Endogenous Information Acquisition and Countercyclical Uncertainty

Jess Benhabib* Xuewen Liu† Pengfei Wang‡

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Abstract

We introduce endogenous information acquisition for firms facing idiosyncratic demand shocks in an otherwise standard monopolistically competitive model. We show that when firms acquire information optimally, economic volatility is time-varying and countercyclical, measured either by firm-level dispersion or by time-series aggregate volatility. Uncertainty becomes endogenous, so that changes in uncertainty and economic activity can both be driven by TFP or by financial friction shocks. When the economy is hit by a negative aggregate TFP shock, the real wage and hence the labor cost of information acquisition will go down, so more firms will optimally choose to acquire information. As firms have more precise information on the demand for their products, their production will become more responsive to their true demand, leading to a higher dispersion across firms in employment, production and sale revenues. If the demand shocks of firms have a common component besides idiosyncratic components, more information acquisition and thus more precise information will also lead firms to respond more strongly to the common shock, resulting in higher time-series aggregate volatility. We also find that financial friction shocks such as borrowing constraint shocks can drive countercyclical volatility under a weaker condition than the TFP shock.

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*Department of Economics, New York University, 19 West 4th Street, New York, NY 10012, USA. Office: (212) 998-8971, Fax: (212) 995-4186. Email: jess.benhabib@nyu.edu.

†Department of Finance, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7679. Email: xuewenliu@ust.hk

‡Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7612. Email: pfwang@ust.hk

1 Introduction

Recessions are often associated with increases in uncertainty, measured either by cross-sectional firm-level dispersion of production or by aggregate time-series volatility in GDP growth. In a recent influential paper, Bloom (2009) shows that dispersion and volatility are countercyclical, suggesting that "uncertainty shocks" and time-varying volatility may be quantitatively important features of business cycles. Indeed, this finding has led many economists and policymakers to conjecture that uncertainty shocks may have been an important driving force of business cycles in general and of the recent Great Recession in particular.[1]

Two most popular measures of uncertainty are cross-sectional firm-level dispersion and time-series aggregate volatility of aggregate output (see Jurado, Ludvigson and Ng (2014), Kozeniauskas, Orlik and Veldkamp (2014), Orlik and Veldkamp (2014), among others). The theoretical literature typically models uncertainty as an exogenous change in volatility of firms' idiosyncratic shocks or of aggregate productivity or other shocks to the economy. Various mechanisms have been proposed to explain why an increase in uncertainty can generate large falls in output, employment, investment, consumption and productivity growth.

Although some of the economic uncertainty is undoubtedly the result of exogenous shocks, in this paper we will explore endogenously determined uncertainty. Recent empirical work (e.g., Bachmann and Bayer (2013, 2014), Bachmann, Elstner and Sims (2013)) finds that uncertainty appears to be an outcome of recessions. Furthermore, some measures of volatility do appear to respond to economic policies.[2] If uncertainty is purely exogenous and technological, recession is then simply a result of the optimal responses of economic agents to the increases in uncertainty, and policy intervention may not desirable. On the other hand, if uncertainty is endogenous, policies that directly aim to reduce uncertainty may even be harmful as uncertainty can be a symptom rather than the cause of the recession. In short, understanding how uncertainty may endogenously arise is important for both positive and normative analysis.

The purpose of this paper is to establish a simple theory of endogenous uncertainty. For this purpose, we introduce endogenous information acquisition in an otherwise standard Dixit-Stiglitz monopolistic competition model. The final goods producer buys inputs from a continuum of intermediate goods firms that engage in monopolistic competition. Intermediate goods firms (entrepreneurs) have imperfect information about their i.i.d. idiosyncratic demand shocks. Nevertheless, before an intermediate goods firm produces, it can use some labor (working hours) to acquire information about the demand shock to its product. Information acquisition enables a firm to earn

[1] For example, the Federal Open Market Committee minutes repeatedly emphasize uncertainty as a key factor driving the 2001 and 2007-2009 recessions (see, e.g., Bloom et al. (2012)).

[2] Tan and Kohili (2011) find that the "quantitative easing" policy by Federal Reserve had a significant negative impact on the stock market volatility.

higher expected profits because more precise information allows it to align its production more closely with the demand for its product. When a firm decides whether to acquire information or not, it makes a trade-off between the benefits and costs. When a positive TFP shock generates a boom in the economy, the gain from information acquisition increases as the expected profit goes up, but the labor cost of information acquisition also increases as wages and the cost of information also go up. We show that under reasonable specifications of preferences, the cost outruns the benefit in a boom, leading to less information acquisition. As firms have less precise information, their production then becomes less responsive to their idiosyncratic demand shock. Hence, their production or sales revenue becomes less dispersed. By contrast, firm-level dispersion in production or sales revenue increases during a recession, so that measured uncertainty is countercyclical.

We then extend our baseline model to study countercyclical aggregate volatility. We assume that the demand shock to an intermediate goods firm has a common component besides an idiosyncratic component. When a firm acquires information, it receives a noisy signal about the weighted average of these two components. Because the common demand shock does not cancel out in aggregation, it drives aggregate volatility. As before, when the economy is in a boom due to a positive TFP shock, firms have less incentive to acquire information and hence their information (signal) becomes less precise, and consequently the signal is less important to firms' production decisions. This means that firms' production is less responsive to both idiosyncratic and aggregate demand shocks. As in the baseline model, firms' production is more homogeneous and hence the measured uncertainty at the firm level declines. At the same time, the aggregate production also becomes less responsive to the common demand shock, leading to a lower aggregate volatility in the aggregate production. In short, a higher (lower) TFP leads to a lower (higher) aggregate volatility.

Finally, we generalize our analysis along several dimensions to illustrate that the information acquisition mechanism provides a robust prediction of countercyclical uncertainty, both at the firm level and at the aggregate level. We first change firms' signal by allowing it to contain a common sentiment/confidence shock. Angeletos and La'O (2013a) and Benhabib, Wang and Wen (2014) formally show that sentiment-driven fluctuations are consistent with rational expectation equilibrium. Our information acquisition mechanism in this setting also implies that uncertainty increases during recessions. Moreover, our results do not change if firms' signals contain information about preference shocks rather than aggregate demand shocks. Furthermore, our results on countercyclical aggregate volatility can be the result of only financial shocks (instead of TFP shocks). This may explain why measured uncertainty often increases during periods of financial turbulence.

We do not have direct empirical evidence on the countercyclicality of the quantity of information acquired. We note, however, that the total cost of information acquisition as a fraction of GDP can be procyclical in our model.[3,4]

Relation to the Literature. We add to the fast growing literature that follows Bloom (2009) to study the interaction between uncertainty and economic activities. Many recent papers have extended Bloom's partial equilibrium analysis to general equilibrium. Since an increase in stochastic volatility typically generates a comovement problem between consumption and investment in a frictionless economy, various frictions have been explored. For example, Christiano, Motto and Rostagno (2014), Arellano, Bai and Kehoe (2012), and Gilchrist, Sim and Zakrajsek (2014) introduce financial frictions and show that financial frictions can propagate uncertainty shocks. Basu and Bundick (2012) study uncertainty shocks in a new-Keynesian model. Bloom et al. (2012) introduce microeconomic rigidities, such as adjustment costs in capital and labor. In contrast to our model, these papers treat uncertainty shocks as exogenous and focus on their impact on the economy. Our paper is closely related to the work of Bachmann and Moscarini (2012), who consider the endogenous uncertainty in cross-sectional measures of dispersion.[5] In their model, firms face uncertainty about the elasticity of their demand but can learn gradually from the volume of sales. They show that bad economic times are the best times to price-experiment. Hence information acquisition is also countercyclical, as in our model, and generates countercyclical cross-sectional measures of dispersion. In our model, the firms know their demand elasticity but need to learn the position of their demand curve, which gives rise to a simpler learning problem and allows us to study both firm-level dispersion and time-series aggregate volatility.[6]

The rest of the paper is organized as follows. Section 2 presents the baseline model, highlighting the basic mechanism and showing the implications on firm-level dispersion. Section 3 extends the baseline model by allowing financial frictions. Section 4 generalizes the baseline model to study both firm-level dispersion and aggregate volatility. Section 5 concludes.

[3] See the comments following Proposition 1.

[4] Quelch (2009), for example, notes that in the US, as a result of the 2008-2009 recession, "spending on market research has dipped for four consecutive quarters... Most big consumer marketers are seeking to shave 10 to 20% off of research budgets. At the same time that marketers must pare down research expenditures, they face added pressure to secure high-quality data and insights." See also the decline in business activity measured by the Research Industry Index of the Marketing Research Association during the 2008-09 recession at http://www.marketingresearch.org/research-industry-index.

[5] Cui (2012) and D'Erasmo and Moscoso-Boedo (2012) also study endogenous cross-sectional dispersion and use different approaches from ours. Fajgelbaum, Taschereau-Dumouchel and Schaal (2014) propose a theory of endogenous time-series aggregate uncertainty, with the mechanism of the "wait-and-see" effect together with agents learning from the actions of others.

[6] Among others, Reis (2006), Angeletos and Pavan (2007), Hellwig and Veldkamp (2009), Vives (2013) and Colombo, Femminis and Pavan (2014) study information acquisition and efficiency. In business-cycle models, Angeletos and La'O (2013b), Llosa and Venkateswaran (2013) and Mackowiak and Wiederholt (2013) contrast the equilibrium acquisition of information with the efficient acquisition of information. The main focus of our paper, on the other hand, is on the implication of endogenous information acquisition on macroeconomic countercyclical uncertainty.

2 The Baseline Model

2.1 Model Setup

The economy is populated by a large representative household that has a continuum of identical workers and a continuum of entrepreneurs, with unit measure of each. The household derives utility from leisure and from consumption of a composite final good produced with a continuum of differentiated intermediate goods. Workers supply labor to entrepreneurs in a competitive labor market. Entrepreneur j is the monopolist of differentiated intermediate good j. The demand for each intermediate good j is affected by an idiosyncratic demand shock ϵ_{jt} and by aggregate demand driven by an aggregate productivity shock A_t. At the beginning of each period, after observing the aggregate productivity shock A_t, entrepreneur j decides whether to acquire information regarding ϵ_{jt}, and then produces accordingly. At the end of each period, the workers and entrepreneurs pool their wage and profit income for the household.

The Representative Household The household maximizes its utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \psi N_t - \psi N_{et} \right] \quad \text{for } \gamma > 1, \tag{1}$$

where C_t is the consumption of the household, N_t is the total working hours of workers, and N_{et} is the total working hours entrepreneurs spend on information acquisition (to be specified later). The budget constraint for the household is

$$P_t C_t \leq W_t N_t + \Pi_t, \tag{2}$$

where the consumption goods price, P_t, is normalized to unity (the numeraire price), W_t is the real wage, and Π_t denotes total profit income earned by entrepreneurs. The first-order condition reveals that the household's consumption depends on its (rational) expectation of real wage,[7] that is,

$$W_t = \psi C_t^{\gamma}. \tag{3}$$

The Final Goods Producer The consumption good is produced by a competitive final goods firm facing competitive factor markets according to the Dixit-Stiglitz aggregate production function:

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1. \tag{4}$$

[7] In our baseline model, there is no aggregate uncertainty, so W_t is deterministic and is perfectly foreseen under rational expectations.

The final goods producer maximizes its profit:

$$\max_{y_{jt}} \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int_0^1 p_{jt} y_{jt} dj.$$

The first-order condition with respect to input y_{jt} is

$$y_{jt} = p_{jt}^{-\theta} \epsilon_{jt} Y_t, \tag{5}$$

which shows that the demand for intermediate goods j is affected by both idiosyncratic shock ϵ_{jt} and aggregate demand Y_t. We assume that ϵ_{jt} is independent across time and goods and $\log \epsilon_{jt} \equiv \varepsilon_{jt} \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$, so the mean of ϵ_{jt} is $\mathbb{E}\epsilon_{jt} = 1$. Denote $\tau_\varepsilon = 1/\sigma_\varepsilon^2$.

Intermediate Goods Producers Entrepreneurs are the producers of intermediate goods. Entrepreneur j is the monopolist of goods j with production function

$$y_{jt} = A_t n_{jt}. \tag{6}$$

Entrepreneur j produces y_{jt} to maximize his profit under demand uncertainty driven by ϵ_{jt}. To reduce the uncertainty before production, entrepreneur j can spend m working hours to acquire some information about ϵ_{jt} (for example, via a market survey). If he chooses to do so, he receives a signal given by $s_{jt} = \varepsilon_{jt} + e_{jt}$, where $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$, so the precision of the signal is $\tau_e = 1/\sigma_e^2$. If the entrepreneur does not acquire information, he knows only the prior, unconditional distribution of ε_{jt}.

An informed entrepreneur j chooses y_{jt} to maximize his expected profit

$$y_{jt} = y(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[p_{jt} y_{jt} - W_t n_{jt} | s_{jt}\right] \tag{7}$$

with constraints (5) and (6). Here $\mathbb{E}(\cdot|s_{jt})$ is the conditional expectation operator over ϵ_{jt}. Denote the realized profit for an informed entrepreneur by $\pi(\epsilon_{jt}, s_{jt}) = p_{jt}(\epsilon_{jt}, y_{jt}) y_{jt} - W_t n_{jt}(y_{jt})$. Likewise, an uninformed entrepreneur j solves

$$\tilde{y}_{jt} = \arg\max_{\tilde{y}_{jt}} \mathbb{E}\left[p_{jt} \tilde{y}_{jt} - W_t n_{jt}\right] \tag{8}$$

with constraints $\tilde{y}_{jt} = p_{jt}^{-\theta} \epsilon_{jt} Y_t$ and $\tilde{y}_{jt} = A_t n_{jt}$. Here $\mathbb{E}$ is simply the unconditional expectation operator over ϵ_{jt}. Denote the realized profit for an uninformed entrepreneur by $\tilde{\pi}_t(\epsilon_{jt}) = p_{jt}(\epsilon_{jt}, \tilde{y}_{jt}) \tilde{y}_{jt} - W_t n_{jt}(\tilde{y}_{jt})$.

Information Acquisition In order to acquire a signal s_{jt}, an entrepreneur needs to spend a fixed amount of time m. The fixed cost in terms of consumption units is then $mW_t = m\psi C_t^\gamma$. The ex ante expected profit for a firm acquiring information is

$$\pi_t^I = \mathbb{E}_{\epsilon_{jt}, s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt})\right] = \mathbb{E}_{s_{jt}}\mathbb{E}_{\epsilon_{jt}|s_{jt}}\left[\pi(\epsilon_{jt}, s_{jt})|s_{jt}\right].$$

The ex ante expected profit for a firm not acquiring information is

$$\pi_t^U = \mathbb{E}_{\epsilon_{jt}}\left[\tilde{\pi}_t(\epsilon_{jt})\right]$$

As entrepreneurs are identical ex ante, all of them will acquire information if $\pi_t^I - m\psi C_t^\gamma > \pi_t^U$ and none will acquire information if $\pi_t^I - m\psi C_t^\gamma < \pi_t^U$. If $\pi_t^I - m\psi C_t^\gamma = \pi_t^U$, entrepreneurs are indifferent in acquiring information or not. Denote by λ_t the fraction of entrepreneurs who acquire information. We must have

$$\begin{cases} \pi_t^I - m\psi C_t^\gamma > \pi_t^U & \text{if } \lambda_t = 1 \\ \pi_t^I - m\psi C_t^\gamma = \pi_t^U & \text{if } \lambda_t \in (0,1) \\ \pi_t^I - m\psi C_t^\gamma < \pi_t^U & \text{if } \lambda_t = 0 \end{cases} \tag{9}$$

Timeline We summarize the sequence of actions by consumers and firms, the information structure, and the rational expectations equilibria of our baseline model.

1. At the beginning of each period, after observing A_t, an entrepreneur makes his decision on whether to acquire a signal about ϵ_{jt}. Signal s_{jt} is obtained if he pays a constant cost m in terms of working hours; otherwise no signal is obtained.

2. Based on signal s_{jt} and real wage W_t, an informed entrepreneur decides how much labor n_{jt} to hire to produce his intermediate good. An uninformed entrepreneur chooses n_{jt} based on the prior of ϵ_{jt}.

3. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

The formal definition of equilibrium in our baseline model is as follows.

Definition 1 *An REE is a sequence of aggregate allocations* $\{C(A_t), Y(A_t), N(A_t), \Pi(A_t), \lambda(A_t)\}$*, individual productions* $y_{jt} = y(A_t, s_{jt})$ *for informed entrepreneurs and* $y_{jt} = \tilde{y}(A_t)$ *for uninformed entrepreneurs, and prices* $\{W(A_t), p(s_{jt}, \epsilon_{jt})\}$*, such that for each realization of* A_t*, (i)* $C(A_t)$ *and* $N(A_t)$ *maximize households' utility given the equilibrium price* $W_t = W(A_t)$ *and aggregate profit* $\Pi(A_t)$*; (ii) equation (5) maximizes the final goods firm's profit given shocks* ϵ_{jt} *and equilibrium prices* $p(s_{jt}, \epsilon_{jt})$*; (iii) given* W_t *and signal* s_{jt}*,* $y(A_t, s_{jt})$ *maximizes the expected profit of* *an informed entrepreneur and $\tilde{y}(A_t)$ maximizes the expected profit of an uninformed entrepreneur; (iv) A $\lambda(A_t)$ fraction of entrepreneurs acquire information about their ϵ_{jt}, so $\Pi(A_t) = \lambda_t \pi_t^I + (1-\lambda_t)\pi_t^U$; and (v) all markets clear, namely, $C(A_t) = Y(A_t)$ and $N_t = \int_0^1 \frac{y_{jt}}{A_t} dj$.*

We now proceed to characterize the equilibrium.

2.2 Characterization of Equilibrium

First, we work out a firm's optimal production given its information acquisition decision. Next, we aggregate all firms' production to obtain the aggregate output Y_t as a function of λ_t and A_t. Then, we compare π_t^I and π_t^U to solve firms' information acquisition problem, which yields another function involving λ_t and Y_t. Finally, we use these two functions to determine Y_t and λ_t simultaneously as functions of A_t.

Equilibrium Y_t for a Given λ_t Substituting (5) and (6) into (7), we have

$$y_{jt}(s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[\left(y_{jt}^{1-\frac{1}{\theta}}(\epsilon_{jt}Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A_t}y_{jt}\right)|s_{jt}\right] \tag{10}$$

for an informed entrepreneur. This yields

$$y_{jt} = y(A_t, s_{jt}) = \left[\left(1-\frac{1}{\theta}\right)\frac{A_t}{W_t}Y_t^{\frac{1}{\theta}}\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt})\right]^{\theta}, \tag{11}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt})\right]^{\theta} = \exp\left[\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt} + \frac{1}{2}\frac{1-\theta}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right]$. Similarly, we obtain the production for an uninformed entrepreneur

$$\tilde{y}_{jt} = \tilde{y}(A_t) = \left[\left(1-\frac{1}{\theta}\right)\frac{A_t}{W_t}Y_t^{\frac{1}{\theta}}\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta}, \tag{12}$$

where $\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}})\right]^{\theta} = \exp(\frac{1}{2}\frac{1-\theta}{\theta}\frac{1}{\tau_\varepsilon})$. Since a λ_t fraction of firms produce according to (11) and $1-\lambda_t$ of them produce according to (12), the aggregate production defined in equation (4) becomes

$$Y_t = \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A_t}{W_t}\right)^{\theta} Y_t \left[\int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj + \int_{\lambda_t}^1 \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj\right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

The labor demand is simply given by $n_{jt} = y_{jt}/A_t$. Hence labor market clearing gives

$$N_t = \frac{1}{A_t}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A}{W_t}\right)^{\theta} Y_t \left[\int_0^{\lambda_t}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj + \int_{\lambda_t}^1 \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj\right]. \tag{14}$$

Exploiting the law of iterated expectations, we can find

$$\int_0^{\lambda_t} \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta-1} dj + \int_{\lambda_t}^1 \epsilon_{jt}^{\frac{1}{\theta}} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta-1} dj = \int_0^{\lambda_t} \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta} dj + \int_{\lambda_t}^1 \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right)^{\theta} dj$$

Hence, (13) can be transformed into

$$W_t = (1 - \frac{1}{\theta}) \left(A_t z_t \right) \tag{15}$$

while (13) and (14) together yield

$$Y_t = N_t \left(A_t z_t \right), \tag{16}$$

where $z_t = z(\lambda_t)$ is given by

$$z(\lambda_t) = \left[\lambda_t \exp\left(-\frac{1}{2} \frac{\theta - 1}{\theta} \frac{1}{\tau_\varepsilon + \tau_e} \right) + (1 - \lambda_t) \exp\left(-\frac{1}{2} \frac{\theta - 1}{\theta} \frac{1}{\tau_\varepsilon} \right) \right]^{\frac{1}{\theta - 1}}, \tag{17}$$

which is the *endogenous* TFP. Denoting $\overline{z} = \exp\left(-\frac{1}{2} \frac{1}{\theta} \frac{1}{\tau_\varepsilon + \tau_e} \right)$ and $\underline{z} = \exp\left(-\frac{1}{2} \frac{1}{\theta} \frac{1}{\tau_\varepsilon} \right)$, we have

$$z(\lambda_t) = \left[\lambda_t \overline{z}^{\theta - 1} + (1 - \lambda_t) \underline{z}^{\theta - 1} \right]^{\frac{1}{\theta - 1}}. \tag{18}$$

It is easy to see that $z'(\lambda_t) > 0$, and $z(\lambda_t = 1) = \overline{z}$ and $z(\lambda_t = 0) = \underline{z}$. That is, if more firms acquire information, the aggregate production becomes more efficient. In fact, efficient allocation requires more resources to be allocated toward firms with higher realized ϵ_{jt}, that is, efficient production should be contingent on realized ϵ_{jt}. So, more precise information about ϵ_{jt} achieved through information acquisition helps improve allocative efficiency.

Equations (15) and (16) are intuitive. (16) implies that despite heterogeneity among firms originating in idiosyncratic demand shocks, our economy works as if there existed a representative firm with productivity $A_t z(\lambda_t)$. (15) means that real wage is proportional to labor productivity $A_t z(\lambda_t)$ where the proportion $1 - \frac{1}{\theta}$ is the share of labor cost in aggregate output (i.e., the average profit-to-revenue ratio is $\frac{1}{\theta}$).

In equilibrium, (3) becomes $W_t = \psi Y_t^{\gamma}$, which together with (15) yields aggregate output as a function of λ_t and A_t:

$$Y_t = \left(1 - \frac{1}{\theta} \right)^{\frac{1}{\gamma}} \left(\frac{A_t z(\lambda_t)}{\psi} \right)^{\frac{1}{\gamma}}. \tag{19}$$

This implies that the aggregate production increases with λ_t.

Equilibrium λ_t under Information Acquisition We now turn to firms' information acquisition problem for another relationship between λ_t and Y_t. Exploiting the law of iterated expectations, we obtain the ex ante expected profit for an informed entrepreneur,

$$\pi_t^I = \frac{1}{\theta} Y_t z_t^{1-\theta} \overline{z}^{\theta-1},$$

and the expected profit for an uninformed entrepreneur,

$$\pi_t^U = \frac{1}{\theta} Y_t z_t^{1-\theta} \underline{z}^{\theta-1}.$$

In fact, the following relationship holds:

$$\frac{1}{\theta} = \frac{\lambda_t \pi_t^I + (1-\lambda_t)\,\pi_t^U}{Y_t};$$

namely, the average profit-to-revenue ratio in the economy is $\frac{1}{\theta}$. It is easy to see $\pi_t^I > \pi_t^U$. So informed entrepreneurs always enjoy a higher expected profit. In other words, information is valuable to firms. However, acquiring information is costly. If $\lambda_t \in (0,1)$, (9) implies

$$\pi_t^I - \pi_t^U = \frac{1}{\theta} Y_t z_t^{1-\theta} \left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1} \right) = m\psi Y_t^{\gamma} \tag{20}$$

in equilibrium. Rearranging terms yields the second equilibrium relationship between Y_t and λ_t:

$$\frac{1}{\theta} \left(z(\lambda_t) \right)^{1-\theta} Y_t^{1-\gamma} \left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1} \right) = m\psi. \tag{21}$$

Under the assumption $\gamma > 1$, equation (21) defines λ_t as a decreasing function of Y_t. The LHS of equation (21) is the benefit of acquiring information in utility units, and the RHS is the utility loss from foregoing leisure. When Y_t increases, the benefit decreases (under $\gamma > 1$), leading to weaker incentives to acquire information. Put slightly differently, the middle part of (20) is the benefit of information acquisition in consumption units, and the RHS is the cost. When Y_t increases, the cost outruns the benefit because of the strong labor supply effect under $\gamma > 1$.

Full Equilibrium Equations (19) and (21) jointly determine Y_t and λ_t. Figure 1 gives a diagrammatic analysis of these equilibrium variables.

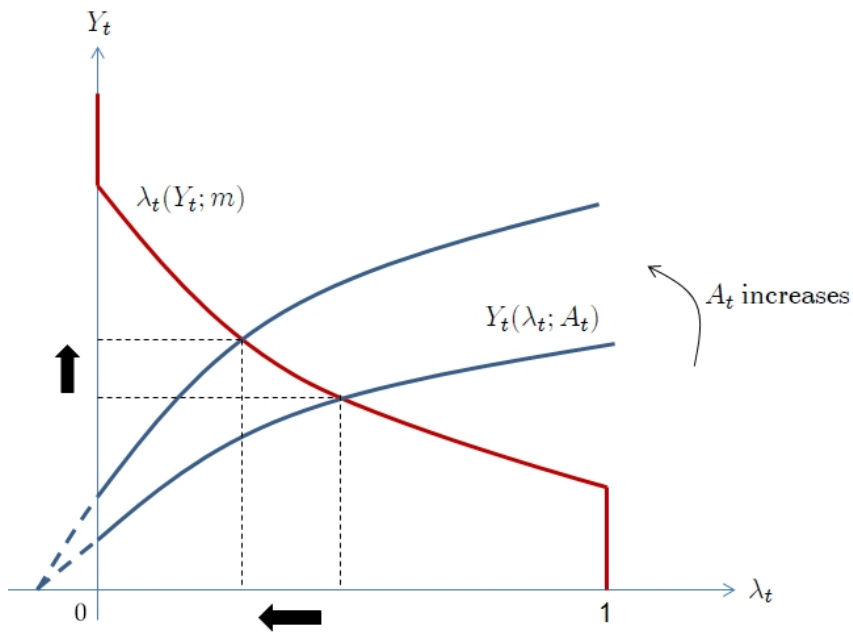

Figure 1: Equilibrium with Information Acquisition

In Figure 1, $Y_t(\lambda_t; A_t)$ is given by equation (19) while $\lambda_t(Y_t; m)$ is given by (21). The vertical lines of $\lambda_t = 1$ and $\lambda_t = 0$ correspond to corner solutions in (9). When output Y_t is below a threshold, all entrepreneurs will acquire information. In other words, $\lambda_t(Y_t; m)$ becomes vertical at $\lambda_t = 1$. Likewise, if output Y_t is very large, the labor supply effect is so strong that no entrepreneur acquires information, in which case $\lambda_t(Y_t; m)$ becomes vertical again at $\lambda_t = 0$. When output is in the intermediate range, an increase in output reduces the incentive to acquire information, so $\lambda_t(Y_t; m)$ is downward sloping. With the help of this graph it is easy to see how the equilibrium responds to a change in A_t. When A_t increases output Y_t increases while λ_t decreases. In other words, information acquisition is countercyclical, which will be shown to have important implications on firm-level volatility in Section 2.3.

To obtain a complete characterization of Y_t and λ_t, we first obtain the endogenous TFP z_t by solving the system of equations (19) and (21). To simplify notation, we normalize $\frac{1}{\theta-1}(1-\frac{1}{\theta})^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}} = 1$ without loss of generality. It follows that

$$\log z_t = \begin{cases} \log \overline{z} & \text{if } \log A_t < \log \underline{A} \\ -\frac{\gamma}{\theta\gamma-1}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right) - \frac{\gamma-1}{\theta\gamma-1}\log A_t & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ \log \underline{z} & \text{if } \log A_t > \log \overline{A} \end{cases}, \tag{22}$$

where $\log \underline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\overline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$ and $\log \overline{A} = \frac{\gamma}{1-\gamma}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\underline{z}^{\frac{\theta\gamma-1}{\gamma}}\right)$. Note that z_t is decreasing in A_t when $\gamma - 1 > 0$. Then, by (19) the equilibrium output is given by

$$\log Y_t = \log(\theta-1) + \frac{1}{\gamma}(\log A_t + \log z_t)$$
$$= \begin{cases} \left[\log(\theta-1) + \frac{1}{\gamma}\log\overline{z}\right] \quad + \frac{1}{\gamma}\log A_t & \text{if } \log A_t < \log \underline{A} \\ \left[\log(\theta-1) - \frac{1}{\theta\gamma-1}\log\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)\right] + \frac{\theta-1}{\theta\gamma-1}\log A_t & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ \left[\log(\theta-1) + \frac{1}{\gamma}\log\underline{z}\right] \quad + \frac{1}{\gamma}\log A_t & \text{if } \log A_t > \log \overline{A} \end{cases}, \quad (23)$$

and by (18) the fraction of informed entrepreneurs is

$$\lambda_t = \frac{z_t^{\theta-1} - \underline{z}^{\theta-1}}{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}$$
$$= \begin{cases} 1 & \text{if } \log A_t < \log \underline{A} \\ \frac{\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)^{-\frac{(\theta-1)\gamma}{\theta\gamma-1}}\exp\left(-\frac{(\theta-1)(\gamma-1)}{\theta\gamma-1}\log A_t\right)-\underline{z}^{\theta-1}}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}} & \text{if } \log \underline{A} \le \log A_t \le \log \overline{A} \\ 0 & \text{if } \log A_t > \log \overline{A} \end{cases}. \quad (24)$$

Figure 2 depicts equilibrium $\lambda_t(A_t; m)$ and $\log Y_t(A_t; m)$.

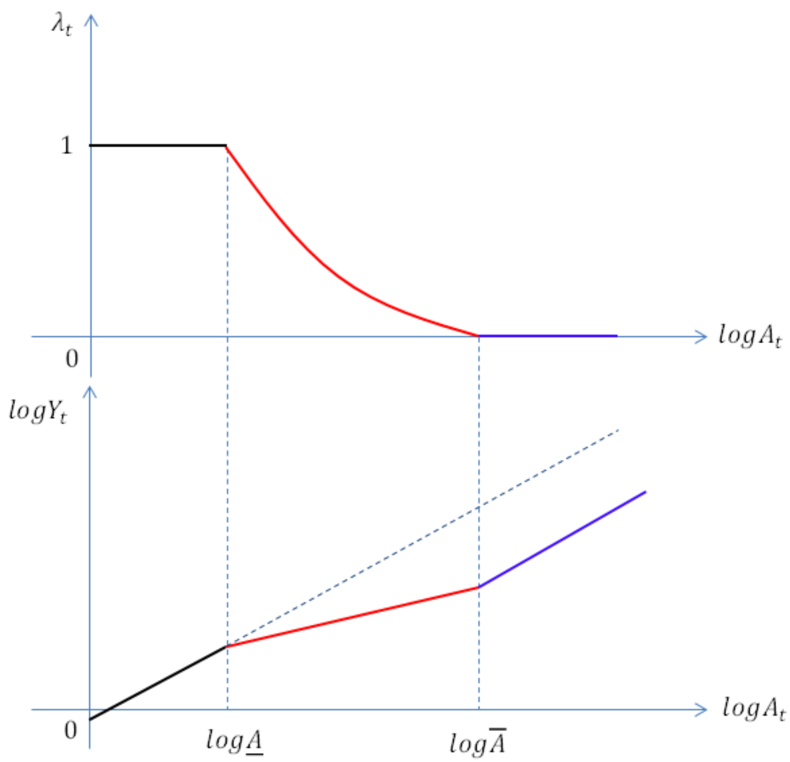

Figure 2: Equilibrium λ_t and $\log Y_t$ as a function of $\log A_t$

Once we have λ_t and Y_t, it is straightforward to obtain the rest of the variables. Based on (16) and (23), we have the aggregate labor[8]

$$N_t = \left(1 - \frac{1}{\theta}\right)\left(\frac{1}{\psi}\right) Y_t^{1-\gamma}. \quad (25)$$

We can apply equation (15) to obtain real wage W_t, equations (11) and (12) to get firms' production y_{jt}, and equation (5) to get price p_{jt}. The following proposition summarizes the equilibrium.

Proposition 1 *In equilibrium with information acquisition,* $\log Y_t(A_t; m)$ *is continuous and increasing in* A_t *(under* $\theta\gamma - 1 > 0$*), while* $\lambda_t(A_t; m)$ *is continuous and decreasing in* A_t *(under* $\gamma - 1 > 0$*).*

Proof. *See Appendix.* ■

Proposition 1 highlights the countercyclicality of information acquisition under the plausible parameter condition $\gamma - 1 > 0$, that is, information acquisition has a smoothing effect. A higher initial TFP shock increases aggregate output but a higher aggregate output also implies lower incentives for firms to acquire information.[9]

Note however that whereas the proportion of firms acquiring information (λ_t) is countercyclical, the total expenditure ($\lambda_t m W_t$) on information acquisition in the economy is procyclical for $A_t \leq \left(\frac{1}{\gamma}\right)^{\frac{\theta\gamma-1}{(\theta-1)(\gamma-1)}} \overline{A}$ (see the proof of Proposition 1 in Appendix). That is, when A_t increases, the effect of the increase in wages dominates the effect of the decrease in λ_t.

2.3 Endogenous Uncertainty

After obtaining the expression of λ_t and Y_t, we now calculate firm-level dispersion. We show that firm-level dispersion, often as a measure of economic uncertainty, endogenously increases during a recession.

Firm-level Dispersion (Individual Volatility) For given A_t and m, we first examine the standard deviation of production (in logs) at the firm level. As $\log y_{jt} = \log n_{jt} + \log A_t$, employment at the firm level has the same standard deviation as production. If no firms acquire information, they will all produce the same amount. If all firms acquire information, their production will vary according to the signals they obtain, in which case the standard deviation of production is positive. This suggests that more information acquisition will lead to an increase in measured uncertainty.

[8] In our baseline model, labor supply is countercyclical for TFP shocks. This is because there is no investment in our model. With investment, in response to a positive TFP shock, labor supply would be procyclical as labor would flow out of leisure to the production of investment goods. In our extended model with financial friction shocks (Section 3), we will show that labor supply is procyclical.

[9] In our baseline model, the (average) markup is constant and the aggregate profit is procyclical. In our extended model with financial friction shocks (Section 3), we will show that the (average) markup is countercyclical.

We obtain the dispersion of production across intermediate goods firms in the following closed-form:

$$SD\left(\log y_{jt}\right)=\begin{cases}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log A_t<\log \underline{A}\\ \left[\lambda_t\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\right)+\frac{\lambda_t-\lambda_t^2}{4\theta^2}\frac{1}{\tau_\varepsilon}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^2\right]^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log \underline{A}\leq\log A_t\leq\log\overline{A}\\ 0 & \text{if } \log A_t>\log\overline{A}\end{cases}, \tag{26}$$

where we calculate volatility as $SD\left(x\right)=\sqrt{\mathbb{E}^2\left(x\right)-\left[\mathbb{E}\left(x\right)\right]^2}$. The second line in (26) is increasing in $\lambda_t\in[0,1]$ under parameter condition $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\leq 4\theta^2$. The requirement of a parameter condition is due to the *mixed* distribution of $\log y_{jt}$ in our baseline model: a fraction $1-\lambda_t$ of firms form a mass-point for the distribution of production, while the remaining fraction λ_t is non-atomic.[10] We can assume that $\sigma_\varepsilon<2\theta$, that is, the standard deviation of idiosyncratic demand is not large. Note that θ is typically assumed to be about $\theta=10$ in new Keynesian models, so this assumption will hold under plausible calibrations. If $\sigma_\varepsilon<2\theta$, the parameter condition is satisfied.

Another commonly used measure of uncertainty is the dispersion of sales revenue across firms. The sales revenue of firm j is

$$\log\left(y_{jt}p_{jt}\right)=(1-\frac{1}{\theta})\log y_{jt}+\frac{1}{\theta}\left(\log\epsilon_{jt}+\log Y_t\right).$$

As Y_t is common across firms, it does not affect the standard deviation of $\log\left(y_{jt}p_{jt}\right)$. If no firms acquire information, we have $SD\left[\log(y_{jt}p_{jt})\right]=\frac{1}{\theta}\sigma_\varepsilon$. More concretely,

$$SD\left[\log\left(y_{jt}p_{jt}\right)\right]=\begin{cases}\left[\left(\frac{1}{\theta}\right)^2+\frac{\theta^2-1}{\theta^2}\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right]^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log A_t<\log\underline{A}\\ \left[\begin{array}{c}\frac{1}{\theta^2}+\lambda_t\frac{\theta^2-1}{\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)\\ +\lambda_t\left(1-\lambda_t\right)\frac{1}{\tau_\varepsilon}\left[\frac{1}{2}\frac{\theta-1}{\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)\right]^2\end{array}\right]^{\frac{1}{2}}\sigma_\varepsilon & \text{if } \log\underline{A}\leq\log A_t\leq\log\overline{A}\\ \frac{1}{\theta}\sigma_\varepsilon & \text{if } \log A_t>\log\overline{A}\end{cases}. \tag{27}$$

The second line in (27) is increasing in $\lambda_t\in[0,1]$ if and only if $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\leq 4\theta^2\frac{\theta+1}{\theta-1}$. This condition is automatically satisfied under the assumption that $\sigma_\varepsilon<2\theta$. Proposition 2 summarizes the prediction of our baseline model on firm-level dispersion.

Proposition 2 *Suppose $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\leq 4\theta^2$. The dispersion of production and that of sales revenue across intermediate goods firms are countercyclical (under $\gamma-1>0$). A positive TFP shock leads to a higher GDP accompanied by a lower dispersion across firms, i.e., dispersion is lower in a boom*

[10] To obtain explicit solutions, we have chosen a setup of the binary information choice: a fraction λ_t of firms acquire information and obtain a signal with a fixed precision (τ_e). If we allow all firms to acquire information and the signal to be a *continuous* function of entrepreneurs' effort (see Section 4), the dispersion, $SD\left[\log\left(y_{jt}\right)\right]$, will be decreasing in A_t without requiring parameter conditions.

and higher in a recession.

Proof. *See Appendix.* ■

Proposition 2 predicts that the measured uncertainty is countercyclical, consistent with a large empirical literature. Intuitively, in a boom with a higher aggregate output, a lower fraction of firms acquire information and thus their production is less responsive to their true demand shocks, which leads to lower dispersion across firms.

2.4 Welfare Implications

We can now study the welfare implications in our model. Total social welfare is measured by

$$U(C_t) - \psi N_t - \psi m \lambda_t \tag{28}$$

where $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, $C_t = Y_t$, and Y_t and N_t are both functions of λ_t given by (19) and (25), respectively. The optimal λ_t for the social planner under the constrained second-best equilibrium is given by the first-order condition of (28):

$$\frac{dU(Y_t)}{dY_t}\frac{dY_t}{d\lambda_t} - \psi\frac{dN_t}{d\lambda_t} = \psi m. \tag{29}$$

Applying (19) we have

$$\begin{aligned}
\frac{dU(Y_t)}{dY_t}\frac{dY_t}{d\lambda_t} &= Y_t^{-\gamma}\frac{dY_t}{dz_t}\frac{dz_t}{d\lambda_t} \\
&= Y_t^{-\gamma} \times \underbrace{\frac{1}{\gamma}\frac{Y_t}{z_t}}_{\frac{dY_t}{dz_t}} \times \underbrace{\frac{1}{\theta-1} z_t^{2-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dz_t}{d\lambda_t}} \\
&= \underbrace{Y_t^{-\gamma}}_{\frac{dU(Y_t)}{dY_t}} \times \underbrace{\frac{1}{\theta-1}\frac{1}{\gamma} Y_t z_t^{1-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dY_t}{d\lambda_t}}
\end{aligned}$$

and applying (25) we have

$$\psi\frac{dN_t}{d\lambda_t} = \underbrace{\left(1-\frac{1}{\theta}\right)(1-\gamma)Y_t^{-\gamma}}_{\psi\frac{dN_t}{dY_t}} \times \underbrace{\frac{1}{\theta-1}\frac{1}{\gamma} Y_t z_t^{1-\theta}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}_{\frac{dY_t}{d\lambda_t}}.$$

So we can write (29) as

$$\left[\frac{1}{(\theta-1)\gamma} + \frac{\gamma-1}{\gamma}\frac{1}{\theta}\right] z_t^{1-\theta} Y_t^{1-\gamma}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = m\psi. \tag{30}$$

Equations (19) and (30) jointly give the constrained second-best equilibrium. Note that (30) parallels (21). So the constrained second-best equilibrium can be depicted by a similar graph as in Figure 1. The $Y_t(\lambda_t; A_t)$ curve, given by (19), is the same in both the market equilibrium and the constrained second-best equilibrium. However, since $\frac{1}{(\theta-1)\gamma} + \frac{\gamma-1}{\gamma}\frac{1}{\theta} > \frac{1}{\theta}$, the $\lambda_t(Y_t, m)$ curve in the constrained second-best equilibrium is always above its market equilibrium counterpart.

Proposition 3 *The equilibrium λ_t is lower in the competitive equilibrium than under the second best (when $1 - \theta\gamma < 0$). That is, too few individual firms acquire information in the competitive equilibrium.*

Proof. *See Appendix.* ■

The intuition behind Proposition 3 is as follows. When an individual firm acquires information, it has two externalities, to other firms' profit and to the consumer surplus. We can decompose and quantify these two externalities (see the proof of Proposition 3 in Appendix). The first externality is negative and the second is positive. The overall effect is that the positive externality to consumer surplus exceeds the negative externality to other firms' profit when $1 - \theta\gamma < 0$, so too few individual firms acquire information in the competitive equilibrium compared with the second-best equilibrium.

3 The Model with Financial Frictions

The recent financial crisis has shown that financial factors continue to be an important source of business cycle fluctuations (see, e.g., Quadrini (2011) and Jermann and Quadrini (2012)). Our goal in the section is to show that besides TFP shocks, financial shocks can also generate countercyclical information acquisition by firms, and therefore also generate countercyclical firm-level dispersion. We now extend our baseline model by introducing simple shocks to borrowing constraints and financial costs.

3.1 Borrowing Constraint Shocks

The tightness of borrowing constraints may have been a major cause of the recent financial crisis. We can show that shocks to the tightness of borrowing constraints can generate countercyclical information acquisition by firms, and have implications to firm-level dispersion.

We assume that firms face borrowing constraints in financing their working capital to hire workers. Specifically, firm j has the following constraint:

$$W_t n_{jt} \leq \zeta_t \mathbb{E}_t[p_{jt} y_{jt} | s_{jt}] \tag{31}$$

where ζ_t measures the tightness of the borrowing constraint. We assume $\zeta_t < 1 - \frac{1}{\theta}$ so (31) will be binding. Banks provide loans to firms in proportion to their expectation of the firms' sales revenues, in the spirit of Kiyotaki and Moore (1997).[11] If a firm acquires/collects information, it obtains a "verifiable" signal in that it has concrete, observable and verifiable evidence (e.g., documentation) about the signal. So the firm can provide the documentation to banks in applying for loans. In contrast, if a firm does not acquire/collect information, banks can only extend loans to the firm based on the prior of the firm's revenue.

Based on its signal s_{jt}, an informed firm j solves

$$\Pi_{jt} = \max_{y_{jt}} \mathbb{E}\left[y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A_t} y_{jt} | s_{jt} \right],$$

with constraint $\frac{W_t}{A_t} y_{jt} \leq \zeta_t \mathbb{E}_t[y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} | s_{jt}]$. Because $\zeta_t < 1 - \frac{1}{\theta}$, the constraint is binding in the optimization. The optimal production is then given by

$$y_{jt} = \left\{ \zeta_t \frac{A_t}{W_t} Y_t^{\frac{1}{\theta}} \mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right\}^{\theta}.$$

Similarly, the production for an uninformed firm is given by

$$\tilde{y}_{jt} = \left\{ \zeta_t \frac{A_t}{W_t} Y_t^{\frac{1}{\theta}} \mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}} \right] \right\}^{\theta}.$$

With the same definition of $z(\lambda_t)$ as in (17), equation (15) for the real wage changes to

$$W_t = \zeta_t A_t z(\lambda_t).$$

Again, by equation (3), we obtain

$$Y_t = \zeta_t^{\frac{1}{\gamma}} \left(\frac{A_t}{\psi} \right)^{\frac{1}{\gamma}} (z(\lambda_t))^{\frac{1}{\gamma}}, \tag{32}$$

which parallels equation (19) in the baseline model. Finally, we obtain the aggregate labor

$$N_t = \frac{1}{\psi} \zeta_t^{\frac{1}{\gamma}} \left(\frac{A_t}{\psi} \right)^{\frac{1-\gamma}{\gamma}} (z(\lambda_t))^{\frac{1-\gamma}{\gamma}}. \tag{33}$$

To derive a relationship between λ_t and Y_t, we again turn to the firms' information acquisition problem. We first obtain the ex ante expected profit for an informed entrepreneur,

$$\pi_t^I = (1 - \zeta_t) Y_t z_t^{1-\theta} \overline{z}^{\theta-1},$$

[11] To focus on the borrowing constraint, we abstract away from the interest rate by assuming it to be zero.

and the expected profit for an uninformed entrepreneur,

$$\pi_t^U = (1-\zeta_t) Y_t z_t^{1-\theta} \underline{z}^{\theta-1}.$$

Then, equation (21) changes to

$$(1-\zeta_t)\,(z(\lambda_t))^{1-\theta}\, Y_t^{1-\gamma}\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right) = m\psi, \tag{34}$$

for $\lambda_t \in (0,1)$.

Figure 3 depicts the equilibrium. Curve $Y_t(\lambda_t; A_t, \zeta_t)$ is given by (32) while curve $\lambda_t(Y_t; A_t, \zeta_t)$ is given by (34). When ζ_t changes, there are two opposite forces acting on Y_t. First, an increase in ζ_t relaxes the borrowing constraint of firms, which induces higher employment and hence higher output; that is, the curve $Y_t(\lambda_t; A_t, \zeta_t)$ shifts up. Second, an increase in ζ_t also reduces the profit margin of firms, leading to lower incentives for firms to acquire information *ceteris paribus*; that is, the curve $\lambda_t(Y_t; A_t, \zeta_t)$ shifts to the left. This tends to reduce aggregate output. It turns out that the first effect dominates the second effect if $\zeta_t < 1 - \frac{1}{\theta}$.

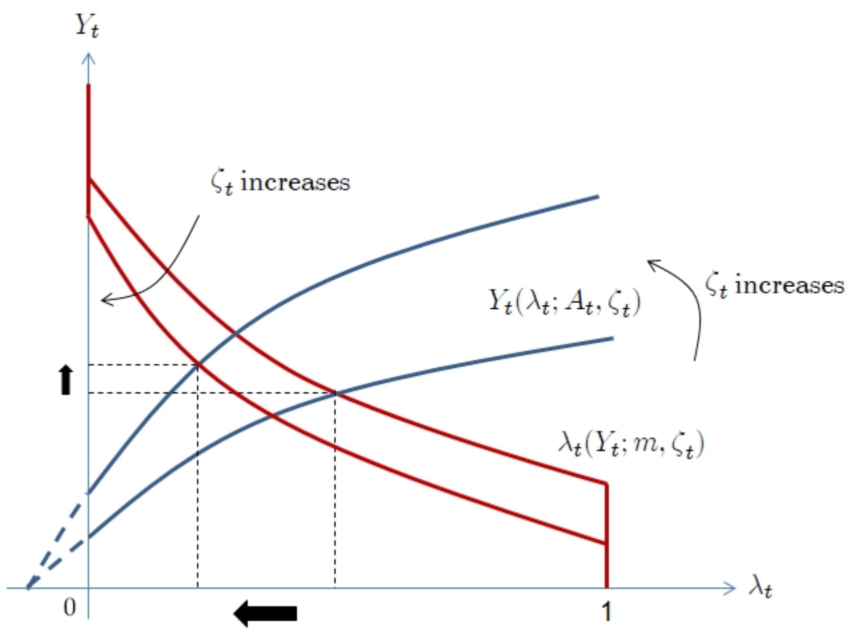

Figure 3: Equilibrium with Information Acquisition and Borrowing Constraints

For explicit solutions to Y_t and λ_t, equations (32) and (34) together yield

$$Y_t = \zeta_t^{\frac{\theta-1}{\theta\gamma-1}} \left(\frac{A}{\psi}\right)^{\frac{\theta-1}{\theta\gamma-1}} \left[\frac{(1-\zeta_t)\left(\overline{z}^{\theta-1} - \underline{z}^{\theta-1}\right)}{m\psi}\right]^{\frac{1}{\theta\gamma-1}}. \tag{35}$$

Y_t is increasing in ζ_t under $\zeta_t < 1 - \frac{1}{\theta}$.[12] We can now easily show that $z(\lambda_t)$ is decreasing in ζ_t. Since $z'(\lambda_t) > 0$, λ_t is decreasing in ζ_t as well. Also, it is easy to check the aggregate profit being

$$\Pi_t = (1 - \zeta_t) Y_t.$$

Proposition 4 *In equilibrium with information acquisition and borrowing constraints,* $\log Y_t(A_t; m, \zeta_t)$ *is increasing in* ζ_t *(under* $\theta\gamma > 1$*) while* $\lambda_t(A_t; m, \zeta_t)$ *is decreasing in* ζ_t *(under* $\gamma > \max\{\frac{1}{\theta}, 1-\zeta_t\} = 1 - \zeta_t$ *or equivalently* $\zeta_t \in \left(1 - \gamma, 1 - \frac{1}{\theta}\right)$*). Also, when* $\theta\gamma - 1 > 0$*, aggregate profit* Π_t *is increasing in* ζ_t *if and only if* $\zeta_t < \frac{\theta-1}{\theta\gamma+\theta-1}$.

Proof. *See Appendix.* ■

Compared with the TFP shock in Proposition 1 in the baseline model, the parameter condition is relaxed for the borrowing constraint shock in Proposition 4. In fact, even if $\gamma = 1$ (i.e., curve $\lambda_t(Y_t; m, \zeta_t)$ in Figure 3 becomes a vertical line), $Y_t(A_t; m, \zeta_t)$ would still be increasing in ζ_t and $\lambda_t(A_t; m, \zeta_t)$ would still be decreasing in ζ_t as long as $\zeta_t \in \left(0, 1 - \frac{1}{\theta}\right)$. If $\gamma < 1$, the result holds for $\zeta_t \in \left(1 - \gamma, 1 - \frac{1}{\theta}\right)$.

Proposition 5 *Suppose* $\frac{\tau_e}{\tau_\varepsilon + \tau_e}\frac{1}{\tau_\varepsilon} \leq 4\theta^2$*. The dispersion of production and that of sales revenue across intermediate goods firms are countercyclical. A shock that tightens the financial constraint (i.e., a lower* $\zeta_t \in \left(1 - \gamma, 1 - \frac{1}{\theta}\right)$*) leads to a lower GDP accompanied by a higher firm-level dispersion, i.e., dispersion is higher in a recession and lower in a boom.*

Proof. *See Appendix.* ■

So like the TFP shock, the borrowing constraint shocks can also generate countercyclical measured uncertainty at the firm level (under weaker conditions). In contrast to the TFP shocks, countercyclical measured uncertainty can arise under financial shocks even with weak income effect on labor supply. When $\gamma < 1$, real wage can display weak procyclicality, consistent with some findings on aggregate real wage behavior.[13]

Finally, based on (32) and (33), when $\lambda_t \in (0,1)$ we have

$$N_t = \zeta_t^{\frac{\theta+\gamma-2}{\theta\gamma-1}} (1-\zeta_t)^{\frac{1-\gamma}{\theta\gamma-1}} \left(\frac{1}{\psi}\right) \left(\frac{A_t}{\psi}\right)^{\frac{(\theta-1)(1-\gamma)}{\theta\gamma-1}} \left[\frac{\overline{z}^{\theta-1} - \underline{z}^{\theta-1}}{m\psi}\right]^{\frac{1-\gamma}{\theta\gamma-1}}.$$

Under $\theta\gamma > 1$, N_t is increasing in ζ_t.[14] So unlike the TFP shock, the financial shock ζ_t can also explain the procyclical movement in labor. In addition, borrowing constraint shocks can generate

[12] Mathematically, the term $\zeta_t^{\frac{\theta-1}{\theta\gamma-1}} (1-\zeta_t)^{\frac{1}{\theta\gamma-1}}$ is increasing in ζ_t iff $\zeta_t < 1 - \frac{1}{\theta}$.

[13] A typical finding in the literature is that aggregate real wage displays little procyclicality but microdata reveals substantial procyclicality. See Barsky and Solon (1989) for an example.

[14] Mathematically, the term $\zeta_t^{\frac{\theta+\gamma-2}{\theta\gamma-1}} (1-\zeta_t)^{\frac{1-\gamma}{\theta\gamma-1}}$ is increasing in ζ_t iff $\zeta_t < 1 + \frac{\gamma-1}{\theta-1}$. Considering $\zeta_t < 1 - \frac{1}{\theta}$, the condition $\zeta_t < 1 + \frac{\gamma-1}{\theta-1}$ is automatically true if $1 - \frac{1}{\theta} < 1 + \frac{\gamma-1}{\theta-1}$ or equivalently $\theta\gamma > 1$.

procyclical profits and countercyclical markups considering that $\frac{d\Pi_t}{d\zeta_t} > 0$ and the profit-to-revenue ratio in the economy is $1 - \zeta_t$. Similar to the case of TFP shocks, the total expenditure ($\lambda_t m W_t$) on information acquisition in the economy is procyclical for a lower range of ζ_t.

3.2 Financial Cost Shocks

Our second approach for modelling financial frictions is to assume that firms need to borrow from financial intermediaries for working capital before sales revenue is realized. Financial intermediaries charge a gross interest rate $R_t > 1$ for the loan. More specifically, if firm j hires n_{jt} and the real wage is W_t, the firm needs to borrow $W_t n_{jt}$ from the financial intermediaries to pay the workers before its sale revenue is realized. It then makes a repayment of $R_t W_t n_{jt}$ to the financial intermediaries after it has received its revenues from sales. Hence, $R_t - 1$ is the net interest rate margin or the external financial premium for the financial intermediaries, which has been documented to be countercyclical (see, e.g., Dueker and Thornton (1997), Graeve (2008), Aliaga-Diaz and Olivero (2009) and Olivero (2010)). The countercyclical external financial premium also plays an essential role in the theory of financial accelerator pioneered by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) where financial frictions can amplify business fluctuations by increasing the cost of external funds. Since our purpose is to study the consequences of an increase in the interest rate margin for uncertainty, we will treat R_t as exogenous.[15]

An informed firm's production problem changes to

$$\Pi_{jt} = \max_{Y_{jt}} \mathbb{E}\left[y_{jt}^{1-\frac{1}{\theta}} (\epsilon_{jt} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A_t} R_t y_{jt} | s_{jt} \right],$$

where $R_t > 1$ is the (gross) interest rate on loan. The optimal production becomes

$$y_{jt} = y(A_t, s_{jt}) = \left[\left(1 - \frac{1}{\theta}\right) \frac{A_t}{W_t R_t} Y_t^{\frac{1}{\theta}} \mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}) \right]^{\theta}.$$

Similarly, the optimal production for an uninformed firm is

$$\tilde{y}_{jt} = \tilde{y}(A_t) = \left[\left(1 - \frac{1}{\theta}\right) \frac{A_t}{W_t R_t} Y_t^{\frac{1}{\theta}} \mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}) \right]^{\theta}.$$

With the same definition of $z(\lambda_t)$ as in (17), real wage becomes

$$W_t = (1 - \frac{1}{\theta}) A_t \frac{z(\lambda_t)}{R_t}.$$

[15] For modeling financial intermediaries in macroeconomic models, see Gertler and Kiyotaki (2012) and Miao and Wang (2014). In these models, a decline in the net worth of the financial intermediaries raises the interest rate margins and causes a contraction of the real economy.

Then we can use equation (3) to obtain the aggregate output

$$Y_t = \left(1 - \frac{1}{\theta}\right)^{\frac{1}{\gamma}} \left(\frac{A_t z(\lambda_t)}{\psi R_t}\right)^{\frac{1}{\gamma}}. \tag{36}$$

The total labor is given by

$$N_t = \left(1 - \frac{1}{\theta}\right)^{\frac{1}{\gamma}} \left(\frac{1}{\psi R_t}\right)^{\frac{1}{\gamma}} [A_t z(\lambda_t)]^{\frac{1-\gamma}{\gamma}}. \tag{37}$$

Intuitively, $R_t > 1$ creates a distortion between the marginal product of labor and real wage. When R_t increases, the distortion becomes larger and hence reduces labor and output.

The firms' problem of information acquisition is the same as in the baseline case. That is, equation (21) still holds. Equations (36) and (21) jointly determine Y_t and λ_t as functions of R_t. It is easy to show that Y_t decreases with R_t while λ_t increases with R_t. So in a recession, more firms will acquire information. As in Section 2.3, the dispersion of sales revenue or production across firms will increase in a recession, so the variation in R_t can also generate countercyclical firm-level dispersion.

Proposition 6 *In the equilibrium with information acquisition and financial costs,* $\log Y_t(A_t; m, R_t)$ *is decreasing in* R_t *(under* $\theta\gamma - 1 > 0$*) while* $\lambda_t(A_t; m, R_t)$ *is increasing in* R_t *(under* $\gamma - 1 > 0$*). Furthermore, the dispersion of production and that of sales revenue across intermediate goods firms are countercyclical under* $\frac{\tau_e}{\tau_\varepsilon + \tau_e} \frac{1}{\tau_\varepsilon} \leq 4\theta^2$.

Proof. *See Appendix.* ■

The intuition again can be illustrated by a diagram, which is Figure 4, similar to Figure 1.

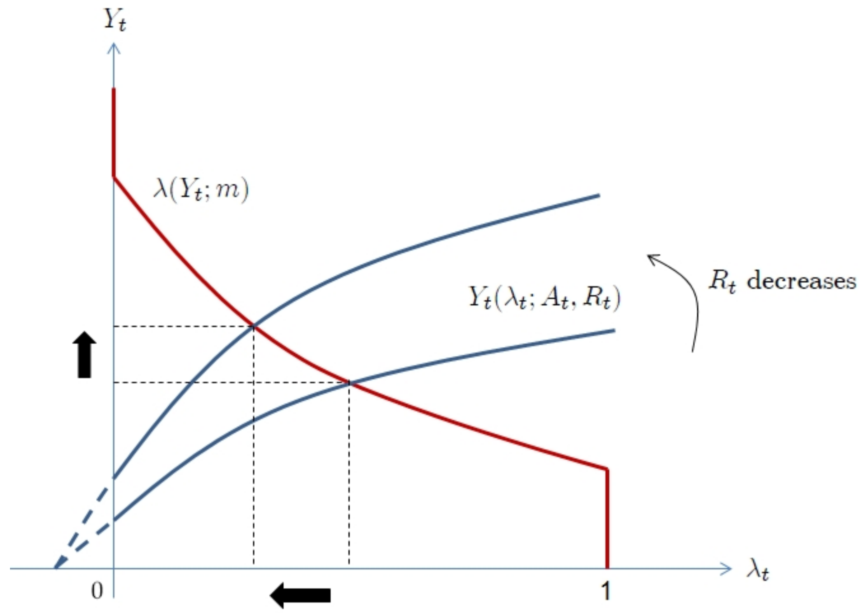

Figure 4: Equilibrium with Information Acquisition and Financial Costs

We can also find the explicit solution of total labor:

$$N_t = (\psi R_t)^{\frac{2-\theta-\gamma}{\theta\gamma-1}} A_t^{\frac{(\theta-1)(1-\gamma)}{\theta\gamma-1}} \left(1-\frac{1}{\theta}\right)^{\frac{\theta+\gamma-2}{\theta\gamma-1}} \left[\frac{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}{\theta m\psi}\right]^{\frac{1-\gamma}{\theta\gamma-1}}$$

When $\gamma > \max\left(2-\theta, \frac{1}{\theta}\right)$, N_t is decreasing in R_t. So, unlike the aggregate TFP shock, labor and output exhibit comovement if they are driven by R_t. Also, it is easy to check that under financial costs shocks, profits are procyclical and markups are countercyclical. Similar to the case of TFP shocks, the total expenditure ($\lambda_t m W_t$) on information acquisition in the economy is procyclical for a higher range of R_t.

Before we close this section, we summarize the comparison of the effects of TFP shocks and the two types of financial friction shocks in Table 1.

	TFP shock	Financial Cost Shock	Borrowing Constraint Shock
Information Acquisition	*countercyclical*	*countercyclical*	*countercyclical*
Parameter Condition	$\gamma > 1$	$\gamma > 1$	$\gamma > 1 - \zeta_t$
Aggregate Employment	*countercyclical*	*procyclical*	*procyclical*
Aggregate Profit	*procyclical*	*procyclical*	*procyclical*
Average Markup	*constant*	*countercyclical*	*countercyclical*

Table 1: Comparison of Different Types of Shocks

4 Aggregate Volatility

In this section, we show that the information acquisition channel can also generate countercyclical aggregate volatility, an empirical fact that is also documented in the empirical literature. We will first show countercyclical aggregate volatility in a simple case. Then we will generalize this simple case along several dimensions to show that the insight carries through to more general setups.

4.1 Aggregate Demand Shocks

The aggregate production function is

$$Y_t = \left[\int_0^1 (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \, y_{jt}^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1, \tag{38}$$

where the demand shock to firm j now also has an aggregate component, ϕ_t. We assume that $\log \phi_t \equiv \eta_t \sim \mathcal{N}(-\frac{1}{2}\sigma_\eta^2, \sigma_\eta^2)$. The idiosyncratic demand shock is the same as before, that is, $\log \epsilon_{jt} \equiv \varepsilon_{jt} \sim \mathcal{N}(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2)$. Like our benchmark model, an entrepreneur can obtain an informative signal about η_t and ε_{jt} after paying some costs. Specifically, entrepreneur j obtains signal

$$s_{jt} = \omega \eta_t + (1-\omega)\,\varepsilon_{jt} + e_{jt} \quad \text{with } e_{jt} \sim \mathcal{N}(0, \frac{1}{\tau_{je}}),$$

if he spends $m_j = h(\tau_{je})$ working hours to acquire information at the beginning of each period. The cost function of information acquisition is $h(\cdot)$, an increasing function.

It is assumed that $\eta_t, \varepsilon_{jt}, e_{jt}$ and A_t are independent. We also make the following assumption:

Assumption 1 *:* $0 < \omega < \frac{(\theta-1)\sigma_\varepsilon^2}{(\theta-1)\sigma_\varepsilon^2 + \theta(\gamma-1)\sigma_\eta^2}$.

Assumption 1 says that ω is not too large, that is, the signal mostly contains information regarding idiosyncratic demand shocks. Notice that this assumption easily holds if γ is close to unity or σ_η^2 is relatively small compared with σ_ε^2. We will explain the role of this assumption later below.

We normalize by the wage $W_t = 1$ instead of the final good price P_t, and assume that firms hire workers based on *nominal* wage before the actual production and trades take place, as in Benhabib, Wang and Wen (2014). Firms of course have to form expectations of the actual prices and hence the *real* wage when they undertake production. All random variables are independent across time.

The timing of events in this extension model is as follows:

1. At the beginning of each period, A_t, η_t and $\{\varepsilon_{jt}\}_0^1$ are realized. Households have full information regarding A_t, η_t and $\{\varepsilon_{jt}\}_0^1$.

2. After observing A_t, an entrepreneur makes his decision on information acquisition. Signal s_{jt} is obtained if he pays cost $m_j = h(\tau_{je})$ in terms of working hours. The precision of the signal increases with the cost.

3. Based on signal s_{jt} and nominal wage $W_t = 1$, an entrepreneur decides how much labor n_{jt} to hire in producing his intermediate goods. An entrepreneur has to optimally forecast the real wage W_t/P_t based on his signal and A_t.

4. Goods markets open. Goods are exchanged at market clearing prices. The final consumption is realized.

4.1.1 Equilibrium with Exogenous Information

Before we turn to the case of endogenous information acquisition, we first analyze the equilibrium under exogenous information. This will facilitate our analysis and help us understand the role played by information acquisition in the next two subsections. Specifically, in this subsection, we assume that precision of the signals received by all entrepreneurs is exogenous and the same, namely, $\tau_{je} = 1/\sigma_e^2$ for all j.

Since households have perfect information, the representative household's consumption problem, (3), is replaced by

$$P_t = \frac{1}{\psi} C_t^{-\gamma} \tag{39}$$

Production Decision Entrepreneur j solves

$$y_{jt} = y(A_t, s_{jt}) = \arg\max_{y_{jt}} \mathbb{E}\left[p_{jt} y_{jt} - n_{jt} | s_{jt}\right]$$

subject to

$$p_{jt} = P_t \cdot \left(\frac{(\phi_t \epsilon_{jt}) Y_t}{y_{jt}} \right)^{\frac{1}{\theta}} \tag{40}$$

and $y_{jt} = A_t n_{jt}$. Notice that we have normalized $W_t = 1$. Using equation (39) and $Y_t = C_t$ in equilibrium, we obtain

$$y_{jt} = y(A_t, s_{jt}) = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left\{ \mathbb{E}_t \left[\left(Y_t^{\frac{1}{\theta} - \gamma} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \right) | s_{jt} \right] \right\}^{\theta}. \tag{41}$$

The aggregate output is hence given by

$$Y_t = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left[\int_0^1 (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \left(\mathbb{E} \left[Y_t^{\frac{1}{\theta} - \gamma} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} | s_{jt} \right] \right)^{\theta - 1} dj \right]^{\frac{\theta}{\theta - 1}}. \tag{42}$$

Equations (41) and (42) jointly determine the aggregate and individual equilibrium outputs. Once we obtain Y_t and y_{jt}, we can first use (39) to compute the aggregate price P_t and then use (40) to compute the individual price p_{jt}. We use the guess-and-verify strategy to obtain Y_t and y_{jt}. We have the following proposition.

Proposition 7 *Aggregate production is given by*

$$\log Y_t = \log \bar{Y} + \frac{1}{\gamma} \log A_t + \kappa \eta_t, \tag{43}$$

where $\bar{Y}$ depends on σ_ε^2, θ, ψ, ω, γ, σ_e^2 and σ_η^2 and

$$\kappa = \frac{1}{\theta - 1} + \omega \frac{(1-\omega)\,\sigma_\varepsilon^2 - \omega \frac{\theta(\gamma-1)}{\theta-1} \sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2}. \tag{44}$$

The individual production is given by

$$\log y_{jt} = \log \bar{y} + \frac{1}{\gamma} \log A_t + \frac{[(1-\theta\gamma)\kappa + 1]\omega\sigma_\eta^2 + (1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2 + (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2} s_{jt}, \tag{45}$$

where $\bar{y}$ depends only on σ_ε^2, θ, ψ, ω, γ, σ_e^2 and σ_η^2.

Proof. *See Appendix.* ■

We now study the implication of information precision on aggregate uncertainty and firm-level uncertainty. We measure aggregate uncertainty as the conditional standard deviation of aggregate output (in logs). It is hence given by

$$SD(\log Y_t | A_t) = \kappa \sigma_\eta. \tag{46}$$

Note that κ is decreasing in σ_e^2 under Assumption 1. It follows that measured aggregate uncertainty increases with the information precision, $1/\sigma_e^2$.

Here we explain the role of Assumption 1. Mathematically, this assumption guarantees that the second term in κ is positive, that is, $(1-\omega)\,\sigma_\varepsilon^2 - \omega \frac{\theta(\gamma-1)}{\theta-1} \sigma_\eta^2 > 0$. Economically, this assumption ensures that firms' production *increases* with the signal s_{jt}, so that more precise information increases, rather than decreases, aggregate volatility. When $\gamma > 1$, firms' production levels are *strategic substitutes* under full information. An increase in η_t increases real wage, leading firms to reduce production despite higher demand. If ω is too large, then the signal contains information mainly about the aggregate demand shock η_t. So a higher signal induces firms to reduce production, which works as an offsetting and dampening force, decreasing the aggregate volatility. In contrast, if ω is small, the signal mainly contains information regarding idiosyncratic demand shocks. Idiosyncratic demand shocks do not affect real wage, so a higher signal induces firms to produce more.

In short, when ω is not too large so that Assumption 1 is satisfied, more precise information makes firms respond more strongly and *positively* to high signals, increasing aggregate volatility.

For a given realization of η_t, heterogeneity of ϵ_{jt} and e_{jt} across firms also generates individual volatility, i.e., firm-level dispersion. The standard deviations of production and sales revenue of firms are respectively

$$SD\left(\log y_{jt}|A_t\right) = \sqrt{\left\{\frac{(1-\omega)\,\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2}\right\}^2 \left[(1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2\right]}$$

and

$$SD\left[\log\left(p_{jt}y_{jt}\right)|A_t\right] = \sqrt{\begin{array}{c}\left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\left(1-\omega\right)+\frac{1}{\theta}\right]^2\sigma_\varepsilon^2 \\ +\left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\right]^2\sigma_e^2\end{array}}$$

When $\theta\gamma\omega^2\sigma_\eta^2 < (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2$ (which is satisfied under sufficient condition $\omega < \frac{\sigma_\varepsilon}{\sigma_\varepsilon+\sqrt{\theta\gamma}\sigma_\eta}$), $SD\left(\log y_{jt}|A_t\right)$ and $SD\left[\log\left(p_{jt}y_{jt}\right)|A_t\right]$ are both decreasing in σ_e^2. Intuitively, more precise information makes firms' production more responsive to, and more aligned with, their idiosyncratic demand shocks, increasing firm-level dispersion.

To summarize, we have the following lemma.

Lemma 1 *Under the assumption* $0 < \omega < \min\left\{\frac{(\theta-1)\sigma_\varepsilon^2}{(\theta-1)\sigma_\varepsilon^2+\theta(\gamma-1)\sigma_\eta^2}, \frac{\sigma_\varepsilon}{\sigma_\varepsilon+\sqrt{\theta\gamma}\sigma_\eta}\right\}$, *aggregate volatility and firm-level dispersion both increase with the information precision,* $1/\sigma_e^2$.

Proof. *See Appendix.* ■

So far, information precision τ_{je} is exogenous. In the next two subsections, we will endogenize τ_{je} and show that it is a function of A_t.

4.1.2 Endogenous Information: Discrete Information Acquisition

We first consider discrete information acquisition. Specifically, we assume that the level of information precision τ_{je} is discrete, i.e., $\tau_{je} \in \{\tau_e = 1/\sigma_e^2, 0\}$, and the level of effort m_j is also discrete, i.e., $m_j \in \{m, 0\}$. More concretely, if an entrepreneur spends a *constant* $m_j = m$ working hours to acquire information at the beginning of each period, he obtains the signal

$$s_{jt} = \omega\eta_t + (1-\omega)\,\varepsilon_{jt} + e_{jt} \quad \text{with } e_{jt} \sim \mathcal{N}(0, \sigma_e^2).$$

If an entrepreneur does not acquire information ($m_j = 0$), he receives no information or equivalently a useless signal

$$s_{jt} = \omega\eta_t + (1-\omega)\,\varepsilon_{jt} + \widetilde{e}_{jt} \quad \text{with } \widetilde{e}_{jt} \sim \mathcal{N}(0, +\infty).$$

In this subsection we assume that A_t only takes one of two values, $A_t \in \{A_H, A_L\}$, where $A_H > A_L$. We construct A_H and A_L such that the information acquisition of the firms is symmetric. Specifically, all firms acquire information in equilibrium after observing $A_t = A_L$ but none of them do so after observing $A_t = A_H$. That is, we specify parameters to highlight the mechanism that information precision chosen by entrepreneurs is endogenous and is decreasing in A_t. We examine the regimes of $A_t = A_H$ and $A_t = A_L$, in order.

The Case of $A_t = A_H$ As before, we solve firms' information acquisition problem by backward induction. We first work out firms' optimal production decision given their information precision. We then compare the cost and benefit of information acquisition to determine firms' optimal information acquisition decision.

The production for an informed entrepreneur, $y(A_t, s_{jt})$, is given by equation (41). His expected profit is given by $\pi(A_t, s_{jt}) = \frac{1}{\theta-1} y(A_t, s_{jt})/A_t$. Likewise, the production for an uninformed entrepreneur is given by

$$\tilde{y}_{jt} = \tilde{y}(A_t) = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left\{\mathbb{E}_t \left[\left(Y_t^{\frac{1}{\theta}-\gamma} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}}\right)\right]\right\}^{\theta}$$

and his expected profit is given by $\tilde{\pi}(A_t) = \frac{1}{\theta-1} \tilde{y}(A_t)/A_t$.

If $\lambda_t = 0$ in equilibrium, the aggregate output is given by

$$Y_t = \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{A_t}{\psi}\right)^{\theta} \left[\int_0^1 (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \left(\mathbb{E}\left[Y_t^{\frac{1}{\theta}-\gamma} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj\right]^{\frac{\theta}{\theta-1}}.$$

Since no firms acquire information, we can directly apply (43) in Proposition 7 to solve Y_t by setting $\sigma_e^2 = +\infty$. It follows that

$$\log Y_t = \log \bar{Y}_H + \frac{1}{\gamma} \log A_H + \frac{1}{\theta - 1} \eta_t, \tag{47}$$

where $\bar{Y}_H$ is a constant provided in Appendix.

It remains to show that indeed no firm has the incentive to acquire information. Using the expression of aggregate output in (47), we obtain the ex ante expected profit for an informed entrepreneur

$$\pi_{t,H}^I = \mathbb{E}_{s_{jt}} \pi(A_H, s_{jt}) = \frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_H^{1-\theta\gamma} \overline{V}_H A_H^{\frac{1}{\gamma}-1},$$

where $\overline{V}_H = \mathbb{E}_{s_{jt}} \left\{\mathbb{E}\left[\phi_t^{\frac{1-\gamma}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} | s_{jt}\right]\right\}^{\theta}$. Likewise, the profit for an uninformed entrepreneur is given by

$$\pi_{t,H}^U = \tilde{\pi}(A_H) = \frac{1}{\theta - 1} \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_H^{1-\theta\gamma} \underline{V}_H A_H^{\frac{1}{\gamma}-1},$$

where $\underline{V}_H = \left\{ \mathbb{E} \left[\phi_t^{\frac{1-\gamma}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} \right] \right\}^{\theta}$. Since $\theta > 1$, by Jensen's inequality, we have[16]

$$\begin{aligned} \overline{V}_H &= \mathbb{E}_{s_{jt}} \left\{ \mathbb{E} \left[\phi_t^{\frac{1-\gamma}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right\}^{\theta} \\ &> \left\{ \mathbb{E}_{s_{jt}} \mathbb{E} \left[\phi_t^{\frac{1-\gamma}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} | s_{jt} \right] \right\}^{\theta} = \left\{ \mathbb{E} \left[\phi_t^{\frac{1-\gamma}{\theta-1}} \epsilon_{jt}^{\frac{1}{\theta}} \right] \right\}^{\theta} = \underline{V}_H. \end{aligned}$$

If no firms have incentive to acquire information, we must have $\pi_{t,H}^I - \pi_{t,H}^U < m$, that is,

$$\frac{1}{\theta-1} \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} A_H^{\frac{1}{\gamma}-1} \bar{Y}_H^{1-\theta\gamma} (\overline{V}_H - \underline{V}_H) < m. \tag{48}$$

Notice that under $\gamma > 1$ the LHS of (48) is decreasing in A_H and approaches zero when A_H approaches infinity. This condition will be satisfied as long as A_H is sufficiently high. So indeed, no firm will acquire information if A_H is high enough.

The Case of $A_t = A_L$ By construction, all entrepreneurs have incentive to acquire information in this case. If $\lambda_t = 1$ in equilibrium, again by applying Proposition 7, the aggregate output is

$$\log Y_t = \log \bar{Y}_L + \frac{1}{\gamma} \log A_L + \kappa \eta_t,$$

where the constant $\bar{Y}_L = \bar{Y}$ is provided in the Appendix.

We now show that if A_L is low enough, all firms will acquire information. The ex ante expected profit for an informed entrepreneur is

$$\begin{aligned} \pi_{t,L}^I &= \frac{1}{\theta-1} \frac{\mathbb{E}_{s_{jt}} y(A_L, s_{jt})}{A_L} \\ &= \frac{1}{\theta-1} \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_L^{1-\theta\gamma} \overline{V}_L A_L^{\frac{1}{\gamma}-1}, \end{aligned}$$

where $\overline{V}_L = \mathbb{E}_{s_{jt}} \left\{ \mathbb{E} \left[\phi_t^{\kappa \frac{1-\theta\gamma}{\theta}} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} | s_{jt} \right] \right\}^{\theta}$. Similarly, the ex ante expected profit for an uninformed entrepreneur is

$$\pi_{t,L}^U = \frac{1}{\theta-1} \left(1 - \frac{1}{\theta}\right)^{\theta} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_L^{1-\theta\gamma} \underline{V}_L A_L^{\frac{1}{\gamma}-1},$$

[16] More specifically, $\overline{V}_H = \exp \begin{pmatrix} -\frac{1}{2} \left[\chi_H \sigma_\eta^2 + \sigma_\varepsilon^2 \right] + \frac{1}{2} \frac{1}{\theta} \left[\chi_H^2 \sigma_\eta^2 + \sigma_\varepsilon^2 \right] \\ + \frac{1}{2} \frac{\theta-1}{\theta} \frac{\left[\chi_H \omega \sigma_\eta^2 + (1-\omega) \sigma_\varepsilon^2 \right]^2}{\omega^2 \sigma_\eta^2 + (1-\omega)^2 \sigma_\varepsilon^2 + \sigma_e^2} \end{pmatrix}$ and $\underline{V}_H = \exp\left(-\frac{1}{2} \left[\chi_H \sigma_\eta^2 + \sigma_\varepsilon^2 \right] + \frac{1}{2} \frac{1}{\theta} \left[\chi_H^2 \sigma_\eta^2 + \sigma_\varepsilon^2 \right]\right)$, where $\chi_H = \theta \frac{1-\gamma}{\theta-1}$. Since $\theta > 1$, we can easily see that $\overline{V}_H > \underline{V}_H$.

where $\underline{V}_L = \left\{ \mathbb{E}\left[\phi_t^{\kappa \frac{1-\theta\gamma}{\theta}} (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \right] \right\}^{\theta}$. Again, by Jensen's inequality, we have $\overline{V}_L > \underline{V}_L$.[17] If all firms acquire information, we must have $\pi_{t,L}^I - \pi_{t,L}^U > m$, that is,

$$\frac{1}{\theta - 1}\left(1 - \frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta} A_L^{\frac{1}{\gamma}-1} \bar{Y}_L^{1-\theta\gamma} (\overline{V}_L - \underline{V}_L) > m. \tag{49}$$

Notice that under $\gamma > 1$ the LHS of (49) approaches infinity when A_L approaches zero. Hence, the above condition holds if A_L is small enough.

To summarize, we have shown that the equilibrium information precision is $1/\sigma_e^2$ for $A_t = A_L$ and is 0 for $A_t = A_H$. Also, if $A_H >> A_L$, it is easy to see that $\mathbb{E}[\log Y_t | A_t = A_L] < \mathbb{E}[\log Y_t | A_t = A_H]$. By Lemma 1, therefore, this establishes that both aggregate volatility and firm-level volatility are countercyclical. The intuition is the same as in our benchmark model. In a boom, the cost of information acquisition increases faster than the benefit because of the strong labor supply effect under $\gamma > 1$. We have the following Lemma.

Lemma 2 *The economy exhibits both higher aggregate volatility and higher firm-level dispersion in a recession.*

Proof. *See Appendix.* ■

4.1.3 Endogenous Information: Continuous Information Acquisition

We now relax the binary choice of information acquisition and make it a continuous choice. Specifically, after spending $h(\tau_e; b)$ working hours at the beginning of the period, entrepreneur j receives a noisy signal:[18]

$$s_{jt} = \omega\eta_t + (1-\omega)\,\varepsilon_{jt} + e_{jt} \quad \text{with } e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$$

Signal precision, $\tau_e = 1/\sigma_e^2$, is endogenous, continuous and effort-dependent. Here b measures the efficiency of information acquisition, and $\frac{\partial h}{\partial \tau_e} > 0$, $\frac{\partial^2 h}{\partial \tau_e^2} > 0$ and $\frac{\partial^2 h}{\partial \tau_e \partial b} > 0$. Since firms are ex ante identical, they will choose the same level of information precision. We denote by τ_e^* the information precision for all entrepreneurs $\ell \neq j$. Given τ_e^*, we now characterize entrepreneur $j's$ information acquisition problem. We first characterize aggregate output for a given information precision τ_e^*.

Equilibrium Y_t for a Given τ_e^* Given their information choice τ_e^*, entrepreneurs $\ell \neq j$

[17] More specifically, $\overline{V}_L = \exp\begin{pmatrix} -\frac{1}{2}\left(\chi_L\sigma_\eta^2 + \sigma_\varepsilon^2\right) + \frac{1}{2}\frac{1}{\theta}\left(\chi_L^2\sigma_\eta^2 + \sigma_\varepsilon^2\right) \\ +\frac{1}{2}\frac{\theta-1}{\theta}\frac{\left[\chi_L\omega\sigma_\eta^2 + (1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2} \end{pmatrix}$ and $\underline{V}_L = \exp\left(-\frac{1}{2}\left[\chi_L\sigma_\eta^2 + \sigma_\varepsilon^2\right] + \frac{1}{2}\frac{1}{\theta}\left[\chi_L^2\sigma_\eta^2 + \sigma_\varepsilon^2\right]\right)$, where $\chi_L = \kappa(1-\theta\gamma) + 1$.

[18] Rigorously, we should write $e_{jt} \sim \mathcal{N}(0, \sigma_{je}^2)$, rather than $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$. But we will confirm that in a symmetric equilibrium, all firms will choose the same level of information precision. To reduce notational clutter, we will use $\tau_e = 1/\sigma_e^2$ for firm j and τ_e^* for firms $\ell \neq j$, and in equilibrium $\tau_e = \tau_e^*$.

decide to produce

$$y(A_t, s_{\ell t}) = \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{A_t}{\psi}\right)^\theta \left\{\mathbb{E}_t\left[\left(Y_t^{\frac{1}{\theta}-\gamma}\left(\phi_t \epsilon_{\ell t}\right)^{\frac{1}{\theta}}\right)|s_{\ell t}\right]\right\}^\theta.$$

The aggregate output is hence given by

$$Y_t = \left(1 - \frac{1}{\theta}\right)^\theta \left(\frac{A_t}{\psi}\right)^\theta \left[\int_0^1 \left(\phi_t \epsilon_{\ell t}\right)^{\frac{1}{\theta}} \left(\mathbb{E}\left[Y_t^{\frac{1}{\theta}-\gamma}\left(\phi_t \epsilon_{\ell t}\right)^{\frac{1}{\theta}}|s_{\ell t}\right]\right)^{\theta-1} d\ell\right]^{\frac{\theta}{\theta-1}}.$$

As in the previous subsection, by applying Proposition 7, we can solve the aggregate output as

$$\log Y_t = \log \bar{Y}_C + \frac{1}{\gamma}\log A_t + \kappa \eta_t, \tag{50}$$

where $\bar{Y}_C$ is a constant not dependent on A_t and ϕ_t but dependent on $1/\tau_e^*$, which is an exogenous constant for entrepreneur j (see the expression of $\bar{Y}_C$ in Appendix). The coefficient κ is given by

$$\kappa = \kappa(\tau_e^*) = \frac{1}{\theta-1} + \omega \frac{(1-\omega)\,\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + 1/\tau_e^*},$$

which is also an exogenous constant for entrepreneur j.

Information Acquisition Decision Given τ_e^*, we now consider entrepreneur $j's$ information choice $\tau_e = 1/\sigma_e^2$. For a given σ_e^2, entrepreneur j's production, $y(A_t, s_{jt})$, based on his signal is given by (41). Hence, if he chooses the precision level $\tau_e = 1/\sigma_e^2$, his ex ante expected profit is

$$\begin{aligned}\pi(\tau_e; A_t, \tau_e^*) &= \frac{1}{\theta-1}\mathbb{E}_{s_{jt}}\frac{y(A_t, s_{jt})}{A_t} \\ &= \frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta \bar{Y}_C^{1-\theta\gamma} A_t^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)),\end{aligned}$$

where

$$\begin{aligned}V(\tau_e, \kappa(\tau_e^*)) &= \mathbb{E}_{s_{jt}}\left\{\mathbb{E}_t\left[\left(\phi_t^{\kappa\frac{1-\theta\gamma}{\theta}}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}\right)|s_{jt}\right]\right\}^\theta \\ &= \exp\left\{\left[-\frac{1}{2}\left(\chi\sigma_\eta^2 + \sigma_\varepsilon^2\right) + \frac{1}{2}\frac{1}{\theta}\left(\chi^2\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right] + \frac{1}{2}\frac{\theta-1}{\theta}\frac{\left[\chi\omega\sigma_\eta^2 + (1-\omega)\,\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2}\right\}\end{aligned}$$

and $\chi = (1-\theta\gamma)\kappa(\tau_e^*)+1$. Notice that $V(\tau_e, \kappa(\tau_e^*))$ is increasing in $\tau_e = 1/\sigma_e^2$ for $\theta > 1$. $V(\tau_e, \kappa(\tau_e^*))$ is concave in τ_e when τ_e is sufficiently large since V is bounded from above when $\tau_e \to +\infty$. Firm $j's$ information acquisition decision on τ_e is given by

$$\underset{\tau_e}{Max}\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^\theta \left(\frac{1}{\psi}\right)^\theta \bar{Y}_C^{1-\theta\gamma} A_t^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)) - h(\tau_e, b) \tag{51}$$

So the first-order condition implies

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}\frac{\partial V(\tau_e,\kappa(\tau_e^*))}{\partial\tau_e}=\frac{\partial h(\tau_e,b)}{\partial\tau_e}. \tag{52}$$

This defines an implicit mapping between A_t and τ_e. Figure 5 illustrates the effect of A_t on τ_e.

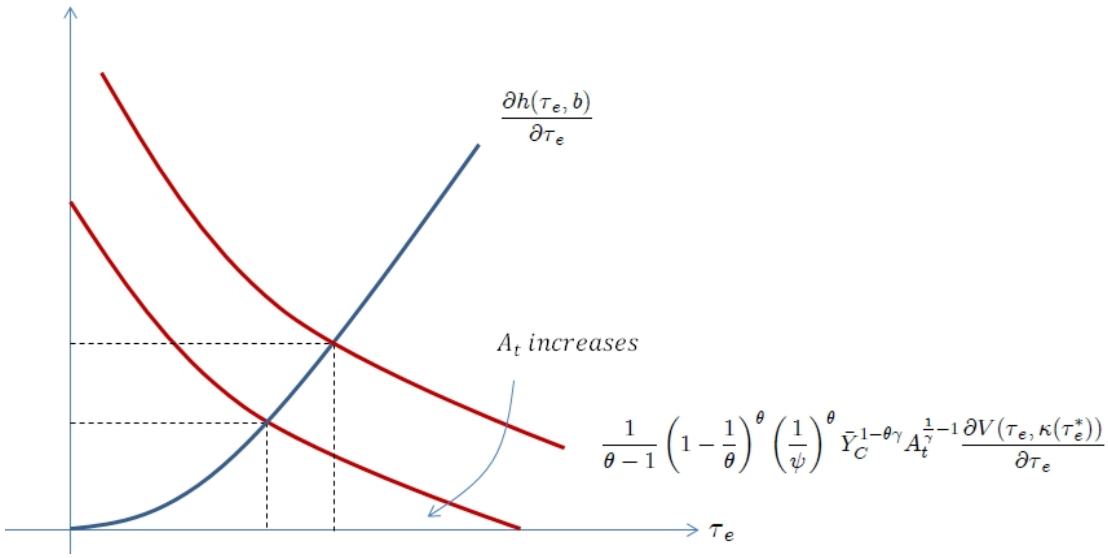

Figure 5: Optimal τ_e Chosen by Firm j for a Given τ_e^*

The LHS of equation (52) captures the marginal benefit of increasing the precision of information, while the RHS measures the marginal cost. The marginal benefit is decreasing in τ_e and is graphed by the downward sloping curve, and the marginal cost is increasing in τ_e and is graphed by the upward slopping curve. The intersection of these two curves gives the optimal τ_e. The LHS is decreasing in A_t under $\gamma > 1$ due to the strong labor supply effect. So when A_t increases, the LHS decreases for any level τ_e, leading to a decline in the optimal τ_e as the figure illustrates.

In a symmetric equilibrium, $\tau_e = \tau_e^*$. So we have

$$\frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_C^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial\tau_e}=\frac{\partial h(\tau_e,b)}{\partial\tau_e}, \tag{53}$$

where $\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial\tau_e} > 0$ is the partial derivative of V with respect to its first argument. (53) then defines a mapping between τ_e and A_t:

$$\tau_e = f(A_t)$$

Unfortunately, we are not able to derive an explicit function for f. However, the property of the LHS of (53) largely follows that of $\frac{\partial V_1(\tau_e,\kappa(\tau_e))}{\partial\tau_e}$. Therefore, as illustrated in Figure 5, we have $f'(A_t) < 0$, the case in which we are interested, at least for some parameter choices. Also, $\mathbb{E}[\log Y_t|A_t] = \log\bar{Y}_C + \frac{1}{\gamma}\log A_t + \kappa\left(-\frac{1}{2}\sigma_\eta^2\right)$ increases with A_t at least under some parameter choices. Figure 6 gives a numerical example for functions $\sigma_e(A_t)$ and $\mathbb{E}[\log Y_t|A_t]$ under plausible parameter values, $\theta = 10$, $\gamma = 2$, $\psi = 1$, $\sigma_\varepsilon = 10\%$ or $\tau_\varepsilon = 100$, $\sigma_\eta = 1\%$ or $\tau_\eta = 10000$, $\omega = 0.5$, and $h(\tau_e, b) = \frac{1}{10\tau_\eta^2}\tau_e^2$.

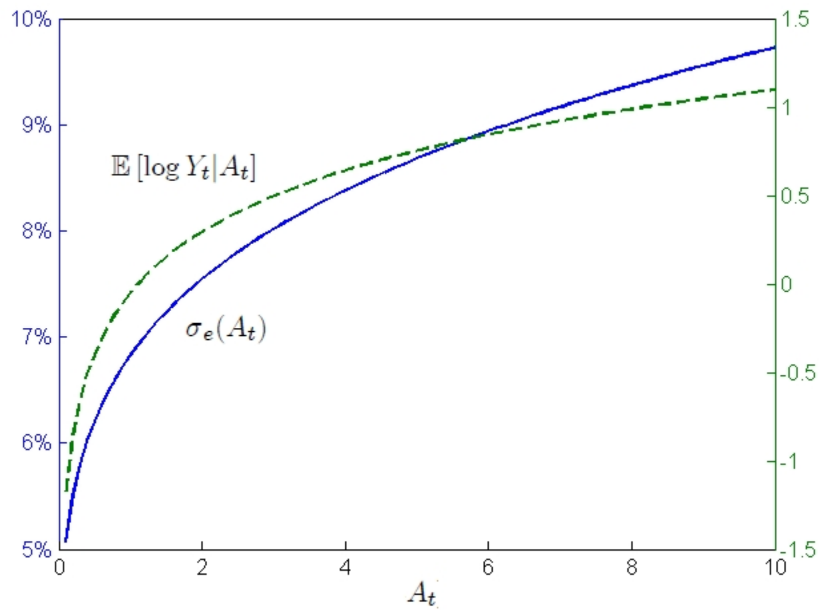

Figure 6: A Numerical Example for Functions $\sigma_e(A_t)$ (left) and $\mathbb{E}[\log Y_t|A_t]$ (right)

Because $\mathbb{E}[\log Y_t|A_t]$ is increasing in A_t while $\tau_e = 1/\sigma_e^2$ is decreasing in A_t, Lemma 1 implies the following proposition.

Proposition 8 *Under the assumption* $0 < \omega < \min\left\{\frac{(\theta-1)\sigma_\varepsilon^2}{(\theta-1)\sigma_\varepsilon^2+\theta(\gamma-1)\sigma_\eta^2}, \frac{\sigma_\varepsilon}{\sigma_\varepsilon+\sqrt{\theta\gamma}\sigma_\eta}\right\}$, *aggregate volatility and firm-level dispersion are both countercyclical.*

Proof. *See Appendix.* ■

4.2 Alternative Information Structures

In the previous subsections, the aggregate volatility is driven by a common demand shock. In this subsection, we introduce two alternative information structures, which can also generate countercyclical aggregate volatility as well as countercyclical firm-level dispersion.

First, we introduce common sentiment shocks. The production function of the final consumption goods is still (4), that is,

$$Y_t = \left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad \text{for } \theta > 1.$$

An entrepreneur receives a noisy signal $s_{jt} = \omega\eta_t + (1-\omega)\varepsilon_{jt} + e_{jt}$ about ϵ_{jt}, where the common shock η_t across firms is interpreted as the *sentiment shock*, $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ and $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$.

Denote $\tau_\eta = 1/\sigma_\eta^2$ and $\tau_e = 1/\sigma_e^2$. To obtain the precision level τ_e, an entrepreneur needs to spend $h(\tau_e, b)$ working hours. With this setting, we can find aggregate output

$$Y_t = \bar{Y}_S \cdot A_t^{\frac{1}{\gamma}} \exp\left(\omega \frac{(1-\omega)\,\sigma_\varepsilon^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2}\eta_t\right),$$

where $\bar{Y}_S$ is a constant independent of A_t and η_t. As in the case of common demand shocks, the endogenous $\tau_e = 1/\sigma_e^2$ is decreasing in A_t when $\gamma - 1 > 0$. Therefore, as long as $0 < \omega < 1$, aggregate volatility is countercyclical. Also, when $\omega < 1$, firm-level dispersion is countercyclical.

Second, we introduce common preference shocks. Specifically, the utility function of the representative household is assumed to be

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\phi_t \frac{C_t^{1-\gamma}}{1-\gamma} - \psi N_t - \psi N_{et}\right] \quad \text{for } \gamma > 1,$$

where ϕ_t is the *preference shock* with $\log \phi_t \equiv \eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$. The production function of the final consumption goods is still (4). An entrepreneur receives a noisy signal $s_{jt} = \omega\eta_t + (1-\omega)\,\varepsilon_{jt} + e_{jt}$ about η_t and ε_{jt}, where $e_{jt} \sim \mathcal{N}(0, \sigma_e^2)$. Denote $\tau_\eta = 1/\sigma_\eta^2$ and $\tau_e = 1/\sigma_e^2$. To obtain the precision level τ_e, an entrepreneur needs to spend $h(\tau_e, b)$ working hours. With this setting, we can find aggregate output

$$Y_t = \bar{Y}_P \cdot A_t^{\frac{1}{\gamma}} \exp\left(\omega \frac{\theta\omega\sigma_\eta^2 + (1-\omega)\,\sigma_\varepsilon^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\,\sigma_\varepsilon^2 + \sigma_e^2}\eta_t\right),$$

where $\bar{Y}_P$ is a constant independent of A_t and η_t. Similar to the case of common demand shocks, the endogenous $\tau_e = 1/\sigma_e^2$ is decreasing in A_t when $\gamma - 1 > 0$. Therefore, as long as $\omega > 0$, aggregate volatility is countercyclical. Also, when $\omega < 1$, firm-level dispersion is countercyclical. All the proofs in this subsection are similar to those in Sections 4.1. To avoid redundancy, we omit the proofs.

The common feature of the different information structures is that an imperfectly observed common shock drives aggregate volatility and gives rise to confusion (signal extraction) for firms, whether the shock originates in preferences, demand or sentiments. Of course, the signal extraction will depend on the precise way in which the shocks are introduced, originating in preferences, in the output aggregator for demand, or directly in the signal as sentiments. In this sense sentiment and various preference or aggregator shocks work through similar mechanisms. Interestingly, the demand shock requires lower ω for countercyclicality for the reasons discussed in Sections 4.1, while the other shocks do not. In general, our model provides a particular perspective based on endogenous information acquisition for the sources of time-varying countercyclical aggregate volatility.

4.3 Financial Frictions

In this subsection we show that borrowing constraint shocks can also generate countercyclical firm-level and aggregate volatility, under weaker conditions than TFP shocks.

As in Section 3.1, we assume that firms face borrowing constraints in financing their working capital used to hire workers:

$$W_t n_{jt} \leq \zeta_t \, \mathbb{E}_t[p_{jt} y_{jt} | s_{jt}]$$

where ζ_t measures the tightness of the borrowing constraint. We assume $\xi_t < 1 - \frac{1}{\theta}$ so the constraint is binding. Other specifications are the same as those in Section 4.1.3.

Denote by τ_e^* the information precision for all entrepreneurs $\ell \neq j$. Given τ_e^*, we characterize first aggregate output and then entrepreneur $j's$ information acquisition problem.

Equilibrium Y_t for a Given τ_e^* Given their information choice τ_e^*, entrepreneurs $\ell \neq j$ decide to produce

$$y_{\ell t} = y(A_t, s_{\ell t}) = \zeta_t^\theta \left(\frac{A_t}{\psi}\right)^\theta \left\{ \mathbb{E}\left[\left(Y_t^{\frac{1}{\theta} - \gamma} \cdot (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} \right) | s_{\ell t} \right] \right\}^\theta . \tag{54}$$

By aggregating, we find the aggregate output

$$\log Y_t = \log \bar{Y}_B + \frac{1}{\gamma} \log A_t + \kappa \eta_t + \frac{1}{\gamma} \log \zeta_t, \tag{55}$$

where κ is the same as the one in Section 4.1.3, that is,

$$\kappa = \kappa(\tau_e^*) = \frac{1}{\theta - 1} + \omega \frac{(1-\omega)\,\sigma_\varepsilon^2 - \omega \frac{\theta(\gamma-1)}{\theta-1} \sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2 \sigma_\varepsilon^2 + \sigma_e^2}, \tag{56}$$

and $\bar{Y}_B = \left(1 - \frac{1}{\theta}\right)^{-\frac{1}{\gamma}} \bar{Y}_C$ and $\bar{Y}_C$ is given in Section 4.1.3.

Information Acquisition Decision Given τ_e^*, we now consider entrepreneur $j's$ information choice $\tau_e = 1/\sigma_e^2$. If entrepreneur j chooses precision level $\tau_e = 1/\sigma_e^2$, his ex ante expected profit is

$$\begin{aligned} \pi(\tau_e; A_t, \tau_e^*) &= \frac{1-\zeta_t}{\zeta_t} \mathbb{E}_{s_{jt}} \frac{y(A_t, s_{jt})}{A_t} \\ &= (1-\zeta_t) \left(\frac{1}{\psi}\right)^\theta \bar{Y}_B^{1-\theta\gamma} (\zeta_t A_t)^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)), \end{aligned}$$

where $V(\tau_e, \kappa(\tau_e^*))$ is the same as in Section 4.1.3. Firm $j's$ information acquisition decision for τ_e is given by

$$\underset{\tau_e}{Max}\, (1-\zeta_t)\, \zeta_t^{\frac{1}{\gamma}-1} \left(\frac{1}{\psi}\right)^\theta \bar{Y}_B^{1-\theta\gamma} A_t^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)) - h(\tau_e, b). \tag{57}$$

The first-order condition implies

$$(1-\zeta_t)\,\zeta_t^{\frac{1}{\gamma}-1}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}_B^{1-\theta\gamma}A_t^{\frac{1}{\gamma}-1}\frac{\partial V(\tau_e,\kappa(\tau_e^*))}{\partial\tau_e}=\frac{\partial h(\tau_e,b)}{\partial\tau_e}. \tag{58}$$

Note that

$$\frac{d\left[(1-\zeta_t)\,\zeta_t^{\frac{1}{\gamma}-1}\right]}{d\zeta_t}<0 \ \text{ iff } \gamma>1-\zeta_t.$$

Hence, when $\gamma>1-\zeta_t$, τ_e is decreasing in ζ_t, similar to the case of τ_e decreasing in A_t as illustrated in Figure 5. Notice that condition $\gamma>1-\zeta_t$ is weaker than condition $\gamma>1$.

A symmetric equilibrium across firms implies

$$\tau_e^*=\tau_e. \tag{59}$$

The system of equations (58)-(59) gives the endogenous τ_e, which is a function of ζ_t.

Parallel to Section 4.1.3, Lemma 1 implies the following proposition.

Proposition 9 *Under the assumption* $0<\omega<\min\left\{\frac{(\theta-1)\sigma_\varepsilon^2}{(\theta-1)\sigma_\varepsilon^2+\theta(\gamma-1)\sigma_\eta^2},\frac{\sigma_\varepsilon}{\sigma_\varepsilon+\sqrt{\theta\gamma}\sigma_\eta}\right\}$, *aggregate volatility and firm-level dispersion are both countercyclical when* $\gamma>1-\zeta_t$ *or equivalently* $\zeta_t\in\left(1-\ \gamma,1-\frac{1}{\theta}\right)$, *i.e., measured uncertainty is higher when* $\zeta_t\in\left(1-\ \gamma,1-\frac{1}{\theta}\right)$ *is lower.*

Proof. *See Appendix.* ■

From Proposition 9, even if $\gamma<1$, there exists a nonempty set $\zeta_t\in\left(1-\ \gamma,1-\frac{1}{\theta}\right)$ in which the variation in ζ_t generates countercyclical aggregate volatility and countercyclical firm-level dispersion. As we discussed before, the real wage can display smaller procyclicality under financial shocks in the case of $\gamma<1$.

5 Conclusion

In the large and growing recent literature on economic uncertainty, two important issues seem to have received particular attention. The first concerns the measurement of economic uncertainty. Two popular proxies in measuring uncertainty in the empirical literature are cross-sectional firm-level dispersion and time-series aggregate volatility. From the theoretical perspective, however, cross-sectional firm-level dispersion and time-series aggregate volatility are different concepts. We study both in the context of a model of imperfect information.

The second issue is about the causality between economic uncertainty and macroeconomic activity. While some researchers propose that the direction of causality runs from the second moment (uncertainty) to the first moment (macroeconomic activity), through mechanisms such as the traditional "wait-and-see" effect and the rise in the cost of capital due to the concave-payoffs of debt contracts, some empirical findings, nevertheless, suggest that the direction of causality might go the other way round.

In this paper, we address both questions and build a unified framework to explain countercyclical firm-level dispersion and countercyclical aggregate volatility. We provide a theoretical framework to model time-varying firm-level volatility and aggregate volatility, and to explain how such volatility can be countercyclical across business cycles.

We introduce endogenous information acquisition for firms facing demand shocks in an otherwise standard monopolistically competitive model. Countercyclical information acquisition arises naturally in our model with standard preference and technology specifications. The precision of the information about demand shocks optimally acquired by firms varies across business cycles. The variation in turn leads to different degrees of responsiveness to the idiosyncratic and aggregate demand shocks by firms, and results in time-varying countercyclical firm-level dispersion and aggregate volatility. Interestingly, in order to generate countercyclical aggregate volatility, the informativeness of the signal about aggregate demand shocks has to be sufficiently low.

The common aggregate shock may be introduced in different ways, originating in preferences or directly in the signals as sentiments, rather than in demand shocks for intermediate inputs. We can also replace TFP shocks with financial shocks. Our insights carry over to these cases, at times under even weaker conditions than the benchmark cases with demand and TFP shocks.

Appendix

A Proofs

Proof of Proposition 1 First, we have

$$\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \mathbb{E}_{s_{jt}}\left[\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta}\right]$$

and

$$\int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj = \mathbb{E}_{\epsilon_{jt},s_{jt}}\left[\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}\right]$$

By the law of iterated expectations, $\mathbb{E}_{s_{jt}}\left[\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta}\right] = \mathbb{E}_{\epsilon_{jt},s_{jt}}\left[\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}\right]$. Hence, $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj$. Similarly, $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj = \int \epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj$.

Second, we calculate $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj$. Then all key variables can be expressed with this term. We have $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta} dj = \int \left(\mathbb{E}\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)|s_{jt}\right]\right)^{\theta} dj$. By the property of log-normal distribution, we further obtain

$$\begin{aligned} & \int \left(\mathbb{E}\left[\exp\left(\frac{1}{\theta}\varepsilon_{jt}\right)|s_{jt}\right]\right)^{\theta} dj \\ = & \int \left[\exp\left(\frac{1}{\theta}\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)+\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}\right]+\frac{1}{2}\left(\frac{1}{\theta}\right)^2\frac{1}{\tau_\varepsilon+\tau_e}\right)\right]^{\theta} dj \\ = & \exp\left[-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right] = \bar{z}^{\theta-1}. \end{aligned}$$

Also, when $\tau_e = 0$, it corresponds to $\int \left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta} dj = \exp\left[-\frac{1}{2}\frac{\theta-1}{\theta}\frac{1}{\tau_\varepsilon}\right] = \underline{z}^{\theta-1}$.

Third, using the above results, we obtain

$$\begin{aligned} z_t &= \left[\int_0^{\lambda_t}\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1} dj + \int_{\lambda_t}^{1}\epsilon_{jt}^{\frac{1}{\theta}}\left(\mathbb{E}\left[\epsilon_{jt}^{\frac{1}{\theta}}\right]\right)^{\theta-1} dj\right]^{\frac{1}{\theta-1}} \\ &= \left[\lambda_t\bar{z}^{\theta-1}+(1-\lambda_t)\,\underline{z}^{\theta-1}\right]^{\frac{1}{\theta-1}}, \end{aligned}$$

which proves equation (18).

Fourth, we calculate the ex ante expected profit for a firm. For an informed firm, its realized profit is

$$\begin{aligned}\pi(\epsilon_{jt},s_{jt}) &= p_{jt}\left(\epsilon_{jt},y_{jt}\right)y_{jt}-W_t n_{jt}\left(y_{jt}\right)\\ &= \left(1-\frac{1}{\theta}\right)^{\theta-1}\left(\frac{A_t}{W_t}\right)^{\theta-1}Y_t\left\{\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt})\right]^{\theta-1}\epsilon_{jt}^{\frac{1}{\theta}}-\left(1-\frac{1}{\theta}\right)\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt})\right]^{\theta}\right\}\end{aligned}$$

Hence, we obtain

$$\begin{aligned}\pi_t^I &= \mathbb{E}_{\epsilon_{jt},s_{jt}}\left[\pi(\epsilon_{jt},s_{jt})\right]=\mathbb{E}_{s_{jt}}\mathbb{E}_{\epsilon_{jt}|s_{jt}}\left[\pi(\epsilon_{jt},s_{jt})|s_{jt}\right]\\ &= \left(1-\frac{1}{\theta}\right)^{\theta-1}\left(\frac{A_t}{W_t}\right)^{\theta-1}Y_t\cdot\frac{1}{\theta}\mathbb{E}_{s_{jt}}\left(\left[\mathbb{E}(\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt})\right]^{\theta}\right)\\ &= \frac{1}{\theta}Y_t z_t^{1-\theta}\overline{z}^{\theta-1},\end{aligned}$$

where the last line is obtained by using $\frac{A_t}{W_t}=\frac{1}{1-\frac{1}{\theta}}\frac{1}{z_t}$ in (15). Similarly, we can find the ex ante expected profit for an uninformed firm

$$\pi_t^U=\frac{1}{\theta}Y_t z_t^{1-\theta}\underline{z}^{\theta-1}.$$

Fifth, by (23), $\log Y_t(A_t;m)$ is continuous and increasing in A_t under $\theta\gamma-1>0$. By (24), $\lambda_t(A_t;m)$ is continuous and decreasing in A_t under $\gamma-1>0$.

Finally, the total expenditure on information acquisition is $\lambda_t m W_t=m\psi\lambda_t Y_t^{\gamma}$, where λ_t is given by equation (24). We hence obtain

$$m\psi\lambda_t Y_t^{\gamma}\propto\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)^{\frac{\theta\gamma-\gamma}{1-\theta\gamma}}A_t^{\frac{\theta-1}{\theta\gamma-1}}-\underline{z}^{\theta-1}A_t^{\frac{(\theta-1)\gamma}{\theta\gamma-1}}.$$

The first-order condition implies that $\frac{\partial(\lambda_t m W_t)}{\partial A_t}>0$ is equivalent to

$$\left(\frac{m}{\overline{z}^{\theta-1}-\underline{z}^{\theta-1}}\right)^{\frac{\theta\gamma-\gamma}{1-\theta\gamma}}\frac{\theta-1}{\theta\gamma-1}A_t^{\frac{\theta-1}{\theta\gamma-1}-1}-\underline{z}^{\theta-1}\frac{(\theta-1)\gamma}{\theta\gamma-1}A_t^{\frac{(\theta-1)\gamma}{\theta\gamma-1}-1}>0,$$

or $A_t<\left(\frac{1}{\gamma}\right)^{\frac{\theta\gamma-1}{(\theta-1)(\gamma-1)}}\overline{A}$. Note that the set of $(\underline{A},\left(\frac{1}{\gamma}\right)^{\frac{\theta\gamma-1}{(\theta-1)(\gamma-1)}}\overline{A})$ is not empty.

Proof of Proposition 2 Denote the type of firms acquiring information by I and the type of those that do not by U. First, we examine the dispersion of sales across intermediate goods firms. We find

$$y_{jt}^I\equiv y(A_t,s_{jt})=\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A}{W_t}\right)^{\theta}Y_t\exp\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)+\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}+\frac{1}{2\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right].$$

We have

$$\begin{aligned}
\mathbb{E}\left[\left(\log y_{jt}^{I}\right)^{2}\right] &= Var\left[\log y_{jt}^{I}\right]+\left[\mathbb{E}\left(\log y_{jt}^{I}\right)\right]^{2} \\
&= \left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)^{2}\left(\sigma_\varepsilon^2+\sigma_e^2\right)+\left(-\frac{1}{2}\sigma_\varepsilon^2+\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right)^{2} \\
&= \left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)\left(\frac{1}{\tau_\varepsilon}\right)+\left(-\frac{1}{2}\frac{1}{\tau_\varepsilon}+\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right)^{2},
\end{aligned}$$

and similarly

$$\mathbb{E}\left[\left(\log y_{jt}^{U}\right)^{2}\right]=\left(-\frac{1}{2}\sigma_\varepsilon^2+\frac{1}{2}\frac{1}{\theta}\sigma_\varepsilon^2\right)^{2},$$

where $y_{jt}^{U}\equiv\tilde{y}(A_t)$. Thus,

$$\begin{aligned}
Var\left[\log y_{jt}\right] &= \mathbb{E}\left[\left(\log y_{jt}\right)^{2}\right]-\left[\mathbb{E}\left(\log y_{jt}\right)\right]^{2} \\
&= \left\{\begin{array}{c}\lambda_t\mathbb{E}\left[\left(\log y_{jt}^{I}\right)^{2}\right] \\ +\left(1-\lambda_t\right)\mathbb{E}\left[\left(\log y_{jt}^{U}\right)^{2}\right]\end{array}\right\}-\left[\begin{array}{c}\lambda_t\mathbb{E}\left(\log y_{jt}^{I}\right) \\ +\left(1-\lambda_t\right)\mathbb{E}\left(\log y_{jt}^{U}\right)\end{array}\right]^{2} \\
&= \lambda_t\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\right)+\frac{\lambda_t-\lambda_t^2}{4\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\right)^{2},
\end{aligned}$$

which is a quadratic equation with respect to λ_t. It is increasing in $[0,1]$ if and only if $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\geq\frac{1}{4\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\right)^{2}$, that is, $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\leq 4\theta^2$.

Similarly, we can work out the dispersion of output values. It follows that

$$\log\left(y_{jt}^{I}p_{jt}^{I}\right)=\log\left[\left(1-\frac{1}{\theta}\right)^{\theta-1}\left(\frac{A}{W_t}\right)^{\theta-1}Y_t\right]+\frac{1}{\theta}\varepsilon_{jt}+\frac{\theta-1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) \\ +\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}+\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\end{array}\right]$$

and

$$\log\left(y_{jt}^{U}p_{jt}^{U}\right)=\log\left[\left(1-\frac{1}{\theta}\right)^{\theta-1}\left(\frac{A}{W_t}\right)^{\theta-1}Y_t\right]+\frac{1}{\theta}\varepsilon_{jt}+\frac{\theta-1}{\theta}\left[-\frac{1}{2}\sigma_\varepsilon^2+\frac{1}{2}\frac{1}{\theta}\sigma_\varepsilon^2\right].$$

Thus,

$$Var\left[\log\left(y_{jt}p_{jt}\right)\right]=\mathbb{E}\left[\left(\log\left(y_{jt}p_{jt}\right)\right)^{2}\right]-\left[\mathbb{E}\left(\log\left(y_{jt}p_{jt}\right)\right)\right]^{2}$$

$$=\left(\frac{1}{\theta}\right)^{2}\left(\frac{1}{\tau_\varepsilon}\right)+\lambda_t\frac{\theta^2-1}{\theta^2}\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)\left(\frac{1}{\tau_\varepsilon}\right)+\left(1-\lambda_t\right)\lambda_t\left(\frac{1}{2}\frac{1}{\theta}\frac{\theta-1}{\theta}\right)^{2}\left[\left(\frac{\tau_e}{\tau_\varepsilon+\tau_e}\right)\left(\frac{1}{\tau_\varepsilon}\right)\right]^{2},$$

which is increasing in $\lambda_t\in[0,1]$ if and only if $\frac{\tau_e}{\tau_\varepsilon+\tau_e}\frac{1}{\tau_\varepsilon}\leq 4\theta^2\frac{\theta+1}{\theta-1}$.

Proof of Proposition 3 Equations (19) and (21) jointly give

$$\frac{1}{\theta-1}(1-\frac{1}{\theta})^{\frac{1}{\gamma}}\left(\frac{A}{\psi}\right)^{\frac{1}{\gamma}-1}\left[\lambda_t\overline{z}^{\theta-1}+(1-\lambda_t)\,\underline{z}^{\theta-1}\right]^{\frac{1-\gamma\theta}{(\theta-1)\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)=\psi m. \tag{A1}$$

Equations (19) and (30) jointly give

$$\left[\frac{1}{(\theta-1)\,\gamma}+\frac{\gamma-1}{\gamma}\frac{1}{\theta}\right]\left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}-1}\left(\frac{A}{\psi}\right)^{\frac{1}{\gamma}-1}\left[\lambda_t\overline{z}^{\theta-1}+(1-\lambda_t)\,\underline{z}^{\theta-1}\right]^{\frac{1-\gamma\theta}{(\theta-1)\gamma}}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)=\psi m. \tag{A2}$$

Notice that the LHS of (A2) is always greater than the LHS of (A1) while their RHS are the same. Also, when $1-\theta\gamma<0$, the LHS of either (A1) or (A2) is a decreasing function with λ. So the optimal λ given by (A1) must be lower than that given by (A2).

The intuition behind Proposition 3 is as follows. When an individual firm acquires information, it has two externalities: to other firms' profit and to the consumer surplus. Actually we can decompose and quantify these two externalities. Concretely, the social welfare can be decomposed into

$$\begin{aligned}
&\frac{C_t^{1-\gamma}}{1-\gamma}-\psi N_t-\psi(m\lambda)\\
=&\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{P_t}{W_t}\psi Y_t\right)+\psi\left(\frac{P_tY_t-W_tN_t}{W_t}\right)-\psi(m\lambda)\\
=&\underbrace{\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{P_t}{W_t}\psi Y_t\right)}_{\text{Consumer surplus}}+\underbrace{\psi\left(\frac{\Pi_t}{W_t}\right)}_{\text{Aggregate firm profits}}-\underbrace{\psi(m\lambda)}_{\text{Information acquisition costs}}\\
=&\left(\frac{Y_t^{1-\gamma}}{1-\gamma}-\frac{1}{\psi Y_t^{\gamma}}\psi Y_t\right)+\psi\left(\frac{\Pi_t}{W_t}-m\lambda\right)\\
=&\left(\frac{\gamma}{1-\gamma}(Y_t(\lambda))^{1-\gamma}\right)+\psi\left(\frac{\Pi_t(Y_t(\lambda),\lambda)}{W_t(Y_t(\lambda))}-m\lambda\right).
\end{aligned} \tag{A3}$$

In the above, $\Pi_t(Y_t(\lambda),\lambda)$ is the aggregate profit of all firms in the economy, which can be verified to be

$$\Pi_t(Y_t(\lambda),\lambda)=\frac{1}{\theta-1}(1-\frac{1}{\theta})^{\theta}\left(\frac{A}{\psi}\right)^{\theta-1}Y_t^{1-\gamma(\theta-1)}\left(z(\lambda_t)\right)^{\theta-1}.$$

In the above decomposition, we normalize all the output values by W_t, that is, we essentially use the wage as the numeraire. An individual firm's optimal information acquisition problem is given by the first-order condition, taking Y_t as given

$$\underbrace{\psi\frac{\partial\frac{\Pi_t(Y_t,\lambda)}{W_t}}{\partial\lambda}}_{\text{Private value}}=\psi m.$$

In maximizing the social welfare, the social planner takes Y_t as endogenous and thus the first-order condition is

$$\underbrace{\frac{\partial\left(\frac{\gamma}{1-\gamma}Y_t^{1-\gamma}\right)}{\partial Y_t}\frac{dY_t}{d\lambda}}_{\substack{\text{Externality to consumer surplus}\\ >0}} + \underbrace{\psi\frac{\partial\frac{\Pi_t(Y_t,\lambda)}{W_t(Y_t)}}{\partial Y_t}\frac{dY_t}{d\lambda}}_{\substack{\text{Externality to other firms' profit}\\ <0}} + \psi\underbrace{\frac{\partial\frac{\Pi_t(Y_t,\lambda)}{W_t}}{\partial\lambda}}_{\text{Private value}} = \psi m.$$

The overall effect is that the positive externality to consumer surplus exceeds the negative externality to other firms' profit, so too few individual firms acquire information in the competitive equilibrium compared with the second best.

Proof of Proposition 4 The equilibrium is determined by the following joint equations:

$$\begin{cases} Y_t = \zeta_t^{\frac{1}{\gamma}}\left(\frac{z(\lambda_t)A_t}{\psi}\right)^{\frac{1}{\gamma}} \\ (1-\zeta_t)\left(z(\lambda_t)\right)^{1-\theta}Y_t^{1-\gamma}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)=\psi m \end{cases}.$$

Combining these two, we have

$$Y_t = \zeta_t^{\frac{\theta-1}{\theta\gamma-1}}(1-\zeta_t)^{\frac{1}{\theta\gamma-1}}\left(\frac{A}{\psi}\right)^{\frac{\theta-1}{\theta\gamma-1}}\left[\frac{1}{m}\cdot\frac{1}{\psi}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)\right]^{\frac{1}{\theta\gamma-1}}.$$

Therefore, when $\theta\gamma-1>0$, Y_t is increasing in ζ_t if and only if

$$\frac{d\left(\zeta_t(1-\zeta_t)^{\frac{1}{\theta-1}}\right)}{d\zeta_t}>0,$$

or $\zeta_t<1-\frac{1}{\theta}$. Also, $\Pi_t=(1-\zeta_t)Y_t$, which is increasing in ζ_t if and only if

$$\frac{\partial\Pi_t}{\partial\zeta_t}=\frac{\partial\left[\zeta_t^{\frac{\theta-1}{\theta\gamma-1}}(1-\zeta_t)^{\frac{1}{\theta\gamma-1}+1}\right]}{\partial\zeta_t}>0,$$

or $\zeta_t<\frac{\theta-1}{\theta\gamma+\theta-1}$. Hence, when $\theta\gamma-1>0$, Π_t is increasing in ζ_t if and only if $\zeta_t<\frac{\theta-1}{\theta\gamma+\theta-1}$.

Finally, we analyze λ_t as a function of ζ_t. We have

$$(z(\lambda_t))^{\frac{\theta\gamma-1}{\gamma}}=\frac{1-\zeta_t}{m\psi}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)\left(\frac{A_t\zeta_t}{\psi}\right)^{\frac{1-\gamma}{\gamma}},$$

where the RHS is decreasing in ζ_t if and only if $\gamma>1-\zeta_t$. Therefore, when $\theta\gamma-1>0$ and $\gamma>1-\zeta_t$, λ_t is decreasing in ζ_t. That is, when $\gamma>\max\{\frac{1}{\theta},1-\zeta_t\}=1-\zeta_t$ (considering that

$\zeta_t < 1 - \frac{1}{\theta}$), λ_t is decreasing in ζ_t.

Proof of Proposition 5 We have

$$\log y_{jt}^I = \theta \log\left(\frac{\zeta_t}{1-\frac{1}{\theta}}\right) + \theta \log\left[\left(1-\frac{1}{\theta}\right)\frac{A}{W_t}Y_t^{\frac{1}{\theta}}\right] + \left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt} + \frac{1}{2\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right]$$

and

$$\log\left(y_{jt}^I p_{jt}^I\right) = \log\left[\left(\frac{\zeta_t}{1-\frac{1}{\theta}}\right)^{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta-1}\left(\frac{A}{W_t}\right)^{\theta-1}Y_t\right] + \frac{1}{\theta}\varepsilon_{jt} + \frac{\theta-1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\ +\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}+\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\end{array}\right].$$

Therefore, $Var\left[\log y_{jt}^I\right]$ and $Var\left[\log\left(y_{jt}^I p_{jt}^I\right)\right]$ are the same as those in the proof of Proposition 2. A similar conclusion to that of Proposition 2 applies.

Proof of Proposition 6 The equilibrium is determined by the following joint equations:

$$\begin{cases} Y_t = \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{z(\lambda_t)A}{\psi R_t}\right)^{\frac{1}{\gamma}} \\ \frac{1}{\theta}\left(z(\lambda_t)\right)^{1-\theta}\frac{1}{\psi}Y_t^{1-\gamma}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right) = m \end{cases}.$$

So

$$Y_t = \left(1-\frac{1}{\theta}\right)^{\frac{\theta-1}{\theta\gamma-1}}\left(\frac{A}{\psi R_t}\right)^{\frac{\theta-1}{\theta\gamma-1}}\left[\frac{1}{\theta}\frac{1}{\psi}\frac{1}{m}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)\right]^{\frac{1}{\theta\gamma-1}}$$

and

$$\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}-1}\frac{1}{\psi}\left(\frac{A}{\psi R_t}\right)^{\frac{1}{\gamma}-1}\left(z(\lambda_t)\right)^{\frac{1-\gamma\theta}{\gamma}} = m\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)^{-1}.$$

Therefore, λ_t is increasing in R_t under $\gamma - 1 > 0$, and Y_t is decreasing in R_t under $\theta\gamma - 1 > 0$.

The total labor is

$$\begin{aligned} N_t &= \left(1-\frac{1}{\theta}\right)\left(\frac{1}{\psi R_t}\right)Y_t^{1-\gamma} \\ &= \left(\psi R_t\right)^{\frac{2-\theta-\gamma}{\theta\gamma-1}}A_t^{\frac{(\theta-1)(1-\gamma)}{\theta\gamma-1}}\left(1-\frac{1}{\theta}\right)^{\frac{\theta+\gamma-2}{\theta\gamma-1}}\left[\frac{1}{\theta\psi m}\left(\overline{z}^{\theta-1}-\underline{z}^{\theta-1}\right)\right]^{\frac{1-\gamma}{\theta\gamma-1}}. \end{aligned}$$

Finally, we have

$$Var\left(\log y_{jt}^I\right) = Var\left[\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) + \frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt} + \frac{1}{2\theta}\frac{1}{\tau_\varepsilon+\tau_e}\right]$$

and

$$Var\left(\log\left(y_{jt}^I p_{jt}^I\right)\right) = Var\left(\frac{1}{\theta}\varepsilon_{jt} + \frac{\theta-1}{\theta}\left[\begin{array}{c}\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_e}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\ +\frac{\tau_e}{\tau_\varepsilon+\tau_e}s_{jt}+\frac{1}{2}\frac{1}{\theta}\frac{1}{\tau_\varepsilon+\tau_e}\end{array}\right]\right).$$

Therefore, $Var\left[\log y_{jt}^{I}\right]$ and $Var\left[\log\left(y_{jt}^{I}p_{jt}^{I}\right)\right]$ are the same as those in the proof of Proposition 2. A conclusion similar to that of Proposition 2 applies.

Proof of Proposition 7 We conjecture that $Y_t = \bar{Y}A_t^{\kappa(1)}\phi_t^{\kappa}$. Substituting the conjectured Y_t into (41), it follows that

$$\begin{aligned} y_{jt} &= \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A_t}{\psi}\right)^{\theta}\left\{\mathbb{E}_t\left[\left(Y_t^{\frac{1}{\theta}-\gamma}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}\right)|s_{jt}\right]\right\}^{\theta} \\ &= \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}^{1-\theta\gamma}A_t^{\theta+\kappa(1)(1-\theta\gamma)}\left\{\exp\left[\begin{array}{c}\frac{[(1-\theta\gamma)\kappa+1]\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}s_{jt}+\theta x \\ +\frac{1}{2}\theta Var\end{array}\right]\right\}, \end{aligned}$$

where

$$x = \left[\begin{array}{c}\left(\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right)\left(-\frac{1}{2}\sigma_\eta^2\right) \\ +\frac{1}{\theta}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\end{array}\right] - \frac{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}\left(1-\omega\right)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+\left(1-\omega\right)^2\sigma_\varepsilon^2+\sigma_e^2}\left[\begin{array}{c}\omega\left(-\frac{1}{2}\sigma_\eta^2\right) \\ +\left(1-\omega\right)\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\end{array}\right]$$

$$Var = \left\{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right]^2\sigma_\eta^2+\frac{1}{\theta^2}\sigma_\varepsilon^2\right\} - \frac{\left\{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}\left(1-\omega\right)\sigma_\varepsilon^2\right\}^2}{\omega^2\sigma_\eta^2+\left(1-\omega\right)^2\sigma_\varepsilon^2+\sigma_e^2}.$$

Substituting $Y_t = \bar{Y}A_t^{\kappa(1)}\phi_t^{\kappa}$ into both sides of (42), we have

$$\begin{aligned} &\bar{Y}A_t^{\kappa(1)}\phi_t^{\kappa} \\ =& \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{A_t}{\psi}\right)^{\theta}\left[\int_0^1\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}\left(\mathbb{E}\left[\left[\bar{Y}A_t^{\kappa(1)}\phi_t^{\kappa}\right]^{\frac{1}{\theta}-\gamma}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}dj\right]^{\frac{\theta}{\theta-1}} \\ =& \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}^{1-\theta\gamma}A_t^{\kappa(1)(1-\theta\gamma)+\theta}\left[\int_0^1\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}\left(\mathbb{E}\left[\phi_t^{\kappa\left(\frac{1}{\theta}-\gamma\right)}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}|s_{jt}\right]\right)^{\theta-1}dj\right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Notice that

$$\mathbb{E}\left[\left(\exp\left[\left(\frac{1}{\theta}-\gamma\right)\kappa\eta_t+\frac{1}{\theta}\left(\varepsilon_{jt}+\eta_t\right)\right]\right)|s_{jt}\right] = \exp\left[\begin{array}{c}\frac{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}s_{jt}+x \\ +\frac{1}{2}Var\end{array}\right].$$

Hence,

$$\begin{aligned} &\bar{Y}A_t^{\kappa(1)}\phi_t^{\kappa} \\ =& \left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}^{1-\theta\gamma}A_t^{\kappa(1)(1-\theta\gamma)+\theta} \\ &\cdot\left[\int_0^1\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}\left(\exp\left[\begin{array}{c}\frac{\left[\left(\frac{1}{\theta}-\gamma\right)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\left[\omega\eta_t+\left(1-\omega\right)\varepsilon_{jt}+e_{jt}\right]+x \\ +\frac{1}{2}Var\end{array}\right]\right)^{\theta-1}dj\right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Comparing coefficients, we have $\kappa_{(1)} = \frac{1}{\gamma}$ and

$$\frac{\left[(1-\theta\gamma)\kappa+1\right]\omega\sigma_\eta^2+(1-\omega)\,\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2}\omega+\frac{1}{\theta-1}=\kappa,$$

that is, $\kappa = \frac{1}{\theta-1} + \omega\frac{(1-\omega)\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}$. Also,

$$\begin{aligned}
\bar{Y} &= \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}}\left[\exp\left(x+\frac{1}{2}Var\right)\right]^{\frac{1}{\gamma}}\left[\int_0^1 \epsilon_{jt}^{\frac{1}{\theta}}\exp\left(\begin{array}{c}\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}\\ \left[(1-\omega)\,\varepsilon_{jt}+e_{jt}\right]\end{array}\right)dj\right]^{\frac{1}{(\theta-1)\gamma}}\\
&= \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}}\left[\exp\left(x+\frac{1}{2}Var\right)\right]^{\frac{1}{\gamma}}\left[\exp\left(\begin{array}{c}\left(\begin{array}{c}\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}(1-\omega)\\ +\frac{1}{\theta}\end{array}\right)\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\ +\frac{1}{2}\left(\begin{array}{c}\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}(1-\omega)\\ +\frac{1}{\theta}\end{array}\right)^2\sigma_\varepsilon^2\\ +\frac{1}{2}\left(\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}\right)^2\sigma_e^2\end{array}\right)\right]^{\frac{1}{(\theta-1)\gamma}}.
\end{aligned}$$

After working out $\bar{Y}$, $\kappa_{(1)}$ and κ, we also have the solution to y_{jt}, that is,

$$\log y_{jt} = \log\bar{y}+\frac{1}{\gamma}\log A_t+\frac{[(1-\theta\gamma)\kappa+1]\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2}s_{jt}, \tag{A4}$$

where

$$\bar{y}=\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta}\bar{Y}^{1-\theta\gamma}\left\{\exp\left[\theta x+\frac{1}{2}\theta Var\right]\right\}.$$

Proof of Lemma 1 We calculate the standard deviations of production and sales revenue of firms. By (A4),

$$\begin{aligned}
SD\left(\log y_{jt}|A_t\right) &= SD\left(\frac{\left[(1-\theta\gamma)\kappa+1\right]\omega\sigma_\eta^2+(1-\omega)\,\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2}s_{jt}|A_t\right)\\
&= SD\left(\frac{\kappa-\frac{1}{\theta-1}}{\omega}s_{jt}|A_t\right)\\
&= SD\left(\frac{(1-\omega)\,\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2}s_{jt}|A_t\right)\\
&= \sqrt{\left\{\frac{(1-\omega)\,\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2}\right\}^2\left[(1-\omega)^2\,\sigma_\varepsilon^2+\sigma_e^2\right]}.
\end{aligned}$$

By the first-order condition, $SD\left(\log y_{jt}|A_t\right)$ is decreasing in σ_e^2 when $\theta\gamma\omega^2\sigma_\eta^2 < (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2$. Also,

$$
\begin{aligned}
& SD\left[\log\left(p_{jt}y_{jt}\right)|A_t\right] \\
= \; & SD\left[\log\left(\epsilon_{jt}^{\frac{1}{\theta}} y_{jt}^{1-\frac{1}{\theta}}\right)|A_t\right] \\
= \; & SD\left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2} s_{jt} + \frac{1}{\theta}\varepsilon_{jt}|A_t\right] \\
= \; & SD\left(\begin{array}{c} \left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2}(1-\omega) + \frac{1}{\theta}\right]\varepsilon_{jt} \\ + \left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2} e_{jt} \end{array}\right) \\
= \; & \sqrt{\begin{array}{c} \left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2}(1-\omega) + \frac{1}{\theta}\right]^2 \sigma_\varepsilon^2 \\ + \left[\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2}\right]^2 \sigma_e^2 \end{array}}.
\end{aligned}
$$

Note that $SD\left[\log\left(p_{jt}y_{jt}\right)|A_t\right] = \sqrt{\begin{array}{c} Var\left[\log y_{jt}|A_t\right] \\ +2\frac{1}{\theta}\left(1-\frac{1}{\theta}\right)\frac{(1-\omega)\sigma_\varepsilon^2 - \omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2 + (1-\omega)^2\sigma_\varepsilon^2 + \sigma_e^2}(1-\omega)\sigma_\varepsilon^2 + \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 \end{array}}$. So if $Var\left[\log y_{jt}|A_t\right]$ is decreasing in σ_e^2, then $SD\left[\log\left(p_{jt}y_{jt}\right)|A_t\right]$ must be decreasing in σ_e^2.

Proof of Lemma 2 We can directly apply (43) in Proposition 7 to solve Y_t by setting $\sigma_e^2 = +\infty$. It follows that

$$
\begin{aligned}
\bar{Y}_H &= \bar{Y}|_{\sigma_e^2=+\infty} \\
&= \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}}\left[\exp\left(\begin{array}{c} \left[\begin{array}{c} \left(\left(\frac{1}{\theta}-\gamma\right)\frac{1}{\theta-1}+\frac{1}{\theta}\right)\left(-\frac{1}{2}\sigma_\eta^2\right) \\ +\frac{1}{\theta}\left(-\frac{1}{2}\sigma_\varepsilon^2\right) \end{array}\right] \\ +\frac{1}{2}\left\{\left[\left(\frac{1}{\theta}-\gamma\right)\frac{1}{\theta-1}+\frac{1}{\theta}\right]^2\sigma_\eta^2 + \frac{1}{\theta^2}\sigma_\varepsilon^2\right\} \end{array}\right)\right]^{\frac{1}{\gamma}} \\
&\quad \cdot\left[\exp\left(\frac{1}{2}\sigma_\varepsilon^2\left[-\frac{1}{\theta}+\left(\frac{1}{\theta}\right)^2\right]\right)\right]^{\frac{1}{(\theta-1)\gamma}}.
\end{aligned}
$$

Similarly, $\bar{Y}_L = \bar{Y}$.

We can calculate $\overline{V}_L$ and $\underline{V}_L$ as well as $\overline{V}_H$ and $\underline{V}_H$. It follows that

$$
\begin{aligned}
\overline{V}_L &= \mathbb{E}_{s_{jt}}\left\{\mathbb{E}\left[\phi_t^{\kappa\frac{1-\theta\gamma}{\theta}}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}|s_{jt}\right]\right\}^\theta \\
&= \mathbb{E}_{s_{jt}}\left\{\mathbb{E}\left[\exp\left(\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\eta_t + \frac{1}{\theta}\varepsilon_{jt}\right)|\omega\eta_t + (1-\omega)\varepsilon_{jt} + e_{jt}\right]\right\}^\theta.
\end{aligned}
$$

Notice that

$$\begin{aligned}
&\mathbb{E}\left[\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\eta_t+\frac{1}{\theta}\varepsilon_{jt}|\omega\eta_t+(1-\omega)\varepsilon_{jt}+e_{jt}\right] \\
=\;& \frac{\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\left(\omega\eta_t+(1-\omega)\varepsilon_{jt}+e_{jt}\right) \\
&+\left[\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\left(-\frac{1}{2}\sigma_\eta^2\right)+\frac{1}{\theta}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\right] \\
&-\frac{\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\left[\omega\left(-\frac{1}{2}\sigma_\eta^2\right)+(1-\omega)\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\right],
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{VAR}\left[\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\eta_t+\frac{1}{\theta}\varepsilon_{jt}|\omega\eta_t+(1-\omega)\varepsilon_{jt}+e_{jt}\right] \\
=\;& \left[\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)^2\sigma_\eta^2+\left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2\right]-\frac{\left[\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}.
\end{aligned}$$

Using the property of log-normal distribution we obtain

$$\overline{V}_L=\exp\begin{pmatrix} -\frac{1}{2}\left[(\kappa(1-\theta\gamma)+1)\sigma_\eta^2+\sigma_\varepsilon^2\right]+\frac{1}{2}\frac{1}{\theta}\left[(\kappa(1-\theta\gamma)+1)^2\sigma_\eta^2+\sigma_\varepsilon^2\right] \\ +\frac{1}{2}\left(1-\frac{1}{\theta}\right)\frac{\left[(\kappa(1-\theta\gamma)+1)\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\end{pmatrix},$$

and similarly,

$$\begin{aligned}
\underline{V}_L &= \left\{\mathbb{E}_{s_{jt}}\mathbb{E}\left[\phi_t^{\kappa\frac{1-\theta\gamma}{\theta}}\left(\phi_t\epsilon_{jt}\right)^{\frac{1}{\theta}}|s_{jt}\right]\right\}^\theta \\
&= \left\{\mathbb{E}_{s_{jt}}\mathbb{E}\left[\exp\left(\left(\kappa\frac{1-\theta\gamma}{\theta}+\frac{1}{\theta}\right)\eta_t+\frac{1}{\theta}\varepsilon_{jt}\right)|\omega\eta_t+(1-\omega)\varepsilon_{jt}+\sigma_e e_{jt}\right]\right\}^\theta \\
&= \exp\left(-\frac{1}{2}\left[(\kappa(1-\theta\gamma)+1)\sigma_\eta^2+\sigma_\varepsilon^2\right]+\frac{1}{2}\frac{1}{\theta}\left[(\kappa(1-\theta\gamma)+1)^2\sigma_\eta^2+\sigma_\varepsilon^2\right]\right).
\end{aligned}$$

So $\overline{V}_L=\exp\begin{pmatrix}-\frac{1}{2}\left(\chi_L\sigma_\eta^2+\sigma_\varepsilon^2\right)+\frac{1}{2}\frac{1}{\theta}\left(\chi_L^2\sigma_\eta^2+\sigma_\varepsilon^2\right) \\ +\frac{1}{2}\frac{\theta-1}{\theta}\frac{\left[\chi_L\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+\sigma_e^2}\end{pmatrix}$ and $\underline{V}_L=\exp\begin{pmatrix}-\frac{1}{2}\left[\chi_L\sigma_\eta^2+\sigma_\varepsilon^2\right] \\ +\frac{1}{2}\frac{1}{\theta}\left[\chi_L^2\sigma_\eta^2+\sigma_\varepsilon^2\right]\end{pmatrix}$, where $\chi_L=\kappa(1-\theta\gamma)+1$. Notice that since $\theta>1$, $\overline{V}_L>\underline{V}_L$.

Considering $\overline{V}_H=\mathbb{E}_{s_{jt}}\left\{\mathbb{E}\left[\phi_t^{\frac{1-\gamma}{\theta-1}}\epsilon_{jt}^{\frac{1}{\theta}}|s_{jt}\right]\right\}^\theta$ and $\underline{V}_H=\left\{\mathbb{E}\left[\phi_t^{\frac{1-\gamma}{\theta-1}}\epsilon_{jt}^{\frac{1}{\theta}}\right]\right\}^\theta$, we only need to replace $\chi_L=\kappa(1-\theta\gamma)+1$ in $\overline{V}_L$ and $\underline{V}_L$ with $\chi_H=\frac{1-\gamma}{\theta-1}\theta$ to obtain $\overline{V}_H$ and $\underline{V}_H$, respectively.

Proof of Proposition 8 The proof is very similar to the proofs of Proposition 7 and Lemma 1. $\bar{Y}_C$ is similar to $\bar{Y}$:

$$\bar{Y}_C = \left(1-\frac{1}{\theta}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\psi}\right)^{\frac{1}{\gamma}}\left[\exp\left(x_C+\frac{1}{2}Var_C\right)\right]^{\frac{1}{\gamma}}\left[\exp\left(\begin{array}{c}\left(\begin{array}{c}\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}(1-\omega)\\+\frac{1}{\theta}\end{array}\right)\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\\+\frac{1}{2}\left(\begin{array}{c}\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}(1-\omega)\\+\frac{1}{\theta}\end{array}\right)^2\sigma_\varepsilon^2\\+\frac{1}{2}\left(\frac{\theta-1}{\theta}\frac{\kappa-\frac{1}{\theta-1}}{\omega}\right)^2\frac{1}{\tau_e^*}\end{array}\right)\right]^{\frac{1}{(\theta-1)\gamma}},$$

where

$$\begin{aligned}
x_C &= \left[\begin{array}{c}\left((\frac{1}{\theta}-\gamma)\kappa+\frac{1}{\theta}\right)\left(-\frac{1}{2}\sigma_\eta^2\right)\\+\frac{1}{\theta}\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\end{array}\right]-\frac{\left[(\frac{1}{\theta}-\gamma)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e^*}\left[\begin{array}{c}\omega\left(-\frac{1}{2}\sigma_\eta^2\right)\\+(1-\omega)\left(-\frac{1}{2}\sigma_\varepsilon^2\right)\end{array}\right]\\
Var_C &= \left\{\left[(\frac{1}{\theta}-\gamma)\kappa+\frac{1}{\theta}\right]^2\sigma_\eta^2+\frac{1}{\theta^2}\sigma_\varepsilon^2\right\}-\frac{\left\{\left[(\frac{1}{\theta}-\gamma)\kappa+\frac{1}{\theta}\right]\omega\sigma_\eta^2+\frac{1}{\theta}(1-\omega)\sigma_\varepsilon^2\right\}^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e^*}\\
\kappa &= \kappa(\tau_e^*)=\frac{1}{\theta-1}+\omega\frac{(1-\omega)\sigma_\varepsilon^2-\omega\frac{\theta(\gamma-1)}{\theta-1}\sigma_\eta^2}{\theta\gamma\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e^*}.
\end{aligned}$$

$V(\tau_e,\kappa(\tau_e^*))$ is similar to $\overline{V}_L$ with $\kappa(\tau_e)$ being replaced by $\kappa(\tau_e^*)$, that is,

$$V(\tau_e,\kappa(\tau_e^*))=\exp\left(\begin{array}{c}-\frac{1}{2}\left[(\kappa(1-\theta\gamma)+1)\sigma_\eta^2+\sigma_\varepsilon^2\right]+\frac{1}{2}\frac{1}{\theta}\left[(\kappa(1-\theta\gamma)+1)^2\sigma_\eta^2+\sigma_\varepsilon^2\right]\\+\frac{1}{2}\left(1-\frac{1}{\theta}\right)\frac{\left[(\kappa(1-\theta\gamma)+1)\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e}\end{array}\right).$$

So,

$$\begin{aligned}
\frac{\partial V(\tau_e,\kappa(\tau_e^*))}{\partial\tau_e} \;=\; & \exp\left(\begin{array}{c}-\frac{1}{2}\left[(\kappa(1-\theta\gamma)+1)\sigma_\eta^2+\sigma_\varepsilon^2\right]+\frac{1}{2}\frac{1}{\theta}\left[(\kappa(1-\theta\gamma)+1)^2\sigma_\eta^2+\sigma_\varepsilon^2\right]\\+\frac{1}{2}\left(1-\frac{1}{\theta}\right)\frac{\left[(\kappa(1-\theta\gamma)+1)\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2\right]^2}{\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e}\end{array}\right)\\
&\cdot\left\{-\frac{1}{2}\left(1-\frac{1}{\theta}\right)\frac{\left[(\kappa(1-\theta\gamma)+1)\omega\sigma_\eta^2+(1-\omega)\sigma_\varepsilon^2\right]^2}{\left[\omega^2\sigma_\eta^2+(1-\omega)^2\sigma_\varepsilon^2+1/\tau_e\right]^2}\right\}\left(-\frac{1}{\tau_e^2}\right).
\end{aligned}$$

Proof of Proposition 9 It is easy to obtain

$$y_{\ell t}=\zeta_t^\theta\left(\frac{A_t}{\psi}\right)^\theta\left\{\mathbb{E}\left[\left(Y_t^{\frac{1}{\theta}-\gamma}\cdot(\phi_t\epsilon_{jt})^{\frac{1}{\theta}}\right)|s_{\ell t}\right]\right\}^\theta.$$

Substituting $y_{\ell t}$ into $Y_t = \left[\int_0^1 (\phi_t \epsilon_{jt})^{\frac{1}{\theta}} y_{\ell t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$ and comparing coefficients yields $\bar{Y}_B = \left(1 - \frac{1}{\theta}\right)^{-\frac{1}{\gamma}} \bar{Y}_C$. We find that the ex ante expected profit for firm j is

$$\begin{aligned}\pi(\tau_e; A_t, \tau_e^*) &= \frac{1-\zeta_t}{\zeta_t} \mathbb{E}_{s_{jt}} \frac{y(A_t, s_{jt})}{A_t} \\ &= (1-\zeta_t)\left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_B^{1-\theta\gamma} (\zeta_t A_t)^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)),\end{aligned}$$

where $V(\tau_e, \kappa(\tau_e^*))$ is the same as in the proof of Proposition 8.

Note that $\bar{Y}_B = \left(1 - \frac{1}{\theta}\right)^{-\frac{1}{\gamma}} \bar{Y}_C$. Comparing the objective function of (57)

$$\underset{\tau_e}{Max}\, (1-\zeta_t)\, \zeta_t^{\frac{1}{\gamma}-1} \left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_B^{1-\theta\gamma} A_t^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)) - h(\tau_e, b)$$

with the objective function of (51)

$$\underset{\tau_e}{Max} \frac{1}{\theta-1}\left(1-\frac{1}{\theta}\right)^{\theta}\left(\frac{1}{\psi}\right)^{\theta} \bar{Y}_C^{1-\theta\gamma} A_t^{\frac{1}{\gamma}-1} V(\tau_e, \kappa(\tau_e^*)) - h(\tau_e, b),$$

the only difference lies in their constant coefficients (noting that $\frac{d\left[(1-\zeta_t)\zeta_t^{\frac{1}{\gamma}-1}\right]}{d\zeta_t} < 0$ iff $\gamma > 1 - \zeta_t$ is the counterpart of $\frac{d\left(A_t^{\frac{1}{\gamma}-1}\right)}{dA_t} < 0$ iff $\gamma > 1$), so the properties in the proof of Proposition 8 carry over to Proposition 9.

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