Persson-Tabellini (AER 1994)

OLG Model: Agents live two periods

1. Utility (d_t^i is second period consumption)

$$v_t^i = U(c_{t-1}^i, d_t^i)$$

2. Budget constraint:

a)

$$c_{t-1}^i + k_t^i = y_{t-1}^i$$

b) $\theta < 1$ is the tax rate, k_t is average capital, θk_t is the average tax.

$$d_t^i = r_t((1-\theta_t)k_t^i + \theta_t k_t)$$

3) e^i is stochastic endowment with zero mean,with density $F(e^i, k,)$ $\quad e_L \leq e \leq \bar{e}$, and creates the income variation:

$$y_{t-1}^i = (w_{t-1} + e_{t-1}^i)k_{t-1}$$

$$c_{t-1}^i = (w_{t-1} + e_{t-1}^i)k_{t-1} - k_t^i$$

Note external effect from last period capital. At

$t-1$ voters choose θ_t then save k_t^i. Only the young vote. Ignore time consistency. Assume homothetic utility so that the rate of return r and θ determines the ratio of consumptions.

$$\frac{d_t^i}{c_{t-1}^i} = D(r_t, \theta_t) \equiv D, \quad D_r > 0 \quad D_\theta < 0$$

Note

$$U(c_{t-1}^i, r_t((1-\theta_t)(y_{t-1}^i - c_{t-1}^i) + \theta_t k_t))$$

$$= U\begin{pmatrix} (w_{t-1} + e_{t-1}^i)k_{t-1} \\ -k_t^i, r((1-\theta_t)(y_{t-1}^i - c_{t-1}^i) + \theta_t k_t) \end{pmatrix}$$

$$FOC \ : \quad \frac{U_c}{U_d} = r(1-\theta_t)$$

If homothetic, $\frac{U_c}{U_d}$ depends on ratios:

$$U = cU\left(1, \frac{d}{c}\right), \quad U_c = U - cU_d \frac{d}{c^2}$$

$$U_c = U\left(1, \frac{d}{c}\right) - \left(\frac{c}{d}\right) U_d\left(1, \frac{d}{c}\right)$$

$$\frac{\partial U}{\partial d} = cU_d c^{-1} = U_d(1, \frac{d}{c})$$

4.

$$d_t^i = r_t((1-\theta_t)(y_{t-1}^i - c_{t-1}^i) + \theta_t k_t)$$

$$= r_t\left((1-\theta_t)\left(y_{t-1}^i - \frac{d_t^i}{D(r_t,\theta_t)}\right) + \theta_t k_t\right)$$

$$d_t^i = \frac{r_t D[(1-\theta_t)y_{t-1}^i + \theta_t k_t]}{D + r_t(1-\theta_t)}$$

5.

$$c_{t-1}^{i} = \frac{d_t^i}{D} = \frac{r_t[(1-\theta_t)y_{t-1}^{i} + \theta_t k_t]}{D + r_t(1-\theta_t)}$$

For average individual $c_{t-1}^{i} = c_{t-1},\ taking\ e^{i} = 0$:

$$c_{t-1}^{i} = \frac{d_t^i}{D} = \frac{r_t[(1-\theta_t)y_{t-1}^{i} + \theta_t(y_{t-1} - c_{t-1})]}{D + r_t(1-\theta_t)}$$

$$= \frac{r_t[y_{t-1} - \theta_t c_{t-1}]}{D + r_t(1-\theta_t)}$$

and solving:

$$c_{t-1}^{i} = \frac{r_t[y_{t-1} - \theta_t c_{t-1}]}{D + r_t(1-\theta_t)}$$

$$c_{t-1}\left[1 + \frac{r_t\theta_t}{D + r_t(1-\theta_t)}\right] = c_{t-1}\left[\frac{D + r_t}{D + r_t(1-\theta_t)}\right]$$

$$= \frac{r_t y_{t-1}}{D + r_t(1-\theta_t)}$$

$$c_{t-1} = \frac{r_t y_{t-1}}{D + r_t} = \frac{r_t w_{t-1} k_{t-1}}{D + r_t}$$

Thus, average capital evolves as $k_t = y_{t-1} - c_{t-1}$:

$$
\begin{aligned}
k_t &= y_{t-1} - c_{t-1} = y_{t-1}\left(1 - \frac{r_t}{D + r_t}\right) \\
&= (w_{t-1} + e^i_{t-1})k_{t-1}\left(1 - \frac{r_t}{D + r_t}\right) \\
&= w_{t-1}k_{t-1}\left(1 - \frac{r_t}{D + r_t}\right) = w_{t-1}\left(\frac{D}{D + r_t}\right)k_{t-1}
\end{aligned}
$$

6. Growth

$$g = \frac{k_t}{k_{t-1}} - 1 = w_{t-1}\frac{D}{D + r_t} - 1 = G(w_{t-1}, r_t, \theta_t)$$

$$g_\theta = w_{t-1}\frac{D_\theta(D + r_t) - DD_\theta}{(D + r_t)^2} = \frac{D_\theta r_t}{(D + r_t)^2} < 0$$

$$g_r = w_{t-1}\frac{D_r(D + r_t) - D(D_r + 1)}{(D + r_t)^2} = \frac{D_r r_t - 1}{(D + r_t)^2}$$

$$g_w = \frac{D}{D + r_t} > 0$$

7. Now Politics: What does individual i prefer?

$$\frac{\partial v_t^i}{\partial \theta_t} = \frac{\partial U(c_{t-1}^i, r((1-\theta_t)k_t^i + \theta_t k_t))}{\partial \theta_t}$$

$$= U_d[k_t - k_t^i + \theta_t \frac{dk_t}{d\theta_t}]r$$

Note:

$$U((w_{t-1} + e_{t-1}^i)k_{t-1} - k_t^i, r((1-\theta_t)k_t^i + \theta_t k_t))$$

$$\frac{\partial U}{\partial k_t^i}\frac{dk_t^i}{d\theta_t} = 0 \quad from \quad foc, wrt\ k_t^i\ or \quad c_{t-1}^i : \qquad \frac{\partial U}{\partial k_t^i} = 0$$

$$\frac{\partial v_t^i}{\partial \theta_t} = U_d[k_t - k_t^i + \theta_t \frac{dk_t}{d\theta_t}]r$$

So there are two effects: Redistribution is beneficial if $k_t^i < k_t$, but also hurts because in the aggregate. $\frac{dk_t}{d\theta_t} < 0$ because taxation , on average discourages accumulation. .

Utility:

$$
\begin{aligned}
v_t^i &= c_{t-1}^i U(1, D(r_t, \theta_t)) \\
&= \frac{r_t[(1-\theta_t)y_{t-1}^i + \theta_t k_t]}{D + r_t(1-\theta_t)} U(1, D(r_t, \theta_t)) \\
&= \frac{r_t \begin{bmatrix} (1-\theta_t)w_{t-1} + (1-\theta_t)e_{t-1}^i \\ +\theta_t\left(w_{t-1}\frac{D}{D+r_t}\right) \end{bmatrix} k_{t-1}}{D + r_t(1-\theta_t)} U(1, D(r_t, \theta_t \\
&= \left(\frac{r_t}{D + r_t(1-\theta_t)}\right) \\
&\quad \begin{pmatrix} \left(1 - \theta_t\left(1 - \frac{D}{D+r_t(1-\theta_t)}\right)\right) w_{t-1} \\ +(1-\theta_t)\varepsilon_t^i \end{pmatrix} k_{t-1} U(1, D(r_t, \theta_t
\end{aligned}
$$

MEDIAN VOTER i CHOOSES θ :

$$Max_{\theta}\ v_t^i$$

$$\theta^* = \theta(w_{t-1}, r_t, e^m)$$

Now e^i has zero mean. Let e^m be the median. We have to show

$$\theta^* \gtreqless 0 \quad \text{as} \quad e^m \lesseqgtr 0$$

If median voter has low $e^m < 0$, he wants the rest to be taxed and to recive subsidy. If median voter has high $e^m > 0$, he wants lump-sum tax to subsidize according to skill.

To show this, derive expression for $k_t - k_t^i$. From budget constraints;

$$d_t^i = r_t((1-\theta_t)k_t^i + \theta_t k_t)$$

$$r_t\theta_t k_t = d_t^i - r_t(1-\theta_t)k_t^i$$

$$r_t\theta_t k_t = Dc_{t-1}^i - r_t(1-\theta_t)k_t^i$$

$$r_t\theta_t k_t = D((w_{t-1} + e_{t-1}^i)k_{t-1} - k_t^i) - r_t(1-\theta_t)k_t^i$$

$$r_t\theta_t k_t = D((w_{t-1} + e_{t-1}^i)k_{t-1}) - (r_t(1-\theta_t) + D)k_t^i$$

SO

$$r_t\theta_t k_t - D((w_{t-1} + e_{t-1}^i)k_{t-1})$$

$$= \left(\begin{array}{c} r_t\theta_t \\ -D\left((w_{t-1} + e_{t-1}^i)\frac{k_{t-1}}{k_t}\right) \end{array} \right) k_t$$

$$= -(r_t(1-\theta_t) + D)k_t^i$$

$$
\begin{aligned}
\left(r_t\theta_t - D\left(w_{t-1}\frac{k_{t-1}}{k_t}\right)\right)k_t &= De^i_{t-1}k_{t-1} \\
&\quad -(r_t(1-\theta_t)+D)k^i_t \\
\left(r_t\theta_t - D\left(w_{t-1}\frac{D+r_t}{w_{t-1}D}\right)\right)k_t &= De^i_{t-1}k_{t-1} \\
&\quad -(r_t(1-\theta_t)+D)k^i_t \\
(r_t\theta_t - (D+r_t))k_t &= De^i_{t-1}k_{t-1} \\
&\quad -(r_t(1-\theta_t)+D)k^i_t \\
(-r_t(1-\theta_t)-D)k_t &= De^i_{t-1}k_{t-1} \\
&\quad -(r_t(1-\theta_t)+D)k^i_t \\
k^i_t - k_t &= \frac{D}{(r_t(1-\theta_t)+D)}k_{t-1}e^i_{t-1}
\end{aligned}
$$

NOW note:

$$k_t = \frac{w_{t-1}D}{r_t + D}k_{t-1}; \qquad \frac{\partial k_t}{\partial \theta_t} = \frac{w_{t-1}r_t D_\theta}{(r_t + D)^2}k_{t-1} < 0$$

$$\frac{\partial v_t^i}{\partial \theta_t} = r_t U_d\left(k_t - k_t^i + \theta_t \frac{\partial k_t}{\partial \theta_t}\right) = 0$$

$$\rightarrow \frac{\theta_t w_{t-1} r_t D_\theta}{(r_t + D)^2} - \frac{D}{(r_t(1-\theta_t) + D)}e_{t-1}^i = 0$$

SO for median voter

$$\frac{\theta_t w_{t-1} r_t D_\theta}{(r_t + D)^2} - \frac{D}{(r_t(1-\theta_t) + D)}e_{t-1}^m = 0$$

implies, since $D_\theta < 0$,

$$sign\ \theta_t = -sign\ e_{t-1}^m$$

EXAMPLE

$$U^i = (c_{t-1}^i)^{\alpha}(d_t^i)^{1-\alpha}$$

$$\frac{U_1^i}{U_2^i} = \frac{\alpha(c_{t-1}^i)^{\alpha-1}(d_t^i)^{1-\alpha}}{(1-\alpha)(c_{t-1}^i)^{\alpha}(d_t^i)^{-\alpha}} = r_t(1-\theta_t)$$

$$\frac{d_t^i}{c_{t-1}^i} = \frac{1-\alpha}{\alpha}r_t(1-\theta_t) = D$$

$$g = \frac{w_{t-1}D}{r_t + D} = w_{t-1}\frac{(1-\alpha)(1-\theta_t)}{1-\theta_t+\alpha\theta_t}$$

$$D_\theta = -\frac{1-\alpha}{\alpha}r_t < 0$$

For computations, note:

$$\frac{\theta_t w_{t-1} r_t D_\theta}{(r_t + D)^2} - \frac{D}{(r_t(1-\theta_t)+D)}e_{t-1}^m = 0$$

$$(r_t(1-\theta_t)+D) = 1-\alpha$$

$$(r_t + D)^2 = r^2\alpha^{-2}(1+\alpha(1-\theta))$$

Then

$$-e^m = \frac{\theta_t w_{t-1}\alpha}{(1-(1-\alpha)\theta_t)^2}$$

Also

$$\frac{\partial \theta_t}{\partial e_m} = \frac{(1-(1-\alpha)\theta_t)^2}{2e^m(1-(1-\alpha)\theta_t)(1-\alpha) - \alpha w_{t-1}} < 0$$

$$if\ e^m < 0$$

$$\frac{\partial \theta_t}{\partial w_{t-1}} = \frac{-\alpha\theta_t}{2e^m(1-(1-\alpha)\theta_t)(1-\alpha) - \alpha w_{t-1}} > 0$$

$$if\ e^m < 0$$

SO

$$g = \frac{k_t}{k_{t-1}} - 1 = w_{t-1}\frac{D}{D + r_t} - 1$$

$$= G(w_{t-1}, r_t, \theta_t(w, r, e_m))$$

$$\frac{dg}{de_m} = g_\theta \frac{d\theta}{de_m} > 0 \qquad if\ e^m < 0$$

$$\frac{dg}{dw_{t-1}} = g_w \frac{d\theta}{dw_{t-1}} > 0 \qquad if\ e^m < 0$$

Growth increases with equality when $e_m < 0$.

Discussion:

Why do the poor grow more slowly (do they?)?

From latter part of Kuznet's Curve, inequality and growth are negatively correlated. P-T suggest lower inequality→lower tax→higher growth. Emprically lower inequality→lower tax (?), but there may be effective franchise issues. P-T empirics suggest a negative effect of ineq. on growth (ineq measured by the share of top 20% of pop.) Holds for postwar democracies (since there is voting), and since 1830. Causality issues: ineq prior to growth.

What are the channels? Effect of ineq. on investment? Why not test directly.