Sentiments, Financial Markets, and Macroeconomic Fluctuations

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Motivations

1. The financial sector plays a central role in a modern economy
   - evident by the great recession of 2007-2009

2. How can the financial sector influence the aggregate real economy?
   - two potential channels
     - financing channel (blood system)
     - information channel (nervous system)

3. An exploding financial accelerator literature so far on financing channel (B&G (1989), K&M (1997))
   - both theoretically and empirically relevant
   - some puzzles

This paper: Information channel (the feedback effect from financial markets to the real economy)
Figure 1: Credit to the US non-financial corporate sector (Source: Adrian, Colla and Shin (2012))
Further Motivation

1. Theoretical

Growing micro literature studies feedback effects from financial markets to the real side of the economy (Bond, Edmans and Goldstein (2012))

- Partial equilibrium
- Typically, a firm manager learns information about his own firm

2. Empirical

Investor sentiment in financial markets can affect asset prices, in particular, aggregate (macro)-level asset prices, in turn impacting corporate financing and investment (Baker and Wurgler (2007), Lamont and Stein (2006))

- Behavioral approach

This paper: In a general equilibrium, rational-expectations framework, the sentiment-driven asset prices can influence real activities and shape macroeconomic fluctuations.
1. The informational role of financial markets in allocating resources can be impaired by investors' sentiments or sunspots

- **Unlike the conventional view** that prices can efficiently allocate economic resources in a free market by signaling relevant information to economic actors (Hayek (1945), Grossman and Stiglitz (1980))

2. The sentiment-driven asset prices in turn may influence *real activities* and shape *macroeconomic fluctuations*. 
Main results

1. One-country economy
   Financial information frictions generate sentiment-driven fluctuations in asset prices and self-fulfilling business cycles.
   - formalize the linkage of the Keynesian notion of "beauty contests" and "animal spirits"
   - Implications for asymmetric and non-linear asset prices

2. Two-country economy
   Explain global recessions and the cross-country comovement puzzles

3. Dynamic OLG economy
   - Explain persistent business cycle fluctuations
   - Implications for asset prices over business cycles
The model setup (1)

Investors

Period 0

$P_0 \leftarrow R_1 \leftarrow N_1 \leftarrow R_2 \leftarrow A_2$

Logic:

Workers

Period 1

Period 2

$K_{j0} = 1$

($P_0 = ?$)

$N_{i1} = ?$

$Y_1 = A_1 K_1^\alpha N_1^{1-\alpha}$

$Y_2 = A_2 K_2^\alpha N_2^{1-\alpha}$

$K_1 R_1$

$N_1 W_1$

$N_1 W_1 \cdot R_2 (A_2)$

savings

consumption

consumption
The model setup (2)

Firm and technology
1. Production in periods 1 and 2, with production function \( Y_t = A_t K_t^\alpha N_t^{1-\alpha} \)
2. Assume \( A_1 = 1 \) and \( \log A_2 = a_2 \sim N\left(-\frac{1}{2} \sigma_a^2, \sigma_a^2\right) \). \( a_2 \) is realized in period 2

Financial market investors:
1. live in periods 0 and 1, but consume in period 1
2. each investor is endowed with \( K_0 = 1 \) unit of capital in period 0
3. Investors trade capital in the financial market in period 0 with price \( P_0 \)
4. Investor j receives a private signal \( s_{j0} = a_2 + \varepsilon_j \) in period 0

Workers:
1. live and supply labor in periods 1 and 2, but consume in period 2
2. Utility function: \( U_i = C_i2 - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} \)

Information structure:
\( \Omega_{j0} = \{P_0, a_2 + \varepsilon_j, z + \delta_j\} \quad \Omega_{i1} = \{R_1, W_1, P_0\} = \Omega_1 \)
The model setup (3): timeline

**Period 0**
- Investors receive signal about $a_2$ and sentiment shock $Z$
- Investors trade capital with financial price $P_0$

**Period 1**
- Investors supply capital
- Workers supply labor
- Firm produces
- Investors obtain capital income and consume
- Workers obtain labor income and save

**Period 2**
- Workers supply capital and labor
- Firm produces
- Workers obtain labor and capital income, and consume

Mathematical expressions:

- $Y_1 = f(A_1, K_1, N_1)$
- $Y_2 = f(A_2, K_2, N_2)$
- $K_0 = 1$
- $A_1 = 1$
- $K_1 = 1$
- $N_1 W_1$
- $N_2 = 1$
Equilibrium (1): steps

Investors’ problem

- Investors are risk neutral. They trade capital in period 0 to maximize

\[ a_2 = \log(A_2): \text{fundamental shock to productivity in period 2} \]
\[ \varepsilon_j: \text{noise in idiosyncratic signal} \]
\[ z: \text{sentiment shocks, drawn from standard normal distribution} \]

- Risk neutral implies \( P_0 = E[R_1|a_2 + \varepsilon_j, P_0, z] \)
- \( R_1: \text{The return of capital in period 1.} \)
- The investors need to forecast the labor supply of workers in period 1 when trading capital in period 0.
Equilibrium (2): steps

Workers’ problem

- Households consume in period 2, the last period. They solve

- $R_2$: The interest rate from period 1 to period 2:
- $W_1$: Wage in period 1
- The workers try to learn the fundamental shock $A_2$ from the price of capital in period 0 based on their beliefs about how price incorporates sentiment shocks and fundamental shocks
- the “forecasting the forecasts of others” problem.
First Order Conditions

• Assuming interiority,

The first-order condition of the investors' problem in (2) is

\[ 0 = \mathbb{E}[R_1 - P_0 | \Omega_0] \] (6)

and the first-order condition of the workers' problem in (5) is

\[ \psi N_{i_1}^{\gamma} = W_1 \mathbb{E}[R_2 | \Omega_1]. \] (7)
Equilibrium (3): steps

Solving key endogenous variable $N_1$

1. Given $K_1 = 1$ and $N_1$, we have
   \[ R_1 \equiv \alpha K_1^{\alpha-1}N_1^{1-\alpha} = \alpha N_1^{1-\alpha}, \]
   \[ W_1 \equiv (1 - \alpha)K_1^{\alpha}N_1^{-\alpha} = (1 - \alpha)N_1^{-\alpha}. \]

2. The capital in period 2 is given by the labor income in period 1. Hence we have
   \[ K_2 = W_1 N_1 = (1 - \alpha)N_1^{1-\alpha}. \]

3. We then express $R_2$ in term of $N_1$:
   \[ R_2 = \alpha A_2 K_2^{\alpha-1} = A_2 \alpha \left[(1 - \alpha)N_1^{1-\alpha}\right]^{\alpha-1}. \]

4. In a symmetric equilibrium where $N_{i1} = N_1$, equation (7) becomes
   \[ \psi N_1^\gamma = (1 - \alpha)N_1^{-\alpha}E[A_2 \alpha \left[(1 - \alpha)N_1^{1-\alpha}\right]^{\alpha-1} | \Omega_{i1}] \]
   or
   \[ N_1^{\gamma + 1 - (1 - \alpha)\alpha} = \psi^{-1} \alpha (1 - \alpha) E[A_2 | \Omega_1]. \]

5. We normalize $\psi^{-1} \alpha (1 - \alpha) = 1$ and denote $\theta = \frac{1}{\gamma + 1 - (1 - \alpha)\alpha}$ and thus obtain
   \[ N_1 = \left\{ E[A_2 | \Omega_1] \right\}^{\theta} = \left\{ E[A_2 | \Omega_{i1}] \right\}^{\theta}. \]

6. Finally, the price $P_0$ should be consistent with the investors’ rational expectations, namely
   \[ P_0 = \alpha E\left[N_1^{1-\alpha} | \Omega_{i0}\right] = \alpha E\left[N_1^{1-\alpha} | a_2 + \varepsilon_j, P_0\right]. \]
Equilibrium (4): intuition

The two-way feedback between financial market and real economy

How sentiment shock \( z \) affects the equilibrium outcome?
Equilibrium: core

Two feedback equations:

Endogenous dividend: \( N_1 = \{\mathbb{E}[A_2|P_0]\}^\theta \)

REE price:
\[
P_0 = \alpha \mathbb{E} [N_1^{1-\alpha}|a_2, z, P_0]
\]

\[
p(a_2, z) = (1 - \alpha) \theta \log \mathbb{E}[\exp(a_2)|p(a_2, z)]
\]

(with \( \log P_0 = \log \alpha + p(a_2, z) \))
Equilibrium: outcome

Three types of *rational expectations* equilibria

1) **Fully-revealing Equilibrium**
   financial price $P_0$ perfectly incorporates information about $a_2$

2) **Non-revealing Equilibrium**
   financial price $P_0$ does not contain information about $a_2$ at all

3) **Sentiment-Driven Equilibrium**
   financial price $P_0$ contains both information about $a_2$ and sentiment shock
Proposition 1  There exists a fully revealing equilibrium in which

\[ \log P_0 = \log \alpha + (1 - \alpha) \theta a_2, \]

and

\[ \log N_1 = \theta a_2. \]
Proposition 2. There exists a non-revealing equilibrium in which

\[ \log P_0 = \log \alpha \]

and

\[ \log N_1 = 0. \]
Equilibrium (6): sentiment-driven equilibrium

**Proposition 3** There exists a continuum of equilibria indexed by $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4} \sigma^2$, in which the price $P_0$ is given by

$$\log P_0 = \bar{p} + \log \alpha + (1 - \alpha) (\phi a_2 + z)$$  \hspace{1cm} (17)

and

$$\log N_1 = \bar{n} + \phi a_2 + z,$$  \hspace{1cm} (18)

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2 \sigma^2 - 4 \sigma_z^2}}{2 \sigma}$$  \hspace{1cm} (19)

and $\bar{p} = \bar{n} = 0$.

**Relationship and intuition:**

[Diagram showing fully-revealing, sentiment-driven, and non-revealing equilibria]
One-country model: summary

1. Modelling the relationship between sentiments, financial markets, macroeconomic activity

2. Formalizing the linkage of the Keynesian notion of "beauty contests" and "animal spirits"

3. Clarifying the difference between financial market sentiments and noisy trading
One-country model: more implications

Non-linear asymmetric equilibrium

1. Price is more informative when fundamentals are strong

2. A small decline in fundamentals can trigger asset price collapses

\[
\log P_0 = \begin{cases} 
\log \alpha + (1 - \alpha) \theta a_2 & \text{if } a_2 \geq 0 \\
\log \alpha + p(a_2, z) & \text{if } a_2 < 0 
\end{cases}
\]

\[
p(a_2, z) = (1 - \alpha) \theta \log \mathbb{E}[\exp(a_2)|p(a_2, z)] < 0
\]

\[
p(a_2, z) = \theta (1 - \alpha) \log \left[ \frac{\exp(a_2) + \exp(z)}{2} \right]
\]
A Two-Country Model

Motivation facts
1. Cross-country comovement puzzles (Backus et al. (1995) and Baxter (1995))
2. Global recessions and synchronized contractions in output and asset prices (Perri and Quadrini (2013))

Model setup
1. two symmetric countries ℓ=h and f, linked by international trade
2. the final goods in home country is produced with home and foreign intermediate (material) goods

\[ Y_t = \left( \frac{X_{ht}}{\eta} \right) ^{\eta} \left( \frac{X_{ht}^*}{1-\eta} \right) ^{1-\eta} \]
3. The production of the intermediate goods in home country is

\[ X_t = A_t K_t^\alpha N_t^{1-\alpha} \]
Figure 4: Productions in the home country and the foreign country
A Two-Country Model: Equilibrium

Intuition
The financial prices in either country is observed by the other country. Both prices are subject to sentiment shocks, which create *international contagion*.

Equilibrium

**Proposition 5** Equilibrium $N_1$ and $N_1^*$ jointly satisfy

\[
N_1 = N_1^{*\omega} \left\{ \mathbb{E}[A_2^\eta A_2^{1-\eta} | P_0, W_1, R_1, P_0^*] \right\}^\lambda,
\]

\[
N_1^* = N_1^\omega \left\{ \mathbb{E}[A_2^{*\eta} A_2^{1-\eta} | P_0^*, W_1^*, R_1^*, P_0] \right\}^\lambda,
\]

where $P_0$ and $P_0^*$ are given by

\[
P_0 = \alpha \mathbb{E} \left[ \left( N_1^{1-\alpha} \right)^\eta \left( N_1^{*1-\alpha} \right)^{1-\eta} | P_0, a_2 + \epsilon_j, P_0^*, z \right],
\]

\[
P_0^* = \alpha \mathbb{E} \left[ \left( N_1^{1-\alpha} \right)^\eta \left( N_1^{*1-\alpha} \right)^{1-\eta} | P_0^*, a_2^* + \epsilon_j^*, P_0, z^* \right],
\]

respectively.
Proposition 6  There exists a fully-revealing equilibrium in which the capital prices are given by
\[ p_0 = \log P_0 = \log \alpha + \pi_h a_2 + \pi_f a^*_2, \]
\[ p_0^* = \log P_0^* = \log \alpha + \pi_h a^*_2 + \pi_f a_2. \]

and the labor supplies are
\[ n_1 = \log N_1 = \theta_h a_2 + \theta_f a^*_2, \]
\[ n_1^* = \log N_1^* = \theta_h a^*_2 + \theta_f a_2. \]

where \( \pi_h = \frac{\lambda(1-\alpha)}{1-\omega^2} \left[ 2 (1 - \eta) \eta \omega + \eta^2 + (1 - \eta)^2 \right], \)
\( \pi_f = \frac{\lambda(1-\alpha)}{1-\omega^3} \left[ (\eta^2 + (1 - \eta)^2) \omega + 2 (1 - \eta) \eta \right], \)
\( \theta_h = \frac{\lambda}{1-\omega^2} [(1 - \eta) \omega + \eta] \) and \( \theta_f = \frac{\lambda}{1-\omega^2} [\eta \omega + (1 - \eta)]. \)

The comovement is weak when \( \eta \) is close to 1
\[ y_1 = \log Y_1 = (1 - \alpha) [\eta n_1 + (1 - \eta) n_1^*] \]
\[ y_1^* = \log Y_1^* = (1 - \alpha) [\eta n_1^* + (1 - \eta) n_1] \]
A Two-Country Model: Sentiment-Driven Comovement

Proposition 7 There exists a continuum of equilibria indexed by \(0 \leq \sigma_z^2 \leq \frac{\lambda^2 \sigma^2}{16(1-\omega)^2}\), in which

\[
\log P_0 = \log P_0^* = \bar{p} + (1 - \alpha) [\phi(a_2 + a_2^*) + \sigma_z(z + z^*)]
\]

\[
\log N_1 = \log N_1^* = \bar{n} + \phi(a_2 + a_2^*) + \sigma_z(z + z^*)
\]

\[
\log Y_1 = \log Y_1^* = (1 - \alpha) [\phi(a_2 + a_2^*) + \sigma_z(z + z^*)]
\]

where

\[
\phi = \frac{\lambda \sigma^2 \pm \sqrt{(\lambda \sigma^2)^2 - 16 (1 - \omega)^2 \sigma^2 \sigma_z^2}}{4 (1 - \omega) \sigma_a^2}
\]

and

\[
\bar{n} = \frac{\lambda}{2 (1 - \omega)} \left\{ \left[ \eta^2 + (1 - \eta)^2 - 1 \right] + \frac{\phi^2 \sigma^2}{2 \phi^2 \sigma_a^2 + 2 \sigma_z^2} \right\} \sigma_a^2
\]

\[
\bar{p} = \log \alpha + (1 - \alpha) \bar{n}.
\]

Output as well as employment in period 1 in the two countries becomes fully synchronized.

Intuition: investors' perceived synchronization across economies can lead to the actual synchronization
The OLG Model

Two additional insights:
1. Persistence in business cycles
2. Asset prices over business cycles

Model setup (static $\Rightarrow$ dynamic)

1. Investors: capitalists (entrepreneurs) - the old generation of workers
2. Workers
3. A bond market is introduced: bonds are traded between workers
4. Agents are risk-averse (Epstein-Zin preferences)

$$U(N_t, C_{t+1}) = -\psi \frac{N_t^{1+\gamma}}{1+\gamma} + (\mathbb{E}C_t^\rho)^{\frac{1}{\rho}}$$
The OLG Model (timeline)

Timeline  In each period $t$, there are four stages:

Stage 1: The old generation of workers become capitalists (entrepreneurs) and a new generation of workers is born. Capitalists and workers are informed of the history of $A^t = \{A_\tau\}_{\tau=0}^t$. Only capitalists receive private signals about $A_{t+1}$ to be realized in the next period.

Stage 2: Capitalists trade capital among themselves in a financial market before production based on their private signals on $A_{t+1}$, the history information $A^t$, the sentiment shock $z_t$, and the capital price $P_t$.

Stage 3: Based on their capital stock, wage $w_t$ and productivity $A_t$, capitalists hire workers and produce. Workers decide their labour supply. Workers obtain information about $A_{t+1}$ through prices $P_t$ and $w_t$.

Stage 4: Capitalists consume and then die. Workers save their labour income for the next period as their capital. The economy repeats stage 1 to 4 in the next period.
The OLG Model (equilibrium)

Proposition 8 There exists a continuum of equilibria indexed by \( 0 \leq \sigma_z^2 \leq \frac{\hat{\theta}^2}{4} \sigma_\alpha^2 \), in which the price \( p_t \) is given by

\[
p_t = \left[ \log \alpha + (1 - \alpha) n^c \right] + \left[ 1 + (1 - \alpha) \varphi \right] a_t + (1 - \alpha) (\pi - 1) k_t + (1 - \alpha) (\phi a_{t+1} + \sigma_z z_t),
\]

and

\[
n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + \sigma_z z_t)
\]

\[
k_{t+1} = \log(1 - \alpha) + a_t + \alpha k_t + (1 - \alpha) n_t
\]

\[
y_t = a_t + \alpha k_t + (1 - \alpha) n_t
\]

where

\[
\pi = \frac{1}{(1 - \alpha) \theta} - \frac{\alpha}{1 - \alpha} - \sqrt{\left[ \frac{1}{(1 - \alpha) \theta} - \frac{\alpha}{1 - \alpha} \right]^2 - 4 \left( \frac{\alpha}{1 - \alpha} \right)^2},
\]

\[
\varphi = \frac{\alpha \theta + (1 - \alpha) \pi \theta}{1 - (1 - \alpha)^2 \pi \theta},
\]

\[
\phi = \frac{\hat{\theta} \pm \sqrt{\hat{\theta}^2 - \frac{4 \sigma_z^2}{\sigma_\alpha^2}}}{2}
\]

A sentiment shock \( z_t \) has a persistent effect on output and employment.
Asset prices over business cycles (1)

Closed-form Risk Premium:
Corollary 2 Suppose sentiment volatility has a seasonal cycle of \((\sigma_{z,t}, \sigma_{z,t+1}, \sigma_{z,t+2}, \sigma_{z,t+3}, \sigma_{z,t+4}, \ldots) = (\sigma_{z(1)}, \sigma_{z(2)}, \sigma_{z(3)}, \sigma_{z(4)}, \sigma_{z(1)}, \ldots)\). The equilibrium of the OLG model exists. The risk-premium has a seasonal cycle, which is \((\Delta r_t, \Delta r_{t+1}, \Delta r_{t+2}, \Delta r_{t+3}, \Delta r_{t+4}, \ldots) = (\Delta r_{(1)}, \Delta r_{(2)}, \Delta r_{(3)}, \Delta r_{(4)}, \Delta r_{(1)}, \ldots)\), where \(\Delta r_{(1)} > \Delta r_{(2)} = \Delta r_{(3)} > \Delta r_{(4)}\).

Time-series implications: calendar effect

![Graph showing time-series implications](image)
Asset prices over business cycles (3)

Markov process of regime change:
\[
\begin{pmatrix}
q_{F,F} & 1 - q_{F,F} \\
q_{N,F} & 1 - q_{N,F}
\end{pmatrix}
\]

**Corollary 3** Suppose the fully-revealing equilibrium and the non-revealing equilibrium switch with a Markov process of (55). The equilibrium of the OLG model exists. The risk premium, $\Delta r_t$, follows the Markov process, in which $\Delta r_t$ is higher in the state of the non-revealing equilibrium and lower in the state of the fully-revealing equilibrium (under a sufficient condition that $|q_{F,N} - q_{N,F}|$ is not too high).

Markov process of risk aversion:
\[
\begin{pmatrix}
\xi_{H,H} & 1 - \xi_{H,H} \\
\xi_{L,H} & 1 - \xi_{L,H}
\end{pmatrix}
\]

**Corollary 4** Suppose the risk aversion $\rho$ follows a Markov process of (56). The equilibrium of the OLG model exists. The risk premium, $\Delta r_t$, follows the Markov process, in which $\Delta r_t$ is higher in the regime of $\rho = \rho_L$ and is lower in the regime of $\rho = \rho_H$ (under a sufficient condition that $|\xi_{H,H} - \xi_{L,H}|$ is not too high). In particular, the variation of the risk premium across the two regimes is increasing in sentiment volatility ($\sigma_z$).