Private Information and Sunspots in Sequential Asset Markets

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- The efficient markets hypothesis states that prices on traded assets reflect all publicly available information.
- In their classic work Grossman and Stiglitz (1980) discussed a model where some agents can obtain private information about asset returns, and can trade on the basis of that information.
- If however the rational expectations equilibrium price reveals the information about the asset, and information collection is costly, then agents have no incentive to collect the information before they observe the price and trade.
- But then prices no longer reflect the information about the asset, and markets are no longer efficient.
- A large empirical and theoretical literature has since then explored the informational efficiency of markets under private information.

- We study the possibility of rational expectations sunspot equilibria driven by non-fundamentals in asset markets with private information.
- We have no noise traders: all agents in our model are Bayesian optimizers.
- We follow the work of Cass and Shell (1983) where sunspot equilibria are not randomizations over multiple fundamental equilibria.
- We combine this approach with more recent approches based on informational frictions that arise naturally in our context.
- We show that sunspots can drive equilibria and market prices do not perfectly reveal the asset returns to uninformed traders.

- In our simplest benchmark model short term traders have noisy information about the return or dividend yield of the asset, but hold and trade the asset before its return is realized at maturity.
- The returns to short-term traders consist of capital gains.
- Investors, on the other hand, who may not have private information about the returns or dividend yields but observe past and current prices, purchase and hold the asset for its final dividend return.

- We show that under such a market structure, in addition to equilibria where equilibrium prices fully reveal asset returns as in Grossman and Stiglitz (1980), there also exists a continuum of equilibria with prices driven by sunspot shocks.
- Furthermore the sunspot or sentiment shocks generate persistent fluctuations in the price of the risky asset that look to the econometrician like a random walk in an efficient market driven by fundamentals.

Related Literature

- Cass and Shell (1983), Aumann (1987), Aumann Peck and Shell (1988), Maskin and Tirole (1987) and Peck and Shell (1991).
- Allen, Morris and Shin (2006), Angeletos, Lorenzoni and Pavan (2010), Albagli, Hellwig and Tsyvinski (2011), Veldkamp (2006), Vives (2008, 2014), Cespa and Vives (2014), Golosov, Lorenzoni, Tysvinski (2014).
- Angeletos and La'O (2012) and Benhabib, Wang and Wen (Forthcoming)

Benchmark Model

- We start with a three period benchmark model with a continuum of short-term traders and long-term investors.
- We index the short-term trader by j and the long-term investor by i.
- In period 0 there is a continuum of short-term traders of unit mass endowed with 1 unit of an asset, a Lucas tree. This tree yields a dividend D in period 2. We assume that

$$\log D = \theta. \tag{1}$$

  where $\theta \sim N\left(-\frac{1}{2}\sigma_\theta^2, \sigma_\theta^2\right)$.
- In period 1 each short-term trader sells the asset and receives utility before D is realized in period 2.
- This short-term trader, maybe because he is involved in creating and structuring the asset, receives a signal s_j in period 0

$$s_j = \theta + e_j \tag{2}$$

  where $e_j \sim N\left(0, \sigma_e^2\right)$, independent of θ.

- So the short-term trader j in period 0 solves

$$\max_{x_{j0}, B_{j0}} \mathbb{E}[C_{j1} | P_0, s_j] \tag{3}$$

with the budget constraints

$$P_0 x_{j0} + B_{j0} = P_0 + w \tag{4}$$

$$C_{j1} = P_1 x_{j0} + B_{j0.} \tag{5}$$

where w is his endowment or labor income, x_{j0}, is the quantity of the asset and B_{j0} is a safe bond that he carries over to period 1.

- We assume, at the moment, that there is no restriction on B_{j0}. Therefore using the budget constraint we can rewrite the short-term trader j's problem as

$$\max_{x_{j0} \in (-\infty, +\infty)} \mathbb{E}[P_0 + w + (P_1 - P_0) x_{j0} | P_0, s_j] \tag{6}$$

- There is a continuum of investors of unit mass in period 1, each endowed with w, who trade with the short-term traders and enjoy consumption in period 2 when the dividend D is realized.
- These investors solve a similar problem, but have no direct information about the dividend of the Lucas tree, except through the prices they observe. Hence an investor i in period 1, solves

$$\max_{x_{i1}, B_{i1}} \mathbb{E}[C_{i2}|P_0, P_1] \tag{7}$$

with the budget constraints

$$P_1 x_{i1} + B_{i1} = w \tag{8}$$

$$C_{i2} = D x_{i1} + B_{i1.} \tag{9}$$

where w is his endowment, x_{i1} is their asset purchase, and B_{i1} is his bond holdings carried over to period 2.
- The objective function (7) can be written as

$$\max_{x_{i1} \in (-\infty, +\infty)} \mathbb{E}[w + (D - P_1) x_{i1} | P_0, P_1], \tag{10}$$

after substituting out B_{i1} from the budget constraints.

An equilibrium is a pair of prices $\{P_0, P_1\}$ such that x_{j0} solves problem (6) and x_{i1} solves problem (10), and markets clear. Formally we define our equilibrium as follows:

Definition

An equilibrium is individual portfolio choices $x_{j0} = x(P_0, s_j)$ for the short-term traders in period 0, $x_{i1} = y(P_0, P_1)$ for the long-term investors in period 1, and two price functions $\{P_0 = P_0(\theta), P_1 = P_1(\theta)\}$ that jointly satisfy market clearing and individual optimization,

$$\int x_{j0}\, dj = 1 = \int x_{i1}\, di, \tag{11}$$

$$P_0 = \mathbb{E}[P_1 | P_0, s_j] \tag{12}$$

for all $s_j = \theta + e_j$, and

$$P_1 = \mathbb{E}\left[D | P_0, P_1\right], \tag{13}$$

where expectations are Bayesian optimal.

- Equation $\int x_{j0} dj = 1 = \int x_{i1} di$ gives market clearing.
- $P_0 = \mathbb{E}[P_1 | P_0, s_j]$ gives the first order conditions for an interior optimum for the short term trader.
- Equation $P_1 = \mathbb{E}\left[D | P_0, P_1\right]$, the first order condition for the long term investors, says that the price that the long term investor is willing to pay is equal to their Bayesian updating of the dividend.
- Under these interior first order conditions our risk-neutral agents are indifferent about the amount of the asset they carry over, so for simplicity we may assume a symmetric equilibrium with

$$x_{i0} = x = 1, \text{ and } x_{i1} = x = 1$$

- Hence the market clearing condition holds automatically. In what follows, we only need to check equations (12) and (13) to verify an equilibrium

Proposition: $P_0 = P_1 = \exp(\theta)$ is always an equilibrium.
Proof: The proof is straightforward. It is easy to check that both (12) and (13) are satisfied.

- In this case, the market price fully reveals the fundamental values.
- Whatever their individual signal, traders in period 0 will be happy to trade at $P_0 = \exp(\theta)$, which reveals the dividend to investors in period 1.
- Even though each trader j in period 0 gets a noisy private signal s_j about θ, which may be high or low, these traders ignore their signal because in maximizing their utility they only care about the price at which they can sell next period.
- If each short term trader believes the price in the next period depends on θ, competition in period 0 will then drive the market price exactly to $\exp(\theta)$. (How? See Implementability Section.)

- For a given market price, the expected payoff of holding one additional asset will be $\mathbb{E}\left\{[\exp(\theta) - P_0] \,|P_0, s_j\right\}$.
- As long as $\log P_0 \neq \theta$, traders with low signals would want to short the risky asset while other traders with high signals would want to go long on the risky asset.
- An equilibrium can only be reached when the market price has efficiently aggregated all private information in such a way that idiosyncratic signals cannot provide any additional profits based on private information: namely $\log P_0 = \int s_j dj = \theta$.
- Since price fully reveals the dividend, the long term investors will be happy to pay $\log P_1 = \theta$ in the next period.

There is however a second equilibrium where the market price reveals no information about dividends.

- **Proposition:** $P_0 = P_1 = 1$ is always an equilibrium.
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- **Proof:** Both first order conditions (12) and (13) are satisfied. It is clear that with $P_0 = P_1 = 1$, investors in period 1 obtain no information about the dividend as the prices simply reflect the unconditional expectation of the dividends in period 2.
- Again in the above equilibrium, the short-term traders "optimally" ignore their private signals. If the short-term traders believe that the price in the next period is independent from θ, then their private signal s_j is no long relevant for their payoff, and these signals become irrelevant.

- We now assume the traders in period 0 also receive some sentiment or sunspot shock $z \sim N(0,1)$ which they believe will drive prices.

Definition

An sentiment-driven equilibrium is given by optimal portfolio choices $x_{j0} = x(P_0, s_j, z)$ for the short-term trader in period 0, $x_{i1} = y(P_0, P_1)$ for the long-term investors in period 1, and two price functions $\{P_0 = P_0(\theta, z), P_1 = P_1(\theta, z)\}$ that jointly satisfy market clearing and individual optimization,

$$\int x_{j0} dj = 1 = \int x_{i1} di, \tag{14}$$

$$P_0 = \mathbb{E}[P_1 | P_0, s_j, z] \tag{15}$$

for all $s_j = \theta + e_j$ and z, and

$$P_1 = \mathbb{E}\left[D | P_0, P_1\right], \tag{16}$$

where expectations are Bayesian optimal.

Proposition

Proposition: *There exists an continuum of sentiment driven equilibria indexed by* $0 \le \sigma_z \le \frac{1}{4}\sigma_\theta^2$, *with* $x_{i0} = 1 = x_{j1}$ *and the prices in two periods given by*

$$\log P_1 = \log P_0 = \phi\theta + \sigma_z z, \tag{17}$$

where

$$0 \le \phi = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\sigma_z^2}{\sigma_\theta^2}} \le 1. \tag{18}$$

Proof: Note that since the prices are the same in both periods, $P_0 = \mathbb{E}[P_1|P_0, s_j, z]$ is satisfied automatically. We only need to check if equation $P_1 = \mathbb{E}\left[D|P_0, P_1\right]$ is satisfied. Taking logs :

$$\begin{aligned}
\log P_1 &= \phi\theta + \sigma_z z \qquad (19)\\
&= \log \mathbb{E}\{\exp(\theta|\phi\theta + \sigma_z z)\},\\
&= \mathbb{E}\left[\theta|\phi\theta + \sigma_z z\right] + \frac{1}{2} var(\theta|\phi\theta + \sigma_z z)\\
&= -\frac{1}{2}\sigma_\theta^2 + \frac{\phi\sigma_\theta^2}{\phi^2\sigma_\theta^2 + \sigma_z^2}\left[\phi\theta + \sigma_z z + \frac{\phi}{2}\sigma_\theta^2\right]\\
&\quad + \frac{1}{2}\left[\sigma_\theta^2 - \frac{\left(\phi\sigma_\theta^2\right)^2}{\phi^2\sigma_\theta^2 + \sigma_z^2}\right].
\end{aligned}$$

which follows from the property of the normal distribution. Comparing terms, coefficients of $\phi\theta + \sigma_z z$ yields

$$\frac{\phi\sigma_\theta^2}{\phi^2\sigma_\theta^2 + \sigma_z^2} = 1. \qquad (20)$$

Solving equation (20) yields the expression of ϕ in the Proposition above.

- In this case traders in period 0 get a common sunspot shock z.
- The investors in period 1, in forming their expectation of D conditional on the prices, believe prices are affected by the sunspot z.
- However they have a signal extraction problem distinguishing θ from z.
- For example, a low z will induce pessimistic expectations for the period 0 traders, who will pay a low price for the asset and expect a low price next period.
- The investor in period 1 will observe the period 0 price and infer that in part, this must be due to a low dividend yield, which will lead him to also pay a low price in period 1, thus confirming the expectations of the period 0 trader.

- For equilibria to be possible for every realization of the sunspot z, the variance of z that enters the signal extraction problem of the investor in period 1 must lie in the interval given in the above Proposition.
- Investors' first order conditions will then be satisfied in equilibrium, generating a continuum of sunspot equilibria indexed by σ_z^2.

Alternative Information Structures

For expositional convenience, we denote Ω_0 and Ω_1 as the information sets of a particular short-term trader and the investor, respectively. The equilibrium conditions can then be written as

$$P_0 = \mathbb{E}[P_1|\Omega_0], \tag{21}$$

and

$$P_1 = \mathbb{E}\left[D|\Omega_1\right]. \tag{22}$$

We can now proceed to study alternative of the information sets Ω_0 and Ω_1.

Heterogenous but Correlated Sentiments

- If each short-term trader receives a noisy sentiment or sunspot shock z, then the information set Ω_0 for a particular trader becomes $\Omega_0 = \{P_0, \theta + e_j, z + \varepsilon_j\}$, where ε_j are drawn from a normal distribution with mean of 0 and variance of σ_ε^2 and $cov(e_j, \varepsilon_j) = 0$.
- Note again the sentiment or sunspot shocks are correlated across traders due to the common component z. Furthermore $\Omega_1 = \{P_0, P_1\}$ is the same as in our benchmark model of the previous section. In this case there again exist an continuum of sunspot equilibria indexed by the sunspot's variance as in the Proposition above.

The Investors and Market Signals on the Dividend and on Sunspots

- We first relax the assumption that only the short-term investor receives information about the dividend through a private signal.
- We allow both the short term trader and the investor to receive private information on the dividend θ. The information set is

$$\Omega_0 = \{P_0, \theta_0 + e_j, z + \varepsilon_j\} \text{ and } \Omega_1 = \{P_0, P_1, \theta_1 + v_i\}$$

- Here $s_{j0} = \theta_0 + e_j$ is the private signal on the dividend received by a trader j in the first period, and $s_{i1} = \theta_1 + v_i$ is the signal of the investor i in the second period.
- We assume that $cov(\theta_0, \theta) > 0$ and $cov(\theta_1, \theta) > 0$, but $cov(\theta_0, \theta_1) = 0$. For example,

$$\theta = \alpha\theta_0 + (1 - \alpha)\theta_1$$

with $0 < \alpha < 1$, but $cov(\theta_0, \theta_1) = 0$ satisfies these assumptions.
- Without loss of generality we assume $\theta = \theta_0 + \theta_1$, that $\theta_0 \sim N\left(-\frac{1}{2}\sigma^2_{\theta_0}, \sigma^2_{\theta_0}\right)$, that $\theta_1 \sim N\left(-\frac{1}{2}\sigma^2_{\theta_0}, \sigma^2_{\theta_0}\right)$ and $v_i \sim N\left(0, \sigma^2_v\right)$.

Proposition: The exists an continuum of sunspot equilibria indexed by $0 \leq \sigma_z \leq \frac{1}{4}\sigma^2_{\theta_0}$, where equilibrium prices are given by

$$\log P_0 = \phi\theta_0 + \sigma_z z, \quad (23)$$

$$\log P_1 = \phi\theta_0 + \sigma_z z + \theta_1, \quad (24)$$

where ϕ is given by

$$0 \leq \phi = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\sigma_z^2}{\sigma^2_{\theta_0}}} \leq 1. \quad (25)$$

- In this case, the market efficiently aggregates the private information of long term investors.
- The price in period 1 has to incorporate all the private information of the long term investors, otherwise some investors with high/low signals on the underlying dividend would attempt to profit by shorting/longing the assets.
- However, the period 1 price only partially incorporates the private information of the short term traders regarding the underlying dividend.
- These short-term traders benefit only from potential capital gains, and are risk neutral.
- As long as the expected return is equal to the risk free rate, they will not care whether the prices are driven by sunspots or by fundamentals and sunspot equilibria will continue to exist.

- We now further generalize the information structure by allowing the investors to receive signals on the sunspots as well as the dividends observed by the short term traders. We assume that

$$\theta = \theta_0 + d + \theta_1 \qquad\qquad z = z_0 + \xi + z_1.$$

- The information sets of the short term trader and the investor are

$$\Omega_0 = \{P_0, \theta_0 + d + e_j,\ z_0 + \xi + \varepsilon_j\}, \quad \Omega_1 = \{P_0, P_1,\ \theta_1 + d + v_i,\ z_1 + \xi + \zeta_i\}$$

- Private signals of short-term traders and investors are correlated.
- We are also allowing the investors, just like the short-term traders, to observe a noisy sunspot signal, correlated with the sunspot signals received by short-term traders. Here d and ξ are common information both for short term traders and long term investors.
- θ_0 and z_0 are in the private information sets of the short-term traders. Investors only learn about them from observing the market price.
- We assume z_0, ξ and z_1 are drawn from standard normal distributions and ζ_i are drawn from a normal distribution with mean of 0 and variance of σ_ζ^2.

- We now can show that there exists an continuum of sunspot equilibria indexed by $0 \leq \sigma_z \leq \frac{1}{4}\sigma_{\theta_0}^2$, with the prices

$$\log P_0 = \phi\theta_0 + \sigma_z z_0 + d, \quad (26)$$
$$\log P_1 = \phi\theta_0 + \sigma_z z_0 + d + \theta_1, \quad (27)$$

where ϕ is given by $0 \leq \phi = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\sigma_z^2}{\sigma_{\theta_0}^2}} \leq 1$.

- A conclusion we can draw is that the market is in general not fully efficient in aggregating the information of the short-term traders, even if the investors receive sunspot and dividend signals correlated with the private signals of short-term traders.
- As long as the short-term traders as a whole have some private information, there exists sunspot equilibria.

Alternative Market Structures

- In our benchmark model, only short term traders are present in period 0 market. We first relax that assumption. Suppose now that both short term traders and long term investors are present in period 0, but only long term investors are present in period 1. Our results are robust to incorporating investors with private information in the early stages of trading.

Multiple Assets and Price Co-Movements

- It is widely known that the traditional asset pricing models cannot explain why asset prices have a high covariance relative to the covariance of their fundamentals. (See Pindyck and Rotemberg, 1993; Barberis, Shleifer and Wurgler, 2005, and Veldkamp (2006).)
- We show that asset prices driven by the sentiment or sunspot shocks can exhibit high co-movements even if their underlying fundamentals are uncorrelated.
- The model is similar to the benchmark model above, but with multiple assets.

- For simplicity we consider two assets, a and b. The two assets yield final dividends in period 2 given by:

$$\log D_{2\ell} = \theta_\ell, \text{for } \ell = a, b. \tag{28}$$

- We assume that $\theta_\ell \sim N\left(-\frac{1}{2}\sigma_\theta^2,\ \sigma_\theta^2\right), \ell = a, b$. To highlight the co-movement, we assume that $cov(\theta_a, \theta_b) = 0$. For simplicity we consider representative agents in each period. The trader in period 0 solves

$$\max_{x_{0a}, x_{0b}} \sum_{\ell=a,b} \left\{\mathbb{E}[P_{1\ell}|\theta_a, \theta_b, P_{0a}, P_{0b}] - P_{0\ell}\right\} x_{0\ell}, \tag{29}$$

where x_{0a}, x_{0b} are the asset holdings of the trader for asset a and b, respectively. Here $P_{1\ell}$ and $P_{0\ell}$ are the asset $\ell's$ price in period 0.

- The investor in period 1 solves

$$\max_{\substack{x_{1a}\in(-\infty,+\infty),\\ x_{1b}\in(-\infty,+\infty)}} \sum_{\ell=a,b} \left\{\mathbb{E}[D_{2\ell}|P_{1a},P_{1b},P_{0a},P_{0b}] - P_{1\ell}\right\} x_{1\ell}, \tag{30}$$

where $x_{1\ell}$ are are the asset holding of investor in period 1 for asset a and b.

- The first order conditions are:

$$\begin{aligned} P_{0\ell} &= \mathbb{E}[P_{1\ell}|\theta_a,\theta_b,P_{0a},P_{0b}], && (31)\\ P_{1\ell} &= \mathbb{E}[D_{2\ell}|P_{1a},P_{1b},P_{0a},P_{0b}]. && (32)\end{aligned}$$

Since the agents are risk neutral, they will be indifferent in buying the asset or not. We will focus on the symmetric equilibrium again, namely an equilibrium with $x_{0\ell}=1, x_{1\ell}=1$ for $\ell=a$ and b.

Proposition: There exists a fully revealing equilibrium with $\log P_{0\ell} = \log P_{1\ell} = \theta_\ell$.

- Notice that in the fully revealing equilibrium the correlation of asset prices is zero and there is no co-movement of asset prices.

Proposition: There exists a continuum of equilibria with prices fully synchronized among assets. The asset prices take the form

$$\log P_{0\ell} = \log P_{1\ell} = \phi(\theta_a + \theta_b) + \sigma_z z + \frac{1}{2}\phi\sigma_\theta^2, \tag{33}$$

for $\ell = a$ and b. Here we have $0 \leq \phi \leq \frac{1}{2}$ and

$$\sigma_z^2 = \phi(1 - 2\phi)\sigma_\theta^2. \tag{34}$$

- The intuition is straightforward. The same traders trade the two assets, and therefore the asset prices can be determined by the same information set. If prices are driven not only by fundamentals but also by sentiments, then the sentiment shocks of the traders will drive both asset prices.

- It is straightforward to extend the information structure to allow any degree of co-movement.
- For example, we can assume that the total dividend of asset ℓ is given by $\theta_\ell + d_\ell$ and the information sets are

  $\Omega_0 = \{\theta_a + d_a, \theta_b + d_b, P_{0a}, P_{0b}, z\}$ and $\Omega_1 = \{d_a, d_b, P_{0a}, P_{0b}, P_{1a}, P_{2b}\}$

  Here d_ℓ is the common information of the dividend observed by both short term traders and long term investors. We assume that $cov(d_a, d_b) = 0$ and $\sigma_d^2 = cov(d_a, d_a)$. Hence the total dividend of these two assets are not correlated.
- Then we can construct equilibria with prices

$$\log P_{0\ell} = \log P_{1\ell} = \phi(\theta_a + \theta_b) + \sigma_z z + \frac{\phi \sigma_\theta^2}{2} + d_\ell. \qquad (35)$$

  where ϕ and σ_z are given by the Proposition above. If $\sigma_d^2 > 0$, then the asset prices do not co-move perfectly with each other.
- When $\sigma_d^2 / \sigma_\theta^2$ approaches infinity $\left(\sigma_\theta^2 \to 0\right)$, the correlation between the asset prices becomes zero. Therefore we can always set the value of σ_d^2 to fit the observed covariance of asset prices.

Multi-Period Assets

- We now extend our model to multiple periods.
- Suppose an asset created in period 0 yields a return or dividend only in period $T + 1$. Between period 0 and $T - 1$, a continuum of short term traders can trade the asset each period. Short term traders in each period hold the asset only for making capital gains.
- Given prices private signals that short-term traders receive do not matter, so we focus on a representative trader in each period. The final dividend is given by

$$\log D_{T+1} = \left(\sum_{t=0}^{T} \theta_t \right)$$

- Again we assume that $\theta_t \sim N\left(-\frac{1}{2}\sigma_\theta^2, \sigma_\theta^2\right)$ so the unconditional mean of D_{T+1} is given by 1.

- Denote the information set of traders in period $t = 0, 1, ..T$ as Ω_t, and their asset holding from period t to $t+1$ as x_t. Their maximization problem can be written as

$$\max_{x_t \in (-\infty, +\infty)} \left[\mathbb{E}\left(P_{t+1} | \Omega_t\right) - P_t\right] x_t, \tag{36}$$

for $t = 0, 1, ..T - 1$.
- Investors who purchase the asset in period T solve

$$\max_{x_T \in (-\infty, +\infty)} \left[\mathbb{E}\left(D_{T+1} | \Omega_T\right) - P_T\right] x_T. \tag{37}$$

- We assume short term traders in period t know θ_t, which may be interpreted as trader t's the private information regarding the final dividend of the underlying asset.
- Since all the agents observe the past and current price, their information Ω_t is given by $\Omega_t = \{\theta_t, z_t\} \cup \{\cup_{\tau=0}^{t} P_\tau\}$, where z_t are i.i.d draws from the standard normal distribution representing sentiment shocks of traders born period t as in our benchmark model.

Definition

An equilibrium is a set of price functions $\{P_t\}_{t=0}^{T}$ such that $x_t = 1$ solves (36) for $\tau = 0, 1, ..T - 1$ and $x_T = 1$ solves (37), where equilibrium conditions for individual optimization are given by

$$\mathbb{E}\left(P_{t+1}|\Omega_t\right) = P_t, \text{for } t = 0, 1, ..T - 1 \tag{38}$$

and

$$\mathbb{E}\left(D_{T+1}|\Omega_T\right) = P_T. \tag{39}$$

Proposition: $P_t = \exp(\sum_{\tau=0}^{t} \theta_\tau)$ for $t = 0, 1, ...T$ is always an equilibrium.

- In the above equilibrium, the price will eventually converge to the true fundamental price. The market is dynamically efficient in the sense that all private information is revealed sequentially by the market prices.

- However as in the benchmark model, the above is not the only equilibrium. Assume traders at each t also condition their expectations of the price P_t some sunspot shock z_t.
- We assume that θ_t, z_t are only observed by the traders at t. We have the following Proposition regarding equilibrium price.

Proposition: There exists a continuum of sentiment-driven or sunspot equilibria indexed by $0 \leq \sigma_z \leq \frac{1}{4}\sigma_\theta^2$, with the price in period t given by

$$\log P_t = \sum_{\tau=0}^{t} (\phi\theta_\tau + \sigma_z z_\tau), \tag{40}$$

for $t = 0, 1, 2, ... T - 1$ and

$$\log P_T = \theta_T + \sum_{\tau=0}^{T-1} (\phi\theta_\tau + \sigma_z z_\tau). \tag{41}$$

and

$$0 \leq \phi = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\sigma_z^2}{\sigma_\theta^2}} \leq 1. \tag{42}$$

- As in our benchmark model, the asset price only efficiently incorporate the investors' information.
- Prices are also driven by the sentiment of the short term traders.
- Note that equation (40) implies that the asset price follows a random walk in the sentiment-driven equilibria (look at the difference in prices).
- If an econometrician studies the asset price driven by the sentiment shocks in our model, he will conclude that asset prices are unpredictable, even though the sentiment shocks can generate permanent deviations of asset prices from their fundamental value.

Implementability

- So far we relied on rational expectations to characterize equilibrium prices and showed that fully revealing REE exist.
- Vives (2014) however raises the issue that a fully revealing rational expectations equilibrium may not be implementable.
- If short term traders ignore their private signal at fully revealing equilibrium prices, we are faced with the problem of how fully revealing equilibrium prices incorporating these private signals are in fact realized in the market. Since Vives' critique applies in our model, we study implementability of our equilibria.
- We explore two alternative approaces to "in equilibrium" Implementability.
- For simplicity we focus on our baseline three-period model the equilibria in other extended models can be implemented in the same way.

- First we note that the sentiment-driven equilibrium with $\log P_1 = \log P_0 = \phi\theta + \sigma_z z$ exists even if the signals are perfect, namely when $\sigma_e^2 = 0$ and $\sigma_\varepsilon^2 = 0$. To fix ideas, we first consider this simple special case.
- To construct well-defined demand functions with risk neutral agents, we assume that the trading is limited by $0 \le x_{j0} \le \bar{x}$, where $\bar{x} > 1$ is the maximum asset holding for all agents.

- Short-term traders believe the next period price is $\log P_1 = \phi\theta + \sigma_z z$, and submit demand functions to a market auctioneer:

$$x_{j0} = \left\{ \begin{array}{ll} \bar{x} & \text{if } \log P_0 < \phi\theta + \sigma_z z \\ 1 & \text{if } \log P_0 = \phi\theta + \sigma_z z \\ 0 & \text{if } \log P_0 > \phi\theta + \sigma_z z \end{array} \right\}, \tag{43}$$

- The only price that can clear the market is $\log P_0 = \phi\theta + \sigma_z z$. Note since the agents know θ and z perfectly in this case, the equilibrium price is implementable.

- After observing the price $\log P_0 = \phi\theta + \sigma_z z$, the long term investors submit their demand function to an market auctioneer according to

$$x_{i1} = \left\{ \begin{array}{ll} \bar{x} & \text{if } \log P_1 < \log P_0 = \phi\theta + \sigma_z z \\ 1 & \text{if } \log P_1 = \log P_0 = \phi\theta + \sigma_z z \\ 0 & \text{if } \log P_1 > \log P_0 = \phi\theta + \sigma_z z \end{array} \right\} \tag{44}$$

- Again the only price will clear the market, consistent with first order conditions is $\log P_1 = \phi\theta + \sigma_z z$. Since the long term trader can observe the first period price, the long term trader's demand function is well defined.
- This establishes that equilibrium with $\log P_1 = \log P_0 = \phi\theta + \sigma_z z$ is an indeed implementable rational expectation equilibrium.

- Now we turn to the case with $\sigma_e^2 > 0$.
- We assume that z is drawn from normal distribution with mean 0 and variance σ_z. Note that a fully revealing equilibrium with $\log P_0 = \log P_1 = 0$ that does not depend on sentiments is always implementable.
- Vives (2014) assumes that the valuation of assets by each trader has a common as well as a private component. Each trader receives a private signal that bundles the common and private valuations together.
- In that case, although the price fully reveals the common component, private signals are still useful for providing information on private valuations. As a result, the demand functions for the traders depend on private signals. The demand functions and information contained in the private signals can then be aggregated to obtain the equilibrium price.

- First we show that the mechanism proposed by Vives (2014) can also implement the equilibria in our economy.
- Suppose that the asset also yields a random idiosyncratic utility u_j drawn from a normal distribution with mean 0 and variance of σ_u^2 for the short term trader in period 1.
- One can think for example that the asset is a firm generating profits in both periods, and that the profits depend on the ability of the owner.
- If the short term traders' abilities are idiosyncratic and the long term investors' ability is homogenous, then we can generate payoffs for short-term traders that depend on idiosyncratic components.
- The short term traders receive signals $s_j = \theta + u_j + e_j$, and they solve the following problem:

$$\max_{x_{j0}} \mathbb{E}[(P_1 - P_0 + u_j)\, x_{j0} | \theta + u_j + e_j, P_0, z], \tag{45}$$

- The problem of the long term investors is the same as before.

- Then the price functions

$$p_0 = \log(P_0 - \bar{u}) = \phi\theta + \sigma_z z \tag{46}$$

$$\log P_1 = p_0 = \phi\theta + \sigma_z z \tag{47}$$

clear the market for any realization of the θ and z where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal and $\bar{u}$ solves

$$1 - \Phi(\bar{u}\frac{\sqrt{\sigma_u^2 + \sigma_e^2}}{\sigma_u^2}) = \frac{1}{\bar{x}}, \tag{48}$$

where short-term traders and investors have demand functions:

$$x_{j0} = \left\{ \begin{array}{ll} \bar{x} & \text{if } \frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}(s_j - \frac{p_0 - \sigma_z z}{\phi}) > \bar{u} \\ 1 & \text{if } \frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}(s_j - \frac{p_0 - \sigma_z z}{\phi}) = \bar{u} \\ 0 & \text{if } \frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}(s_j - \frac{p_0 - \sigma_z z}{\phi}) < \bar{u} \end{array} \right\} \tag{49}$$

$$x_{i1} = \left\{ \begin{array}{ll} \bar{x} & \text{if } \log P_1 < p_0 = \phi\theta + \sigma_z z \\ 1 & \text{if } \log P_1 = p_0 = \phi\theta + \sigma_z z \\ 0 & \text{if } \log P_1 > p_0 = \phi\theta + \sigma_z z \end{array} \right\} \tag{50}$$

- Alternatively, to implement the sentiment driven equilibria with a noisy signal, we can also follow the approach of Golosov, Lorenzoni and Tsyvinski (2014) by assuming decentralized trading in period 0.
- In particular, we can divide the initial period into N sub-periods and allow $N \to \infty$: the short term traders randomly meet and trade in the N periods. In so doing they learn information from other traders through their own trading history.
- We show that eventually, at the end of first period, all short-term traders become perfectly informed about θ. We have already shown that the equilibrium can be implemented in this case.
- Following Vives (2008)[1] and Golosov, Lorenzoni and Tsyvinski (2014), we construct an approach based on "in equilibrium" learning to implement the equilibria in our model.

[1]Chapter 7 of Vives (2008) presents an excellent summary on learning and convergence to a full information equilibrium.

Conclusion

- We study a market where sequential short term traders have private information and earn capital gains by trading a risky asset before it yields dividends, while uninformed investors purchase the asset for its dividend yield, forming expectations based on observed prices.
- In a rational expectation equilibrium, prices based on fundamentals can reveal the information of private traders.
- We show that there are also rational expectations equilibria driven by sunspots that do not fully reveal private information.
- We show that our results on sunspot equilibria are robust to a wide range of on informational assumptions and market structures.
- If an econometrician studies the asset data generated by these sunspot equilibria, they will find that the asset prices follow a random walk that look as if they are generated by an efficient market reflecting fundamental values.
- Future work may more closely explore the connections between sentiment-driven asset prices and macroeconomic fluctuations.