Acemoglu, Aghion, Zilibotti

Production

$$Y_t = \int_0^1 (A_{it})^{1-\alpha}(x_{it})^{\alpha}$$

Profits for monopolist

$$\Pi_{it} = p_{it}x_{it} - x_{it}$$

Usually p_t marginal product,

$$p_{it} = \frac{\partial Y_t}{\partial x_{it}} = \alpha(A_{it})^{1-\alpha}(x_{it})^{\alpha-1}$$

but with limit pricing from competitive fringe, at $\chi > 1$ units of final output instead of 1, monopolists can only charge

$$p_{it} \le \chi$$

Then monopolists employ

$$x_{it} = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_{it}$$

and

$$\Pi_{it} = p_{it}x_{it} - x_{it} = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_{it} \equiv \pi A_{it}$$

Let average productivity be

$$A_t = \int_0^i A_{it} di$$

with $\bar{A}_t$ average world productivity which grows at the rate g: $\bar{A}_t = (1+g)\bar{A}_{t-1}$

Two ways to close productivity gap: Imitation and Innovation.:

$$A_t = \eta \bar{A}_{t-1} + \gamma A_{t-1}$$

Dividing by $\bar{A}_t$:

$$a_t = \frac{1}{1+g}(\eta + \gamma \bar{a}_{t-1})$$

where $a_t = \frac{A_t}{\bar{A}_t}$ ids the distnce to frontier.

There are two types of policies , one enhances imitation, with $\eta \in [\eta_L, \eta_H]$, the other enhances innovation, with $\gamma \in [\gamma_L, \gamma_H]$ Assume frontier growth is governed by

$$1 + g = \eta_L + \gamma_H$$

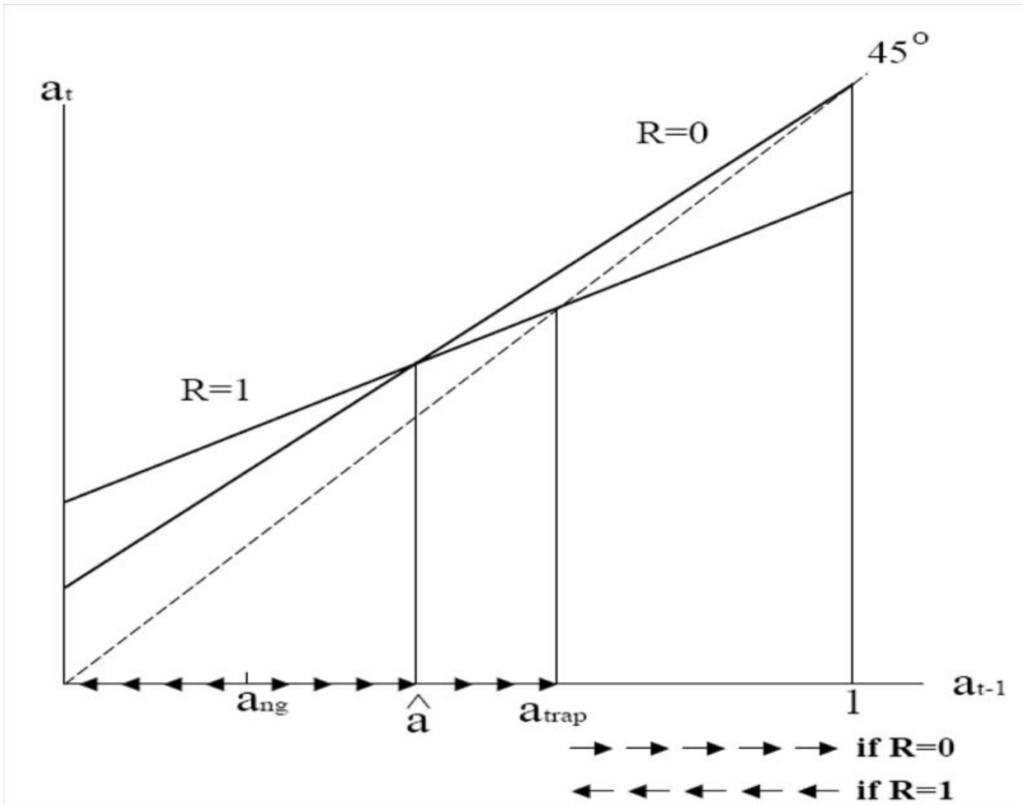
So for innovation based growth becomes:

$$a_t = \frac{1}{1+g}(\eta_L + \gamma_H a_{t-1})$$

which has a stable equilibrium at $a = 1$ for the assumption $1 + g = \eta_L + \gamma_H$. Imitation based growth becomes

$$a_t = \frac{1}{1+g}(\eta_H + \gamma_L a_{t-1})$$

Since $\eta_L > \eta_H$ and $\gamma_L < \gamma_H$ the two curves intersect at $\hat{a} = \frac{\eta_H - \eta_L}{\gamma_H - \gamma_L}$. Will institutions change at $\hat{\alpha}$? Yes under a growth maximizing strategy.



a_t
45°
R=0
R=1
a_{t-1}
a_{ng}
$\hat{a}$
a_{trap}
1
if R=0
if R=1

Managers

They live two periods and are young and old.They manage by

$$A_{it} = \eta(i,t)\bar{A}_{t-1} + \gamma(i,t)A_{t-1}$$

Some managers are talented, others are not.

All young imitate at $\eta(i,t) = \eta,$ and old imitate at $\eta(i,t) = \eta + \varepsilon,$ where ε comes from experience.

However, the talented, a fraction λ of the old, can also innovate at intensity $\gamma(i,t) = \gamma.$ For the untalented $\gamma = 0.$ Talent is revealed in the beginning of second period.

Let managers retain a share μ of profits. Will firm owners keep the untalented managers? Depends: yes if

$$(1-\mu)(\eta+\varepsilon)\pi\bar{A}_{t-1} + \mu\pi\bar{A}_{t-2} > (1-\mu)(\eta+\lambda\gamma a_{t-1})\pi$$
$$- \kappa\bar{A}_{t-1}$$

Note on the left: $\mu\pi\bar{A}_{t-2}$ is the bribe an old manager can give to retain the job. On the right $\lambda\gamma a_{t-1}$ is the productivity adjusted expected innovation gain (where $a_{t-1} = \frac{A_{t-1}}{\bar{A}_{t-1}}$). $\kappa \bar{A}_{t-1}$ is the cost of hiring a new manager.

Dividing by $\bar{A}_{t-1}$

$$(1-\mu)(\eta+\varepsilon)\pi + \frac{\mu\pi}{(1+g)} > (1-\mu)(\eta+\lambda\gamma a_{t-1})\pi - \kappa$$

$$a_{t-1} \leq a_r = \frac{(1-\mu)\varepsilon + \frac{\mu}{1+g}}{(1-\mu)\lambda\gamma}$$

So renewing managers is profitable when a_{t-1} is low so country is far from technology frontier. That is because a_{t-1} multiplies profits that the new manager may obtain: the improvement is low if far below the frontier. Slow catch-up, no leapfrog!

Types of equilibria

1. Note that imitation enhancing policy yields

$$\eta_H = \eta + 0.5\varepsilon, \qquad \gamma_L = 0.5\lambda\gamma$$

This is because only half the firms have old managers and the others are young, and only the young firms (half) hire new managers who with probability λ are talented.

2. Innovation enhancing policy yields

$$\eta_L = \eta + 0.5\lambda\varepsilon,$$

$$\gamma_H = 0.5\lambda\gamma + 0.5(\lambda\gamma + (1 - \lambda)\lambda\gamma)$$

$$\gamma_H = \lambda\gamma(1 + 0.5(1 - \lambda))$$

This is because only half the firms have old talented managers whom they keep with probability λ. In young firms managers are talented with probability λ and are kept on, while a fraction $(1 - \lambda)$ turn out untalented and are replaced with new managers who are talented with probability

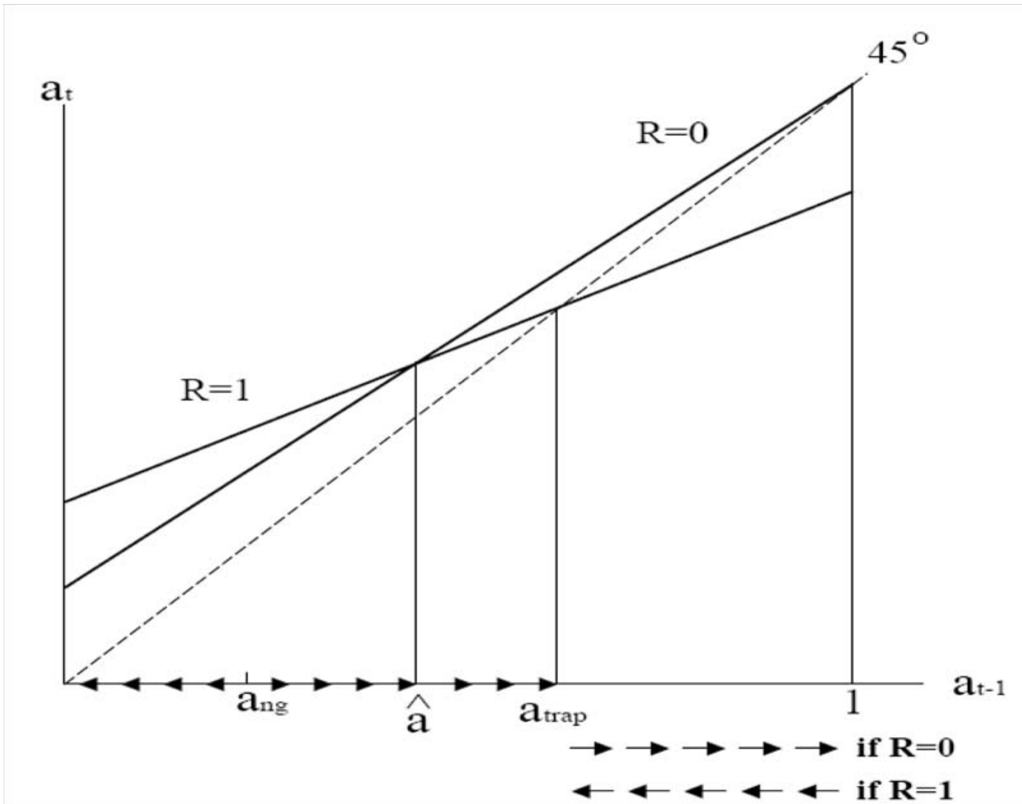
λ.

Now types of equilibria depend on various combinations of the intercept and slopes associated with (η, γ), and determine the threshold $\hat{a}$ at which point strategies switch. Remember the slopes and intercepts of innovation based growth policies are from :

$$a_t = \frac{1}{1+g}(\eta_L + \gamma_H a_{t-1})$$

and imitation based growth policies from

$$a_t = \frac{1}{1+g}(\eta_L + \gamma_H a_{t-1})$$



1. Growth maximizing if $a_r = \hat{a},$ as in the diagram above. But

2.If $a_r < \hat{a},$ switch to innovation enhancing policies too soon because in $(a_r, \hat{a})$ imitation enhancing policies provide faster growth. But eventually you cross $\hat{a}$.

3. If $a_r > \hat{a}$ but the growth rate $\frac{a_t}{a_{t-1}} > 1$ on $(0, a_r)$ then you stay with imitation enhancing policies too long but eventuall you switch to innovation enhancing

policies and converge to frontier.

4. Trap under imitation enhancing policies: never catch up to frontier because $\frac{a_t}{a_{t-1}} < 1$ before a_r, the switching point, and there is an equilibrium before a_r.