda_t)$ in equation (17) increases in λ_t, we have the following corollary.

[17] We owe this point to a referee.

Corollary 3 *Under $\gamma \in \left(\frac{1}{\theta}, 1\right)$, the measured endogenous total factor productivity (TFP) is procyclical.*

Recent studies by Restuccia and Rogerson (2008) have suggested that misallocation of resources across firms can have important effects on aggregate TFP. Hsieh and Klenow (2008) estimate that misallocation leads to a manufacturing TFP loss of 30-50% in China and 40-60% in India. Inspired by these studies, a growing literature has attributed resource reallocation to be an important determinant of TFP. Nevertheless, the literature has pointed to various reasons for resource misallocation, such as borrowing constraints, political influences, trade barriers, and so on.

Corollary 3 indicates that information frictions could be another important source of resource (mis)allocation. Due to ex ante demand uncertainty, ex post resource allocation is not efficient, in the sense that the ex post marginal product of labor is not equalized across firms. More information causes labor to be allocated toward firms with a higher marginal product of labor, resulting in a higher measured TFP. As information acquisition is procyclical, measured TFP is also procyclical. Interestingly, David, Hoppenhayn and Venkateswaran (2015) show that uncertainty in idiosyncratic demand shocks can generate a very significant TFP loss (i.e., ranging from 7 to 10% for China and India). This provides evidence for the key mechanism in our model, that uncertainty-induced reallocation is an important source of endogenous productivity fluctuations.

The procyclicality of TFP is a well-documented fact (see, e.g., Rotemberg and Summers (1990), Basu and Fernald (2001)) but it also poses a long-standing difficulty to business cycle theories based on demand shocks. One traditional explanation is cyclical capital utilization (e.g., Burnside, Eichenbaum and Rebelo (1995), Bai, Rios-Rull and Storesletten (2012)): firms use resources more intensively in booms, so the measured TFP increases. The information acquisition mechanism in our model provides an alternative explanation.[18] Basu and Fernald (2001) find that in booms productive factors are reallocated from firms low in social marginal productivity to firms high in social marginal productivity, as our model suggests.

[18] Our model implies that the aggregate markup is $\frac{\theta-1}{\theta}$, a constant, and so is the measured labor wedge. The latter does not square well with the data. This limitation of our model in this respect is due to the well-known constant price elasticity of the Dixit-Stiglitz aggregator. Many papers have shown that borrowing constraints in terms of working capital can generate countercyclical markups (e.g., Jermann and Quadrini (2012), Benhabib and Wang (2013). In the earlier version of our paper (Benhabib, Liu and Wang (2015)), we introduced a working capital constraint

$$W_t n_{jt} \leq \zeta_t E_t \left[p_{jt} y_{jt} | s_{jt} \right],$$

similar to that in Benhabib and Wang (2013). In that case, shocks to ζ_t generate countercyclical markups and hence countercyclical measured aggregate labor wedges. Moreover, it is also possible to generate countercyclical labor wedges if households face uncertainty and acquire information on ψ (consumption risk). The study of labor wedges is known to be important for understanding labor fluctuations (see Shimer (2009) for a review). But this study is not the focus of our paper and we leave it to future research.

2.5 Welfare Implications

We study the welfare implications of our baseline model. Total social welfare is measured by

$$U(C_t) - \psi N_t - \psi m \lambda_t \tag{26}$$

where $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, $C_t = Y_t$, and Y_t and N_t are both functions of λ_t given by (18) and (23), respectively. The optimal λ_t for the social planner under the constrained second-best equilibrium is given by the first-order condition of (26):

$$\frac{dU(Y_t)}{dY_t}\frac{dY_t}{d\lambda_t} - \psi\frac{dN_t}{d\lambda_t} = \psi m. \tag{27}$$

Applying (18) we have

$$\begin{aligned}
\frac{dU(Y_t)}{dY_t}\frac{dY_t}{d\lambda_t} &= Y_t^{-\gamma}\frac{dY_t}{dz_t}\frac{dz_t}{d\lambda_t} \\
&= Y_t^{-\gamma} \times \underbrace{\frac{1}{\gamma}\frac{Y_t}{z_t}}_{\frac{dY_t}{dz_t}} \times \underbrace{\frac{1}{\theta-1} z_t^{2-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dz_t}{d\lambda_t}} \\
&= \underbrace{Y_t^{-\gamma}}_{\frac{dU(Y_t)}{dY_t}} \times \underbrace{\frac{1}{\theta-1}\frac{1}{\gamma}Y_t z_t^{1-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dY_t}{d\lambda_t}}
\end{aligned}$$

and applying (23) we have

$$\psi\frac{dN_t}{d\lambda_t} = \underbrace{\left(1-\frac{1}{\theta}\right)(1-\gamma)Y_t^{-\gamma}}_{\psi\frac{dN_t}{dY_t}} \times \underbrace{\frac{1}{\theta-1}\frac{1}{\gamma}Y_t z_t^{1-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dY_t}{d\lambda_t}}.$$

So we can write (27) as

$$\left[\frac{1}{(\theta-1)\gamma} + \frac{\gamma-1}{\gamma}\frac{1}{\theta}\right] z_t^{1-\theta} Y_t^{1-\gamma}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = m\psi. \tag{28}$$

Equations (18) and (28) jointly give the constrained second-best equilibrium, noting that (28) parallels (20). Figure 4 depicts the constrained second-best equilibrium versus the market equilibrium. The $Y_t(\lambda_t; A_t)$ curve, given by (18), is the same for both equilibria. However, since $\frac{1}{(\theta-1)\gamma} + \frac{\gamma-1}{\gamma}\frac{1}{\theta} > \frac{1}{\theta}$, the $\lambda_t(Y_t, m)$ curve in the constrained second-best equilibrium is always on the right of its market equilibrium counterpart.

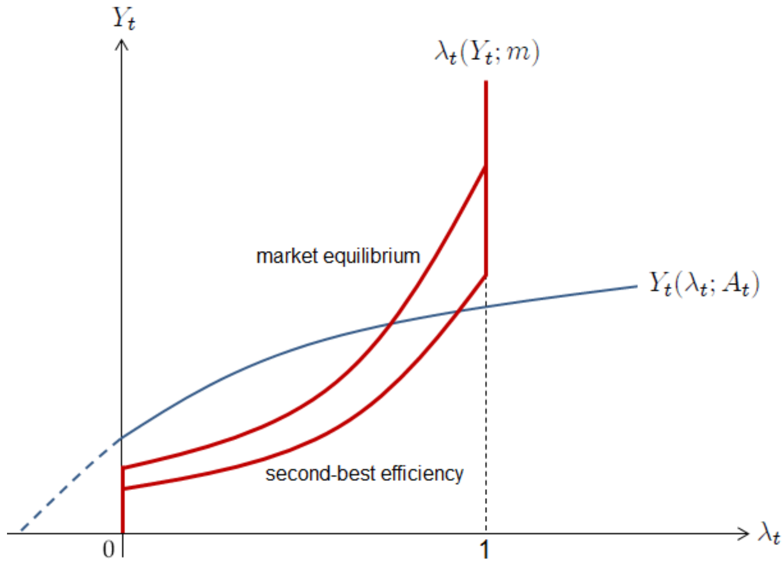

Figure 4: Market Equilibrium versus Constrained Second-best Equilibrium

Proposition 2 *The equilibrium λ_t is lower in the competitive equilibrium than under the second best (when $\theta\gamma - 1 > 0$). That is, too few individual firms acquire information in the competitive equilibrium.*

Proof. *See Appendix A.* ■

The intuition behind Proposition 2 is as follows. When an individual firm acquires information, it has two externalities: to other firms' profit and to the consumer surplus. Actually we can decompose and quantify these two externalities. Concretely, the social welfare can be decomposed into

$$
\begin{aligned}
&\frac{C_t^{1-\gamma}}{1-\gamma} - \psi N_t - \psi(m\lambda_t) \\
=\ & \underbrace{\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \psi P_t Y_t\right)}_{\text{Consumer surplus}} + \underbrace{\psi\left(P_t Y_t - N_t\right)}_{\text{Aggregate firm profits}} - \underbrace{\psi(m\lambda_t)}_{\text{Information acquisition costs}} \\
=\ & \left(\frac{\gamma}{1-\gamma}(Y_t(\lambda_t))^{1-\gamma}\right) + \psi\Pi_t(Y_t(\lambda_t), \lambda_t) - \psi(m\lambda_t),
\end{aligned}
$$

where $\Pi_t(Y_t, \lambda_t) = \frac{1}{\theta-1}(1-\frac{1}{\theta})^{\theta}\frac{1}{\psi}\left(\frac{A_t}{\psi}\right)^{\theta-1} Y_t^{1-\theta\gamma}\left(z(\lambda_t)\right)^{\theta-1}$. The third line above is obtained by applying $P_t = \frac{1}{\psi Y_t^{\gamma}}$ in (3) and $\Pi_t(Y_t, \lambda_t) = \lambda_t \pi_t^I + (1-\lambda_t)\pi_t^U$. An individual firm's optimal information acquisition problem is given by the first-order condition, taking Y_t as given:

$$
\underbrace{\psi\frac{\partial \Pi_t(Y_t, \lambda_t)}{\partial \lambda_t}}_{\text{Private value}} = \psi\left(\pi_t^I - \pi_t^U\right) = \psi m.
$$

In maximizing the social welfare, the social planner takes Y_t as endogenous and thus his first-order condition is

$$\underbrace{\frac{\partial\left(\frac{\gamma}{1-\gamma}Y_t^{1-\gamma}\right)}{\partial Y_t}\frac{dY_t}{d\lambda_t}}_{\substack{\text{Externality to consumer surplus}\\ >0}} + \underbrace{\psi\frac{\partial \Pi_t(Y_t,\lambda_t)}{\partial Y_t}\frac{dY_t}{d\lambda_t}}_{\substack{\text{Externality to other firms' profit}\\ <0}} + \underbrace{\psi\frac{\partial \Pi_t(Y_t,\lambda_t)}{\partial \lambda_t}}_{\text{Private value}} = \psi m. \tag{29}$$

The overall effect is that the positive externality to consumer surplus exceeds the negative externality to other firms' profit, so too few individual firms acquire information in the competitive equilibrium compared with the second best. In fact, we can calculate the sum of the first two terms in (29):

$$\frac{\partial\left(\frac{\gamma}{1-\gamma}Y_t^{1-\gamma}\right)}{\partial Y_t}\frac{dY_t}{d\lambda_t} + \psi\frac{\partial \Pi_t(Y_t,\lambda_t)}{\partial Y_t}\frac{dY_t}{d\lambda_t} = \frac{1}{\theta}Y_t^{-\gamma}\frac{dY_t}{d\lambda_t} > 0,$$

by noting that $\frac{dY_t}{d\lambda_t} > 0$.

3 Idiosyncratic Uncertainty and Aggregate Volatility

In this section, we conduct a simple extension of the baseline model to show that under the mechanism of endogenous information acquisition, countercyclical idiosyncratic uncertainty also manifests as countercyclical aggregate volatility. To do so, we necessarily need to introduce an aggregate (common) shock. We consider the aggregate sentiment shock in the spirit of Angeletos and La'O (2013a). The aggregate sentiment shock is a non-fundamental shock. We show that in a boom due to a positive TFP shock firms optimally choose to acquire more information, so they are less responsive to the non-fundamental sentiment shock, leading to a decrease in the volatility of aggregate output.

Information Structure and Information Acquisition Each entrepreneur receives two signals: x_{jt} and s_{jt}. First, following Angeletos and La'O (2013a), an entrepreneur receives a sentiment-related "public" signal:

$$x_{jt} = \varepsilon_{jt} + \Delta_t, \text{ where } \Delta_t \sim \mathcal{N}(0, \sigma_\Delta^2), \tag{30}$$

where the noise term, Δ_t, is the economy-wide common sentiment shock about *aggregate demand* (denote $\tau_\Delta = 1/\sigma_\Delta^2$). Second, as in the baseline model, s_{jt} is a "private" signal about the idiosyncratic demand shock, i.e.,

$$s_{jt} = \varepsilon_{jt} + e_{jt}, \text{ where } e_{jt} \sim \mathcal{N}(0, \sigma_e^2). \tag{31}$$

As in the baseline model, we consider discrete information acquisition; that is, if entrepreneur j spends m working hours acquiring information, the precision of his private signal s_{jt} would improve.

Timeline In the presence of aggregate shock Δ_t (which is imperfect information for entrepreneurs), the aggregate output Y_t and hence the aggregate price P_t are not deterministic. So when making decisions, entrepreneurs have to form expectations about Y_t (or Δ_t). The timing of events in this extended model is as follows:

1. At the beginning of each period, A_t and Δ_t are realized. The representative household has full information regarding Δ_t.[19]

2. After observing A_t, an entrepreneur makes his decision on whether to acquire information or not.

3. Based on signals, x_{jt} and s_{jt}, and nominal wage $W \equiv 1$, an entrepreneur decides how much labor n_{jt} to hire in producing his intermediate good. An entrepreneur has to optimally forecast the real wage W_t/P_t based on A_t and his signals.

4. Given the production y_{jt}, price p_{jt} adjusts to equate demand and supply according to equation (5).[20]

5. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

3.1 Equilibrium with Exogenous Information

Before we turn to the case of endogenous information acquisition, we first analyze the equilibrium under exogenous information. That is, in this subsection we assume precision $\tau_e = 1/\sigma_e^2$ is exogenously given and symmetric (same) for *all* entrepreneurs.

Since the representative household has perfect information about Δ_t, its consumption problem (or labor supply decision) is still given by (3).[21] An entrepreneur's production decision is still given

[19] Firms may obtain some information from households on Δ_t, for example a second noisy signal on sentiments (see Benhabib, Wang and Wen (2015), section 2.8). However, as long as the signal has some noise, our results would hold. A more formal treatment of this can be in the spirit of the analysis in Benhabib, Wang and Wen (2013) who introduce heterogeneous but correlated sentiments across households, so surveying a subset of households still leaves some sampling noise, assuming of course that surveying the full population of households is too costly or infeasible. For a related setup see also Angeletos, Lorenzoni and Pavan (2010); in their two-stage model, the public information (sentiment) of entrepreneurs in the first stage is not revealed to financial investors in the second stage. Finally, for further discussion of the microfoundations of signals, see section 3.1 of Benhabib, Wang and Wen (2015).

[20] The final goods firm might be informed of $\{\varepsilon_{jt}\}$ just at this stage, later than entrepreneurs making their production decisions in stage 3. Even if the final goods firm knows $\{\varepsilon_{jt}\}$ before entrepreneurs do, the information cannot be perfectly revealed to the entrepreneurs. A formal justification, for example, is that there is a continuum of final goods firms who have heterogeneous demand distribution over $\{\varepsilon_{jt}\}$ and the aggregate demand for any intermediate good j across these final goods firms is still ε_{jt}; so a sample of final goods firms can only have noisy information about $\{\varepsilon_{jt}\}$.

[21] In the presence of aggregate shock Δ_t, the real wage depends on Y_t or Δ_t (but not on idiosyncratic shocks $\{\varepsilon_{jt}\}$).

by (10) with the information set now changed to $\{x_{jt}, s_{jt}\}$; that is,

$$y_{jt} = y(A_t, x_{jt}, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{A_t}{\psi}\right)^\theta \left\{\mathbb{E}_t \left[\left(Y_t^{\frac{1}{\theta}-\gamma} \epsilon_{jt}^{\frac{1}{\theta}}\right) | x_{jt}, s_{jt}\right]\right\}^\theta. \tag{32}$$

Note that Y_t is a function of Δ_t and thus an entrepreneur has uncertainty about Y_t and has to form expectations about it, which is different from the case in the baseline model. The aggregate output is hence given by

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{A_t}{\psi}\right)^\theta \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[Y_t^{\frac{1}{\theta}-\gamma} \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt}\right]\right)^{\theta-1} dj\right]^{\frac{\theta}{\theta-1}}. \tag{33}$$

Equations (32) and (33) jointly determine the aggregate and individual equilibrium outputs. Once we have obtained Y_t and y_{jt}, we can first use (3) to compute the aggregate price P_t and then use (5) to compute the individual price p_{jt}. We use the guess-and-verify strategy to obtain Y_t and y_{jt}. We have the following proposition.

Proposition 3 *Under the information structure of (30) and (31), aggregate production is given by*

$$\log Y_t = \log \bar{Y} + \frac{1}{\gamma} \log A_t + \kappa \Delta_t, \tag{34}$$

where $\bar{Y}$ depends on θ, ψ, γ, σ_ε^2, σ_e^2, and σ_Δ^2, and

$$\kappa = \frac{\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}. \tag{35}$$

The individual production is given by

$$\begin{aligned} &\log y_{jt} \\ = \; &\log \bar{y} + \frac{1}{\gamma} \log A_t + \left[\frac{\tau_\Delta}{\tau_e + \tau_\Delta + \tau_\varepsilon} - (\theta\gamma - 1)\,\kappa \frac{\tau_e + \tau_\varepsilon}{\tau_e + \tau_\Delta + \tau_\varepsilon}\right] x_{jt} + \left[1 + (\theta\gamma - 1)\,\kappa\right] \frac{\tau_e}{\tau_e + \tau_\Delta + \tau_\varepsilon} s_{jt}, \end{aligned} \tag{36}$$

where $\bar{y}$ depends on θ, ψ, γ, σ_ε^2, σ_e^2, and σ_Δ^2.

Proof. *See Appendix A.* ■

3.2 Endogenous Information

We assume that A_t only takes one of two values, $A_t \in \{A_H, A_L\}$, where $A_H > A_L$. We construct A_H and A_L such that the information acquisition of the firms is symmetric. Specifically, all firms acquire information in equilibrium after observing $A_t = A_H$ but none does so after observing $A_t = A_L$. That is, we specify parameters to highlight the mechanism that the level of information precision chosen by entrepreneurs is endogenous and increasing in A_t. To save space, we provide the details of this subsection in the appendix.

We also prove that if $A_H >> A_L$, then $\mathbb{E}[\log Y_t | A_t = A_H] > \mathbb{E}[\log Y_t | A_t = A_L]$. Thus, we have Proposition 4.

Proposition 4 *The economy exhibits procyclical information acquisition (under $\gamma < 1$).*

Proof. *See Appendix A.* ■

The intuition for Proposition 4 is similar to that for the baseline model. Under $\gamma < 1$, the real wage does not increase as fast as aggregate output Y_t. Hence, when Y_t increases, the increase in the benefit of information acquisition (in terms of consumption units) outruns the increase in the associated cost (in terms of real wage), so the incentives to acquire information become stronger.

Proposition 3 and Proposition 4 together also imply that the sentiment shock Δ_t is more important in recessions than in booms. In fact, the coefficient κ of term Δ_t in (34) is decreasing in τ_e or A_t.

We can alternatively assume that information acquisition is symmetric across firms and is a continuous function of A_t. We show in Appendix D that our results in this section (Proposition 4 and Corollary 4 below) are robust to this alternative setup.[22]

3.3 Implications for Measured Uncertainty

First, it is straightforward to show that in equilibrium the residual idiosyncratic uncertainty faced by a firm decreases with information acquisition. In fact,

$$SD(\varepsilon_{jt} | x_{jt}, s_{jt}) = \sqrt{\frac{1}{\tau_e + \tau_\Delta + \tau_\varepsilon}},$$

that is, $SD(\varepsilon_{jt} | x_{jt}, s_{jt})$ is decreasing in τ_e.

Next, we calculate aggregate volatility, a common measure of economic uncertainty. The aggregate volatility in our model comes from the time-varying common sentiment shock Δ_t for a given A_t. We measure aggregate volatility as the unconditional standard deviation of aggregate output (in log). Based on (34), it is given by

$$SD(\log Y_t | A_t) = \sqrt{\left[\frac{\tau_\Delta}{\theta\gamma(\tau_\varepsilon + \tau_e) + \tau_\Delta}\right]^2 \frac{1}{\tau_\Delta}}.$$

[22] Discrete (binary choice) information acquisition and assuming that A_t takes one of two values is for tractability and to obtain analytical solutions. In the presence of an aggregate shock, the model is intractable if we assume asymmetric information acquisition across firms as in the baseline model (i.e., some firms acquire information while others do not). This is because aggregating in (33) is intractable after obtaining y_{jt}. On the other hand, if we assume that information acquisition is symmetric across firms and is a *continuous* function of A_t, as in this extension, we cannot obtain an analytical solution and must rely on numerical simulation.

Clearly, $SD(\log Y_t|A_t)$ is decreasing in τ_e. That is, aggregate volatility decreases under more precise information. Moreover, the residual aggregate uncertainty (or forecast error) faced by a firm is

$$SD\left(\log Y_t|A_t, x_{jt}, s_{jt}\right) = \frac{\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\sqrt{\frac{1}{\tau_e + \tau_\Delta + \tau_\varepsilon}}, \tag{37}$$

which is decreasing in τ_e.

Third, we calculate firm-level dispersion. For a given realization of A_t and Δ_t, heterogeneity of ε_{jt} and e_{jt} across firms generates firm-level dispersion. Based on (36), the standard deviation of production or employment is given by

$$SD\left(\log y_{jt}|A_t, \Delta_t\right) = \sqrt{\left[1 - \frac{\theta\gamma\tau_\varepsilon}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\right]^2 \frac{1}{\tau_\varepsilon} + \left[\frac{\theta\gamma\tau_e}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\right]^2 \frac{1}{\tau_e}},$$

which is increasing in τ_e. The standard deviation of revenue-based TFP (productivity) or sale price is given by

$$SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]|A_t, \Delta_t\right) = \frac{1}{\theta}\sqrt{\left[\frac{\theta\gamma}{\theta\gamma\left(\tau_\varepsilon + \tau_e\right) + \tau_\Delta}\right]^2 \left(\tau_\varepsilon + \tau_e\right)}, \tag{38}$$

which is decreasing in τ_e in the interval $\tau_e \in \left[\frac{\tau_\Delta}{\theta\gamma} - \tau_\varepsilon, \infty\right)$. Note that parameter condition $\frac{\tau_\Delta}{\theta\gamma} - \tau_\varepsilon < 0$ (or $\tau_\varepsilon > \frac{\tau_\Delta}{\theta\gamma}$) is easy to satisfy and we assume such a parameter condition, so (38) is always decreasing in τ_e. Corollary 4 follows.

Corollary 4 *The economy exhibits i) countercyclical idiosyncratic uncertainty, ii) countercyclical aggregate volatility and countercyclical aggregate uncertainty, and iii) procyclical firm-level dispersion in production and countercyclical firm-level dispersion in productivity.*

Proof. *See Appendix A.* ■

We discuss the intuition behind Corollary 4. In our model, aggregate volatility comes from the common sentiment shock Δ_t. When firms acquire information about signal s_{jt} and become more informed of their idiosyncratic demand shock ε_{jt}, they are less responsive to signal x_{jt} and thereby the "common demand shock" Δ_t, which decreases the aggregate volatility. In the extreme case of $\sigma_e^2 = 0$, for example, intermediate goods firms become perfectly informed of their idiosyncratic demand ε_{jt} for $A_t = A_H$ and the aggregate volatility becomes zero. When an individual firm uses signals x_{jt} and s_{jt} to forecast Y_t, the forecast error is also countercyclical because of two joint forces — countercyclical *unconditional* aggregate volatility and procyclical information precision (as seen in (37)). As for firm-level dispersion, like in the baseline model, when firms are more informed of their ε_{jt}, their production is more aligned with their ε_{jt}, thus increasing firm-level dispersion in production and decreasing firm-level dispersion in productivity.

Corollary 4 implies that our model's predictions are consistent with countercyclical idiosyncratic uncertainty, countercyclical aggregate uncertainty, and countercyclical aggregate volatility, but not countercyclical firm-level dispersion in production. In Appendix C, we consider an extension, which can also explain countercyclical cross-sectional dispersion in production. The basic intuition is the following. We assume that there are two layers of production: the final good is produced from a continuum of *sectoral* goods, and each sectorial good is composed of a continuum of differentiated goods with each being produced by a monopoly firm. A firm's "public" signal is the sum of the economy-wide common sentiment shock, the *sector-specific* sentiment shock and the firm's idiosyncratic demand shock. The cross-sectional dispersion comes from the heterogeneity of sector-specific sentiment shocks. When firms acquire more information, they are less responsive to their "public" signal and thus their sector-specific sentiment shock, so the cross-sectional dispersion goes down.

4 Conclusion

In the large and growing recent literature on economic uncertainty, one important issue seems to have received particular attention: the direction of causality between economic uncertainty and macroeconomic activity. While some researchers propose that the causality runs from the second moment (uncertainty) to the first moment (macroeconomic activity) through mechanisms such as the traditional "wait-and-see" effect and the rise in the cost of capital due to the concave payoffs of debt contracts, some empirical findings suggest that the direction of causality might go the other way round.

In this paper, we develop a third approach, suggesting that primitive shocks such as a TFP shock can simultaneously drive uncertainty movements and business cycles and trigger two-way feedback between them.[23] We introduce endogenous information acquisition for firms facing demand shocks in an otherwise standard monopolistically competitive model. The precision of the information about demand shocks optimally acquired by firms varies across business cycles. Procyclical information acquisition arises naturally in our model with standard preference and technology specifications. The endogenous information acquisition affects not only the residual uncertainty (forecast error) faced by the firms in equilibrium but also the efficiency of resource allocation — the endogenous TFP. The prediction of our model is consistent with the observed countercyclical aggregate volatility — the macro-level measured economic uncertainty. Our framework can also be extended to explain countercyclical cross-sectional dispersion — the micro-level measured uncertainty.

[23] Financial friction shocks can have an effect similar to TFP shocks (see Benhabib, Liu and Wang (2015)).

Appendix

A Proofs

Proof of Proposition 1: First, we have

$$\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \mathbb{E}_{s_{jt}}\left[\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta}\right]$$

and

$$\int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj = \mathbb{E}_{\epsilon_{jt},s_{jt}}\left[\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}\right].$$

By the law of iterated expectations, it follows that

$$\begin{aligned}
\mathbb{E}_{\epsilon_{jt},s_{jt}}\left[\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}\right] &= \mathbb{E}_{s_{jt}}\mathbb{E}_{\epsilon_{jt}|s_{jt}}\left[\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}|s_{jt}\right] \\
&= \mathbb{E}_{s_{jt}}\left[\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}\right] \\
&= \mathbb{E}_{s_{jt}}\left[\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta}\right].
\end{aligned}$$

Hence, $\int\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj$. Similarly, $\int\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj = \int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj$.

Thus, we find that

$$\int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj + \int_{\lambda_t}^1 \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj = \int_0^{\lambda_t}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj + \int_{\lambda_t}^1\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj,$$

based on which (13) can be transformed into (15) while (13) and (14) together yield (16).

Second, we calculate $\int\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj$. Then all key variables can be expressed with this term. We have $\int\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \int\left(\mathbb{E}\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)|s_{jt}\right]\right)^{\theta} dj$. By the property of log-normal distribution, we further obtain

$$\begin{aligned}
&\int\left(\mathbb{E}\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)|s_{jt}\right]\right)^{\theta} dj \\
=& \int\left[\exp\left(\frac{1}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)+\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}\right]+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_e}\right)\right]^{\theta} dj \\
=& \exp\left[-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right] = \bar{z}^{\theta-1}. \qquad \text{(A.1)}
\end{aligned}$$

Also, when $\tau_e = 0$, (A.1) corresponds to $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta} dj = \exp\left[-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon} \right] = \underline{z}^{\theta-1}$.

Third, using the above results, we obtain

$$
\begin{aligned}
z_t &= \left[\int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta-1} dj + \int_{\lambda_t}^{1} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta-1} dj \right]^{\frac{1}{\theta-1}} \\
&= \left[\lambda_t \bar{z}^{\theta-1} + (1-\lambda_t)\, \underline{z}^{\theta-1} \right]^{\frac{1}{\theta-1}},
\end{aligned}
$$

which proves equation (17).

Fourth, we calculate the ex ante expected profit for a firm. For an informed firm, its realized profit is

$$
\begin{aligned}
\pi(\epsilon_{jt}, s_{jt}) &= p_{jt}\left(\epsilon_{jt}, y_{jt}\right) y_{jt} - W_t n_{jt}\left(y_{jt}\right) \\
&= \left(1 - \frac{1}{\theta}\right)^{\theta-1} P_t^{\theta} A_t^{\theta-1} Y_t \left\{ \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}) \right]^{\theta-1} \epsilon_{jt}^{\frac{1}{\theta}} - \left(1 - \frac{1}{\theta}\right) \left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}) \right]^{\theta} \right\}.
\end{aligned}
$$

Exploiting the law of iterated expectations, we obtain

$$
\begin{aligned}
\pi_t^I &= \mathbb{E}_{\epsilon_{jt}, s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt}) \right] = \mathbb{E}_{s_{jt}} \mathbb{E}_{\epsilon_{jt}|s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt}) | s_{jt} \right] \\
&= \left(1 - \frac{1}{\theta}\right)^{\theta-1} P_t^{\theta} A_t^{\theta-1} Y_t \cdot \frac{1}{\theta} \mathbb{E}_{s_{jt}} \left(\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}) \right]^{\theta} \right) \\
&= \frac{1}{\theta-1} \frac{1}{A_t} \bar{z}^{\theta-1} z_t^{-\theta} Y_t,
\end{aligned}
$$

where the last line is obtained by applying $\frac{1}{P_t} = (1 - \frac{1}{\theta})\,(A_t z_t)$ in (15). Similarly, we can find the ex ante expected profit for an uninformed firm:

$$
\pi_t^U = \frac{1}{\theta-1} \frac{1}{A_t} \underline{z}^{\theta-1} z_t^{-\theta} Y_t.
$$

Fifth, we work out the expressions of equilibrium variables Y_t and λ_t. The full equilibrium is given by (22) with considering corner solutions in (9). Denote the LHS of (22) by function $G(\lambda_t)$.

i) The case of $\gamma \in \left(\frac{1}{\theta}, 1\right)$

When A_t is lower such that $G(\lambda_t = 0) < \psi m$ (by noting that $G'(\lambda_t) < 0$ for $\theta\gamma > 1$), the unique equilibrium is $\lambda_t = 0$. When A_t is higher such that $G(\lambda_t = 1) > \psi m$, the unique equilibrium is $\lambda_t = 1$. When A_t is in an intermediate range such that equation (22) has an interior solution of λ_t, the unique equilibrium corresponds to the interior solution. Concretely, the expression of the equilibrium variables is given by (A.2) and (A.3):

$$\log Y_t = \log(\theta-1) + \frac{1}{\gamma}(\log A_t + \log z_t)$$
$$= \begin{cases} \left[\log(\theta-1) + \frac{1}{\gamma}\log \underline{z}\right] + \frac{1}{\gamma}\log A_t & \text{if } \log A_t < \log \underline{A} \\ \left[\log(\theta-1) - \frac{1}{\theta\gamma-1}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)\right] + \frac{\theta-1}{\theta\gamma-1}\log A_t & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ \left[\log(\theta-1) + \frac{1}{\gamma}\log \overline{z}\right] + \frac{1}{\gamma}\log A_t & \text{if } \log A_t > \log \overline{A} \end{cases} \quad \text{(A.2)}$$

and

$$\lambda_t = \frac{z_t^{\theta-1} - \underline{z}^{\theta-1}}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}$$
$$= \begin{cases} 0 & \text{if } \log A_t < \log \underline{A} \\ \frac{\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)^{-\frac{(\theta-1)\gamma}{\theta\gamma-1}} \exp\left(\frac{(\theta-1)(1-\gamma)}{\theta\gamma-1}\log A_t\right) - \underline{z}^{\theta-1}}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}} & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ 1 & \text{if } \log A_t > \log \overline{A} \end{cases}, \quad \text{(A.3)}$$

where $\log \underline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\underline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$ and $\log \overline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\overline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$, and to simplify notation we normalize $\frac{1}{\theta-1}(1-\frac{1}{\theta})^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}} = 1$ without loss of generality.

Figure A-1 shows the amplification effect (multiplier effect) discussed in the main text.

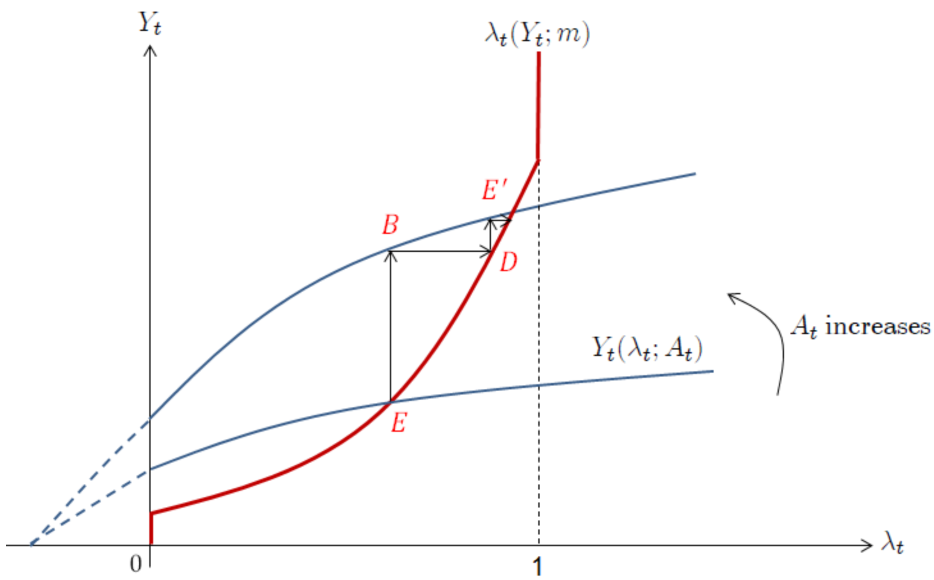

Figure A-1: Amplification Effect under Endogenous Information Acquisition for $\gamma \in \left(\frac{1}{\theta}, 1\right)$

ii) The case of $\gamma \in \left(0, \frac{1}{\theta}\right)$

If equation (22) has an interior solution $\lambda_t \in (0,1)$, there must be three equilibria: besides the equilibrium corresponding to the interior solution, $\lambda_t = 0$ and 1 are also equilibria. In fact, when $\theta\gamma < 1$, $G'(\lambda_t) > 0$. So $G(\lambda_t = 1) > \psi m$. By (9), the corner solution $\lambda_t = 1$ is also an equilibrium.

Similarly, the corner solution $\lambda_t = 0$ is another equilibrium.

When A_t is lower such that $G(\lambda_t = 1) < \psi m$, the unique equilibrium is $\lambda_t = 0$. When A_t is higher such that $G(\lambda_t = 0) > \psi m$, the unique equilibrium is $\lambda_t = 1$. When A_t is in an intermediate range such that equation (22) has an interior solution of λ_t, there are three equilibria.

Concretely, there are three equilibria for $\log \overline{A} \leq \log A_t \leq \log \underline{A}$ (by noting that $\overline{A} < \underline{A}$ under $\theta\gamma < 1$, and $\underline{A}$ and $\overline{A}$ are defined in (A.3)). Besides the equilibrium given by (A.2) and (A.3), the expressions of the two additional equilibria are $\log z_t = \log \underline{z}$ with $\log Y_t = \left[\log(\theta - 1) + \frac{1}{\gamma}\log \underline{z}\right] + \frac{1}{\gamma}\log A_t$ and $\log z_t = \log \overline{z}$ with $\log Y_t = \left[\log(\theta - 1) + \frac{1}{\gamma}\log \overline{z}\right] + \frac{1}{\gamma}\log A_t$. For $\log A_t > \log \underline{A}$ and $\log A_t < \log \overline{A}$, the unique equilibrium is given by (A.2) and (A.3).

iii) The case of $\gamma \in (1, \infty)$

When A_t is lower such that $G(\lambda_t = 1) > \psi m$, the unique equilibrium is $\lambda_t = 1$. When A_t is higher such that $G(\lambda_t = 0) < \psi m$, the unique equilibrium is $\lambda_t = 0$. When A_t is in an intermediate range such that equation (22) has an interior solution of λ_t, the unique equilibrium corresponds to the interior solution. Concretely, the expressions of the equilibrium variables are as follows:

$$
\begin{aligned}
\log Y_t &= \log(\theta - 1) + \frac{1}{\gamma}(\log A_t + \log z_t) \\
&= \begin{cases}
\left[\log(\theta - 1) + \frac{1}{\gamma}\log \overline{z}\right] + \frac{1}{\gamma}\log A_t & \text{if } \log A_t < \log \underline{A} \\
\left[\log(\theta - 1) - \frac{1}{\theta\gamma - 1}\log\left(\frac{m}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}\right)\right] + \frac{\theta-1}{\theta\gamma-1}\log A_t & \text{if } \log \underline{A} \leq \log A_t \leq \log \overline{A} \\
\left[\log(\theta - 1) + \frac{1}{\gamma}\log \underline{z}\right] + \frac{1}{\gamma}\log A_t & \text{if } \log A_t > \log \overline{A}
\end{cases}
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda_t &= \frac{z_t^{\theta-1} - \underline{z}^{\theta-1}}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}} \\
&= \begin{cases}
1 & \text{if } \log A_t < \log \underline{A} \\
\frac{\left(\frac{m}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}\right)^{-\frac{(\theta-1)\gamma}{\theta\gamma-1}} \exp\left(-\frac{(\theta-1)(\gamma-1)}{\theta\gamma-1}\log A_t\right) - \underline{z}^{\theta-1}}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}} & \text{if } \log \underline{A} \leq \log A_t \leq \log \overline{A} \\
0 & \text{if } \log A_t > \log \overline{A}
\end{cases},
\end{aligned}
$$

where $\log \underline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}\overline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$ and $\log \overline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}\underline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$. See also Benhabib, Liu and Wang (2015) for the case of $\gamma \in (1, \infty)$.

Sixth, by (A.2), $\log Y_t(A_t; m)$ is continuous and increasing in A_t under $\frac{\theta-1}{\theta\gamma-1} > 0$ or $\gamma > \frac{1}{\theta}$. By (A.3), $\lambda_t(A_t; m)$ is continuous and increasing in A_t under $\frac{1-\gamma}{\theta\gamma-1} > 0$ or $\frac{1}{\theta} < \gamma < 1$.

Finally, we prove that in the case of multiple (three) equilibria under $\gamma \in \left(\frac{1}{\theta}, 1\right)$, the equilibrium with $\lambda_t = 1$ is the most efficient and the equilibrium with $\lambda_t = 0$ is the least efficient. By (22), in the case of multiple (three) equilibria, the following equation holds:

$$\frac{1}{\theta}\left[\left(1-\frac{1}{\theta}\right)\frac{1}{\psi}\right]^{\frac{1-\gamma}{\gamma}} A_t^{\frac{1-\gamma}{\gamma}} \left(z(\lambda_t)\right)^{\frac{1-\theta\gamma}{\gamma}} \left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)=\psi m.$$

As shown in Section 2.5, the welfare is measured by $U(C_t)-\psi N_t-\psi m\lambda_t$. Hence, the welfare for the equilibrium of $\lambda_t=1$ is

$$\left[\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)\right]\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}} \overline{z}^{\frac{1-\theta\gamma}{\gamma}}\overline{z}^{\theta-1}-\psi m; \tag{A.4}$$

the welfare for the interior equilibrium is

$$\left[\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)\right]\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}}\left[z(\lambda_t)\right]^{\frac{1-\theta\gamma}{\gamma}}\left[\lambda_t\overline{z}^{\theta-1}+(1-\lambda_t)\,\underline{z}^{\theta-1}\right]-\psi m\lambda_t; \tag{A.5}$$

and the welfare for the equilibrium of $\lambda_t=0$ is

$$\left[\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)\right]\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}} \underline{z}^{\frac{1-\theta\gamma}{\gamma}}\underline{z}^{\theta-1}. \tag{A.6}$$

It is easy to show that (A.4)>(A.5)>(A.6). In fact,

$$\begin{aligned}
&\text{(A.4)-(A.5)}\\
>\ &(1-\lambda_t)\left\{\left[\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)\right]\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}}\left[z(\lambda_t)\right]^{\frac{1-\theta\gamma}{\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)-\psi m\right\}\\
>\ &(1-\lambda_t)\left\{\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}}\left[z(\lambda_t)\right]^{\frac{1-\theta\gamma}{\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)-\psi m\right\}=0,
\end{aligned}$$

and

$$\begin{aligned}
&\text{(A.5)-(A.6)}\\
>\ &\lambda_t\left\{\left[\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)\right]\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}}\left[z(\lambda_t)\right]^{\frac{1-\theta\gamma}{\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)-\psi m\right\}\\
>\ &\lambda_t\left\{\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{A_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}}\left[z(\lambda_t)\right]^{\frac{1-\theta\gamma}{\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)-\psi m\right\}=0.
\end{aligned}$$

Even if we consider only the aggregate profit but not the consumer surplus, the equilibrium with $\lambda_t=1$ would still be the most efficient and the equilibrium with $\lambda_t=0$ still the least efficient because $\frac{1}{1-\gamma}-\left(1-\frac{1}{\theta}\right)>\frac{1}{\theta}$. Also, considering that firms are symmetric and have the same level of profit in any given equilibrium, the equilibrium with $\lambda_t=1$ is also Pareto efficient.

Proof of Corollary 2: Denote the type of firms acquiring information by I and the type of those that do not by U. We examine the dispersion of production across intermediate goods firms. We find

$$y_{jt}^{I} \equiv y(A_t, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} Y_t \exp\left[\frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{\tau_e}{\tau_\varepsilon + \tau_e} s_{jt} + \frac{1}{2\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right].$$

We have

$$\begin{aligned}
\mathbb{E}\left[\left(\log y_{jt}^{I}\right)^2\right] &= Var\left[\log y_{jt}^{I}\right] + \left[\mathbb{E}\left(\log y_{jt}^{I}\right)\right]^2 \\
&= \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\right)^2 \left(\sigma_\varepsilon^2 + \sigma_e^2\right) + \left(\varphi - \frac{1}{2}\sigma_\varepsilon^2 + \frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right)^2 \\
&= \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\right)\left(\frac{1}{\tau_\varepsilon}\right) + \left(\varphi - \frac{1}{2}\frac{1}{\tau_\varepsilon} + \frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right)^2,
\end{aligned}$$

where $\varphi = \log\left[\left(1 - \frac{1}{\theta}\right)^{\theta} (P_t A_t)^{\theta} Y_t\right]$. Similarly,

$$\mathbb{E}\left[\left(\log y_{jt}^{U}\right)^2\right] = \left(\varphi - \frac{1}{2}\sigma_\varepsilon^2 + \frac{1}{2}\frac{1}{\theta}\sigma_\varepsilon^2\right)^2,$$

where $y_{jt}^{U} \equiv \tilde{y}(A_t)$. Thus,

$$\begin{aligned}
Var\left[\log y_{jt}\right] &= \mathbb{E}\left[\left(\log y_{jt}\right)^2\right] - \left[\mathbb{E}\left(\log y_{jt}\right)\right]^2 \\
&= \left\{\begin{array}{c} \lambda_t \mathbb{E}\left[\left(\log y_{jt}^{I}\right)^2\right] \\ + (1-\lambda_t)\,\mathbb{E}\left[\left(\log y_{jt}^{U}\right)^2\right] \end{array}\right\} - \left[\begin{array}{c} \lambda_t \mathbb{E}\left(\log y_{jt}^{I}\right) \\ + (1-\lambda_t)\,\mathbb{E}\left(\log y_{jt}^{U}\right) \end{array}\right]^2 \\
&= \lambda_t \left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon}\right) + \frac{\lambda_t - \lambda_t^2}{4\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon}\right)^2,
\end{aligned}$$

which is a quadratic equation with respect to λ_t. It is increasing in $\lambda_t \in [0,1]$ if and only if $\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon} \geq \frac{1}{4\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon}\right)^2$, that is, $\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon} \leq 4\theta^2$.

Denote by p_{jt}^{I} the sale price for an informed firm and by p_{jt}^{U} the sale price for an uninformed firm. So,

$$p_{jt}^{I} = \exp\left\{\varphi_1 + \left(-\frac{1}{\theta}\right)\left[\frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{\tau_e}{\tau_\varepsilon + \tau_e} e_{jt} + \frac{1}{2\theta}\frac{1}{\tau_\varepsilon + \tau_e}\right] + \frac{1}{\theta}\frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_e}\varepsilon_{jt}\right\}$$

and

$$p_{jt}^{U} = \exp\left\{\varphi_1 + \left(-\frac{1}{\theta}\right)\left[\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{1}{2\theta}\sigma_\varepsilon^2\right] + \frac{1}{\theta}\varepsilon_{jt}\right\}$$

where $\varphi_1 = \log\left[P_t \cdot Y_t^{\frac{1}{\theta}} \cdot \left(1-\frac{1}{\theta}\right)^{-1}(P_t A_t)^{-1} Y_t^{-\frac{1}{\theta}}\right]$. We have

$$\mathbb{E}\left[\left(\log p_{jt}^I\right)^2\right] = Var\left[\log p_{jt}^I\right] + \left[\mathbb{E}\left(\log p_{jt}^I\right)\right]^2$$
$$= \left(\frac{1}{\theta}\right)^2\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^2\sigma_e^2 + \left(\frac{1}{\theta}\right)^2\left(\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\right)^2\sigma_\varepsilon^2 + \left\{\varphi_1 + \left(-\frac{1}{\theta}\right)\left[\frac{1}{2\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right]\right\}^2$$

and

$$\begin{aligned}\mathbb{E}\left[\left(\log p_{jt}^U\right)^2\right] &= Var\left[\log p_{jt}^U\right] + \left[\mathbb{E}\left(\log p_{jt}^U\right)\right]^2 \\ &= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left\{\varphi_1 + \left(-\frac{1}{\theta}\right)\left[\frac{1}{2\theta}\frac{1}{\tau_\varepsilon}\right]\right\}^2.\end{aligned}$$

Thus,

$$\begin{aligned}Var\left[\log p_{jt}\right] &= \mathbb{E}\left[\left(\log p_{jt}\right)^2\right] - \left[\mathbb{E}\left(\log p_{jt}\right)\right]^2 \\ &= \lambda_t\left(\frac{1}{\theta}\right)^2\left(\frac{1}{\tau_\varepsilon+\tau_e}\right) + (1-\lambda_t)\left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \lambda_t(1-\lambda_t)\left[\frac{1}{2\theta^2}\frac{1}{\tau_\varepsilon+\tau_e} - \frac{1}{2\theta^2}\frac{1}{\tau_\varepsilon}\right]^2,\end{aligned}$$

which is decreasing in $\lambda_t \in [0,1]$ if and only if $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon} \leq 4\theta^2$.

Because $\frac{p_{jt}y_{jt}}{n_{jt}} = p_{jt}A_t$, we also have $Var\left[\log p_{jt}\right] = Var\left[\log \frac{p_{jt}y_{jt}}{n_{jt}}\right]$.

Proof of Proposition 2: Equations (18) and (20) jointly give

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{A}{\psi}\right)^{\frac{1}{\gamma}-1}\left[\lambda_t\overline{z}^{\theta-1} + (1-\lambda_t)\underline{z}^{\theta-1}\right]^{\frac{1-\gamma\theta}{(\theta-1)\gamma}}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = \psi m. \tag{A.7}$$

Equations (18) and (28) jointly give

$$\left[\frac{1}{(\theta-1)\gamma} + \frac{\gamma-1}{\gamma}\frac{1}{\theta}\right]\left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}-1}\left(\frac{A}{\psi}\right)^{\frac{1}{\gamma}-1}\left[\lambda_t\overline{z}^{\theta-1} + (1-\lambda_t)\underline{z}^{\theta-1}\right]^{\frac{1-\gamma\theta}{(\theta-1)\gamma}}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = \psi m. \tag{A.8}$$

Notice that the LHS of (A.8) is always greater than the LHS of (A.7) while their RHS are the same. Also, when $1-\theta\gamma < 0$, the LHS of either (A.7) or (A.8) is a decreasing function of λ_t. So the optimal λ_t given by (A.7) must be lower than that given by (A.8).

We calculate

$$\begin{aligned}\Pi_t(Y_t,\lambda_t) &= \lambda_t\pi_t^I + (1-\lambda_t)\pi_t^U \\ &= \frac{1}{\theta-1}\frac{1}{A_t}Y_t z_t^{-\theta}\left[\lambda_t\overline{z}^{\theta-1} + (1-\lambda_t)\underline{z}^{\theta-1}\right] \\ &= \frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\frac{1}{\psi}\left(\frac{A_t}{\psi}\right)^{\theta-1} Y_t^{1-\theta\gamma}\left(z(\lambda_t)\right)^{\theta-1},\end{aligned}$$

where the last line above is obtained by applying $z_t = \frac{\theta}{\theta-1}\psi\frac{1}{A_t}Y_t^{\gamma}$ from (18). Then,

$$\begin{aligned}
& \frac{\partial\left(\frac{\gamma}{1-\gamma}Y_t^{1-\gamma}\right)}{\partial Y_t}\frac{dY_t}{d\lambda_t} + \psi\frac{\partial\Pi_t(Y_t,\lambda_t)}{\partial Y_t}\frac{dY_t}{d\lambda_t} \\
= & \left[\gamma Y_t^{-\gamma} + \frac{1}{\theta-1}(1-\frac{1}{\theta})^{\theta}\left(\frac{A_t}{\psi}\right)^{\theta-1}(1-\theta\gamma)\,Y_t^{-\theta\gamma}z_t^{\theta-1}\right]\frac{dY_t}{d\lambda_t} \\
= & \frac{1}{\theta}Y_t^{-\gamma}\frac{dY_t}{d\lambda_t} > 0,
\end{aligned}$$

where the third line is obtained by applying $\frac{A_t}{\psi} = \frac{\theta}{\theta-1}Y_t^{\gamma}\frac{1}{z_t}$ from (18).

Proof of Proposition 3: An individual entrepreneur uses $x_{jt} = \varepsilon_{jt} + \Delta_t$ and $s_{jt} = \varepsilon_{jt} + e_{jt}$ to infer ε_{jt} and uses $x_{jt} = \Delta_t + \varepsilon_{jt}$ and $x_{jt} - s_{jt} = \Delta_t - e_{jt}$ to infer Δ_t. We conjecture that $Y_t = \bar{Y}A_t^{\kappa_{(1)}}\exp(\kappa\Delta_t)$. Substituting the conjectured Y_t into (32) yields

$$\begin{aligned}
y_{jt} &= \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A_t}{\psi}\right)^{\theta}\left\{\mathbb{E}_t\left[\left(Y_t^{\frac{1}{\theta}-\gamma}\epsilon_{jt}^{\frac{1}{\theta}}\right)|x_{jt},s_{jt}\right]\right\}^{\theta} \\
&= \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}^{1-\theta\gamma}A_t^{\theta+\kappa_{(1)}(1-\theta\gamma)}\exp\left[\begin{array}{l}\frac{\frac{1}{\sigma_\Delta^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}x_{jt} + \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}s_{jt} \\ +(1-\theta\gamma)\,\kappa\left[\begin{array}{l}\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}x_{jt} \\ +\frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}(x_{jt}-s_{jt})\end{array}\right] \\ +\theta\mu + \frac{1}{2}\theta Var\end{array}\right],
\end{aligned} \tag{A.9}$$

where

$$\begin{aligned}
\mu &= \frac{1}{2}\left[\left(\frac{1}{\theta}-\gamma\right)\kappa - \frac{1}{\theta}\right]\frac{1}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} \\
Var &= \left\{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa\right]^2 + \left(\frac{1}{\theta}\right)^2\right\}\frac{1}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}.
\end{aligned}$$

Substituting $Y_t = \bar{Y} A_t^{\kappa_{(1)}} \exp(\kappa \Delta_t)$ into the LHS of $Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$ and substituting (A.9) into the RHS gives

$$\begin{aligned}
\bar{Y} A_t^{\kappa_{(1)}} \exp(\kappa \Delta_t) &= \left(1-\frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}^{1-\theta\gamma} A_t^{\theta+\kappa_{(1)}(1-\theta\gamma)} \left\{\exp\left[\theta\mu + \frac{1}{2}\theta Var\right]\right\} \\
&\cdot \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} \exp\left(\frac{\theta-1}{\theta}\left[\begin{array}{c} \frac{\frac{1}{\sigma_\Delta^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} x_{jt} + \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} s_{jt} \\ +(1-\theta\gamma)\kappa\left[\begin{array}{c} \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} x_{jt} \\ + \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} (x_{jt}-s_{jt}) \end{array}\right] \end{array}\right]\right) dj\right]^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

Comparing coefficients, we have $\kappa_{(1)} = \frac{1}{\gamma}$ and

$$\frac{\frac{1}{\sigma_\Delta^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} + (1-\theta\gamma)\kappa \frac{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} = \kappa,$$

that is, $\kappa = \frac{\frac{1}{\sigma_\Delta^2}}{\theta\gamma\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_e^2}\right)+\frac{1}{\sigma_\Delta^2}}$; also,

$$\bar{Y} = \left(\frac{1-\frac{1}{\theta}}{\psi}\right)^{\frac{1}{\gamma}} \left[\exp\left(\mu + \frac{1}{2}Var\right)\right]^{\frac{1}{\gamma}} \left[\exp\left(\begin{array}{l} \left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\left[\begin{array}{c} \frac{\tau_\Delta+\tau_e}{\tau_e+\tau_\Delta+\tau_\varepsilon} \\ -\frac{\tau_\Delta}{\tau_e+\tau_\Delta+\tau_\varepsilon}\frac{(\theta\gamma-1)\tau_\varepsilon}{\theta\gamma(\tau_\varepsilon+\tau_e)+\tau_\Delta} \end{array}\right]\right)\left(-\frac{1}{2}\sigma_\varepsilon^2\right) \\ +\frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\left[\begin{array}{c} \frac{\tau_\Delta+\tau_e}{\tau_e+\tau_\Delta+\tau_\varepsilon} \\ -\frac{\tau_\Delta}{\tau_e+\tau_\Delta+\tau_\varepsilon}\frac{(\theta\gamma-1)\tau_\varepsilon}{\theta\gamma(\tau_\varepsilon+\tau_e)+\tau_\Delta} \end{array}\right]\right)^2 \sigma_\varepsilon^2 \\ +\frac{1}{2}\left(\frac{\theta-1}{\theta}\left[\frac{\theta\gamma\tau_e}{\theta\gamma(\tau_\varepsilon+\tau_e)+\tau_\Delta}\right]\right)^2 \sigma_e^2 \end{array}\right)\right]^{\frac{1}{(\theta-1)\gamma}}.$$

After working out $\bar{Y}$, $\kappa_{(1)}$ and κ, we also have the solution to y_{jt}, that is,

$$\begin{aligned}
&\log y_{jt} \\
&= \log\bar{y} + \frac{1}{\gamma}\log A_t + \left[\begin{array}{c} \frac{\frac{1}{\sigma_\Delta^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} \\ -(\theta\gamma-1)\kappa\frac{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}} \end{array}\right] x_{jt} + \left[\left[1+(\theta\gamma-1)\kappa\right]\frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}\right] s_{jt},
\end{aligned} \tag{A.10}$$

where

$$\bar{y} = \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta} \bar{Y}^{1-\theta\gamma}\exp\left(\theta\mu + \frac{1}{2}\theta Var\right).$$

Proof of Proposition 4: We consider discrete information acquisition; namely, the amount of working hours on information acquisition $\ell \in \{0, m\}$. We assume the following mapping:

$$\sigma_e^2 = \begin{cases} \bar{\sigma}_e^2 & \text{if } \ell = 0 \\ \underline{\sigma}_e^2 & \text{if } \ell = m \end{cases},$$

where $\bar{\sigma}_e^2 > \underline{\sigma}_e^2$; that is, information acquisition makes information about the idiosyncratic demand shock more precise (in the extreme case of $\underline{\sigma}_e^2 = 0$, ε_{jt} becomes perfect information). The baseline model can be regarded as a special case of the current setup of information acquisition with $\bar{\sigma}_e^2 = \infty$.

We examine the regimes of $A_t = A_H$ and $A_t = A_L$.

The Case of $A_t = A_H$ We solve firms' information acquisition problem by backward induction. We first work out firms' optimal production decision given their information precision. We then compare the cost and benefit of information acquisition to determine firms' optimal information acquisition decision.

The production for an informed entrepreneur, $y(A_t, x_{jt}, s_{jt})$, is given by equation (32), and his expected profit is given by $\pi(A_t, x_{jt}, s_{jt}) = \frac{1}{\theta-1} y(A_t, x_{jt}, s_{jt})/A_t$, where $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \underline{\sigma}_e^2)$. For an uninformed entrepreneur, the only difference is that $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \bar{\sigma}_e^2)$.

If $\lambda_t = 1$ in equilibrium, the aggregate output is given by (33). Since all firms acquire information, we can directly apply (34) in Proposition 3 to solve Y_t by setting $\sigma_e^2 = \underline{\sigma}_e^2$. It follows that

$$\log Y_t = \log \bar{Y}_H + \frac{1}{\gamma} \log A_H + \kappa \Delta_t, \tag{A.11}$$

where $\bar{Y}_H$ is the constant $\bar{Y}$ in the proof of Proposition 3 with $\sigma_e^2 = \underline{\sigma}_e^2$, and κ is also given in Proposition 3 with $\sigma_e^2 = \underline{\sigma}_e^2$.

It remains to be shown that indeed all firms have the incentive to acquire information. Using the expression of aggregate output in (A.11), we obtain the ex ante expected profit for an entrepreneur if he acquires information:

$$\pi_{t,H}^I = \mathbb{E}_{x_{jt}, s_{jt}} \pi(A_H, x_{jt}, s_{jt}) = \frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta \bar{Y}_H^{1-\theta\gamma} \overline{V}_H A_H^{\frac{1}{\gamma}-1},$$

where $\overline{V}_H = \mathbb{E}_{x_{jt}, s_{jt}} \left\{ \mathbb{E} \left[\exp\left[\left(\frac{1}{\theta} - \gamma\right) \kappa \Delta_t\right] \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt} \right] \right\}^\theta$ with $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \underline{\sigma}_e^2)$. Note that an individual entrepreneur treats Y_t (or $\bar{Y}_H$ and κ) as given. Likewise, the profit for an entrepreneur if he does not acquire information is given by

$$\pi_{t,H}^U = \frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta \bar{Y}_H^{1-\theta\gamma} \underline{V}_H A_H^{\frac{1}{\gamma}-1},$$

where $\underline{V}_H = \mathbb{E}_{x_{jt},s_{jt}} \left\{ \mathbb{E}\left[\exp\left[\left(\frac{1}{\theta} - \gamma\right) \kappa \Delta_t \right] \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt} \right] \right\}^\theta$ with $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \bar{\sigma}_e^2)$.

We prove that $\overline{V}_H > \underline{V}_H$. In fact, for $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \sigma_e^2)$, we calculate

$$\begin{aligned}
& \mathbb{E}_{x_{jt},s_{jt}} \left\{ \mathbb{E}\left[\exp\left[\left(\frac{1}{\theta} - \gamma\right) \kappa \Delta_t \right] \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt} \right] \right\}^\theta \\
= \; & \mathbb{E}_{\varepsilon_{jt}, e_{jt}, \Delta_t} \exp \left[\begin{array}{c} \frac{\frac{1}{\sigma_\Delta^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\Delta^2} + \frac{1}{\sigma_\varepsilon^2}} (\varepsilon_{jt} + \Delta_t) + \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\Delta^2} + \frac{1}{\sigma_\varepsilon^2}} (\varepsilon_{jt} + e_{jt}) \\ + (1 - \theta\gamma) \kappa \left[\begin{array}{c} \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\Delta^2} + \frac{1}{\sigma_\varepsilon^2}} (\varepsilon_{jt} + \Delta_t) \\ + \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\Delta^2} + \frac{1}{\sigma_\varepsilon^2}} (\Delta_t - e_{jt}) \end{array} \right] + \theta\mu + \frac{1}{2}\theta Var \end{array} \right] \\
= \; & \exp\left[\frac{1}{2} \left[(1 - \theta\gamma) \kappa \right]^2 \sigma_\Delta^2 + \left[(1 - \theta\gamma) \kappa + \frac{1}{2} \left(\frac{1}{\theta} - 1 \right) \left(\left[(1 - \theta\gamma) \kappa \right]^2 + 1 \right) \right] \frac{1}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\Delta^2} + \frac{1}{\sigma_\varepsilon^2}} \right],
\end{aligned}$$

which is an increasing function of $\frac{1}{\sigma_e^2}$ under $\theta > 1$.

If all firms have incentive to acquire information, we must have $\pi_{t,H}^I - \pi_{t,H}^U > m$, that is,

$$\frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta A_H^{\frac{1}{\gamma} - 1} \bar{Y}_H^{1 - \theta\gamma} (\overline{V}_H - \underline{V}_H) > m. \tag{A.12}$$

Notice that under $\gamma < 1$ the LHS of (A.12) is increasing in A_H and approaches infinity when A_H approaches infinity. This condition will be satisfied as long as A_H is sufficiently high. So indeed, all firms will acquire information if A_H is high enough.

The Case of $A_t = A_L$ By construction, no entrepreneurs have incentive to acquire information in this case. If $\lambda_t = 0$ in equilibrium, again by applying Proposition 3, the aggregate output is

$$\log Y_t = \log \bar{Y}_L + \frac{1}{\gamma} \log A_L + \kappa \eta_t,$$

where $\bar{Y}_L$ is the constant $\bar{Y}$ in the proof of Proposition 3 with $\sigma_e^2 = \bar{\sigma}_e^2$, and κ is also given in Proposition 3 with $\sigma_e^2 = \bar{\sigma}_e^2$.

We now show that if A_L is low enough, no firms will acquire information. The ex ante expected profit for an entrepreneur if he acquires information is

$$\pi_{t,L}^I = \frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta \bar{Y}_L^{1 - \theta\gamma} \overline{V}_L A_L^{\frac{1}{\gamma} - 1},$$

where $\overline{V}_L = \mathbb{E}_{x_{jt},s_{jt}} \left\{ \mathbb{E}\left[\exp\left[\left(\frac{1}{\theta} - \gamma\right) \kappa \Delta_t \right] \epsilon_{jt}^{\frac{1}{\theta}} | x_{jt}, s_{jt} \right] \right\}^\theta$ with $s_{jt} \sim \mathcal{N}(\varepsilon_{jt}, \underline{\sigma}_e^2)$. Note that an individual entrepreneur treats Y_t (or $\bar{Y}_L$ and κ) as given. Similarly, the ex ante expected profit for an entrepreneur if he does not acquire information is given by

$$\pi_{t,L}^{U}=\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_{L}^{1-\theta\gamma}\underline{V}_{L}A_{L}^{\frac{1}{\gamma}-1},$$

where $\underline{V}_L=\mathbb{E}_{x_{jt},s_{jt}}\left\{\mathbb{E}\left[\exp\left[\left(\frac{1}{\theta}-\gamma\right)\kappa\Delta_t\right]\epsilon_{jt}^{\frac{1}{\theta}}|x_{jt},s_{jt}\right]\right\}^{\theta}$ with $s_{jt}\sim\mathcal{N}(\varepsilon_{jt},\bar{\sigma}_e^2)$. Again, we have $\overline{V}_L>\underline{V}_L$. If no firms acquire information, we must have $\pi_{t,L}^{I}-\pi_{t,L}^{U}<m$, that is,

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}A_{L}^{\frac{1}{\gamma}-1}\bar{Y}_{L}^{1-\theta\gamma}(\overline{V}_L-\underline{V}_L)<m. \tag{A.13}$$

Notice that under $\gamma<1$ the LHS of (A.13) approaches zero when A_L approaches zero. Hence, the above condition holds if A_L is small enough.

To summarize, we have shown that the equilibrium information precision is $1/\bar{\sigma}_e^2$ for $A_t=A_L$ and is $1/\underline{\sigma}_e^2$ for $A_t=A_H$. Also, if $A_H>>A_L$, it is easy to see that $\mathbb{E}\left[\log Y_t|A_t=A_L\right]<\mathbb{E}\left[\log Y_t|A_t=A_H\right]$. Hence, information acquisition is procyclical.

Proof of Corollary 4: Based on (34), it is easy to calculate $SD(\log Y_t|A_t)$ and $SD\left(\log Y_t|A_t,x_{jt},s_{jt}\right)$. Based on (36), the standard deviation of production is given by

$$\begin{aligned}
&SD\left(\log y_{jt}|A_t,\Delta_t\right)\\
=&\sqrt{Var\left[\frac{\frac{1}{\sigma_\Delta^2}+(1-\theta\gamma)\kappa\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_e^2}\right)}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}x_{jt}+\frac{\frac{1}{\sigma_e^2}-(1-\theta\gamma)\kappa\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_\Delta^2}+\frac{1}{\sigma_\varepsilon^2}}s_{jt}\right]}\\
=&\sqrt{\frac{1}{\tau_\varepsilon}-\frac{\theta\gamma}{\theta\gamma\tau_\varepsilon+\theta\gamma\tau_e+\tau_\Delta}\frac{\theta\gamma\left(\tau_\varepsilon+\tau_e\right)+2\tau_\Delta}{\theta\gamma\left(\tau_\varepsilon+\tau_e\right)+\tau_\Delta}}.
\end{aligned} \tag{A.14}$$

It is easy to prove that (A.14) is increasing in τ_e.

The standard deviation of revenue-based TFP (or sale price) is given by

$$\begin{aligned}
&SD\left(\log\left[\left(y_{jt}p_{jt}\right)/n_{jt}\right]|A_t,\Delta_t\right)\\
=&\sqrt{Var\left[\left(-\frac{1}{\theta}\right)\left(-\frac{\theta\gamma\tau_\varepsilon}{\theta\gamma\tau_\varepsilon+\theta\gamma\tau_e+\tau_\Delta}\right)\varepsilon_{jt}+\left(-\frac{1}{\theta}\right)\left(\frac{\theta\gamma\tau_e}{\theta\gamma\tau_\varepsilon+\theta\gamma\tau_e+\tau_\Delta}\right)e_{jt}\right]}\\
=&\sqrt{\left(\frac{1}{\theta}\right)^2\left(\frac{\theta\gamma}{\theta\gamma\tau_\varepsilon+\theta\gamma\tau_e+\tau_\Delta}\right)^2(\tau_\varepsilon+\tau_e)},
\end{aligned}$$

which is decreasing in τ_e in the interval $\tau_e\in\left[\frac{\tau_\Delta}{\theta\gamma}-\tau_\varepsilon,\infty\right)$.

B Price Setting

In this extension, we assume that firms set their selling price first and then produce to meet the market demand.

If we still assume that the idiosyncratic shock is the demand shock ϵ_{jt}, information acquisition will play no role. In fact, in this case, intermediate goods firms always set their sale price based on their expectation of the wage, independent of their signal about ϵ_{jt}. This result originates from the weakness of the Dixit-Stiglitz production function, which generates a constant markup (under perfect information) no matter what the idiosyncratic shock ϵ_{jt} is.

The Setup We alternatively assume that the idiosyncratic shock is the productivity shock. Specifically, the aggregate production function is

$$Y_t = \left[\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1$$

and entrepreneur j is the monopolist of intermediate good j with production function

$$y_{jt} = A_t A_{jt} n_{jt},$$

where A_{jt} is idiosyncratic productivity shock with $\log A_{jt} \equiv a_{jt} \sim \mathcal{N}(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$ (denoting $\tau_a = 1/\sigma_a^2$). If entrepreneur j can spend m working hours to acquire some information about a_{jt}, he receives a signal given by $s_{jt} = a_{jt} + e_{jt}$, where $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$.

The Equilibrium The price set by an informed entrepreneur is given by

$$\max_{p_{jt}} \mathbb{E}\left[p_{jt} y_{jt} - W_t \frac{y_{jt}}{A_t A_{jt}} | s_{jt}\right]$$

where $y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} Y_t$ and $W_t = 1$. The first-order condition yields

$$p_{jt} = p(A_t, s_{jt}) = \frac{\theta}{\theta - 1}\frac{1}{A_t}\mathbb{E}\left[\frac{1}{A_{jt}}|s_{jt}\right]$$

and hence his actual production is

$$y_{jt} = \left(\frac{\theta}{\theta-1}\frac{1}{A_t}\mathbb{E}\left[\frac{1}{A_{jt}}|s_{jt}\right]\right)^{-\theta} P_t^{\theta} Y_t.$$

Similarly, the price set by an uninformed entrepreneur is

$$p_{jt} = p(A_t) = \frac{\theta}{\theta - 1} \frac{1}{A_t} \mathbb{E}\left[\frac{1}{A_{jt}}\right]$$

and his actual production is

$$y_{jt} = \left(\frac{\theta}{\theta - 1} \frac{1}{A_t} \mathbb{E}\left[\frac{1}{A_{jt}}\right]\right)^{-\theta} P_t^{\theta} Y_t.$$

Therefore, by $P_t = \frac{1}{\psi} Y_t^{-\gamma}$, the aggregate output is

$$Y_t = \left[\frac{1}{\psi}\left(1 - \frac{1}{\theta}\right) A_t z\left(\lambda_t\right)\right]^{\frac{1}{\gamma}}, \tag{B.1}$$

where

$$\begin{aligned} z\left(\lambda_t\right) &= \left[\int_0^{\lambda_t} \left(\mathbb{E}\left[\frac{1}{A_{jt}} | s_{jt}\right]\right)^{1-\theta} dj + \int_{\lambda_t}^1 \left(\mathbb{E}\left[\frac{1}{A_{jt}}\right]\right)^{1-\theta} dj\right]^{\frac{1}{\theta-1}} \\ &= \left\{\lambda_t \exp\left[\left(\frac{\theta - 2}{2}\sigma_a^2 - \frac{1}{2}\frac{1}{\tau_a + \tau_e}\right)(\theta - 1)\right] + (1 - \lambda_t) \exp\left[\left(\frac{\theta - 2}{2}\sigma_a^2 - \frac{1}{2}\frac{1}{\tau_a}\right)(\theta - 1)\right]\right\}^{\frac{1}{\theta-1}} \end{aligned}$$

Denoting $\overline{z} = \exp\left(\frac{\theta-2}{2}\sigma_a^2 - \frac{1}{2}\frac{1}{\tau_a+\tau_e}\right)$ and $\underline{z} = \exp\left(\frac{\theta-2}{2}\sigma_a^2 - \frac{1}{2}\frac{1}{\tau_a}\right)$, we have

$$z(\lambda_t) = \left[\lambda_t \overline{z}^{\theta-1} + (1 - \lambda_t)\, \underline{z}^{\theta-1}\right]^{\frac{1}{\theta-1}}.$$

The expected profit for an informed entrepreneur is

$$\pi_t^I = \frac{1}{\theta - 1} \frac{1}{A_t} Y_t z_t^{-\theta} \overline{z}^{\theta-1},$$

and the expected profit for an uninformed entrepreneur is

$$\pi_t^U = \frac{1}{\theta - 1} \frac{1}{A_t} Y_t z_t^{-\theta} \underline{z}^{\theta-1}.$$

If $\lambda_t \in (0, 1)$, (9) implies

$$\pi_t^I - \pi_t^U = \frac{1}{\theta - 1} \frac{1}{A_t} Y_t z_t^{-\theta} \left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = m. \tag{B.2}$$

Since (B.1) is identical to (18) and (B.2) is identical to (19), the equilibrium under the pricing setting is essentially the same as the equilibrium under the quantity setting. The results in Proposition 1 and Lemmas 1 and 2 hold.

C Model Extension in Section 3

Our model can also generate countercyclical cross-sectional dispersion in production by slightly changing the information structure of (30) and (31). Specifically, we assume that there are two layers of production: the final good is produced from a continuum of sectoral goods (indexed by k), and each sectorial good is composed of a continuum of differentiated goods with each being produced by a monopoly firm (indexed by j); that is,

$$Y_t = \left[\int_0^1 y_{kt}^{\frac{\vartheta-1}{\vartheta}} dj\right]^{\frac{\vartheta}{\vartheta-1}} \quad \text{for } \vartheta > 1$$

and

$$y_{kt} = \left[\int_0^1 \epsilon_{jkt}^{\frac{1}{\theta}} y_{jkt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1.$$

The endowed "public" signal in (30) is changed to

$$x_{jkt} = \varepsilon_{jkt} + \Lambda_{kt} + \Delta_t, \tag{C.1}$$

where $\Delta_t \sim \mathcal{N}(0, \sigma_\Delta^2)$ is the economy-wide common sentiment shock, and $\Lambda_{kt} \sim \mathcal{N}(0, \sigma_\Lambda^2)$ is the sector-specific sentiment shock for sector k and Λ_{kt} is independent across sectors, and $\log \epsilon_{jkt} \equiv \varepsilon_{jkt} \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$ is the idiosyncratic demand shock for firm j in sector k. The second signal in (31) is changed to

$$s_{jkt} = \varepsilon_{jkt} + e_{jkt}, \ \text{ where } e_{jkt} \sim \mathcal{N}(0, \sigma_e^2). \tag{C.2}$$

We assume that both ε_{jkt} and e_{jkt} are i.i.d. across firms in any sector k and they are independent of each other. For simplicity, we assume that $\underline{\sigma}_e^2 = 0$; that is, if entrepreneur j in any sector k spends m working hours on information acquisition, he knows ε_{jkt} perfectly.

The cross-sectional dispersion comes from the heterogeneity of sector-specific sentiment shock Λ_{kt}. When firms acquire more information, they are less responsive to signal x_{jkt} and thus Λ_{kt}, so the cross-sectional dispersion goes down.

Corollary 5 *Under the alternative information structure of (C.1) and (C.2), the economy exhibits countercyclical aggregate volatility and countercyclical cross-sectional dispersion in production.*

This extended model implies procyclical firm-level dispersion in production within each sector but countercyclical dispersion in production across sectors. The mechanism generating countercyclical cross-sectional dispersion in production is essentially the same as that generating countercyclical aggregate volatility analyzed in Section 3.3. Without information acquisition (for the case of $A_t = A_L$), ε_{jkt} is imperfect information and firms respond to signal x_{jkt}, which generates cross-sectional dispersion driven by sector-specific sentiment shock Λ_{kt} as well as aggregate volatility driven by economy-wide sentiment shock Δ_t. More concretely, an individual firm's production is given by $y_{jkt}(x_{jkt}, s_{jkt})$, and thus the aggregate (or average) production of sector k is given by $\hat{y}_{kt}(A_t, \Delta_t, \Lambda_{kt}) = \int_j y_{jkt}(\varepsilon_{jkt} + \Lambda_{kt} + \Delta_t, \varepsilon_{jkt} + e_{jkt})$ considering that ε_{jkt} and e_{jkt} are i.i.d. across firms within a sector. The cross-sectional dispersion is hence $SD(\log \hat{y}_{kt} | A_t = A_L, \Delta_t) > 0$. Aggregate output of the economy is given by $Y_t(A_t, \Delta_t)$ considering that Λ_{kt} is also i.i.d. across sectors, so $SD(\log Y_t | A_t = A_L) > 0$. When firms acquire information (for the case of $A_t = A_H$), ε_{jkt} is perfect information, so both aggregate volatility and cross-sectional dispersion disappear. In fact, an individual firm's production is given by $y_{jkt}(\varepsilon_{jkt})$, which implies that $\hat{y}_{kt}(A_t) = \int_j y_{jkt}(\varepsilon_{jkt})$ and $Y_t(A_t)$. So $SD(\log \hat{y}_{kt} | A_t = A_H, \Delta_t) = 0$ and $SD(\log Y_t | A_t = A_H) = 0$.

We can interpret the structure of the above extended model in a different way: the economy has a continuum of multi-product firms as recently emphasized in the trade literature (e.g., Bernard, Redding and Schott (2010)), and each firm produces a continuum of products, with each being run by a different manager. Each manager is endowed with the "public" signal and acquires information about his private signal. In this case, Corollary 5 would imply countercyclical firm-level dispersion in production.

D Continuous Information Acquisition

We now relax the binary choice of information acquisition and make it a continuous choice. Specifically, after spending $h(\tau_e)$ working hours at the beginning of the period, entrepreneur j receives a noisy signal s_{jt} given by (31).[24] Signal precision, $\tau_e = 1/\sigma_e^2$, is endogenous, continuous and effort-dependent. We assume that $\frac{\partial h}{\partial \tau_e} > 0$ and $\frac{\partial^2 h}{\partial \tau_e^2} > 0$. Since firms are ex ante identical, they will choose the same level of information precision. We denote by τ_e^* the information precision for all entrepreneurs $k \neq j$. Given τ_e^*, we now characterize entrepreneur $j's$ information acquisition problem. We first characterize aggregate output for a given information precision τ_e^*.

Equilibrium Y_t for a Given τ_e^* Given their information choice τ_e^*, entrepreneurs $i \neq j$ decide to produce according to (32). The aggregate output is hence given by (34), where $\bar{Y}$ is a constant not dependent on A_t and Δ_t but dependent on $1/\tau_e^*$, which is an exogenous constant for entrepreneur j. The coefficient κ is given by

$$\kappa = \kappa(\tau_e^*) = \frac{\tau_\Delta}{\theta\gamma(\tau_e^* + \tau_\varepsilon) + \tau_\Delta}, \tag{D.1}$$

which is also an exogenous constant for entrepreneur j.

[24] Rigorously, we should write $e_{jt} \sim \mathcal{N}(0, \sigma_{je}^2)$, rather than $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$. But we will confirm that in a symmetric equilibrium, all firms will choose the same level of information precision. To reduce notational clutter, we will use $\tau_e = 1/\sigma_e^2$ for firm j and τ_e^* for firms $i \neq j$, and in equilibrium $\tau_e = \tau_e^*$.

Information Acquisition Decision Given τ_e^*, we now consider entrepreneur $j's$ information choice $\tau_e = 1/\sigma_e^2$. For a given σ_e^2, entrepreneur j's production, $y(A_t, x_{jt}, s_{jt})$, based on his signal is given by (32). Hence, if he chooses the precision level $\tau_e = 1/\sigma_e^2$, his ex ante expected profit is

$$\begin{aligned}\pi(\tau_e; A_t, \tau_e^*) &= \frac{1}{\theta-1}\mathbb{E}_{x_{jt},s_{jt}}\frac{y(A_t,x_{jt},s_{jt})}{A_t} \\ &= \frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}V(\tau_e,\kappa(\tau_e^*)),\end{aligned}$$

where $\bar{Y}_C$ is the constant $\bar{Y}$ in the proof of Proposition 3 with $\sigma_e^2 = 1/\tau_e^*$, and

$$\begin{aligned}V(\tau_e,\kappa(\tau_e^*)) &= \mathbb{E}_{x_{jt},s_{jt}}\left\{\mathbb{E}\left[\exp\left[\left(\frac{1}{\theta}-\gamma\right)\kappa\Delta_t\right]\epsilon_{jt}^{\frac{1}{\theta}}|x_{jt},s_{jt}\right]\right\}^{\theta} \\ &= \exp\left[\begin{array}{l}\frac{1}{2}\left[(1-\theta\gamma)\kappa(\tau_e^*)\right]^2\frac{1}{\tau_\Delta} \\ +\left[(1-\theta\gamma)\kappa(\tau_e^*)+\frac{1}{2}\left(\frac{1}{\theta}-1\right)\left(\left[(1-\theta\gamma)\kappa(\tau_e^*)\right]^2+1\right)\right]\frac{1}{\tau_\Delta+\tau_\varepsilon+\tau_e}\end{array}\right].\end{aligned}$$

Notice that $V(\tau_e,\kappa(\tau_e^*))$ is increasing in τ_e for $\theta > 1$. $V(\tau_e,\kappa(\tau_e^*))$ is concave in τ_e when τ_e is sufficiently large since V is bounded from above when $\tau_e \to +\infty$. Firm $j's$ information acquisition decision on τ_e is given by

$$\underset{\tau_e}{Max}\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}V(\tau_e,\kappa(\tau_e^*)) - h(\tau_e). \tag{D.2}$$

So the first-order condition implies

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}\frac{\partial V(\tau_e,\kappa(\tau_e^*))}{\partial\tau_e} = \frac{\partial h(\tau_e)}{\partial\tau_e}, \tag{D.3}$$

where

$$\begin{aligned}&\frac{\partial V(\tau_e,\kappa(\tau_e^*))}{\partial\tau_e} \\ = \;& V(\tau_e,\kappa(\tau_e^*))\cdot\left[(1-\theta\gamma)\kappa(\tau_e^*)+\frac{1}{2}\left(\frac{1}{\theta}-1\right)\left(\left[(1-\theta\gamma)\kappa(\tau_e^*)\right]^2+1\right)\right]\frac{(-1)}{(\tau_\Delta+\tau_\varepsilon+\tau_e)^2}.\end{aligned}$$

This defines an implicit mapping between A_t and τ_e. Figure D.1 illustrates the effect of A_t on τ_e.

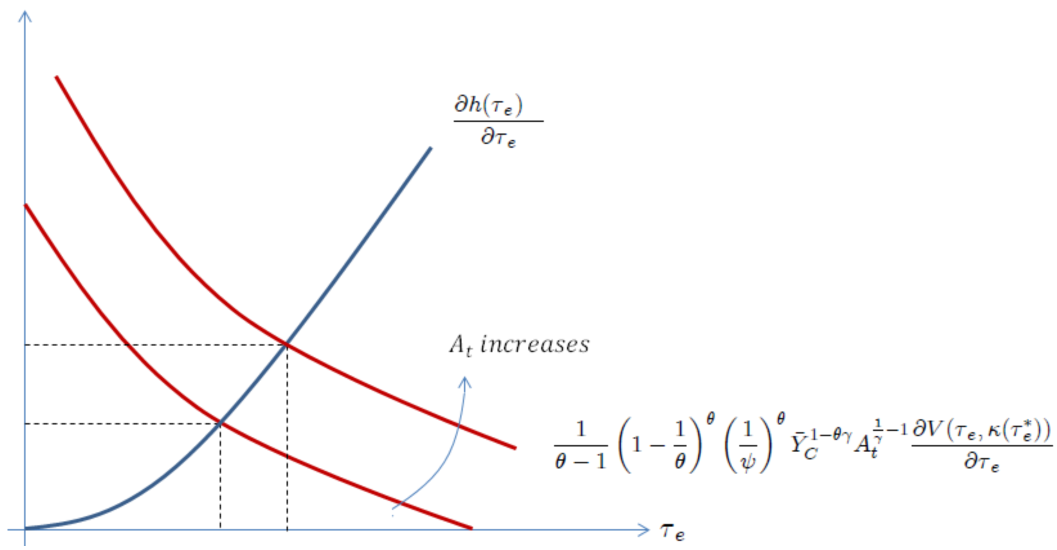

Figure D.1: Optimal τ_e Chosen by Firm j for a Given τ_e^*

The LHS of equation (D.3) captures the marginal benefit of increasing the precision of information, while the RHS measures the marginal cost. The marginal benefit is decreasing in τ_e and is graphed by the downward sloping curve, and the marginal cost is increasing in τ_e and is graphed by the upward sloping curve. The intersection of these two curves gives the optimal τ_e. The LHS is increasing in A_t under $\gamma < 1$. So when A_t increases, the LHS increases for any level τ_e, leading to an increase in the optimal τ_e as the figure illustrates.

In a symmetric equilibrium, $\tau_e = \tau_e^*$. So we have

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial \tau_e} = \frac{\partial h(\tau_e,b)}{\partial \tau_e}, \tag{D.4}$$

where $\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial \tau_e} > 0$ is the partial derivative of V with respect to its first argument. (D.4) then defines a mapping between τ_e and A_t:

$$\tau_e = f(A_t)$$

Unfortunately, we are not able to derive an explicit function for f. However, the property of the LHS of (D.4) largely follows that of $\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial \tau_e}$. In fact, the LHS of (D.4) approaches 0 when $\tau_e \to \infty$, and it is increasing in A_t under $\gamma < 1$. Therefore, as illustrated in Figure D.1, we have $f'(A_t) > 0$, the case in which we are interested, at least for some parameter choices. Also, $\mathbb{E}[\log Y_t|A_t] = \log \bar{Y}_C + \frac{1}{\gamma}\log A_t$ increases with A_t.

Because $\mathbb{E}[\log Y_t|A_t]$ is increasing in A_t and also τ_e is increasing in A_t, information acquisition is procyclical, as in Proposition 4. Corollary 4 also follows.

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