Earnings Inequality and Other Determinants of Wealth Inequality

Jess Benhabib, Alberto Bisin, Mi Luo*
New York University

First draft: December 2016

Abstract: We study the relation between the distribution of labor earnings and the distribution of wealth. We show, both theoretically as well as empirically, that while labor earnings and precautionary savings are important determinants of wealth inequality factors, they cannot by themselves account for the thick tail of (the large top shares in) the observed distribution of wealth. Other determinants, like stochastic returns to wealth, as well as savings rates and rates of returns increasing in wealth need be accounted for.

Key words: wealth distribution; thick tails; inequality

JEL codes: E13, E21, E24

*Thanks to Alexis Toda and Xavier Gabaix.
Corresponding author: alberto.bisin@nyu.edu. We thank the Washington Center for Equitable Growth for their financial support.

1 Earnings and wealth

Increasing income and wealth inequality has led to renewed interest in understanding and explaining wealth and income distributions, and in particular the recent growth in their top shares (Piketty, 2014). The literature has emphasized the role of earnings inequality in explaining wealth inequality. The early work of Aiyagari (1994), which emphasizes the role of precautionary savings, has been very influential in this respect.

However, models of earnings inequality and precautionary savings find it generally difficult to reproduce the thick top tail of the wealth distribution observed in the data. In particular, these models cannot reproduce wealth distributions with substantially larger top shares than earnings distributions, as observed in the available data (see Table 1).

Table 1: Tail Indices for Wealth and Earnings

Country	Wealth	Earnings
US	1.48 - 1.55	2
Sweden	1.63 - 1.85	3
Canada	1.33 - 1.54	2

Notes: Wealth: Vermullen (2014), Table 8, for U.S.; Cowell (2011) for Canada and Sweden. Earnings: Badel, Dayl, Huggett and Nybom (2016).

In this context, for instance, Carroll, Slajek and Tokuo (2014b) note that simulations produce wealth inequality close to the inequality in permanent income fed into the model. Similar results are obtained by De Nardi et al (2016), which adapts earnings data from Guvenen, Karahan, Ozkan, and Song (2015), and most recently by Humber, Krusell, and Smith (2017).

Interestingly, though only suggestively, the cross-country data display essentially no correlation between inequality in earnings and wealth, as measured by Gini coefficients (which can also be considered a proxy for the inverse of tail indices (see Figure 1).

The correlation coefficient is -0.063. We consider this suggestive evidence that earnings inequality does not explain wealth inequality.

Figure 1: Earnings and Wealth Gini

Notes: Wealth: Davies, Sandström, Shorrocks, and Wolff (2011). Earnings: special issue of the *Review of Economic Dynamics* (2010), titled "Cross-sectional facts for macroeconomists," edited by Krueger, Perri, Pistaferri, and Violante.

2 Theory

A simple linear model is useful to highlight several theoretical aspects underlying this difficulty.

Consider a linear individual wealth accumulation equation,

$$w_{t+1} = r_t w_t + y_t - c_t; \tag{1}$$

where w_t, y_t, c_t and r_t are wealth, earnings, consumption and rate of return at time t. We may assume $\{y_t, r_t\}$ are stationary stochastic processes. Consider also a linear consumption function, $c_t = \psi w_t + \chi_t$.[1] We can then write the accumulation equation as

$$w_{t+1} = (r_t - \psi)\, w_t + (y_t - \chi_t)\,. \tag{2}$$

Suppose that r_t and $y_t > 0$ are both random variables, independent and $i.i.d$ over time and independent of w_t. Suppose also that $\chi_t \geq 0$,[2] $0 < E(r_t) - \psi < 1,$ and $prob\,(r_t - \psi > 1) > 0$ for any $t \geq 0$.[3]

Now let $(y_t - \chi_t)$ have a thick right-tail, with tail-index $\beta > 0$. The stationary distribution for w_t can then be characterized by applying a theorem due to Grey (1994), extending results of Kesten (1973), to (2).

Proposition 1 *If* $E\left((r_t - \psi)^{\beta}\right) < 1,$ *and* $E\left((r_t - \psi)^{\gamma}\right) < \infty$ *for some* $\gamma > \beta > 0,$ *then*

[1] Infinitely-lived agent models with stochastic earnings and precautionary savings are concave, but with CRRA (DARA) preferences the consumption function becomes asymptotically linear at high wealth levels; see Benhabib, Bisin and Zhu (2016) for a rigorous exposition and proofs. Linear consumption policies can also be obtained with quadratic preferences.

[2] Note that χ_t will depend on the stochastic properties (i.e. the persistence and variance of its innovations) of the earnings process.

[3] Some additional regularity conditions are required; see Benhabib, Bisin, Zhu (2011) for details.

under some regularity assumptions, the right-tail of the stationary distribution of wealth will be β*. If instead* $E\left((r_t - \psi)^{\gamma}\right) = 1$ *for* $\gamma < \beta$*, then the right-tail index of the stationary distribution of wealth will be* γ*.*

The Proposition makes clear that the right-tail index of the wealth distribution induced by Equation 2 is either γ, which depends on the stochastic properties of returns, or β, the right-tail of $\{y_t - \chi_t\}$). With $\chi_t \geq 0$ the right tail of $\{y_t - \chi_t\}$ will be no thicker than that of than that of $\{y_t\}$. Under our assumptions, for stochastic process describing the accumulation of wealth, the tail index of earnings could not amplify the tail index of the wealth distribution; it's either the accumulation process or the skewed earnings which determine the thickness of the right- tail of the wealth distribution.

3 Empirics

This result is of course obtained under very specific assumptions. However, it does point to the potential difficulty of matching the upper tail of wealth distribution by relying solely on earnings. Furthermore it suggests an explanation how several studieswhich postulate some extraordinarily high earnings states do in fact match the wealth distribution relying only on earnings as a determinant. The theoretical result we highlight suggest that, if earnings were the main determinant of the thickness in the tail of the distribution of wealth, a much thicker distribution of the tail of earnings relative to the tail of actual earnings data would be required to fit the wealth data. For example, Castaneda et al. (2003) estimate the properties of an *awesome state* in a rich overlapping-generation model with various demographic and life-cycle features. It requires the top 0.039% earners have about $1,000$ times the average labor endowment of the bottom 61%, while this ratio, even for the top .01%, is at most of the order of 200 in the World Wealth and Income Database (WWID) by Facundo Alvaredo, Anthony B. Atkinson, Thomas Piketty, Emmanuel Saez, and Gabriel Zucman (since 2011).[4]

[4] We use WWID earnings data, which is not top-coded, for 2014. The argument is not much changed even when considering average income, excluding capital gains.

Similarly, Krueger and Kindermann's (2014) *awesome state* in their Aiyagari-Bewley model requires the top 0.25% earnings to be 400 to 600 larger than the median. Instead, according to the WWID, even the top 0.1% are just about 34 times larger than the median. Finally, Diaz et al. (2003) estimate a top 6% of the population to earn 46 times the labor earnings of the median; while the top 5% in WWID earns about 5 times the median.

We conclude that other determinants of the distribution of wealth need be accounted for. Empirical work by Benhabib, Bisin, Luo (2016) suggest that stochastic earnings, idiosyncratic returns, and savings rates increasing in wealth are all important for explaining wealth distribution. In particular, Benhabib, Bisin, Luo (2016) explicitly estimate the stochastic properties of the Markov process for r_t to match the distribution of wealth. Interestingly, the mean and standard deviation of estimated returns, 2.76% and 2.54%, respectively, closely match those estimated by Fagereng, Guiso, Malacrino and Pistaferri (2015) for the idiosyncratic component of the lifetime rate of return on wealth. Furthermore they also find some empirical evidence for returns increasing in wealth.

References

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E.,(2013). "The Top 1 Percent in International and Historical Perspective," *Journal of Economic Perspectives*, American Economic Association, vol. 27,3-20.

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E., Zuchmn, G., (since 2011). World Wealth and Income Database, http://52.209.180.1/

Aiyagari, S.R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics,*109 (3), 659-684.

Badel, A., Dayl, M., Huggett, M. and M. Nybom (2016). "Top Earners: Cross-Country Facts", mimeo.

Benhabib, J., Bisin, A. and Zhu, S. (2011). "The Distribution of Wealth and Fiscal Policy in Economies with Finitely-Lived Agents" *Econometrica* (79) 2011, 123-157.

Benhabib, J., Bisin, A. and Zhu, S. (2016). "The Wealth Distribution in Bewley Models with Capital Income", *Journal of Economic Theory,* Part A, September 2015, 489-515.

Benhabib, J., Bisin, A. and Luo, M., (2015) "Wealth distribution and social mobility in the US: A quantitative approach," NBER working paper w21721.

Castaneda, A., J. Diaz-Gimenez, and J. V. Rios-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 4, 818-57.

Cowell F.A., (2011): "Inequality Among the Wealthy," mimeo, Centre for Analysis of Social Exclusion, LSE.

Davies, J.B., S. Sandström, A. Shorrocks, E. Wolff (2011): "The Level and Distribution of Global Household Wealth," *Economic Journal,* 121(551), 223-254.
http://onlinelibrary.wiley.com/doi/10.1111/j.1468-0297.2010.02391.x/epdf

De Nardi, M. (2004): "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 743-768.

Diaz, A, Josep Pijoan-Masb, J., and Rios-Rull, J-F, (2003). "Precautionary savings and wealth distribution under habit formation preferences,"*Journal of Monetary Economics* 50, 1257–1291.

Facundo A., A. B. Atkinson, T. Piketty, E. Saez, and G. Zucman (2016), *The World Wealth and Income Database*, http://www.wid.world

Fagereng, A., L. Guiso, D. Malacrino, and L. Pistaferri (2015): "Wealth Return Dynamics and Heterogeneity," mimeo, Stanford University.

Grey, D.R. (1994): "Regular variation in the tail behavior of solutions of random difference equations," *The Annals of Applied Probability*, 4)(1), 169-83.

Guvenen, F., Karahan, F., Ozkan, S. and J. Song (2015): "What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Risk?," National Bureau of Economic Research, Inc. NBER Working Papers 20913.

Hubmer, J., P. Krusell, A. Smith, Jr. (2017): "The Historical Evolution of the Wealth Distribution: A Quantitative-Theoretic Investigation," NBER W.P. 23011.

Kesten, H. (1973): "Random Difference Equations and Renewal Theory for Products of

Random Matrices," *Acta Mathematica,* 131, 207–248.

Krueger, D., F. Perri, L. Pistaferri, G.L. Violante (eds.) (2010). *Review of Economic Dynamics* 13.

Krueger, D. and Kindermann, F., (2014). "High marginal tax rates on the top 1%? Lessons from a life-Cycle model with idiosyncratic income risk," NBER
WP 20601 http://www.nber.org/papers/w20601

Piketty, T. (2014) *Capital in the Twenty-First Century,* Harvard University Press.

Vermeulen, P. (2014), "How fat is the top tail of the wealth distribution?" *European Central Bank*, Working Paper Series no.1692.