ROMER-JPE 1990

$x(i)$ are intermediate goods. Capital is given by:

$$K = n \int_0^A x(i) di$$

Note that the sum of $x(i)$ does not add up to K : there is a proportionality factor n. Final good production:

$$Y = H_y^\alpha L^\beta \int_0^A x(i)^{1-\alpha-\beta} di$$

L is labor and set to unity. Human Capital:

$$H = H_y + H_A$$

Note that total H is fixed. Accumluation of K is given by:

$$\dot{K} = Y - c$$

where c is consumption.

Research Sector:

$$\dot{A} = \delta H_a A$$

Now if P_A is the price of a new design, and w_H is the wage or rental of human capital, equilibrium requires w_H to equal to the marginal product of H_a:

$$w_H = \delta P_a A$$

Maximization in the final good sector:

$$Max \int_0^{\infty} \left(H_y^{\alpha} L^{\beta} x(i)^{1-\alpha-\beta} - p(i)x(i) \right) di$$

This implies:

$$p(i) = (1 - \alpha - \beta) H_y^{\alpha} L^{\beta} x(i)^{-\alpha-\beta}$$

Intermediate Goods

First define profit, not net of cost of aquiring new design at price P_A.

$$\pi = p(x)x - rnx$$

where nx is the amount of capital, or own input tied in the process of production, and therefore generating opportunity cost rnx. (Note that the price of the final good, c or K, is normalized at unity.) Let us maximize the profit of the firm producing intermediate good i :

$$Max_x \; p(x)x - rnx$$

$$= Max_x \, (1 - \alpha - \beta) H_y^{\alpha} L^{\beta} x^{1-\alpha-\beta} - rnx$$

Digression on monopoly pricing:

$$Max_x \;\; p(x)x - wx$$

implies:

$$p'(x)x + p - w = 0$$

or:

$$w = p\left(1 + \frac{p'(x)x}{p}\right) = p(1 - \epsilon)$$

or:

$$\frac{w}{1-\epsilon} \equiv \frac{MC}{1-\epsilon} = p$$

where ϵ is the elasticity of demand (inverse of price elasticity!), and $(1-\epsilon)^{-1}$ is the markup over marginal cost, $\frac{p}{MC}$.

So in the case above we get:

$$\bar{p} = \frac{rn}{1-(\alpha+\beta)} = \frac{MC}{(1 - elasticity\, of\, demand)}$$

Then substitititing we find profits:

$$\pi = \bar{p}\bar{x}(\alpha+\beta)$$

However, the intermediate goods industry producing $x(i)$ is monopolistically competitive, and earns zero profits. Therefore the cost of buying a design to produce $x(i)$ is equal to discounted profits:

$$\int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi(\tau)d\tau = P_A(t)$$

Note that if $P_A(t)$ is constant or at a steady state, differentiating the above with respect to t gives:

$$-\pi(t) + r(t)\int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi(\tau)d\tau = 0$$

$$\pi = r(t)P_A$$

Preferences

$$\int_0^{\infty}(1-\sigma)^{-1}c^{1-\sigma}dt$$

FOC:

$$\dot{c} = c\frac{(r-\rho)}{\sigma}$$

Symmetric solution

All intermediate goods are priced the same and used in the same amount in the symmetric equilibrium.

$$\begin{aligned} Y(H_A, L, x) &= H_y^{\alpha}L^{\beta}\int_0^{A} x(i)^{1-\alpha-\beta}di \\ &= H_y^{\alpha}L^{\beta}A\bar{x}^{1-\alpha-\beta} \\ &= H_y^{\alpha}L^{\beta}A\left(\frac{K}{nA}\right)^{1-\alpha-\beta} \\ &= n^{\alpha+\beta-1}(AH_y)^{\alpha}(AL)^{\beta}(K)^{1-\alpha-\beta} \end{aligned}$$

If A were fixed, the model would imply convergence to the steady state.

The Steady State

At the steady state $\bar{x}$ and H_y are fixed, so $\frac{\dot{A}}{A}$ is fixed. The discounted profits of the intermediate good firm equals the purchase price of the design.

$$P_A = \frac{\pi}{r} = \frac{(\alpha+\beta)\bar{p}\bar{x}}{r}$$

$$= \left(\frac{(\alpha+\beta)}{r}\right)((1-\alpha-\beta))(H_y^{\alpha} L^{\beta} \bar{x}^{1-\alpha-\beta})$$

But also, the wage rate of H is equalized across sectors:

$$w_H = \delta P_A A = \alpha(H_y^{\alpha-1} L^{\beta} \bar{x}^{1-\alpha-\beta})$$

Combining, and setting $L = 1$, :

$$H_y = \frac{\alpha r}{\delta(1-\alpha-\beta)(\alpha+\beta)}$$

Note that A cancels.

Now $H_A = H - H_y,$ so

$$\frac{\dot{A}}{A} = \delta H_A$$

Also, if $\bar{x}$ is fixed, H_y is fixed if r is fixed. (We'll show to get steady state r later below.) Steady state output then grows like A since:

$$Y(H_A, L, x) = H_y^{\alpha} L^{\beta} A \bar{x}^{1-\alpha-\beta}$$

and so does $K = nA\bar{x},$ so $\frac{K}{Y}$ is constant, as is the savings rate:

$$\frac{c}{Y} = 1 - \frac{\dot{K}}{Y} = 1 - \frac{\dot{K}}{K}\frac{K}{Y} = 1 - \delta H_A \frac{K}{Y}$$

The growth rate is given by:

$$g = \frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A$$

$$= \delta(H - H_Y) = \delta H - \frac{\alpha r}{(1 - \alpha - \beta)(\alpha + \beta)}$$

$$= \delta H - \Lambda r$$

But from preferences:

$$g = \frac{\dot{c}}{c} = \frac{(r - \rho)}{\sigma}$$

Combining, this implies:

$$g = \frac{\delta H - \Lambda \rho}{\sigma \Lambda + 1}$$

Interpretations

1. Increasing interest rate r causes g to decline because it affects the allocation of H. This is because marginal product of H depends on P_A which reflects discounted future revenues of the new design.

2. L, n do not affect growth.An increase in L or decrease in n increases the demand for intermediate goods and the marginal productivity of H_y. But higher H_y offsets the increased demand for H_A due to increased profitability of intermediate goods. But this result (exact offset) is not robust to specification. So subsidizing K (or lowering n) or L may affect growth.

3. Increasing H increases g and H_A since $g = \delta H_A = \delta H - \Lambda r$. Note however that for low $H, H_A = 0$, and there is no growth: all H goes to H_y. This happens because the marginal product of H_y is higher than w_H for $H = H_y$. Corner solution.

4. Doubling H doubles growth: scale effect.

5. Subsidy to employment in research raises growth. (Note, if H_y falls, so does $\bar{p}\bar{x}$, through the demand for intermediate goods, and then P_A also falls.)

6. There are two reasons for inefficiency.

a) An additional design increases marginal product of future research as well. But the present P_A does not reflect this externality: it only incorporates discounted profits of current design, not increased productivity of future research through higher A.

b) Monopolistic competition forces a markup of price over marginal cost of x. A new design contributes $H_y^{\alpha} L^{\beta} A \bar{x}^{1-\alpha-\beta}$ but the producer of x as monopolist restricts its supply to raise profits π, which is driven down to zero by competition to buy designs, but distotion remains since price differs from marginal cost. Note however that without profits to pay for designs there would be no research! So this distortion may improve welfare because we started from a distorted situation where there are externalities to research.

7. For indeterminacy in the Lucas Model see:

Jess Benhabib and Roberto Perli, Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth, *Journal of Economic Theory*, 1994, Pages 113-142.

8. For indeterminacy in the Romer Model see

Jess Benhabib, Roberto Perli and Danyang Xie, Monopolistic competition, indeterminacy and growth, *Ricerche Economiche, (1994)* 279-298.

Evans, George W & Honkapohja, Seppo & Romer, Paul, 1998. "Growth Cycles," *American Economic Review*, vol. 88(3), pages 495-515.