1. Subsequent to 1800, per capita income of the leading industrial country grew at a rapid rate, doubling every 40 years.

2. Prior to 1800, living standards differed little across countries and across time.

3. Differences in living standards increased dramatically between 1800 and 1950 as the West grew rich and the rest of the world stagnated or grew slowly.

4. Differences between the West and the East declined after 1950 as most countries in the East started modern economic growth and most grew even faster than most countries in the West.

5. World differences in living standards declined over 1960–88 as growth has been achieved by almost every country and the East narrowed West’s lead.
6. Growth miracles have occurred, but only in countries that were well behind the leader at the time the miracle began.

7. Countries reaching a given level of income at a later date typically double that level in a shorter time.
UNMEASURED INVESTMENTS

- The size of these unmeasured investments relative to GDP is a crucial issue for determining the effects of savings rates on relative steady-state income levels. What is the size of these unmeasured investments?
- There are at least two ways to come up with an estimate of this fraction. The first way follows from the answer to the question, How much could inputs be reduced without reducing measured current output? Eliminating research and development activities, new software purchases and development, personnel offices, the major part of informational technology offices, skill training programs, central headquarters, advertising activities, people searching for better suppliers, and most managers would not lower current output. There have been cases of large reductions in the size of organizations with
little decline in output in the short run.
Let us assume that inputs can be reduced by one third without reducing current measured output when the time period is a year. If so, then two thirds of resources are being used to produce \( c \) and \( x_k \) and one-third is being used to produce \( x_z \). This assumption implies that unmeasured investment, \( x_z \), is one-half of GDP, \( c + x_k \). That is, unmeasured investment is 50 percent of GDP.
Hall and Jones (1999) find significant differences in TFP that are strongly and positively related to the level of development. They report a correlation coefficient between the log of TFP and the log of output per worker equal to 0.89. The difference in TFP implied by this theory between the rich and the middle income is of the order of two. Between the rich and the poor, the factor difference is between four and six.

Another important piece of direct evidence we present against a human capital theory of international incomes derives from studies of relative productivity among the large rich countries in a variety of industries. The United States is overall the world’s most productive nation. It has been since 1890, when it overtook the United Kingdom. However, the United States is not the most productive nation in all
industries in all sectors.
While value added per worker in service sector industries is uniformly higher in the United States, it is not uniformly higher in manufacturing sector industries. In that sector, Japan is more productive than the United States in a number of important industries including the auto and steel industries. Why do we see these large differences as evidence against a schooling capital theory of international incomes? The reason is rather simple. There was no important difference in the schooling of the workforce across industries within a given country. Consequently, if schooling capital differences were the key to understanding income differences, then one country should be the most productive in all industries, and not just a few. This and the other evidence presented in this chapter suggest that we look to something other than human capital to explain international income differences. In
particular, they suggest we look to differences in TFP.
$Y$ is output, $\bar{N}$ is number of workers, $B$ is plant quality, $h$ is hours, $K$ is capital per worker:

$$\frac{Y_t}{\bar{N}} = h_t B_t K_t^{\theta_k}, \quad 0 < \theta_k < 1$$

CRS: There is an optimal plant size: if economy expands number of plants expand. There are no aggregate increasing returns to scale in our economy. Workweeks of different lengths are different commodities, and plants with different qualities are different types of capital. Thus, there is a continuum of different types of both labor and technology capital inputs. Given certain restrictions on technology parameters, there is an optimal plant size, and it is small relative to the economy. As the size of the economy increases, the number rather than the size of plants adjusts. Investment to raise quality from $B$ to $B'$ is
\[ X_{BB'} \]
\[
\frac{\partial X_{BB'}}{\partial B'} > 0, \quad \frac{\partial^2 X_{BB'}}{\partial B'^2} < 0, \quad X_{BB^*} + X_{B^*B'} = X_B
\]

and \( X_{BB'} \) increases in the entry barrier \( \pi \).

\[
X_{BB'} = (1 + \pi) \int_{B}^{B'} \left( \frac{S}{W_t} \right)^\alpha dS
\]

\[
X_{BB'}(\alpha + 1) = (1 + \pi) \left[ \frac{(B')^{\alpha+1} - (B)^{\alpha+1}}{(W_t)^\alpha} \right]
\]

where \( W \) is world knowledge frontier.

\[
W_{t+1} = W_t (1 + \gamma W)^t
\]

\[
\frac{\partial X}{\partial W} < 0 : \text{required investment needed to raise plant quality is lower, the higher is } W.
\]
Let

\[ Z_t = \frac{(1 + \pi)(B_t)^{\alpha+1}}{(1 + \alpha)(W_{t-1})^\alpha} \]

\[ B_t = \left( Z_t (1 + \alpha)(W_{t-1})^\alpha (1 + \pi)^{-1} \right)^{\frac{1}{1+\alpha}} \]

where \( Z \) has the economic interpretation of the value of the sum of a plant’s past investments in quality improvement. In other words, \( Z \) is a plant’s intangible capital stock per worker. In this representation, the stock of date \( t \) intangible capital, \( Z_t \), is measured in terms of the date \( t \) composite-output good, \( Y_t \).
To see this define:

\[ X_Z = X_{BB'} \quad \theta_z = (1 + \alpha)^{-1} \]

\[ (1 - \delta_z) = \left( (1 + \gamma W)^{-1} \right)^{\frac{1 - \theta_z}{\theta_z}} = \left( (1 + \gamma W)^{-1} \right)^\alpha \]

\[ (1 + \gamma)^{1 - \theta_z - \theta_k} = (1 + \gamma W)^{1 - \theta_z} \]

Then

\[ X_{BB'}(\alpha + 1) = (1 + \pi) \left[ \frac{(B')^{\alpha+1} - (B)^{\alpha+1}}{(W_t)^\alpha} \right] \]

\[ X_Z = \frac{(1 + \pi)}{(\alpha + 1)} \left[ \frac{(B_{t+1})^{\alpha+1}}{(W_t)^\alpha} \right] \]

\[ X_Z = \frac{(1 + \pi)}{(\alpha + 1)} \left[ \frac{(B_t)^{\alpha+1}}{(W_{t-1}(1 + \gamma_w))^\alpha} \right] \]

\[ X_Z = \frac{(1 + \pi)}{(\alpha + 1)} \left[ \frac{(B_{t+1})^{\alpha+1}}{(W_t)^\alpha} \right] \]

\[ X_Z = \left( (1 + \gamma_w)^{-1} \right)^\alpha \frac{(1 + \pi)}{(\alpha + 1)} \left[ \frac{(B_t)^{\alpha+1}}{(W_{t-1})^{\alpha}} \right] \]

\[ X_Z = Z_{t+1} - (1 - \delta_z)Z_t \]

where \( \delta_z \) is a measure of the depreciation
of intangible capital
For output we now have

\[
\frac{Y_t}{N} = h_t B_t K_t^\theta_k
\]

\[
= h_t K_t^\theta_k \left\{ \left[ ((1 + \alpha)(1 + \pi))^{-1}(W_{t-1})^\alpha \right] \frac{1}{\alpha+1} (Z_t) \frac{1}{\alpha+1} \right\}
\]

\[
= \left[ (\theta_{z}^{-1}(1 + \pi))^{-1}(W_0(1 + \gamma W)^t)^\alpha \right]^{\theta_z} h_t K_t^\theta_k (Z_t)^{\theta_z}
\]

\[
= (\theta_{z}^{\theta_z}(1 + \pi))^{-\theta_z}(W_0(1 + \gamma W)^t)^{\theta_z \alpha} h_t K_t^\theta_k (Z_t)^{\theta_z}
\]

\[
= (\theta_{z}^{\theta_z}(W_0)^{\theta_z \alpha}) (1 + \pi)^{-\theta_z}((1 + \gamma W)^t)^{\theta_z \alpha} h_t K_t^\theta_k (Z_t)^{\theta_z}
\]

\[
= \mu(1 + \pi)^{-\theta_z}(1 + \gamma)^{(1-\theta_z)t} h_t K_t^\theta_k (Z_t)^{\theta_z}
\]

\[
= \mu(1 + \pi)^{-\theta_z}(1 + \gamma)^{(1-\theta_z-\theta_k)t} h_t K_t^\theta_k (Z_t)^{\theta_z}
\]

It looks like output is subject to exogenous technical change by \((1 + \gamma)^{(1-\theta_z-\theta_k)t}\), whereas this is coming from plant quality \(B\). It is cheaper in terms of output used in investment, the faster is the the growth of the world frontier, since \(\frac{\partial X}{\partial W} < 0\).
Assume \( \alpha > \frac{\theta_k}{1-\theta_k} \) so that \( 1 - \theta_k - \theta_z > 0 \).

Plants optimally choose to maximize:

\[
\operatorname{Max}_{h,k,X} \sum_{0}^{\infty} \bar{N}p_t \left( \frac{Y_t}{N} - w_t(h_t) - r_k K_t - XZ_t \right)
\]

subject to

\[
Z_{t+1} = (1 - \delta_z)Z_t + XZ_t, \quad Z_0 \text{ given}
\]

\[
\frac{Y_T}{N} = \mu(1 + \pi)^{-\theta_z}(1 + \gamma)^{(1-\theta_z-\theta_k)t} h_t k_t^{\theta_k} (Z_t)^{\theta_z}
\]

We can now, assuming firms start and remain identical, aggregate. Punch line:

\[
Y_{AGG,t} = \lambda (1 + \pi)^{-\theta_z}(1 + \gamma)^{(1-\theta_z-\theta_k)t} h_t k_t^{\theta_k} (Z_t)^{\theta_z}
\]

\[
Y_{AGG,t} = \lambda A(\pi)(1 + \gamma)^{(1-\theta_z-\theta_k)t} h_t k_t^{\theta_k} (Z_t)^{\theta_z}
\]

includes \( A(\pi) = (1 + \pi)^{-\theta_z} \), a measure of barriers, differing across countries and affecting the Solow residual.