Benhabib-Gali

$$Y = f(K, HL),\ L = 1,\ H(0) = \phi,$$

$$H(t) = \phi e^{\gamma t},\ \ y_i = \frac{Y_i(t)}{H(t)} = f(\tfrac{K}{H}, L) = f(k, L)$$

$$\frac{y_i(t)}{y_i(0)} = \frac{\frac{Y_i(t)}{H(t)}}{\frac{Y_i(0)}{\phi}} = \frac{\frac{Y_i(t)}{\phi e^{\gamma t}}}{\frac{Y_i(0)}{\phi}} = \frac{Y_i(t)}{Y_i(0)} e^{-\gamma t}$$

$$\ell n\left(\frac{y_i(T)}{y_i(0)}\right) = \ell n\left(\frac{Y_i(T)}{Y_i(0)}\right) - \gamma T$$

$$\left(\frac{1}{T}\right)\ell n\left(\frac{Y_i(T)}{Y_i(0)}\right) = \ell n\left(\frac{y_i(T)}{y_i(0)}\right)\left(\frac{1}{T}\right) + \gamma$$

But since from convergence we have

$$\ell n(y(T)) - \ell n(y(0)) = \ell n\left(\frac{y_i(T)}{y_i(0)}\right)$$
$$= (1 - e^{-\lambda T})(\ell n(y^*) - \ell n(y(0)))$$

we get

$$\left(\frac{1}{T}\right)\ell n\left(\frac{Y_i(T)}{Y_i(0)}\right)$$
$$= \left(\frac{1}{T}\right)(1 - e^{-\lambda T})(\ell n(y^*) - \ell n(y(0))) + \gamma$$

or,if

$$dY_i = \left(\frac{1}{T}\right)\ell n\left(\frac{Y_i(T)}{Y_i(0)}\right), \;\; \pi = \left(\frac{1}{T}\right)(1 - e^{-\lambda T})$$

,

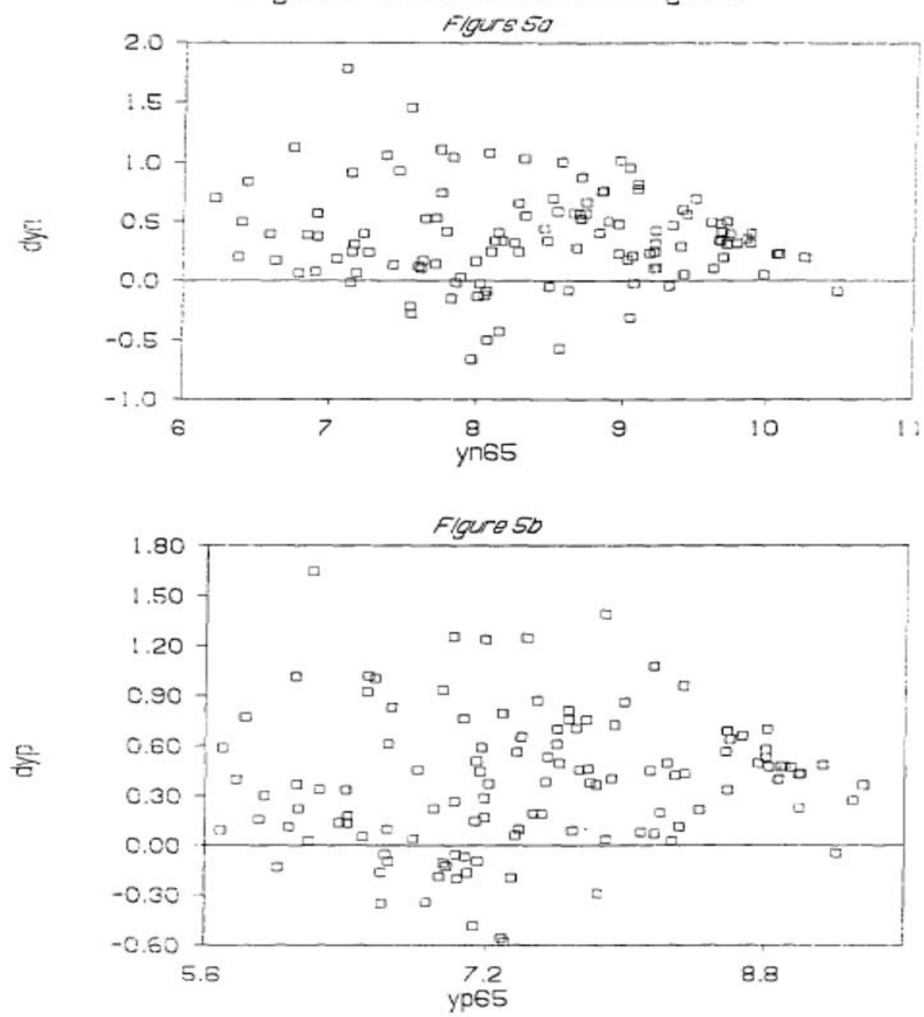
$$dY_i = (\gamma + \pi\ell n(y^*)) - \pi\ell n(y_i(0)) + \varepsilon_i$$

$$dY_i = (\gamma + \pi\ell n(y^*)) - \pi\ell n(Y_i(0)) + \pi\ell n\phi + \varepsilon_i$$

$$dY_i = \kappa - \pi\ell n(Y_i(0)) + \varepsilon_i$$

where ε_i is the error term, and
$\kappa = \gamma + \pi\ell n(y^*) + \pi\ell n\phi$.

Figure 5: unconditional convergence



What if y_i^* differ accross countries, or if initial values $Y_i(0)$ are correlated with technology levels ϕ_i?

Let $\gamma,$ and y^* be common. Note that

$$\begin{aligned} \ell n(y_i(0)) &= \alpha \ell n\left(\frac{k_i(0)}{y_i(0)}\right) + \alpha \ell n(y_i(0)) \\ &= \left(\frac{\alpha}{1-\alpha}\right) \ell n\left(\frac{k_i(0)}{y_i(0)}\right) \end{aligned}$$

Accordingly, write

$$\begin{aligned} dY_i &= (\gamma + \pi \ell n(y^*)) - \pi \ell n(y_i(0)) + \varepsilon_i \\ &= \Gamma - \left(\frac{\pi\alpha}{1-\alpha}\right) \ell n\left(\frac{K_i(0)}{Y_i(0)}\right) + \varepsilon_i \end{aligned}$$

This gets rid of non-common ϕ_i. We estimate

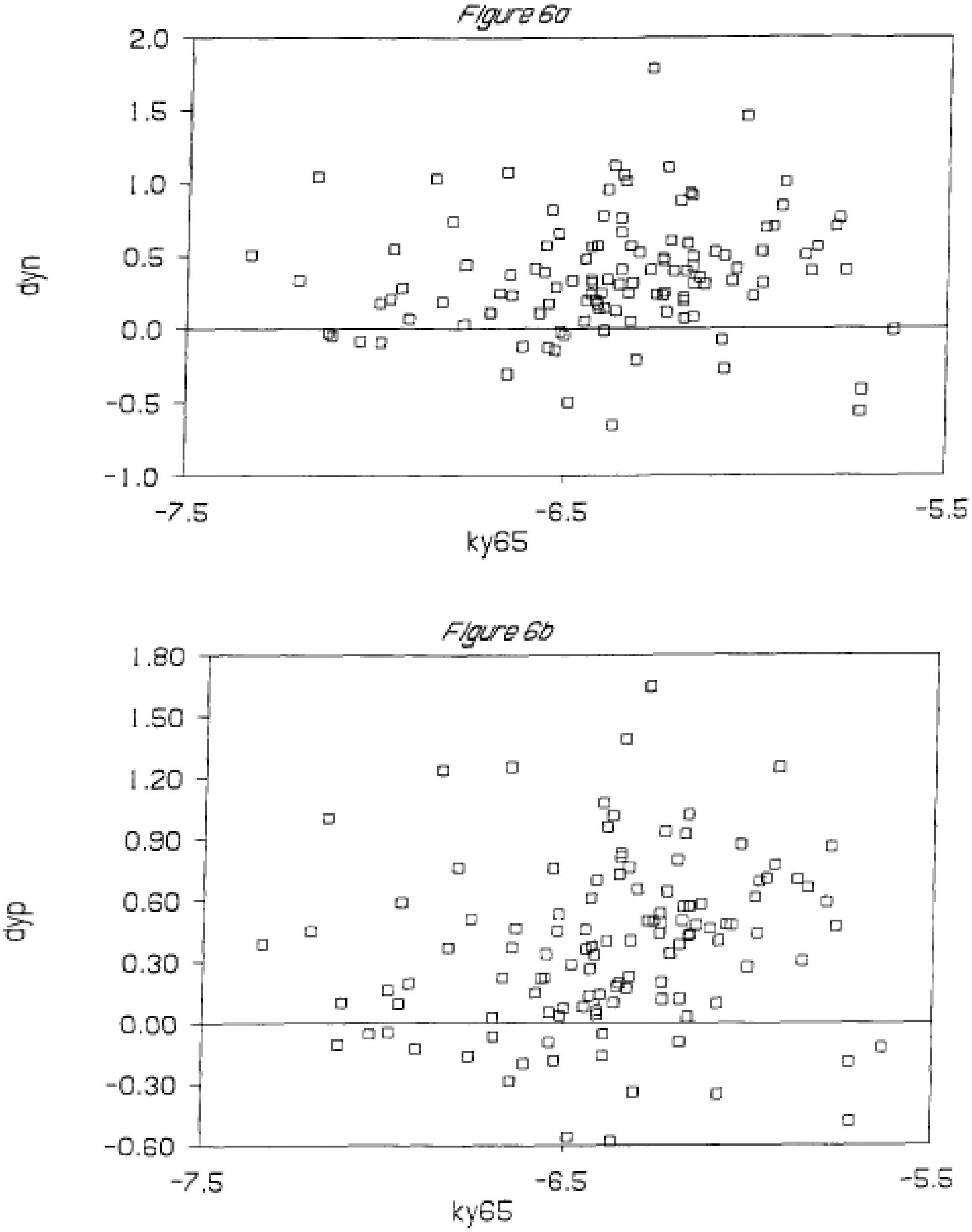
$$dY_i^n = 1.199 + 0.130 \ln\left(\frac{K_i(0}{Y_i^n(0)}\right) + \varepsilon_i$$

$$(0.687) \quad (0.107)$$

$$dY_i^p = 1.475 + 0.172 \ln\left(\frac{K_i(0}{Y_i^n(0)} \right) + \varepsilon_i$$

$$(0.705) \quad (0.110)$$

Figure 6: omega-conditional convergence



But what if $\gamma_i,\ y_i^*$ etc. hidden in Γ are non-common, and what if $\frac{K_i(0)}{Y_i(0)}$ is corrolated with Γ?

Split sample into two equal parts and difference, so that

$$dY_i(t,T) - dY(0,t)) = -\Psi_i(dK_i(0,t) - dY_i(0,t))$$

where

$$dX(a,b) = (1/(a-b))(\ell n(X_i(b)) - \ell n(X_i(a)))$$

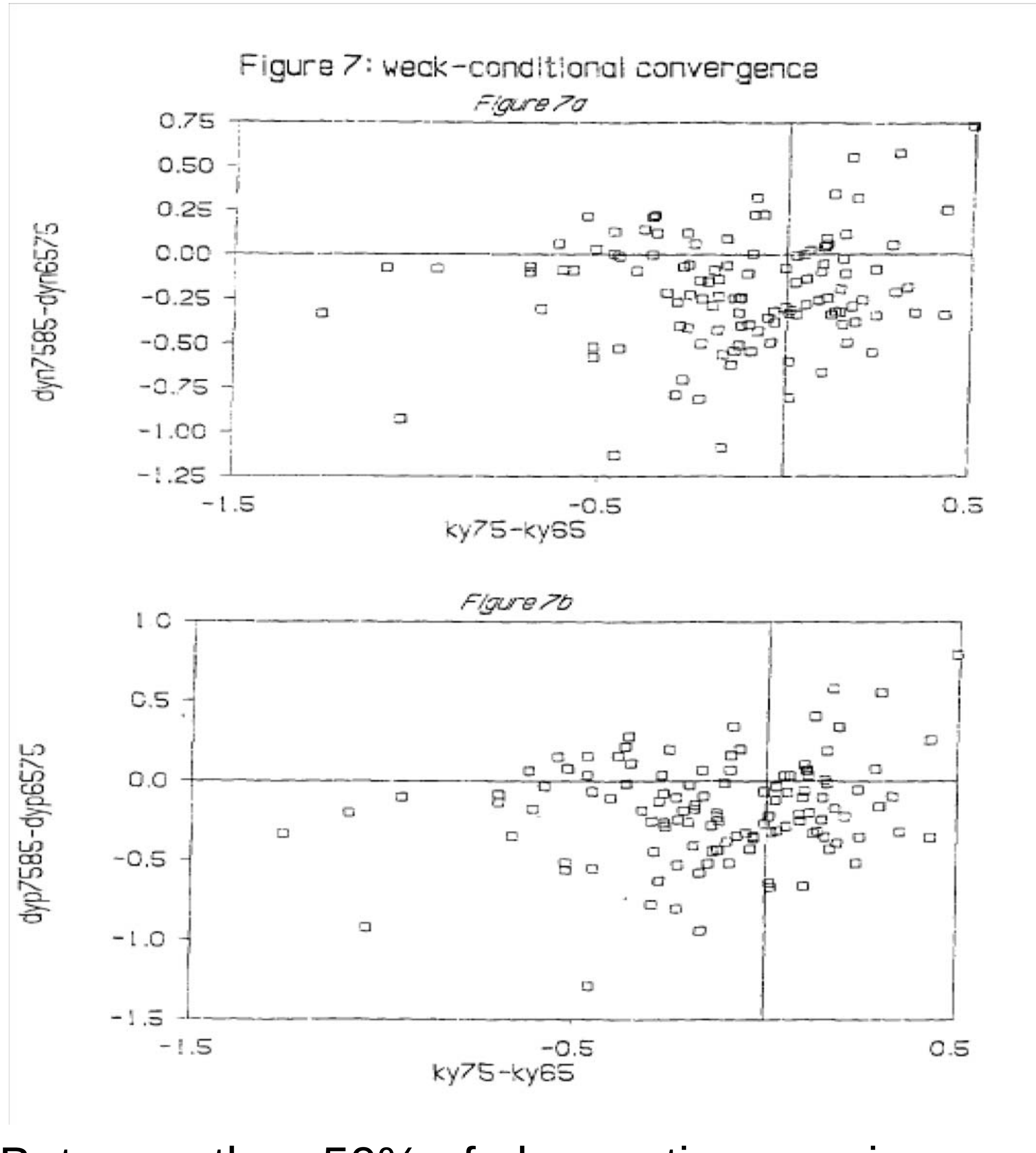
This gets rid of Γ. But what about α and π, inside Ψ_i? Are they common? Then use

$$\sum_i (dY_i(t,T) - dY(0,t)))(dK_i(0,t) - dY_i(0,t))$$

since we expect

$$(dY_i(t,T) - dY(0,t)))(dK_i(0,t) - dY_i(0,t)) < 0$$

$$for\ all\ i$$



But more than 50% of observations are in the southwest and northeast quadrants!

Quah’s Analysis

If M is an transition matrix of incomes (25 year transition)

$$M = \begin{array}{ccccc} 0-.25 & .25-.5 & .5-1 & 1-2 & 2-\infty \\ .97 & .3 & 0 & 0 & 0 \\ .05 & .92 & .04 & 0 & 0 \\ 0 & .04 & .92 & .04 & 0 \\ 0 & 0 & .04 & .94 & .02 \\ 0 & 0 & 0 & .01 & .99 \end{array}$$

The row eigenvector of the transpose of the transition matrix is the ergodic distribution:

$$.24 \quad .18 \quad .16 \quad .16 \quad .27$$

But the thinning middle may be due to differences in underlying parameters affecting savings, technology etc. Lets do it with $\frac{K}{Y}$:

$$M = \begin{array}{ccc} 0-0.8 & 0.8-1.2 & 1.2-\infty \\ 0.7 & 0.24 & 0.06 \\ 0.32 & 0.34 & 0.34 \\ 0.09 & 0.21 & 0.7 \end{array}$$

Ergodic Distribution

$$0.3809 \quad 0.2545 \quad 0.3645$$

Some thinning in the middle.