

# Sentiments, Financial Markets, and Macroeconomic Fluctuations

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# Motivations

1. The financial sector plays a central role in a modern economy
  - *evident* by the great recession of 2007-2009
2. How can the financial sector influence the aggregate real economy?
  - two potential channels
    - financing channel (*blood system*)
    - information channel (*nervous system*)
3. An exploding financial accelerator literature so far on *financing channel* (B&G (1989), K&M (1997))
  - both theoretically and empirically relevant
  - some puzzles

*This paper: Information channel (the feedback effect from financial markets to the real economy)*

# Motivations (1)

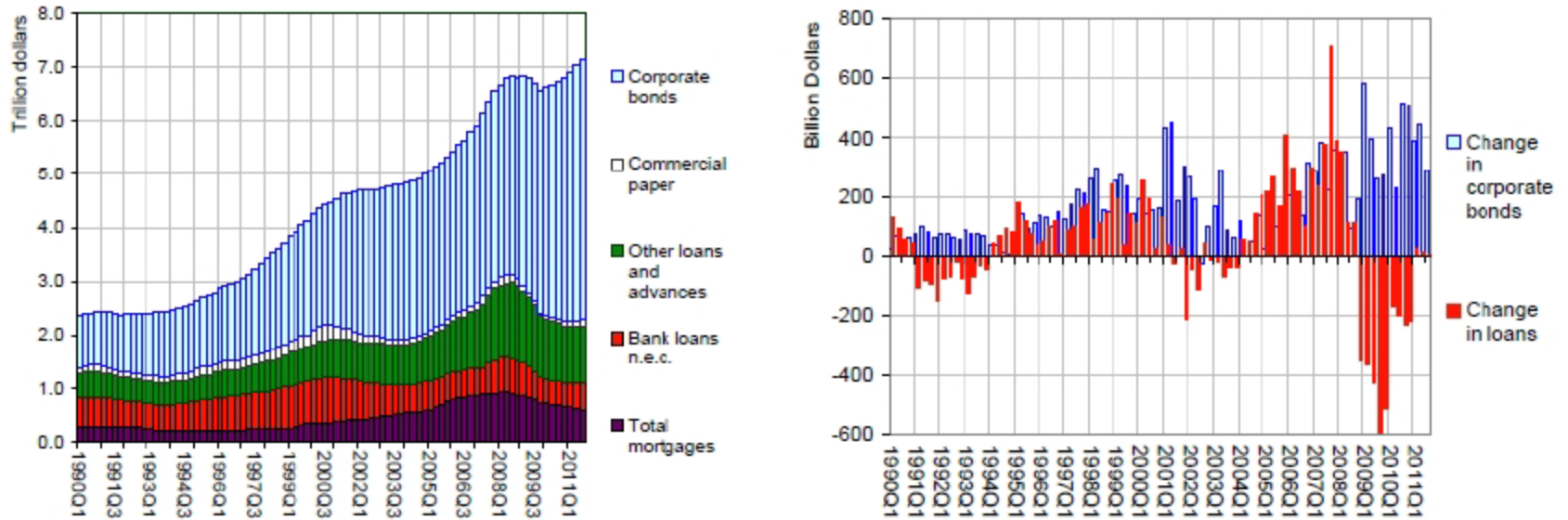


Figure 1: Credit to the US non-financial corporate sector (Source: Adrian, Colla and Shin (2012))

# Further Motivation

## 1. Theoretical

Growing micro literature studies feedback effects from financial markets to the real side of the economy (Bond, Edmans and Goldstein (2012))

- Partial equilibrium
- Typically, a firm manager learns information about his own firm

## 2. Empirical

Investor sentiment in financial markets can affect asset prices, in particular, aggregate (macro)-level asset prices, in turn impacting corporate financing and investment (Baker and Wurgler (2007), Lamont and Stein (2006))

- Behavioral approach

***This paper:*** In a ***general equilibrium, rational-expectations*** framework, the sentiment-driven asset prices can influence ***real activities*** and shape ***macroeconomic fluctuations***.

# Key insights

1. The informational role of financial markets in allocating resources can be impaired by investors' sentiments or sunspots

- **Unlike the conventional view** that prices can efficiently allocate economic resources in a free market by signaling relevant information to economic actors (Hayek (1945), Grossman and Stiglitz (1980))

2. The sentiment-driven asset prices in turn may influence *real activities* and shape *macroeconomic fluctuations*.

# Main results

## 1. One-country economy

Financial information frictions generate sentiment-driven fluctuations in asset prices and self-fulfilling business cycles.

- formalize the linkage of the Keynesian notion of "***beauty contests***" and "***animal spirits***"
- Implications for asymmetric and non-linear asset prices

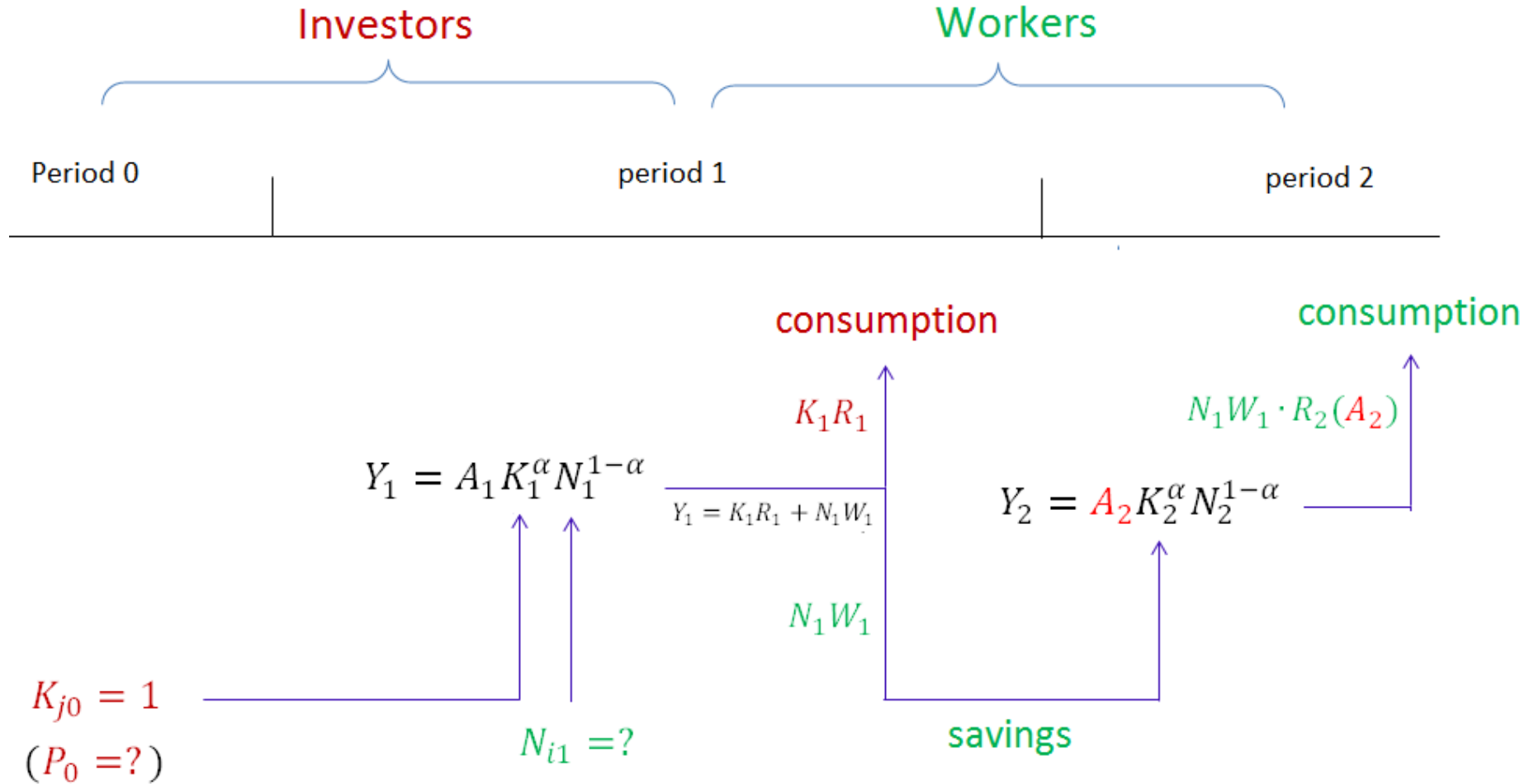
## 2. Two-country economy

Explain global recessions and the cross-country ***comovement*** puzzles

## 3. Dynamic OLG economy

- Explain ***persistent*** business cycle fluctuations
- Implications for asset prices over business cycles

# The model setup (1)



**Logic:**  $P_0 \leftarrow R_1 \leftarrow N_1 \leftarrow R_2 \leftarrow A_2$

# The model setup (2)

## Firm and technology

1. Production in periods 1 and 2, with production function  $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$
2. Assume  $A_1=1$  and  $\log A_2 = a_2 \sim N(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$ .  $a_2$  is realized in period 2

## Financial market investors:

1. live in periods 0 and 1, but consume in period 1
2. each investor is endowed with  $K_0=1$  unit of capital in period 0
3. Investors trade capital in the financial market in period 0 with price  $P_0$
4. Investor  $j$  receives a private signal  $s_{j0}=a_2 + \varepsilon_j$  in period 0

## Workers:

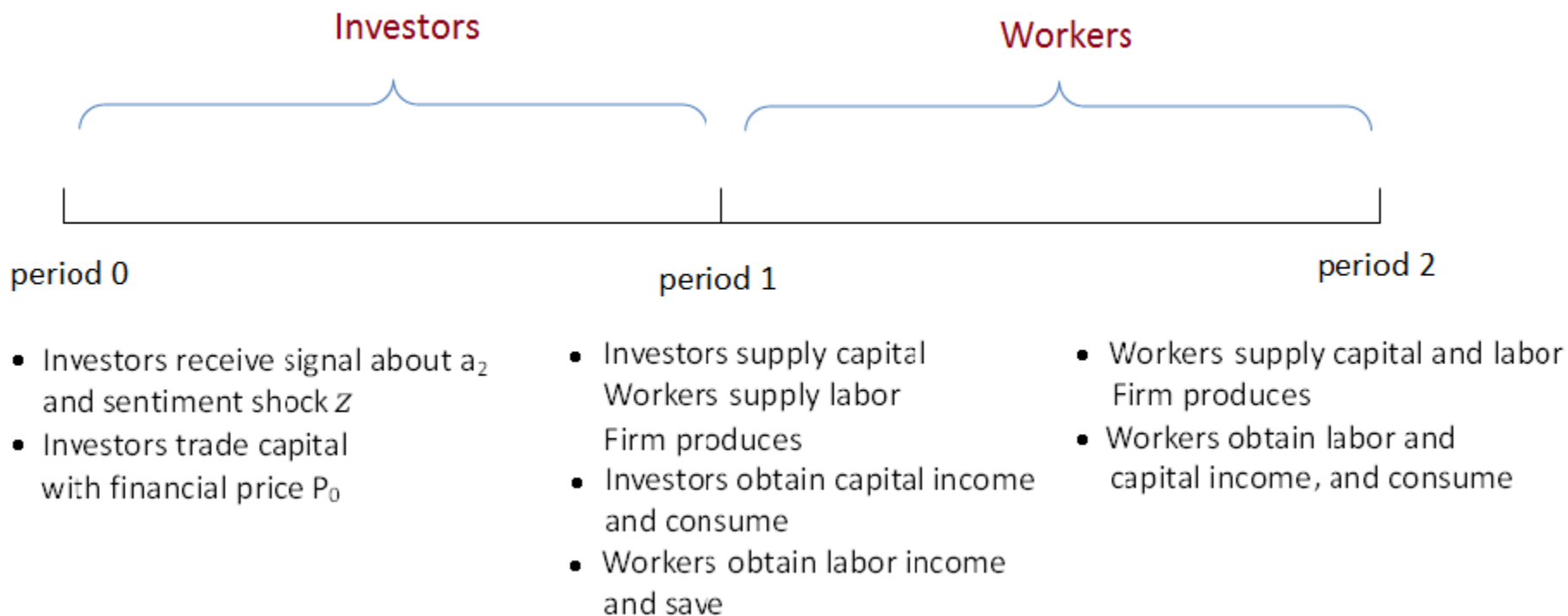
1. live and supply labor in periods 1 and 2, but consume in period 2
2. Utility function:  $U_i = C_{i2} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma}$

## Information structure:

$$\Omega_{j0} = \{P_0, a_2 + \varepsilon_j, z + \delta_j\} \quad \Omega_{i1} = \{R_1, W_1, P_0\} = \Omega_1$$

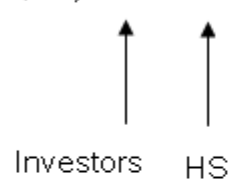


# The model setup (3): timeline



$$K_0 = 1$$

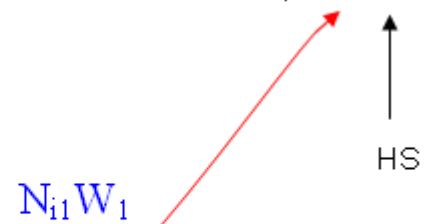
$$Y_1 = f(A_1, K_1, N_1)$$



$$A_1 \equiv 1$$

$$K_1 = 1$$

$$Y_2 = f(A_2, K_2, N_2)$$



$$N_2 \equiv 1$$

# Equilibrium (1): steps

## Investors' problem

- Investors are risk neutral. They trade capital in period 0 to maximize
  - $a_2 = \log(A_2)$ : fundamental shock to productivity in period 2
  - $\varepsilon_j$ : noise in idiosyncratic signal
  - $z$ : sentiment shocks, drawn from standard normal distribution
  - Risk neutral implies  $P_0 = E[R_1 | a_2 + \varepsilon_j, P_0, z]$
  - $R_1$ : The return of capital in period 1.
  - The investors need to forecast the labor supply of workers in period 1 when trading capital in period 0.

# Equilibrium (2): steps

## Workers' problem

- Households consume in period 2, the last period. They solve
  - $R_2$  : The interest rate from period 1 to period 2:
  - $W_1$ : Wage in period 1
  - The workers try to learn the fundamental shock  $A_2$  from the price of capital in period 0 based on their beliefs that how price incorporates sentiment shocks and fundamental shocks
  - **the “forecasting the forecasts of others” problem.**

# First Order Conditions

- Assuming interiority,

The first-order condition of the investors' problem in (2) is

$$0 = \mathbb{E}[R_1 - P_0 | \Omega_0] \quad (6)$$

and the first-order condition of the workers' problem in (5) is

$$\psi N_{i1}^\gamma = W_1 \mathbb{E}[R_2 | \Omega_1]. \quad (7)$$

# Equilibrium (3): steps

## Solving key endogenous variable $N_1$

1. Given  $K_1 = 1$  and  $N_1$ , we have

$$R_1 = \alpha K_1^{\alpha-1} N_1^{1-\alpha} = \alpha N_1^{1-\alpha},$$

$$W_1 = (1 - \alpha) K_1^\alpha N_1^{-\alpha} = (1 - \alpha) N_1^{-\alpha}$$

2. The capital in period 2 is given by the labor income in period 1. Hence we have

$$K_2 = W_1 N_1 = (1 - \alpha) N_1^{1-\alpha}$$

3. We then express  $R_2$  in term of  $N_1$ :

$$R_2 = \alpha A_2 K_2^{\alpha-1} = A_2 \alpha [(1 - \alpha) N_1^{1-\alpha}]^{\alpha-1}$$

4. In a symmetric equilibrium where  $N_{i1} = N_1$ , equation (7) becomes

$$\psi N_1^\gamma = (1 - \alpha) N_1^{-\alpha} \mathbb{E}[A_2 \alpha [(1 - \alpha) N_1^{1-\alpha}]^{\alpha-1} | \Omega_{i1}]$$

or

$$N_1^{\gamma+1-(1-\alpha)\alpha} = \psi^{-1} \alpha (1 - \alpha)^\alpha \mathbb{E}[A_2 | \Omega_1]$$

5. We normalize  $\psi^{-1} \alpha (1 - \alpha)^\alpha = 1$  and denote  $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$  and thus obtain

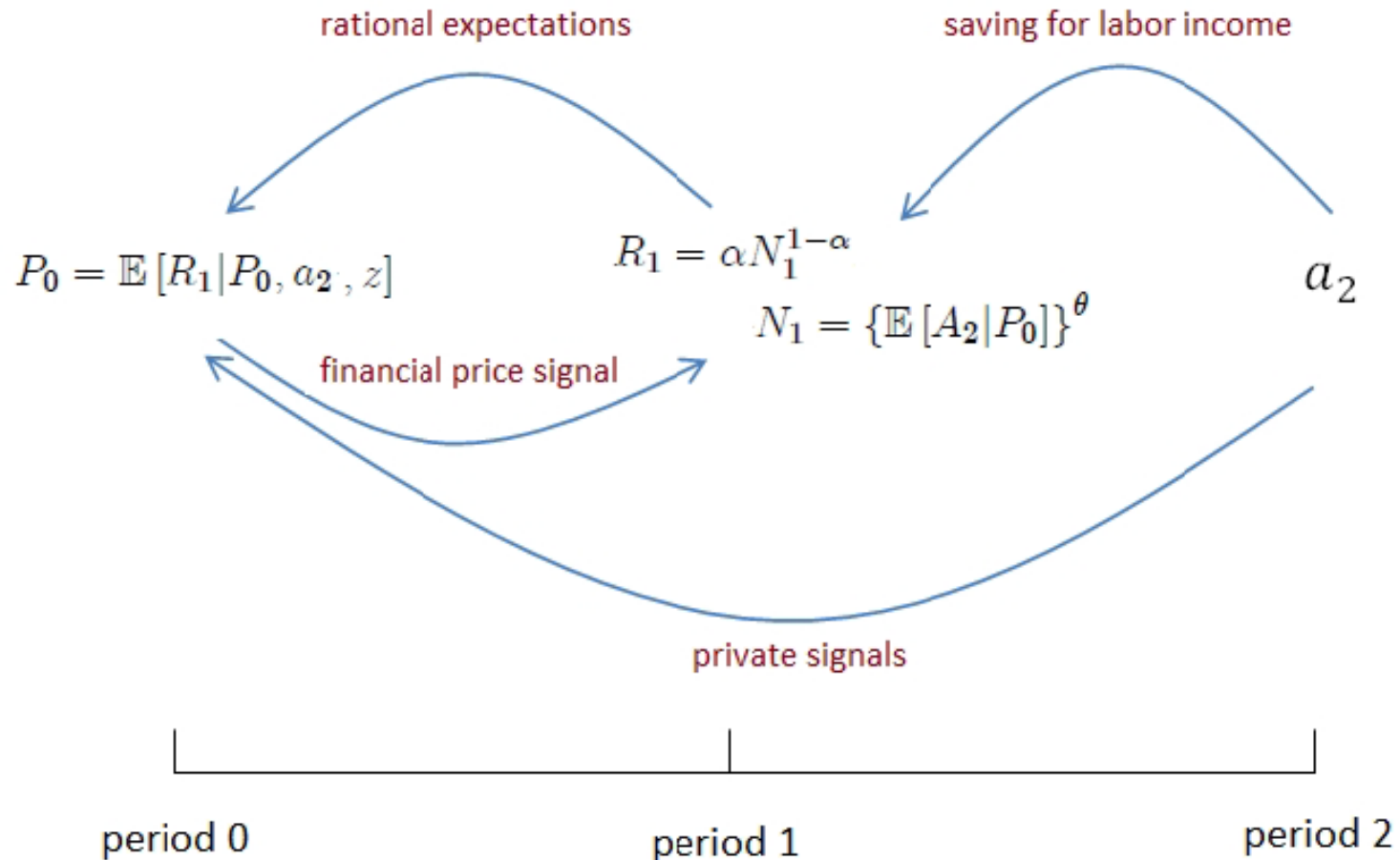
$$N_1 = \{\mathbb{E}[A_2 | \Omega_1]\}^\theta = \{\mathbb{E}[A_2 | P_0]\}^\theta$$

6. Finally, the price  $P_0$  should be consistent with the investors' rational expectations, namely

$$P_0 = \alpha \mathbb{E}[N_1^{1-\alpha} | \Omega_{j0}] = \alpha \mathbb{E}[N_1^{1-\alpha} | a_2 + \varepsilon_j, P_0]$$

# Equilibrium (4): intuition

The two-way feedback between financial market and real economy



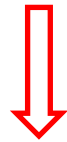
*How sentiment shock  $z$  affects the equilibrium outcome?*

# Equilibrium : core

**Two feedback equations:**

Endogenous dividend:  $N_1 = \{\mathbb{E}[A_2|P_0]\}^\theta$

REE price:  $P_0 = \alpha \mathbb{E}[N_1^{1-\alpha}|a_2, z, P_0]$



$$p(a_2, z) = (1 - \alpha)\theta \log \mathbb{E}[\exp(a_2)|p(a_2, z)]$$

(with  $\log P_0 = \log \alpha + p(a_2, z)$  )

# Equilibrium : outcome

Three types of *rational expectations* equilibria

1) **Fully-revealing Equilibrium**

financial price  $P_0$  perfectly incorporates information about  $a_2$

2) **Non-revealing Equilibrium**

financial price  $P_0$  does not contain information about  $a_2$  at all

3) **Sentiment-Driven Equilibrium**

financial price  $P_0$  contains both information about  $a_2$  and sentiment shock



## Equilibrium (4): fully-revealing equilibrium

**Proposition 1** *There exists a fully revealing equilibrium in which*

$$\log P_0 = \log \alpha + (1 - \alpha)\theta a_2,$$

*and*

$$\log N_1 = \theta a_2.$$

## Equilibrium (5): non-revealing equilibrium

**Proposition 2** *There exists a non-revealing equilibrium in which*

$$\log P_0 = \log \alpha$$

*and*

$$\log N_1 = 0.$$

# Equilibrium (6): sentiment-driven equilibrium

**Proposition 3** *There exists a continuum of equilibria indexed by  $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_\alpha^2$ , in which the price  $P_0$  is given by*

$$\log P_0 = \bar{p} + \log \alpha + (1 - \alpha) (\phi a_2 + z) \quad (17)$$

and

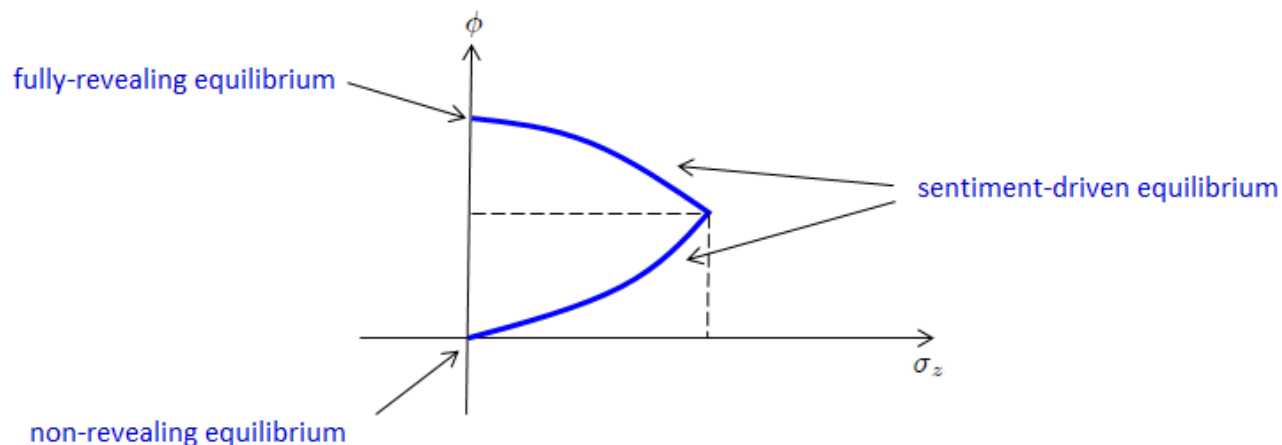
$$\log N_1 = \bar{n} + \phi a_2 + z, \quad (18)$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2 \sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha} \quad (19)$$

and  $\bar{p} = \bar{n} = 0$ .

*Relationship and intuition:*



# One-country model : summary

1. Modelling the relationship between sentiments, financial markets, macroeconomic activity
2. Formalizing the linkage of the Keynesian notion of "***beauty contests***" and "***animal spirits***"
3. Clarifying the difference between financial market sentiments and noisy trading

# One-country model : more implications

## Non-linear asymmetric equilibrium

1. Price is more informative when fundamentals are strong
2. A small decline in fundamentals can trigger asset price collapses

$$\log P_0 = \begin{cases} \log \alpha + (1 - \alpha)\theta a_2 & \text{if } a_2 \geq 0 \\ \log \alpha + p(a_2, z) & \text{if } a_2 < 0 \end{cases}$$

$$p(a_2, z) = (1 - \alpha)\theta \log \mathbb{E}[\exp(a_2) | p(a_2, z)] < 0$$



$$p(a_2, z) = \theta(1 - \alpha) \log \left[ \frac{\exp(a_2) + \exp(z)}{2} \right]$$

# A Two-Country Model

## Motivation facts

1. Cross-country comovement puzzles (Backus et al. (1995) and Baxter (1995))
2. Global recessions and synchronized contractions in output and asset prices (Perri and Quadrini (2013))

## Model setup

1. two symmetric countries  $\ell=h$  and  $f$ , linked by international trade
2. the final goods in home country is produced with *home* and *foreign* intermediate (material) goods

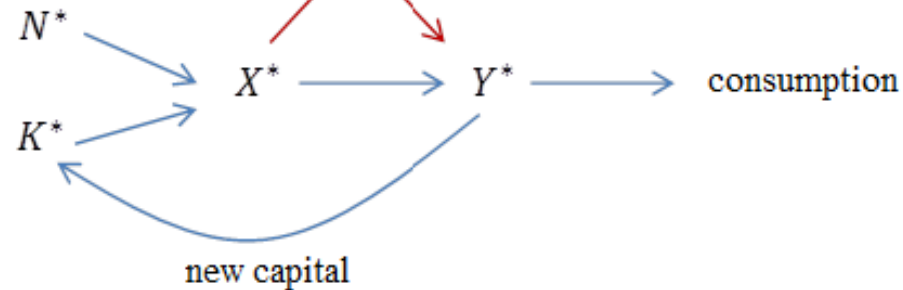
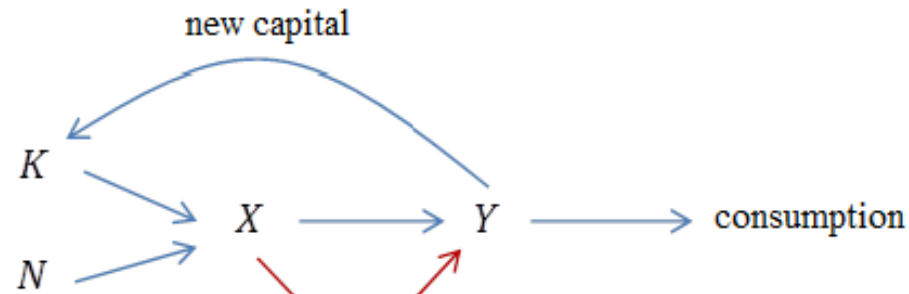
$$Y_t = \left( \frac{X_{ht}}{\eta} \right)^\eta \left( \frac{X_{ht}^*}{1-\eta} \right)^{1-\eta}$$

3. The production of the intermediate goods in home country is

$$X_t = A_t K_t^\alpha N_t^{1-\alpha}$$

# A Two-Country Model

Home country



Foreign country

Figure 4: Productions in the home country and the foreign country

# A Two-Country Model: Equilibrium

## Intuition

The financial prices in either country is observed by the other country. Both prices are subject to sentiment shocks, which create *international contagion*.

## Equilibrium

**Proposition 5** *Equilibrium  $N_1$  and  $N_1^*$  jointly satisfy*

$$N_1 = N_1^{*\omega} \left\{ \mathbb{E}[A_2^\eta A_2^{*1-\eta} | P_0, W_1, R_1, P_0^*] \right\}^\lambda, \quad (40)$$

$$N_1^* = N_1^\omega \left\{ \mathbb{E}[A_2^{*\eta} A_2^{1-\eta} | P_0^*, W_1^*, R_1^*, P_0] \right\}^\lambda, \quad (41)$$

where  $P_0$  and  $P_0^*$  are given by

$$P_0 = \alpha \mathbb{E} \left[ (N_1^{1-\alpha})^\eta (N_1^{*1-\alpha})^{1-\eta} | P_0, a_2 + \varepsilon_j, P_0^*, z \right], \quad (42)$$

$$P_0^* = \alpha \mathbb{E} \left[ (N_1^{*1-\alpha})^\eta (N_1^{1-\alpha})^{1-\eta} | P_0^*, a_2^* + \varepsilon_j^*, P_0, z^* \right], \quad (43)$$

respectively.



# A Two-Country Model: Fully-revealing Equilibrium

**Proposition 6** *There exists a fully-revealing equilibrium in which the capital prices are given by*

$$p_0 = \log P_0 = \log \alpha + \pi_h a_2 + \pi_f a_2^*,$$

$$p_0^* = \log P_0^* = \log \alpha + \pi_h a_2^* + \pi_f a_2$$

*and the labor supplies are*

$$n_1 = \log N_1 = \theta_h a_2 + \theta_f a_2^*,$$

$$n_1^* = \log N_1^* = \theta_h a_2^* + \theta_f a_2.$$

where  $\pi_h = \frac{\lambda(1-\alpha)}{1-\omega^2} [2(1-\eta)\eta\omega + \eta^2 + (1-\eta)^2]$ ,  $\pi_f = \frac{\lambda(1-\alpha)}{1-\omega^2} [(\eta^2 + (1-\eta)^2)\omega + 2(1-\eta)\eta]$ ,  
 $\theta_h = \frac{\lambda}{1-\omega^2} [(1-\eta)\omega + \eta]$  and  $\theta_f = \frac{\lambda}{1-\omega^2} [\eta\omega + (1-\eta)]$ .

***The comovement is weak when  $\eta$  is close to 1***

$$y_1 = \log Y_1 = (1-\alpha) [\eta n_1 + (1-\eta) n_1^*]$$

$$y_1^* = \log Y_1^* = (1-\alpha) [\eta n_1^* + (1-\eta) n_1]$$

# A Two-Country Model: Sentiment-Driven Comovement

**Proposition 7** *There exists a continuum of equilibria indexed by  $0 \leq \sigma_z^2 \leq \frac{\lambda^2 \sigma_\alpha^2}{16(1-\omega)^2}$ , in which*

$$\log P_0 = \log P_0^* = \bar{p} + (1 - \alpha) [\phi(a_2 + a_2^*) + \sigma_z(z + z^*)] \quad (44)$$

$$\log N_1 = \log N_1^* = \bar{n} + \phi(a_2 + a_2^*) + \sigma_z(z + z^*) \quad (45)$$

$$\log Y_1 = \log Y_1^* = (1 - \alpha) [\phi(a_2 + a_2^*) + \sigma_z(z + z^*)]$$

where

$$\phi = \frac{\lambda \sigma_\alpha^2 \pm \sqrt{(\lambda \sigma_\alpha^2)^2 - 16(1 - \omega)^2 \sigma_\alpha^2 \sigma_z^2}}{4(1 - \omega) \sigma_\alpha^2} \quad (46)$$

and

$$\begin{aligned} \bar{n} &= \frac{\lambda}{2(1 - \omega)} \left\{ [\eta^2 + (1 - \eta)^2 - 1] + \frac{\phi^2 \sigma_\alpha^2}{2\phi^2 \sigma_\alpha^2 + 2\sigma_z^2} \right\} \sigma_\alpha^2 \\ \bar{p} &= \log \alpha + (1 - \alpha) \bar{n}. \end{aligned}$$

***Output as well as employment in period 1 in the two countries becomes fully synchronized.***

***Intuition:*** investors' perceived synchronization across economies can lead to the actual synchronization

# The OLG Model

## Two additional insights:

1. Persistence in business cycles
2. Asset prices over business cycles

## Model setup (static ==> dynamic)

1. Investors: capitalists (entrepreneurs) - the old generation of workers
2. Workers
3. A bond market is introduced: bonds are traded between workers
4. Agents are risk-averse (Epstein-Zin preferences)

$$U(N_t, C_{t+1}) = -\psi \frac{N_t^{1+\gamma}}{1+\gamma} + (\mathbb{E} C_{t+1}^\rho)^{\frac{1}{\rho}}$$

# The OLG Model (timeline)

**Timeline** In each period  $t$ , there are four stages:

- Stage 1:** The old generation of workers become capitalists (entrepreneurs) and a new generation of workers is born. Capitalists and workers are informed of the history of  $A^t = \{A_\tau\}_{\tau=0}^t$ . Only capitalists receive private signals about  $A_{t+1}$  to be realized in the next period.
- Stage 2:** Capitalists trade capital among themselves in a financial market before production based on their private signals on  $A_{t+1}$ , the history information  $A^t$ , the sentiment shock  $z_t$ , and the capital price  $P_t$ .
- Stage 3:** Based on their capital stock, wage  $w_t$  and productivity  $A_t$ , capitalists hire workers and produce. Workers decide their labour supply. Workers obtain information about  $A_{t+1}$  through prices  $P_t$  and  $w_t$ .
- Stage 4:** Capitalists consume and then die. Workers save their labour income for the next period as their capital. The economy repeats stage 1 to 4 in the next period.

# The OLG Model (equilibrium)

**Proposition 8** *There exists a continuum of equilibria indexed by  $0 \leq \sigma_z^2 \leq \frac{\hat{\theta}^2}{4}\sigma_\alpha^2$ , in which the price  $P_t$  is given by*

$$p_t = [\log \alpha + (1 - \alpha) n^c] + [1 + (1 - \alpha) \varphi] a_t + (1 - \alpha) (\pi - 1) k_t + (1 - \alpha) (\phi a_{t+1} + \sigma_z z_t),$$

and

$$n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + \sigma_z z_t)$$

$$k_{t+1} = \log(1 - \alpha) + a_t + \alpha k_t + (1 - \alpha) n_t$$

$$y_t = a_t + \alpha k_t + (1 - \alpha) n_t$$

where

$$\pi = \frac{\left[ \frac{1}{(1-\alpha)^2 \theta} - \frac{\alpha}{1-\alpha} \right] - \sqrt{\left[ \frac{1}{(1-\alpha)^2 \theta} - \frac{\alpha}{1-\alpha} \right]^2 - 4 \left( \frac{\alpha}{1-\alpha} \right)^2}}{2},$$

$$\varphi = \frac{\alpha \theta + (1 - \alpha) \pi \theta}{1 - (1 - \alpha)^2 \pi \theta},$$

$$\phi = \frac{\hat{\theta} \pm \sqrt{\hat{\theta}^2 - \frac{4\sigma_z^2}{\sigma_\alpha^2}}}{2}$$

***A sentiment shock  $z_t$  has a persistent effect on output and employment***

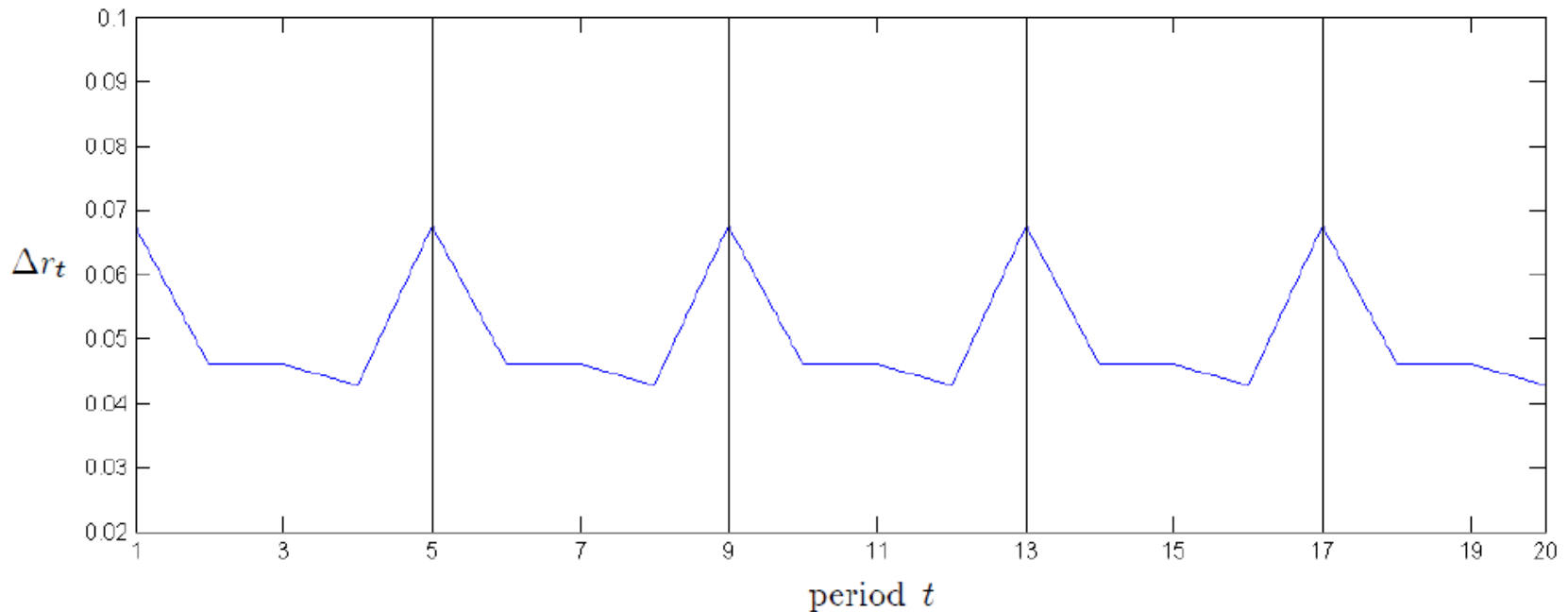
# Asset prices over business cycles (1)

**Closed-form Risk Premium:**

# Asset prices over business cycles (2)

**Corollary 2** *Suppose sentiment volatility has a seasonal cycle of  $(\sigma_{z,t}, \sigma_{z,t+1}, \sigma_{z,t+2}, \sigma_{z,t+3}, \sigma_{z,t+4}, \dots) = (\sigma_{z(1)}, \sigma_{z(2)}, \sigma_{z(3)}, \sigma_{z(4)}, \sigma_{z(1)}, \dots)$ . The equilibrium of the OLG model exists. The risk-premium has a seasonal cycle, which is  $(\Delta r_t, \Delta r_{t+1}, \Delta r_{t+2}, \Delta r_{t+3}, \Delta r_{t+4}, \dots) = (\Delta r_{(1)}, \Delta r_{(2)}, \Delta r_{(3)}, \Delta r_{(4)}, \Delta r_{(1)}, \dots)$ , where  $\Delta r_{(1)} > \Delta r_{(2)} = \Delta r_{(3)} > \Delta r_{(4)}$ .*

## Time-series implications: calendar effect



# Asset prices over business cycles (3)

Markov process of regime change: 
$$\begin{pmatrix} q_{F,F} & 1 - q_{F,F} \\ q_{N,F} & 1 - q_{N,F} \end{pmatrix}$$

**Corollary 3** *Suppose the fully-revealing equilibrium and the non-revealing equilibrium switch with a Markov process of (55). The equilibrium of the OLG model exists. The risk premium,  $\Delta r_t$ , follows the Markov process, in which  $\Delta r_t$  is higher in the state of the non-revealing equilibrium and lower in the state of the fully-revealing equilibrium (under a sufficient condition that  $|q_{F,N} - q_{N,F}|$  is not too high).*

Markov process of risk aversion: 
$$\begin{pmatrix} \xi_{H,H} & 1 - \xi_{H,H} \\ \xi_{L,H} & 1 - \xi_{L,H} \end{pmatrix}$$

**Corollary 4** *Suppose the risk aversion  $\rho$  follows a Markov process of (56). The equilibrium of the OLG model exists. The risk premium,  $\Delta r_t$ , follows the Markov process, in which  $\Delta r_t$  is higher in the regime of  $\rho = \rho_L$  and is lower in the regime of  $\rho = \rho_H$  (under a sufficient condition that  $|\xi_{H,H} - \xi_{L,H}|$  is not too high). In particular, the variation of the risk premium across the two regimes is increasing in sentiment volatility ( $\sigma_z$ ).*