Adverse Selection

ECON 101

- The seminal work of Wilson (1980) shows that in a static model, adverse selection can generate multiple equilibria because of asymmetric information about product quality. The aim of this paper is to analyze how adverse selection in the credit market can give rise to lending externalities to generate multiple equilibria and indeterminacies in an otherwise standard dynamic general equilibrium model of business cycles.

- Time is continuous and proceeds from zero to infinity. There is an infinitely-lived representative household and a continuum of final good producers.

The final goods producers purchase the intermediate goods as input to produce final good, which is then sold to household for consumption and investment.

- The intermediate goods is produced by capital and labor in a competitive market. We assume no distortion in the production of intermediate goods.

- Final goods firms do not have resources to make up-front payments for intermediate good input until production takes place and revenues from sales are realized. They therefore must borrow from the competitive financial intermediates (the lenders) to finance their working capital.
- The loan is risky as the final goods producers may default. We assume that there are two types of producers (borrowers): the honest borrowers have the ability to produce and always pay back the loan after the production, while the dishonest borrowers simply run away with the loan.
- The lenders do not have information about lenders' types. They makes loan to firms by taking the adverse selection problem into consideration.
- We begin by assuming that all trade is anonymous so we exclude the possibility of reputation effects. We later relax this strong assumptions and introduce reutation effects.

Households

- The representative household has a lifetime utility function

$$\int_0^\infty e^{-\rho t} \left[\log\left(C_t\right) - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right] dt \tag{1}$$

  where $\rho > 0$ is the subjective discount factor, C_t is the consumption, N_t is the hours worked, $\psi > 0$ is the utility weight for labor, and $\gamma \geq 0$ is the inverse Frisch elasticity of labor supply.
- The household faces the following budget constraint

$$C_t + I_t \leq R_t u_t K_t + W_t N_t + \Pi_t, \tag{2}$$

  where R_t, W_t and Π_t denote respectively the rental price, wage and total profits from all the firms and financial intermediates.
- u is capacity utilization

- Endogenous capacity utilization rate u_t makes indeterminacy empirically more plausible in models with production externalities. We will show that capacity utilization serves a similar role in our model.
- As is standard in the literature, the depreciation rate of capital increases with the capacity utilization rate according to

$$\delta(u_t) = \delta^0 \frac{u_t^{1+\theta}}{1+\theta}, \tag{3}$$

where $\delta^0 > 0$ is a constant and $\theta > 0$.
- Finally, the law of motion for capital is governed by

$$\dot{K}_t = -\delta(u_t)K_t + I_t. \tag{4}$$

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The households choose a path of consumption X_t, C_t, N_t, u_t, and K_t to maximize the utility function (1), taking R_t, W_t and Π_t as given. The first-order conditions are

$$\frac{1}{C_t} W_t = \psi N_t^{\gamma}, \tag{5}$$

$$\frac{\dot{C}_t}{C_t} = u_t R_t - \delta \left(u_t \right) - \rho, \tag{6}$$

and

$$R_t = \delta^0 u_t^{\theta}. \tag{7}$$

Final goods producers

- There is unit measure of final good producers indexed by i.
- A fraction π of them are dishonest and a fraction $1 - \pi$ are honest.
- Each one of the honest producers is endowed with an indivisible project as in Stiglitz and Weiss(1981), which tranform Φ units of intermediate goods to Φ units of final goods.
- Let P_t be price of the intermediate goods input. Each project then requires ΦP_t of working capital.
- The dishonest producers, however, can claim to be honest and borrow $P_t \Phi$ and then run away with the borrowed funds. They enjoy $P_t \Phi$ profits by doing so.
- Anticipating this adverse selection problem, the financial intermediates will charge a gross interest rate $R_{ft} > 1$ to all borrowers. Hence the profit from borrowing and producing for an honest producer is given by:

$$\Pi_t^h = [1 - R_{ft} P_t]\, \Phi$$

Denote s_t measure of honest producers that invest in their projects:

$$s_t = \begin{cases} 1-\pi & \text{if } R_{ft} < \frac{1}{P_t} \\ \in [0, 1-\pi) & \text{if } R_{ft} = \frac{1}{P_t} \\ 0 & \text{if } R_{ft} > \frac{1}{P_t} \end{cases} . \tag{8}$$

The total demand for intermediate goods input is hence given by

$$X_t = s_t \Phi, \tag{9}$$

Since each of them also produce Φ unit of final goods, the total final good is hence

$$Y_t = s_t \Phi = X_t \tag{10}$$

Intermediate goods

- The intermediate goods is produced by capital and labor with the technology
$$X_t = A\tilde{K}_t^{\alpha} N_t^{1-\alpha}, \tag{11}$$
where $\tilde{K}_t = u_t K_t$ is total capital supply from the households.
- In a competitive market: The profit is $\Pi_t^x = P_t A\tilde{K}_t^{\alpha} N_t^{1-\alpha} - W_t N_t - R_t \tilde{K}_t$ The first order conditions are
$$R_t = P_t \alpha \frac{X_t}{\tilde{K}_t} = P_t \alpha \frac{X_t}{u_t K_t}, \tag{12}$$
$$W_t = P_t (1-\alpha) \frac{X_t}{N_t}. \tag{13}$$
- Notice that $\Pi_t^x = 0$, so $W_t N_t + R_t u_t K_t = P_t X_t$.

Financial Intermediates

The financial intermediates behave competitively. Anticipating Θ_t fraction of loan will be paid back, the interest rate is then given by

$$R_{ft} = \frac{1}{\Theta_t} \tag{14}$$

- So the financial intermediates earn zero profit. The honest producers borrow total $X_t P_t$ working capital loans and the dishonest producers borrows total $\pi\Phi P_t$ working capital loans. Since only the honest producer pay back their loan, so the average payback rates is

$$\Theta_t = \frac{X_t P_t}{\pi\Phi P_t + X_t P_t} = \frac{X_t}{\pi\Phi + X_t}. \tag{15}$$

Equilibrium

- We focus on an interior solution so $R_{ft} = \frac{1}{P_t}$. (We can also assume that there are potential infinite measures of honest producers. A free rate entry condition then implies $R_{ft} = \frac{1}{P_t}$, zero profits).

In equilibrium, the total profits are simply $P_t \Phi$. Hence the total budget constraint becomes

$$C_t + I_t = P_t X_t + \pi P_t \Phi. \tag{16}$$

Since $P_t = \frac{1}{R_{ft}} = \Theta_t = \frac{X_t}{\pi\Phi + X_t}$, the above equation can be futher reduced to

$$C_t + I_t = P_t \left(X_t + \pi\Phi \right) = X_t = Y_t. \tag{17}$$

We then have resource constraint

$$C_t + \dot{K}_t = Y_t - \delta(u_t) K_t. \tag{18}$$

Since

$$P_t\left(X_t + \pi\Phi\right) = X_t = Y_t; \qquad P_t = 1 - \frac{\pi\Phi P_t}{Y}$$

The inverse of markup is hence given by:

$$\phi_t \equiv 1 - \frac{\Pi_t}{Y_t} = 1 - \frac{\pi P_t \Phi}{Y} = P_t = \Theta_t.$$

and factor prices in terms of inverse markups are

$$R_t = \phi_t \cdot \left(\frac{\alpha Y_t}{u_t K_t}\right). \tag{19}$$

$$W_t = \phi_t \cdot \frac{\left(1-\alpha\right) Y_t}{N_t}. \tag{20}$$

The labor mkt., capacity utilization and Euler eqs. then become

$$\psi N_t^{\gamma} = \left(\frac{1}{C_t}\right)(1-\alpha)\phi_t \frac{Y_t}{N_t}, \tag{21}$$

$$R_t = \alpha\phi_t \frac{Y_t}{u_t K_t} = \delta^0 u_t^{\theta} = \frac{(1+\theta)\,\delta\,(u_t)}{u_t} \tag{22}$$

$$\delta\,(u_t) = \frac{1}{(1+\theta)} \frac{\alpha\phi_t Y_t}{K_t} \tag{23}$$

$$\frac{\dot{C}_t}{C_t} = \alpha\phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \tag{24}$$

$$\frac{\dot{C}_t}{C_t} = \alpha\phi_t \frac{Y_t}{K_t}\left(1 - \frac{1}{(1+\theta)}\right) - \rho, \quad (\textit{substituting } \delta(u_t)) \tag{25}$$

$$\frac{\dot{C}_t}{C_t} = \alpha\phi_t \frac{Y_t}{K_t}\left(\frac{\theta}{(1+\theta)}\right) - \rho, \tag{26}$$

Equilibrium

In short, the equilibrium can be characterized by six equations that fully determine the dynamics of the six variables C_t, K_t, Y_t, u_t, N_t and ϕ_t.

$$Y_t = A\left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}. \tag{27}$$

$$\psi N_t^{\gamma} = \left(\frac{1}{C_t}\right)(1-\alpha)\phi_t \frac{Y_t}{N_t}, \tag{28}$$

$$\frac{\dot{C}_t}{C_t} = \alpha \phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \tag{29}$$

$$\alpha \phi_t \frac{Y_t}{u_t K_t} = \delta^0 u_t^{\theta} = \frac{(1+\theta)\,\delta\left(u_t\right)}{u_t} \tag{30}$$

$$C_t + \dot{K}_t = Y_t - \delta(u_t) K_t. \tag{31}$$

$$\phi_t = \frac{Y_t}{\pi \Phi + Y_t} \tag{32}$$

- Equation $\phi_t = \frac{Y_t}{\pi\Phi + Y_t}$ implies that ϕ_t increases with aggregate output. Notice that $\frac{1}{\phi_t} = \frac{Y_t}{R_t u_t K_t + W_t N_t}$ is the aggregate markup in our model economy. Therefore the endogenous markup in our model is countercyclical, which is consistent with the empirical regularity well documented in the literature. The credit spread in our model is $R_{ft} - 1 = \pi\Phi / Y_t$, moving in countercyclical fashion as in the data.
- The countercyclical markup has important implications. For example, it can make hours and the real wage move in the same direction. To see this, suppose N_t increases, so output increases. Then according to Equation $\phi_t = \frac{Y_t}{\pi\Phi + Y_t}$, marginal cost ϕ_t increases as well, which in turn raises the real wage $W_t = \phi_t \cdot \frac{(1-\alpha)Y_t}{N_t}$.
- If the markup is a constant, then the real wage would be proportional to the marginal product of labor and would fall when hours increase.
- Note also that when $\pi = 0$, *i.e.*, there is no adverse selection, Equation $\phi_t = \frac{Y_t}{\pi\Phi + Y_t}$ implies that $\phi_t = 1$, and our model simply collapses into a standard real business cycle model.
- The markup is $1/\phi_t > 1$ if and only if lemon producers obtain an information rent from the information asymmetry on product quality.

Steady State

- To solve the steady state, we first express all other variables in terms of ϕ and then we solve ϕ as a fixed point problem.
- Combining

$$\frac{\dot{C}_t}{C_t} = \alpha \phi_t \frac{Y_t}{K_t} - \delta(u_t) - \rho, \tag{33}$$

$$\delta^0 u^{\theta+1} - \frac{\delta^0 u^{\theta+1}}{1+\theta} = \rho,$$

or $u = \left[\frac{1}{\delta^0}\frac{\rho}{\theta}(1+\theta)\right]^{1+\theta}$.

- Notice that u only depends on δ^0, ρ and θ. Therefore, without loss of generality, we can set $\delta^0 = \frac{\rho}{\theta}(1+\theta)$ so that $u = 1$ at the steady state.
- The depreciation rate at steady state is then $\delta(u) = \rho/\theta$.

Given ϕ, and that at the steady state $\delta(u) = \rho/\theta$,we have,

$$\begin{aligned}
k_y &= \frac{K}{Y} = \frac{\alpha\phi}{\rho + \rho/\theta} = \frac{\alpha\phi\theta}{\rho(1+\theta)}, \\
c_y &= 1 - \delta k_y = 1 - \frac{\alpha\phi}{1+\theta}, \\
N &= \left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}}, \\
Y &= A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\phi\theta}{\rho(1+\theta)} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1-\alpha)\phi}{1 - \frac{\alpha\phi}{1+\theta}} \frac{1}{\psi} \right)^{\frac{1}{1+\gamma}} \equiv Y(\phi).
\end{aligned}$$

So we have $Y = Y(\phi)$.

- Then we can use Equation $\phi_t = \frac{Y_t}{\pi\Phi + Y_t}$ to pin down ϕ from

$$\bar{\Phi} \equiv \pi\Phi = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi) \equiv \Psi(\phi)$$

$$\bar{\Phi} \equiv \pi\Phi = (1-\phi)\, A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\theta}{\rho(1+\theta)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1-\alpha)}{1-\frac{\alpha\phi}{1+\theta}}\frac{1}{\psi}\right)^{\frac{1}{1+\gamma}} \phi^{\frac{\alpha}{1-\alpha}+\frac{1}{1+\gamma}}$$

  where the left-hand side is the total supply of lemon products and the right hand-side is he maximum amount of lemon tcompatible with market equilibrium, given that the average quality is $q = \phi$.
- When $\alpha/(1-\alpha) + \frac{1}{1+\gamma} > 1$, $\Psi(\phi)$ is a non-monotonic function of ϕ since $\Psi(0) = 0$ and $\Psi(1) = 0$. On the one hand, if the average quality is 0, the household demand would be zero, and hence no lemon will be needed. On the other hand, if the average quality is one, *i.e.*, $\phi = q = 1$, then by definition no lemon will be sold.
- So given $\bar{\Phi}$, there may exist two steady state values of ϕ.

- Denote $\Psi^* \equiv \max_{0 \leq \phi \leq 1} \Psi(\phi)$, and $\phi^* \equiv \arg\max_{0 \leq \phi \leq 1} \Psi(\phi)$. Then we have the following lemma regarding the possibility of multiple steady state equilibria.

Lemma

When $0 < \bar{\Phi} < \Psi^*$, there exists two steady states ϕ that solve $\bar{\Phi} = \Psi(\phi)$.

- We also have, from linearizing eq. 32, that is $\phi_t = \frac{Y_t}{\pi\Phi + Y_t}$,

$$\hat{\phi}_t = (1 - \phi)\hat{y}_t \equiv \tau \hat{y}_t, \tag{36}$$

  which states that the percent deviation of the marginal cost is proportional to output.
- We see that one-percent increase in capital directly increases output and the marginal product of labor by a percent and, from Equation (36), reduces the markup by $a\tau$ percent.
- Thanks to its higher marginal productivity, the labor supply also increases. A one-percent increase in labor supply then increases output by b percent. The precise increase in labor supply depends on the Frisch elasticity γ.

This explains why the equilibrium output elasticity with respect to capital, λ_1, depends on parameters a, b and through them on γ and τ.

Local Dynamics

- We denote $\hat{x}_t = \log X_t - \log X$ as the percent deviation from its steady state. We log-linearize the dynmic system: in particular for aggregate output we get

$$\hat{y}_t = \frac{\alpha\theta\hat{k}_t + (1+\theta)(1-\alpha)\hat{n}_t}{1+\theta-(1+\tau)\alpha} \equiv a\hat{k}_t + b\hat{n}_t, \tag{37}$$

  where $a \equiv \frac{\alpha\theta}{1+\theta-(1+\tau)\alpha}$ and $b \equiv \frac{(1+\theta)(1-\alpha)}{1+\theta-(1+\tau)\alpha}$. We assume that $1+\theta-(1+\tau)\alpha > 0$, or equivalently $\tau < \frac{1+\theta}{\alpha} - 1$, to make $a > 0$ and $b > 0$. In general these restrictions are easily satisfied.
- It is worth mentioning that $a + b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} = 1$ if $\tau = 0$. Recall that $\tau = 0$ corresponds to the case without adverse selection. Thus endogenous capacity utilization alone does not generate increasing returns to scale at the aggregate level.
- However, $a + b = \frac{1+\theta-\alpha}{1+\theta-(1+\tau)\alpha} > 1$ if $\tau > 0$; that is, adverse selection combined with endogenous capacity utilization mimics increasing returns to scale.

- On the household side, since both leisure and consumption are normal goods, an increase in consumption has a wealth effect on labor supply. The effect of a change in labor supply on output induced by a change in consumption works through the marginal cost channel, and also depends on τ.
- Again since both a and b increase with τ, output elasticities with respect to capital and consumption are increasing functions of τ.
- We can substitute out $\hat{n}_t$ after lineaizing the labor market equilibrium equation, to express $\hat{y}_t$ as

$$\hat{y}_t = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}\hat{k}_t - \frac{b}{1+\gamma-b(1+\tau)}\hat{c}_t \equiv \lambda_1\hat{k}_t + \lambda_2\hat{c}_t. \quad (38)$$

- In other words, the presence of adverse selection, $\tau > 0$, makes equilibrium output more sensitive to changes in capital and to changes in autonomous consumption, and creates an amplification mechanism for business fluctuations.

We can then characterize the local dynamics as follows:

$$\begin{aligned}
\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} & \\
= \ & \delta \begin{bmatrix} \frac{1+\theta}{\alpha\phi}\lambda_1 - (1+\tau)\,\lambda_1 & \frac{1+\theta}{\alpha\phi}(\lambda_2 - 1) + 1 - (1+\tau)\lambda_2 \\ \theta\left[(1+\tau)\lambda_1 - 1\right] & \theta(1+\tau)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \\
= \ & J \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix},
\end{aligned} \tag{39}$$

where $\lambda_1 = \frac{a(1+\gamma)}{1+\gamma-b(1+\tau)}$, $\lambda_2 = -\frac{b}{1+\gamma-b(1+\tau)}$, and $\delta = \rho/\theta$ is the steady state depreciation rate.

- The local dynamics around the steady state is determined by the roots of J.
- The model exhibits local indeterminacy if both roots of J are negative.

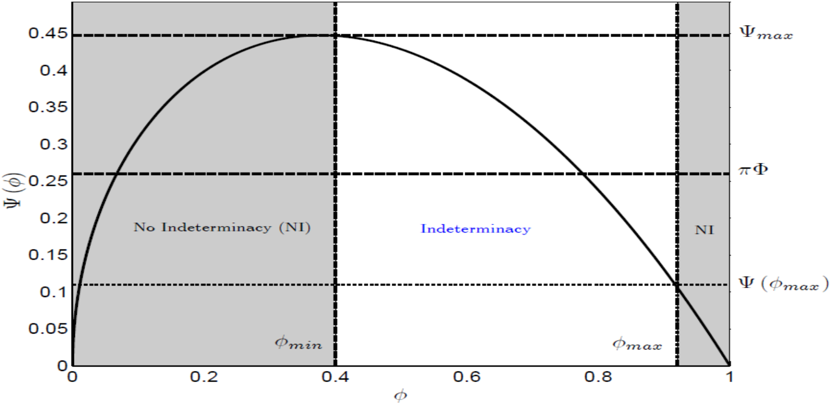
Local Indeterminacy

- Note that the sum of the roots equals the trace of J, and the product of the roots equals the determinant of J. The following lemma specifies the sign for the trace and determinant condition for local indeterminacy.
- Denote $\Psi^* \equiv \max_{0 \leq \phi \leq 1} \Psi(\phi)$, and $\phi^* \equiv \arg\max_{0 \leq \phi \leq 1} \Psi(\phi)$.

Lemma Denote $\tau_{\text{min}} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$ and $\tau_{\text{max}} \equiv 1 - \phi^*$, then Trace$(J) < 0$ if and only if $\tau > \tau_{\text{min}}$, and Det$(J) > 0$ if and only if $\tau_{\text{min}} < \tau < \tau_{\text{max}}$.

Proposition: The model exhibits local indeterminacy around a particular steady state if and only if $\tau_{\text{min}} < \tau < \tau_{\text{max}}$. Equivalently, indeterminacy emerges if and only if $\phi \in (\phi_{\text{min}}, \phi_{\text{max}})$, where $\phi_{\text{min}} \equiv 1 - \tau_{\text{max}} = \phi^*$, and $\phi_{\text{max}} \equiv 1 - \tau_{\text{min}}$.

- Why?



Some Intuition

- To understand this, notice that if $\tau > \tau_{\min}$, we have

$$1 + \gamma - b(1 + \tau) < 1 + \gamma - \frac{(1 + \theta)(1 - \alpha)}{1 + \theta - (1 + \tau_{\min})\alpha}(1 + \tau_{\min}) = 0.$$

- Then the equilibrium elasticity of output with respect to consumption λ_2 becomes positive, namely, an autonomous change in consumption will lead to an increase in output. Since capital is predetermined, labor must increase.
- To induce an increase in labor, the real wage must increase enough to overcome the income effect, which is only possible if the increase in markup is large enough to allow wages to rise enough, while keeping profits non-negative.

Calibration

We now examine the empirical plausibility of self-fulfilling equilibria under calibrated parameter values.
The frequency is a quarter. . We set $\psi = 1.75$ so that $N = \frac{1}{3}$ in the "good" steady state. We set $\overline{\Phi} = \pi\Phi = 0.13$ so that $\underline{\phi} = \bar{\phi}_H = 0.9$, which is consistent with average profit rate in the data.ated $\bar{\phi}_L = 0.011$. If we further set $\pi = 0.1$, *i.e.*, the proportion of dishonest borrowers is around 10%, then $\Phi = 1.3$.
Consequently, based on our calibration and the indeterminacy condition, we conclude that our baseline model does generate self-fulfilling equilibria.

Parameter	value	Description
ρ	0.01	Discount factor
θ	0.3	Utilization elasticity of depreciation
δ	0.033	Depreciation rate
α	0.33	Capital income share
γ	0	Inverse Frisch elasticity of labor supply
ψ	1.75	Coefficient of labor disutility
π	0.1	Proportion of firms that produce lemons
Φ	1.3	Maximum firm capacity

Table 1: Calibration

Our calibration uses a delinquency rate of about 10%, which of in the same magnitude as in the great recession. but is higher than the average of delinquency rate in the data (the average is 3.73% from period 1985 to 2013). Delinquency rates do vary over time however. For example commercial residential mortgages had high delinquency rates during 2009-2013, which spread panic to financial markets through mortgage-backed securities and other derivatives.
Nevertheless we will show in the next subsection, where we introduce reputation effects, that indeterminacy will arise even if there is no equilibrium default.

Global Dynamics in the Benchmark Case

Substituting out Y_t, N_t and u_t from equation (21), the foc for labor market equilibrium, yields

$$C_t = f_0 \cdot g\left(\phi_t\right) \cdot h(K_t) \tag{40}$$

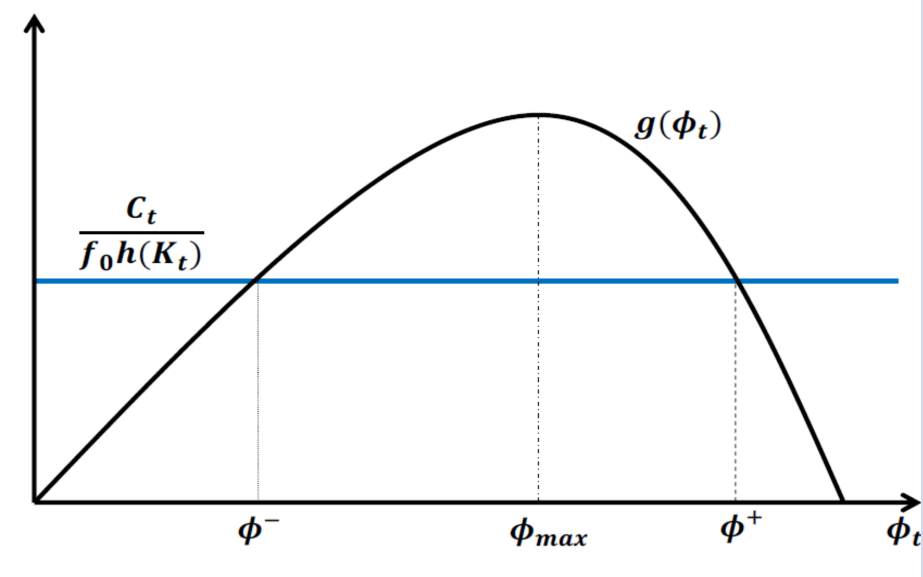
where $f_0 \equiv A^{\frac{1+\gamma}{1-\alpha}} \left(\frac{\alpha}{\delta^0}\right)^{\frac{\alpha(1+\gamma)}{(1+\theta)(1-\alpha)}} \left(\frac{1-\alpha}{\psi}\right)$, $h(K_t) \equiv K_t^{\frac{\alpha\theta(1+\gamma)}{(1+\theta)(1-\alpha)}}$, and

$$g\left(\phi_t\right) \equiv \left[\phi_t^{1-\alpha+\frac{\alpha(1+\gamma)}{1+\theta}} Y\left(\phi_t\right)^{1-\alpha-\left(1-\frac{\alpha}{1+\theta}\right)(1+\gamma)}\right]^{\frac{1}{1-\alpha}}. \tag{41}$$

Equation (41) below explicitly determines C_t as a function of ϕ_t and K_t.

Lemma

For any K_t and $C_t < f_0 \cdot h(K_t) \cdot g\left(\phi_{\max}\right)$, there are two possible $\phi_t = \phi^+\left(\frac{C_t}{f_0 h(K_t)}\right) > \phi_{\max}$ and $\phi_t = \phi^-\left(\frac{C_t}{f_0 h(K_t)}\right) < \phi_{\max}$ that yield the same level of consumption defined by (41).


$g(\phi_t)$
$\frac{C_t}{f_0 h(K_t)}$
ϕ^-
ϕ_{max}
ϕ^+
ϕ_t

To understand the global dynamics we substitute for consumption using equation (41) to obtain two differential equations regarding K_t and ϕ_t:

$$\begin{aligned}
&\left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_t}{1-\phi_t}\right)\frac{\dot{\phi}_t}{\phi_t}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K}_t}{K_t} \\
&= (1-\alpha)\left[\alpha\frac{\theta}{1+\theta}\phi_t\frac{Y_t}{K_t}-\rho\right] \\
\dot{K}_t &= (1-\frac{\alpha\phi_t}{1+\theta})Y_t-C_t
\end{aligned}$$

where $Y_t = \pi\Phi\phi_t/(1-\phi_t)$ and C_t is defined by (41). As we discussed before, the above equations have two steady states. We consider two cases. In the first case, one of the steady state is a sink and the other is a saddle. In the second case, both steady states are saddles.

Transition Dynamics

- For (K, ϕ)

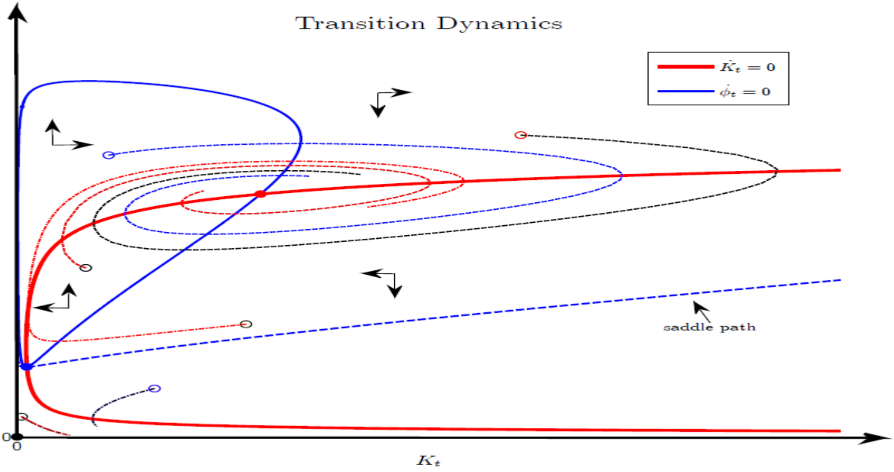


Figure 4: Global Dynamics with One Saddle (High π)

Two Saddles

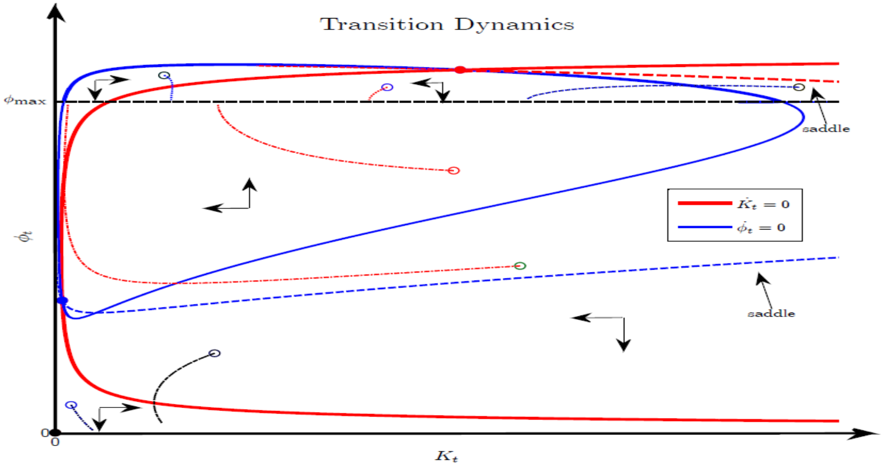
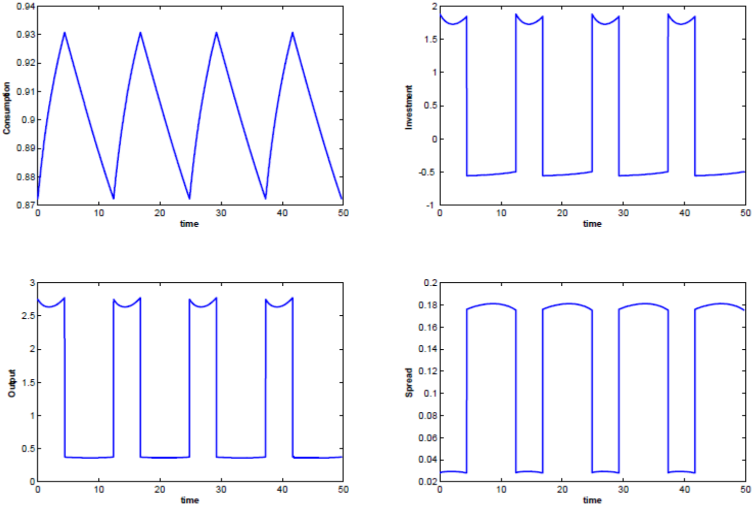


Figure 5: Two Saddles

Deterministic Cycle



Two Saddles and Sunspots

- We can also construct stochastic sunspot equilibrium by allowing ϕ_t to jump randomly.
- We introduce sunspot variables z_t, which takevalues: 1 and 0. We assume that in a short period time interval dt, there is probability λdt for the sunspot variable to change from 1 to 0 and probability ωdt for it to change from 0 to 1.
- We construct the equilibrium ϕ_t as a function of K_t and sunspot z_t, $\phi_t = \phi(K_t, z_t)$ such that $\phi(K_t, 1) > \phi(K_t, 0)$. So the equilibrium ϕ_t will jump when z_t changes its value.
- When $z_t = 1$, the economic confidence is high so the adverse selection is very modest to characterize the normal time.
- But when $z_t = 0$, the economic confidence is low and the adverse selection is very severe to characterize the crisis time.

We use the change of z_t from 1 to 0 to trigger an economic crisis and from 0 to 1 to stop the crisis as economic confidence is restored.

We set $\lambda = 0.01$ and $\omega = 0.025$ as an example, which means that the economy will stays in the normal time with probability probability 0.7143 on average.

As jumps in ϕ_t now become stochastic, consumption is exposed to the jump risk. Equation (41) must be modified to take this risk into account.

Denote $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$, we then have

$$\begin{aligned} & \left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{1t}}{1-\phi_{1t}}\right)\frac{\dot{\phi_{1t}}}{\phi_{1t}}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K_t}}{K_t} \\ = \ & (1-\alpha)\left[\alpha\frac{\theta}{1+\theta}\phi_{1t}\frac{Y_{1t}}{K_t}-\rho+\lambda(\frac{g(\phi_{1t})}{g(\phi_{0t})}-1)\right], \end{aligned}$$

for the normal time. Here last term reflect the $\frac{g(\phi_{1t})}{g(\phi_{0t})} - 1$ reflects the percentage change in consumption due to the jump from ϕ_{1t} to ϕ_{0t} and $Y_{1t} = \frac{\pi \Phi \phi_{1t}}{1-\phi_{1t}}$ is the aggregate output when $\phi_t = \phi_{1t}$. Similarly we have

$$\begin{aligned} & \left(1-\alpha+\frac{\alpha\left(1+\gamma\right)}{1+\theta}\right)\left(\frac{\phi_{\max}-\phi_{0t}}{1-\phi_{0t}}\right)\frac{\dot{\phi_{0t}}}{\phi_{0t}}+\left(\frac{\alpha\theta\left(1+\gamma\right)}{1+\theta}\right)\frac{\dot{K_t}}{K_t} \\ = \ & (1-\alpha)\left[\alpha\frac{\theta}{1+\theta}\phi_{0t}\frac{Y_{0t}}{K_t}-\rho+\omega(\frac{g(\phi_{0t})}{g(\phi_{1t})}-1)\right] \end{aligned}$$

in the crisis time when $z_t = 0$.

It is evident if $\lambda = \omega = 0$, the function $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ are functions defining the saddle paths toward the upper and lower steady state, respectively. By continuity, we know that the existence of these two function for small λ and ω. We solve these two functions by using the collocation method discussed in Miranda and Fackler (2005). More specifically we employ 15 degree Chebychev polynomial of K to approximate these two functions. Once we obtain $\phi_{1t} = \phi(K_t, 1)$ and $\phi_{0t} = \phi(K_t, 0)$ as functions of capital K_t, we can then use equation (**??**) to simulate the dynamic path of capital. Figure below shows a possible dynamic path of the economy.

Sunspot Cycle

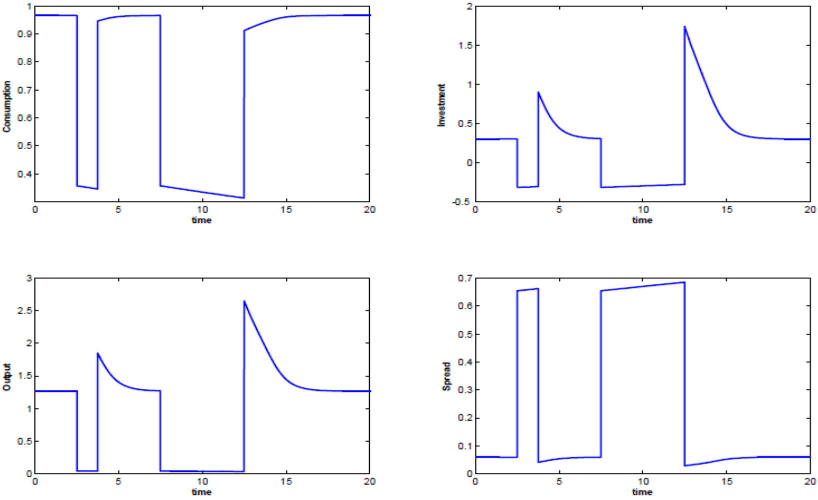


Figure 2: Stochastic Switches between branches

- Assume the economy initially is in the normal time with $z_t = 1$ for a sufficiently long period. So capital, consumption, output, investment do not change. The parameter values we choose yields $K = 10.5427$.
- Due to precuationary saving, this level of capital is higher than the deterministic upper steady state level of capital, as households have an incentive to save to insure the stochastic crash in output.
- The economy stays in this level of capital for 2.5 years, then a crisis triggered by a drop of z_t from 1 to 0, so the spread (on the last window) immediately jumps up as the adverse selection problem in the credit market deteriorates sharply.
- As a result, production and output collapse (depicted on the lower left window).
- Since the time of collapse in output is unpredictable ex ante, consumption also falls immediately (the upper left window).
- Investment falls for two reasons: to partially offset the fall in output to finance consumption, and due to the decline in the effective return as a result of severe adverse selection in the credit market.

The economy stays in the crisis period for about 1 years and then the confidence is restored and the recession is over.

Interestingly output and investment experience an over-shooting when the recession is over and this overshooting is larger if the economy stays in the recession longer.

The longer recession that economy stays in recession, the smaller the capital stock left.

So the return to investment is very high, households opt to work hard and invest more to enjoy the high return from investment.

The figures shows several large boom and bust cycles due to the stochastic jumps in the sunspot variables.

Reputation

- If firms are not anonymous in the market, they may refrain from defaulting and instead may want to build their reputation. Lenders may also refrain from lending to firms with a bad credit history.
- So we examine whether the indeterminacy results obtained in our baseline model survive if such reputational effects are taken into account.
- We follow Kehoe and Levine (1993) closely in modeling reputation. Firms are infinitely-lived, and can choose to default at any time. Firms that default, with some probability, acquire a bad reputation and are excluded from the credit market forever. In equilibrium, the fear of loosing all future profits from production discourages firms from defaulting. We will show that self-fulfilling equilibria still exist even if there are no defaults in equilibrium.

To keep the model analytically tractable, we assume that all firms are owned by a representative entrepreneur. The entrepreneur's utility function is given by

$$U(C_{et}) = \int_0^\infty e^{-\rho_e t} \log(C_{et}) dt, \tag{42}$$

where C_{et} is the entrepreneur's consumption and ρ_e her discount factor. For tractability, we assume that $\rho_e << \rho$ such that the entrepreneur does not accumulate capital. The entrepreneur's consumption equals the firm's profits.

$$C_{et} = \int_0^1 \Pi_t(i) di \equiv \Pi_t, \tag{43}$$

where $\Pi_t(i)$ denotes the profit of firm i.

- Since the only cost of default is the loss of future production opportunities, the price must exceed the marginal cost (also the average cost) of production to be profitable. (At zero profits better to default).
- If the price exceeds the marginal cost, each firm will then have an incentive to produce an infinite amount. To overcome this problem, we will assume firms' production projects are indivisible, as in the benchmark model and that they produce according to the orders that they receive.
- A production project produces a flow of final goods Φ from intermediate goods. Each unit of the final good requires one unit of intermediate goods for its production.
- The project will be carried out only if the firms receive a sale order. Denote the total demand for final good as Y_t. Then fraction $\eta_t = Y_t/\Phi$ of firms will receive the sale orders.
- Again we assume that firms need to borrow to finance their working capital.
- Denote the intermediate goods' price as P_t, they need to borrow $P_t\Phi$.

- To illustrate the reputation problem, let us consider a short time interval from t to $t+dt$. We use V_{1t} (V_{0t}) to denote the value of a firm that receives an order (no orders). We can then formulate V_{1t} recursively as

$$V_{1t} = (1-\phi_t)\Phi dt + e^{-\rho_e dt}\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)[\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}]. \tag{44}$$

  where $\phi_t = P_t$ is the unit production cost. If $1 > \phi_t$, then the firm receives a positive profit from production.

- The second term on the right-hand side is the continuation value of the firms. Since firms are owned by the entrepreneur, the future value is discounted by the marginal utility of the entrepreneur.
- Since there is no default in equilibrium, the gross interest rate for a working capital loan is $R_{ft} = 1$.

- The firm can also choose to default on its working capital. By doing so, the firms obtain instantaneous gain of $\Phi\phi_t$.
- However,default comes with the risk of acquiring a bad reputation. Upon default, the firm acquires a bad reputation in the short time interval between t and $t+dt$ with probability λdt. In that case, the firm will be excluded from production forever.

The payoff for defaulting is hence

$$V_t^d = \Phi dt + e^{-\rho_e dt}(1-\lambda dt)E_t\left(\frac{C_{e,t}}{C_{e,t+dt}}\right)[\eta_{t+dt}V_{1t+dt} + (1-\eta_{t+dt})V_{0t+dt}]. \tag{45}$$

The value of a firm that does not receive any order is given by

$$V_{0t} = e^{-\rho_e dt} E_t \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) [\eta_{t+dt} V_{1t+dt} + (1 - \eta_{t+dt}) V_{0t+dt}]. \qquad (46)$$

- Define $V_t = \eta_t V_{1t} + (1 - \eta_t) V_{0t}$ as the expected value of the firm. The firm has no incentive to produce lemons if and only if $V_{1t} \geq V_t^d$, or

$$\Phi dt \leq (1 - \phi_t)\Phi dt + \lambda dt e^{-\rho_e dt} \left(\frac{C_{e,t}}{C_{e,t+dt}} \right) V_{t+dt}. \qquad (47)$$

- In the limit $dt \to 0$, the incentive compatibility condition becomes $\phi_t \Phi \leq \lambda V_t$.(Under the incentive compatibility condition we can consider one step deviations since $V_{1t},\ V_{0t}$ are then optimal value functions).

- The expected value of the firm, with no default in equilibrium, is given by the present discounted value of all future profits as

$$V_t = \int_0^\infty e^{-\rho_e s} \frac{C_{et}}{C_{es}} \Pi_s ds. \tag{48}$$

- In equilibrium, $\eta_t = \frac{Y_t}{\Phi}$. For simplicity, we assume Φ is big enough such that $\eta_t < 1$ always holds.
- The average profit is then obtained as $\Pi_t = (1 - \phi_t) Y_t$.In turn, since $C_{ej} = \Pi_j$, we have

$$V_t = \frac{(1 - \phi_t) Y_t}{\rho_e}. \tag{49}$$

- The households' budget constraint changes to (no profits from lemons):

$$C_t + I_t \leq R_t u_t K_t + W_t N_t = \phi_t Y_t. \tag{50}$$

- Then the incentive constraint (47), dividing by dt and letting $dt \to 0$, becomes

$$\phi_t \Phi \leq \lambda \frac{(1 - \phi_t) Y_t}{\rho_e}. \tag{51}$$

From the household budget constraint (50), we know that household utility increases with ϕ_t and thus the incentive constraint (51) must be binding. Then Equation (51) can be simplified as

$$\phi_t = \frac{Y_t}{\pi\Phi + Y_t} < 1, \tag{52}$$

where now $\pi \equiv \frac{\rho_e}{\lambda}$. Similar to the baseline model, here firms also receive an information rent. However, the rent in the baseline is derived from hidden information while the rent here arises from hidden action. As indicated in Equation (52), ϕ_t is procyclical and hence the markup is countercyclical. If output is high, the total profit from production is high. Therefore the value of a good reputation is high and the opportunity cost of producing a lemon also increases. This then alleviates the moral hazard problem.

The cost minimization problem again yields the factor prices given by Equation (19) and (20). Since households do not own firms, their budget constraint is modified as

$$C_t + \dot{K}_t = \phi_t Y_t - \delta(u_t) K_t. \tag{53}$$

The equilibrium system of equations is the same as in the baseline model except that Equation (18) is replaced by Equation (53).

The steady state can be computed similarly. The steady state output is given by

$$Y = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha\phi\theta}{\rho\left(1+\theta\right)}\right]^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\alpha}{1-\frac{\alpha}{1+\theta}}\right) \cdot \frac{1}{\psi}\right]^{\frac{1}{1+\gamma}} \equiv Y(\phi), \tag{54}$$

and ϕ can be solved from

$$\bar{\Phi} \equiv \pi\Phi \equiv \Psi(\phi) = \left(\frac{1-\phi}{\phi}\right) \cdot Y(\phi). \tag{55}$$

Unlike the baseline model, the steady state equilibrium is unique. We summarize the result in the following lemma.

LEMMA: If $\alpha < \frac{1}{2}$, a consistently standard calibrated value of α, then the steady state equilibrium is unique for any $\bar{\Phi} > 0$.

We can now study the possibility of self-fulfilling equilibria around the steady state. Since ϕ and $\bar{\Phi}$ form a one-to-one mapping, we will treat ϕ as a free parameter in characterizing the indeterminacy condition. We can then use Equation (55) to back out the corresponding value of $\bar{\Phi}$. The following proposition specifies the condition under which self-fulfilling equilibria arises.

PROPOSITION: Let $\tau = 1 - \phi$. Then indeterminacy emerges if and only if

$$\tau_{\min} < \tau < \min\left\{\frac{1+\theta}{\alpha} - 1, \tau_H\right\} \equiv \tau_{\max},$$

where $\tau_{\min} \equiv \frac{(1+\theta)(1+\gamma)}{(1+\theta)(1-\alpha)+\alpha(1+\gamma)} - 1$, and τ_H is the positive solution to $A_1\tau^2 - A_2\tau - A_3 = 0$, where

$$\begin{aligned}
A_1 &\equiv s\,(1+\theta)\,(2+\alpha+\alpha\gamma)\\
A_2 &\equiv (1+\theta)\,(1+\alpha\gamma) - s\,[(1+\theta)\,(1-\alpha)\,(1-\gamma) + (1+\gamma)\,\alpha]\\
A_3 &\equiv (1+\theta)\,(1-\alpha)\,[s + (1-s)\,\gamma]\,.
\end{aligned}$$

The necessary condition for indeterminacy turns out to be the same as in our baseline model. It is easy to verify that under $\tau > \tau_{\text{min}}$, the labor demand curve slopes upward and is steeper than the labor supply curve. So the intuition for indeterminacy is similar to that in the baseline. Indeterminacy implies that the model exhibits multiple expectation-driven equilibria around the steady state. The steady state equilibrium is now unique however, which suggests that the continuum of equilibria implied by indeterminacy cannot be obtained in static models studied the earlier literature.

So far, the condition to sustain indeterminacy is given in terms of ϕ and τ. The following lemma specifies the underlying condition in terms of ρ_e, λ and Φ.

LEMMA: Indeterminacy emerges if and only if $\frac{\Psi(1-\tau_{\min})}{\Phi} < \frac{\rho_e}{\lambda} < \frac{\Psi(1-\tau_{\max})}{\Phi}$.

- Given the other parameters, a decrease in ρ_e or an increase in λ increases the steady state ϕ. According to the above lemma, it makes indeterminacy less likely. The intuition is straightforward.
- A large λ means the opportunity cost of producing lemon increases, as the firm becomes more likely to be excluded from future production. This alleviates the moral hazard problem, which is the source of indeterminacy.
- Similarly, a decrease of ρ_e means that the entrepreneurs become more patient. So the future profit flow from production is more valuable to them, which again increases the opportunity cost of producing lemons and thus alleviates the moral hazard problem.

Adverse Selection with Heterogeneous Productivity

The households' problems as in the benchmark model and thus the first order conditions are still Equations (5), (6) and (7).
Assume the risk of lending to final good firms $j \in [0, 1]$, are continuous.
Again each final good firms have one production project, which requires Φ unit of intermediate goods.
The loan is risky as the final goods firms' production may not be successful. Assume the final good firm j's output is governed by:

$$y_{jt} = \begin{cases} a_{jt} x_{jt}, & \text{with probability } q_{jt} \\ 0, & \text{with probability } 1 - q_{jt} \end{cases}, \tag{56}$$

Here x_{jt} is the intermediate input for firm j and a_{jt} the firm's productivity.
We assume q_{jt} is IID drawn from a common distribution function $F(q)$ and $a_{jt} = a_{\min} q_{jt}^{-\tau}$: the higher the probability of success, the lower the productivity or the higher cost of production.. Notice that expected productivity is given by $q_{jt} a_{jt} = a_{\min} q_{jt}^{1-\tau}$. We assume that $\tau < 1$, *i.e.*, a firm with a higher success probability enjoys a higher expected productivity.

Denote P_t as the price of intermediate goods. Then the total borrowing is given by $P_t x_{jt}$. Denote R_{ft} be the gross interest rate. Then the final good firm $j's$ profit maximization problem becomes

$$\max_{x_{jt} \in \{0, \Phi\}} q_{jt}[a_{jt} x_{jt} - R_{ft} P_t x_{jt}], \tag{57}$$

Note that, due to limited liability, the final goods firm pays back the working capital loan only if the project is successful. This implies that, given R_{ft} and P_t, the demand for x_{jt} is simply given by

$$x_{jt} = \begin{cases} \Phi & \text{if } a_{jt} > R_{ft} P_t \equiv a_t^* \\ 0 & \text{otherwise} \end{cases} \; ; \; \textit{or equivalently} \tag{58}$$

$$a_{\min} q_{jt}^{-\tau} > a_t^*, \; q_{jt} < \left[\frac{a_t^*}{a_{\min}} \right]^{-\frac{1}{\tau}} = q_t^* = \left[\frac{R_{ft} P_t}{a_{\min}} \right]^{-\frac{1}{\tau}}. \tag{59}$$

This establishes that only firms with risky production opportunities will enter the credit markets, which highlight the adverse selection problem in the financial market. Firms with $q_{jt} > q_t^*$ are driven out from the financial market, despite their higher social expected productivity $q_{jt} a_{jt} = a_{\min} q_{jt}^{1-\tau}$.

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Since financial intermediaries are assumed to be fully competitive, we have

$$R_{ft} P_t \Phi \int_0^{q_t^*} q dF(q) = P_t \Phi \int_0^{q_t^*} dF(q), \tag{60}$$

where the left-hand side is the actual repayment from the final goods firms, and the right-hand side the actual lending. We obtain

$$R_{ft} = \frac{1}{\int_0^{q_t^*} q dF(q) / \int_0^{q_t^*} dF(q)} = \frac{1}{E\left(q | q \leq q_t^*\right)} > 1, \tag{61}$$

where the denominator is average success rate. The above equation says that interest rate decreases with the average success rate.

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The total production of final goods is

$$Y_t = \int_0^1 q_j a_{jt} x_{jt} dF(q) = \Phi \int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q). \tag{62}$$

where the second equality follows equation (58). The total production of intermediate goods is

$$X_t = \Phi \int_0^{q_t^*} dF(q). \tag{63}$$

Finally the intermediate goods are produced according to $X_t = A_t (u_t K_t)^{\alpha} N_t^{1-\alpha}$, where $u_t K_t$ is the capital rented from the households. Equation (62) and then (63) then yields

$$Y_t = \Gamma(q_t^*) A_t (u_t K_t)^{\alpha} N_t^{1-\alpha}, \tag{64}$$

where $\Gamma(q_t^*) = \left[\int_0^{q_t^*} a_{\min} q^{1-\tau} dF(q)\right] / \int_0^{q_t^*} dF(q)$ depends on the threshold q_t^* and the distribution.

The above equation then says the measured TFP is obtained as

$$TFP_t = \frac{Y_t}{(u_t K_t)^{\alpha} N_t^{1-\alpha}} = \Gamma(q_t^*) A_t. \tag{65}$$

$$\Gamma'(q_t^*) = \frac{a_{\min} f(q_t^*) \int_0^{q_t^*} \left[q_t^{*1-\tau} - q^{1-\tau} \right] dF(q)}{\left[\int_0^{q_t^*} dF(q) \right]^2} > 0.$$

So the endogenous TFP increases with the threshold. This is very intuitive as the threshold increases, more firms with high productivity enters the credit market, making resource allocation more efficient. Equation (62) implies that q_t^* increases with Y_t, so we have the following lemma.
Lemma: TFP is endogenous and increase in Y, namely $\frac{\partial \Gamma(q_t^*)}{\partial Y_t} > 0$.
We have therefore established that the endogenous TFP, $\Gamma(q^*)$, is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions.

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We have therefore established that the endogenous TFP, $\Gamma(q^*)$, is procyclical. Notice that the procyclicality of endogenous TFP holds generally for continuous distributions. So without loss of generality, we now assume $F(q) = q^\eta$ for tractability, so the distribution of q^{-1} is a power law. Equations (62) and (64) together yield the aggregate output as

$$
\begin{aligned}
Y_t &= \left(\frac{\eta}{\eta-\tau+1}\right) a_{\min}\Phi^{-\frac{1-\tau}{\eta}} \left(A_t u_t^\alpha K_t^\alpha N_t^{1-\alpha}\right)^{1+\frac{1-\tau}{\eta}}. && (66)\\
&\equiv \phi a_{\min}\Phi^{-\frac{1-\tau}{\eta}} \left(A_t u_t^\alpha K_t^\alpha N_t^{1-\alpha}\right)^{1+\frac{1-\tau}{\eta}}. && (67)
\end{aligned}
$$

The intuition is as follows. Suppose that the total lending from financial intermediaries increases. This creates a downward pressure on interest rate R_{ft}, which increases the cutoff q_t^* according to the definition at Equation (59). Firms with higher q have smaller risk of default. A rise in the cutoff q_t^* therefore reduces the average default rate. If it is strong enough, it can in turn stimulate more lending from the financial intermediaries. Since firms with higher q are also more productive on average, the increased efficiency in re-allocating credit implies that resources are better allocated across firms.

Notice that the aggregate output again exhibits increasing returns to scale. Equation (66) reveals that the degree of increasing returns to scale clearly depends on the adverse selection problem. The degree of increasing returns to scale decreases with τ and η. When $\eta = \infty$, the firms' product quality is homogeneous. Hence there is no asymmetric information and adverse selection. If $\tau = 1$, firms are equally productive in the sense their expected productivity is the same. It therefore does not matter how credits are allocated among firms. Given $\tau < 1$, a smaller η implies that firms are more heterogenous, creating a large asymmetric information problem. Similarly given η, a smaller τ implies that the productivity of firms deteriorates faster with respect to their default risk, making the adverse selection more damaging to resource allocation. We formally state this result in the following proposition.

The reduced-form aggregate production in our model exhibits increasing returns to scale if and only if there exists adverse selection, *i.e.*, $\tau < 1$ and $\eta < \infty$. Also in the benchmark model, both the credit spread, measured by $R_{ft} - 1$, and the expected default risk, $1 - E\left(q \mid q \leq q_t^*\right)$, are countercyclical.

Indeterminacy

It is straightforward to show $W_t = \phi \frac{(1-\alpha)Y_t}{N_t}$ and $R_t = \phi \frac{\alpha Y_t}{u_t K_t}$.. Here

$$\phi = \frac{\eta + 1 - \tau}{\eta + 1}$$

- Together with Equations (5), (6), (7), (66), and (18), we can determine the seven variables, C_t, Y_t, N_t, u_t, K_t, W_t and R_t.
- The steady state can be obtained as in the baseline model. We can express the other variables in terms of the steady state ϕ.
- Since ϕ is unique, unlike in the baseline model, the steady state here is unique. We assume that Φ is large enough so that an interior solution to q^* is always guaranteed.
- We state this result formally in the following corollary.

Corollary: The steady state is unique in the extended model with $F(q) = (q/q_{\max})^{\eta}$.

Proposition: The model is indeterminate if and only if

$$\sigma_{\min} < \sigma < \sigma_{\max} \tag{68}$$

Proposition

where $\sigma \equiv \frac{1-\tau}{\eta}$, $\sigma_{\min} \equiv \left(\frac{1}{\frac{1-\alpha}{1+\gamma} + \frac{\alpha}{1+\theta}} \right) - 1$ *and* $\sigma_{\max} \equiv \frac{1}{\alpha} - 1.$

To better understand the proposition, we first consider how output responds to a fundamental shock, such as a change in A, the true TFP. Holding factor inputs constant, we have

$$1 + \tilde{\sigma} \equiv \frac{d \log Y_t}{d \log A} = (1+\sigma) \left[\frac{1+\theta}{1+\theta-\alpha(1+\sigma)} \right] > 1, \tag{69}$$

The above equations shows that adverse selection and variable capacity utilization can significantly amplify the impact of a TFP shock on output. Let us define $1+\tilde{\sigma}$ as the multiplier of adverse selection. Note that the necessary condition $\sigma > \sigma_{\min}$ can be written as

$$(1+\tilde{\sigma})(1-\alpha) - 1 > \gamma. \tag{70}$$

Empirical Possibility of Indeterminacy To empirically evaluate the possibility of indeterminacy, we set the same value to ρ, θ, δ, α and γ as in Table 1. q_{max} and Φ do not affect the indeterminacy condition, so we do not need to specify their value.We have new parameters in this extended model (τ, η). We use two moments to pin them down and set τ and η to match the steady state markup $\frac{\eta+1-\tau}{\eta+1} = 0.9$. Basu and Fernald (1997) estimate aggregate increasing returns to scale for manufacturing to be around 1.1. So we set $\sigma = 0.1$. This leads to $\tau = 0.55$ and $\eta = 4.5$. We have $\sigma_{\text{min}} = 0.083$ and $\sigma_{\text{max}} \equiv 2$, which meet the indeterminacy conditions. Hence, with these parameters the model exhibits self-fulfilling equilibria. Since the steady state equilibrium is unique, such multiple equilibria must come from the dynamic nature of the model.