Catch-up and Fall-back

Jess Benhabib, NYU
Jesse Perla, NYU
Christopher Tonetti, NYU

Introduction-Questions

- This paper was motivated by some questions raised by this course last year and the papers of Perla-Tonetti and Lucas-Moll, among others.
- How to simultaneously model innovation, the technology frontier and technology diffusion when agents/countries optimally invest in fostering innovation and diffusion.
- In equilibrium will some agents/countries choose to invest in technology diffusion and catch-up, while others choose to invest in innovation? What does the BGP look like?
- Will some agents optimally choose instead to fall back relative to the moving frontier?
- Thus our title: Catch-up and Fall-back through Diffusion and Innovation.

Motivation: Catch-up

- Rapid, sustained growth of some emerging economies: often $5-10\%$ over the last decade.

- Growth can't all be from factor accumulation and innovation. Some must be from technology diffusion and catch-up.
- In 1977, after visiting a Samsung research lab in Korea as a consutant for the British government, Ira Magaziner wrote: "[It] reminded me of a dilapidated high-school science room...They'd gathered color televisions from every major company in the world—RCA, GE, Hitachi—and were using them to design a model of their own." Fifteen years later there would not be a UK owned TV manufacturer.

Motivation: Fall-back?

- Rapid growth of European countries in 1970s and 1980s slowing to stable ratio of GDP relative to the US
- Since mid 1990s, low innovation middle-upper income developed countries decreasing GDP ratio relative to US.
- "A growing productivity shortfall in the EU15's leading economies relative to the world's innovation leader—the United States—was also recorded. While in 1985, European productivity was 95% of the US levels, this figure fell to 85% in 1995. The growth slowdown in Europe can also be explained in terms of lagging innovation. ."

Golden Growth: Restoring the Lustre of the European Economic Model, Gill and Raiser, World Bank
http://www.bruegel.org/fileadmin/bruegel_files/Events/Write_ups/2012
up_golden_growth.pdf

Related Literature

- There is some empirical evidence suggesting rate of productivity growth depends on distance to the frontier (e.g. Benhabib and Spiegel (1994, 2005)).
- Early papers had some shortcomings in that economic variables related to diffusion were not subject to choice (e.g. Nelson and Phelps (1966))
- Later literature introduced models with investment decisions on innovation and "imitation" (e.g. Barro and Sala-i-Martin (1997), Aghion, Howitt, and Mayer-Foulkes (2005)) but had exogenous permanent types of innovators and "imitators"

Contribution

- Highly Stylized model with a distribution of agents endogenously choosing a *portfolio* of both diffusion and innovation investments. Agents are *identical* other than initial productivity.
- How precisely does the initial distribution matter?
- Catch-up or diffusion model that *nests* Nelson-Phelps, Gompertz, Logistic growth.
  - Productivity distribution influences the returns to diffusion through the distance to the frontier
- Two competing forces for the choice of investment intensity:
  - Agents have an incentive to *catch-up* in order to have higher productivity and hence consumption.
  - Agents may also have an incentive to *fall-back* in order to exploit the higher productivity of diffusion from being further behind the frontier

Summary of Model

- Log utility, linear production
- Continuum of agents heterogeneous only over productivity: $z(t)$, with CDF $\tilde{\Phi}(t, z)$
- Frontier, $F(t) \in (0, \infty)$, is highest productivity among agents
- Agents choose optimal portfolio in CRS growth technologies
  - Investments deterministically increase $z(t)$
  - Investment costs paid in forgone consumption
  - Invest s in diffusion and/or γ in innovation
  - Returns to innovation: geometric and autarkic
  - Returns to diffusion: geometric and depend on distance (sort of) to the frontier through catch-up function

Model: Growth Technologies

- Technologies to produce goods could consist of various processes, not all of which are instantly adopted. For example, production may start with an assembly plant and the manufacturing of parts. Associated processes may be gradually adopted as know-how develops and accumulates.
- The productivity of innovation is: σ
- The productivity of diffusion is: $\tilde{D}(t, z; m)$

$$\tilde{D}(t, z; m) \equiv \frac{c}{m}\left(1 - \left(\frac{z(t)}{F(t)}\right)^{m}\right)$$

- $m = 1$: logistic, $m = -1$: Nelson-Phelps, $m = 0$: Gompertz

$$\tilde{D}(t, z; m) \equiv c\left(\frac{F(t)}{z(t)} - 1\right) \quad if \;\; m = -1$$

$$\tilde{D}(t, z; m) \equiv c\left(1 - \frac{z(t)}{F(t)}\right) = c\frac{z(t)}{F(t)}\left(\frac{F(t)}{z(t)} - 1\right) \quad if \;\; m = 1$$

- Note: $\tilde{D}(t, F(t); m) = 0$ for all t, m.

Agent's Problem

The law of motion for productivity is a flow:

$$\frac{\dot{z}}{z} = \sigma\gamma + \tilde{D}(t, z; m)s$$

Given discount rate r, and production parameter B:

$$\max_{s,\gamma,z} \int_0^\infty \left(\ln\left(Bz - sz - \gamma z\right)\right) e^{-rt} dt$$

$$s.t. \quad \frac{\dot{z}}{z} = \sigma\gamma + \tilde{D}(t, z)s$$

$$s \geq 0,\ \gamma \geq 0$$

The Technology Frontier

- Agent's at the frontier: $z(t) = F(t)$
  - $\tilde{D}(t, F(t)) = 0$ for all t
  - Hence, they choose $s = 0$ and $\gamma \geq 0$

- **Assumption 1:** $\sigma B - r > 0$
  - Don't discount future so much that don't want to invest in growing.

- **Solution:** Given Assumption 1, the agent chooses:
  - $\gamma(t) = B - \frac{r}{\sigma}$, for all t
  - $\frac{\dot{z}}{z} = B\sigma - r \equiv g$, for all t
  - $F(t) = F(0)e^{gt}$

Thus, the frontier grows at a constant rate independent of diffusion technology

Followers

Agent's Problem in Relative Productivity

With $F(t)$ and g, do change of variables to $x = \frac{z}{F}$:

$$\frac{\dot{x}(t)}{x(t)} = \sigma\gamma(t) + D(x(t))s(t) - \frac{\dot{F}(t)}{F(t)}$$

Problem of an agent not at the frontier becomes:

$$\max_{s,\gamma,x} \int_0^\infty \left(\ln x + \ln(B - s - \gamma)\right) e^{-rt} dt + \frac{g + r\ln(F(0))}{r^2}$$
$$\frac{\dot{x}}{x} = \sigma\gamma + D(x)s - g$$
$$s \geq 0,\ \gamma \geq 0$$

Productivity of innovation is constant, while productivity of diffusion is constant *conditional* on x

The Diffusion Technology

$D(\cdot)$ is autonomous in relative productivity $x(t) \equiv \frac{z(t)}{F(t)}$

$$D(x) = \frac{c}{m}\left(1 - x^m\right), \text{ for } x \in (0,1)$$

$$D'(x) \equiv \frac{d}{dx} D(x) < 0, \text{ diffusion more productive the further behind}$$

Nelson-Phelps$(m = -1)$, Gompertz$(m \to 0)$, Logistic$(m = 1)$

- $\frac{d}{dm} D(x) < 0 \implies \uparrow m$ means less efficient diffusion

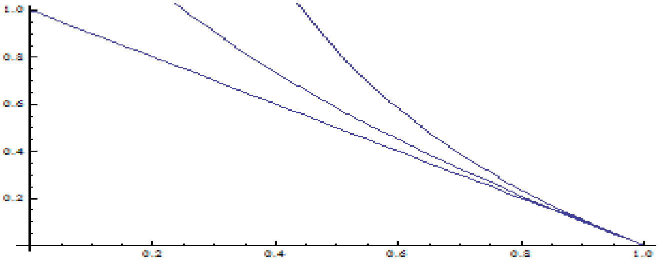


Figure: $D\left(x\right)$ as a function of x for various m

Innovation and Imitation

Simultaneous innovation and diffusion, $s > 0, \gamma > 0$, occurs only at a unique point, x^* (via the bang-bang solution). Equate FOCs:

$$\lambda x^* \sigma = \lambda x^* \frac{c}{m}\left(1 - x^{*m}\right)$$

$$x^* \equiv (1 - \tfrac{\sigma m}{c})^{1/m}$$

Assumption 2: $\frac{\sigma m}{c} < 1$

This assumption assures that $x^* > 0$: the efficiency parameter for technology diffusion, $\frac{c}{m}$ (that also depends on the distance to the technology frontier) is larger than the efficiency parameter for innovation, σ.

$$x^* = \left(1 - \frac{\sigma m}{c}\right)^{1/m}$$

$$x < x^* \implies s(x) \geq 0,\ \gamma(x) = 0$$

$$x > x^* \implies s(x) = 0,\ \gamma(x) > 0$$

Innovation Only

$s = 0, \gamma > 0$ occurs for $x^* < x < 1$
Dynamic and Stationary Solution:

- $\gamma(t) = \frac{B\sigma - r}{\sigma}$ for all t
- $\frac{\dot{x}}{x} = 0 \implies \frac{\dot{z}}{z} = g$
  - Hence, all innovating agents invest the same amount in innovation and grow at the same rate as the frontier
- $\lambda(t) = \frac{1}{x}\frac{1}{r}$
  - Constant shadow price of x, no future value from diffusion

Imitation Only

$s \geq 0, \gamma = 0$ occurs for $0 < x < x^*$
Unique Stationary Solution: $(\bar{x}, \bar{s}, \bar{\lambda})$

$$\bar{\gamma} = B - \frac{r}{\sigma}$$

$$\bar{\lambda} = \frac{1}{\bar{x}} \frac{1}{r + c\bar{s}\bar{x}^m} = \frac{1}{\bar{x}} \left(\frac{1}{r - \bar{s}\bar{x}D'(\bar{x})} \right) < \frac{1}{r}$$

$$\bar{s} = B - \frac{1}{c/m\bar{\lambda}\bar{x}(1 - \bar{x}^m)} = B - \frac{1}{\bar{\lambda}\bar{x}D(\bar{x})} > 0$$

$$\bar{x} = \left[\frac{1}{2} \left(\left(2 - \frac{m\sigma}{c} + \frac{m^2}{c}\left(\sigma - \frac{r}{B}\right)\right) - \mathrm{sign}(m) \left(\left(2 - \frac{m\sigma}{c} + \frac{m^2}{c}\left(\sigma - \frac{r}{B}\right)\right)^2 - 4\left(1 - \frac{m\sigma}{c}\right)\right)^{\frac{1}{2}} \right) \right]^{\frac{1}{m}}$$

Assumption 3: $m \geq -1$ and $\frac{c}{\sigma} < m + 2$
Assumption 3 ensures concavity of the Hamiltonian. Note that

both conditions in Assumption 3 restrict the efficiency of technology diffusion. Concavity may be lost as technology diffusion becomes too easy, with high c or low m.

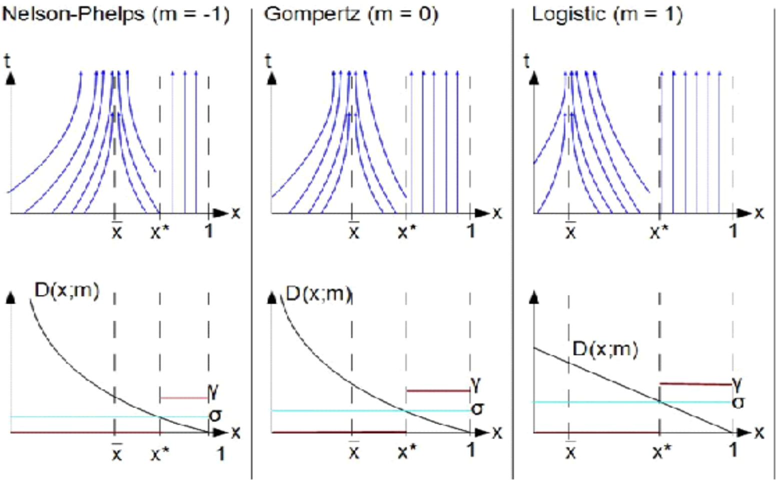
Dynamic Solution

Let Assumptions 1, 2, and 3 hold, then for arbitrary $\Phi(0, x)$

1. There exists a threshold ratio $x^* = \left(1 - \frac{m\sigma}{c}\right)^{1/m}$ such that all agents with $x > x^*$, including the leader, do only research. They set $s = 0$, $\gamma = B - \frac{r}{\sigma} > 0$, and grow at the rate $\sigma B - r$. For $x > x^*$, the distribution $\Phi(t, x) = \Phi(0, x)$ for all t.
2. All agents with initial conditions $x < x^*$ invest only in technology adoption with $s(t) \geq 0$ and $\gamma(t) = 0$. There exists a strictly positive BGP productivity ratio $0 < \bar{x} < x^*$ such that all initial productivity ratios $x < x^*$ converge to $\bar{x}$.

Example Dynamics Given $D(x; m)$ and σ

Nelson-Phelps (m = -1) Gompertz (m = 0) Logistic (m = 1)



Balanced Growth Path

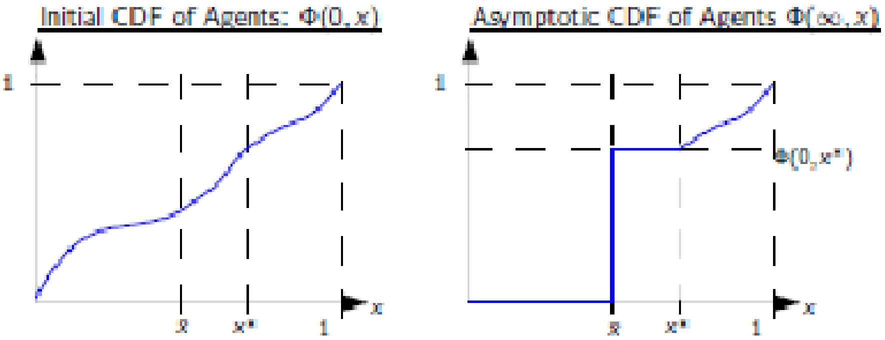
Corollary (2)

A balanced growth path equilibrium is a growth rate $g = \sigma B - r$, a steady state imitator productivity ratio $\bar{x}$, an innovator threshold x^*, and an asymptotic distribution $\Phi(\infty, x)$ such that

1. The growth rate of all agents is $\frac{\dot{z}}{z} = g = \sigma B - r$, so the growth rate $\frac{\dot{x}}{x} = 0$ for all x, t.
2. For arbitrary $\Phi(0, x)$,

$$\Phi(\infty, x) = \begin{cases} 0 & \text{for} \quad 0 \leq x < \bar{x} \\ \Phi(0, x^*) & \text{for} \quad \bar{x} \leq x < x^* \\ \Phi(0, x) & \text{for} \quad x^* \leq x \leq 1 \end{cases}$$

Balanced Growth Path Example



Summary of Comparative Dynamics of m

- Reminder: As $m \uparrow$ diffusion becomes less efficient: $\frac{d}{dm}D(x) < 0$

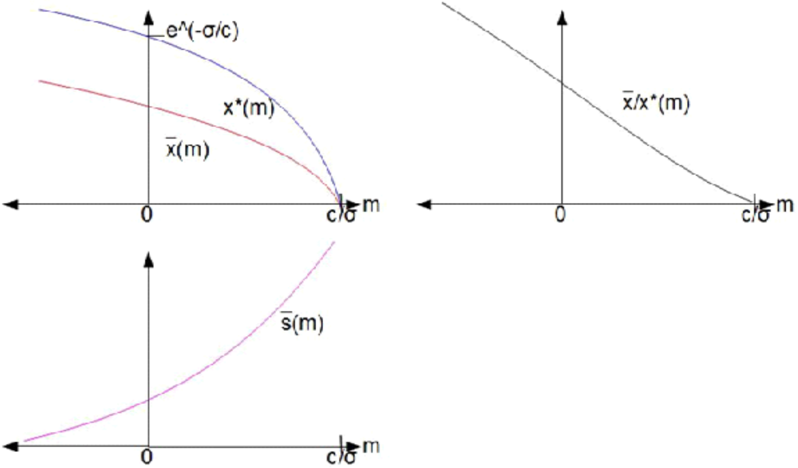
As technology diffusion becomes less efficient:

- A larger mass of agents become innovators and fewer choose to be imitators: $\frac{dx^*}{dm} < 0$.

And under some additional restrictions:

- Imitating agents optimally let the ratio of their productivity to the technology frontier slip lower and they benefit from the higher diffusion rate that comes from falling behind the frontier: $\frac{d\bar{x}}{dm} < 0$.
- Since $\bar{s}$ is increasing in m, imitators increase expenditures that aid technology diffusion even as they fall further behind the technology frontier: $\frac{d\bar{s}}{dm} > 0$

Numerical Example of Equilibrium Given m



Catch-up, Fall-back, and the Shadow Value of x

From the Dynamic Euler equation:

$$r = \frac{\dot{\lambda}}{\lambda} + \frac{1}{x} + (\sigma\gamma + sD(x) - g) + sxD'(x)$$

Euler equation: total benefit of a marginal $x =$ discount rate

- $\frac{\dot{\lambda}}{\lambda}$: Appreciation in the shadow price
- $\frac{1}{x}$: Marginal utility of x
- $(\sigma\gamma + sD(x) - g)$: Marginal product of x
- $sxD'(x) < 0$: Fall-back incentive

Equilibrium shadow price:

$$\bar{\lambda} = \frac{1}{\bar{x}}\left(\frac{1}{r - \bar{s}\bar{x}D'(\bar{x})}\right) < \frac{1}{r}$$

The Ratio of Marginal Productivities

Define $\theta \equiv \frac{\sigma}{c}$:

$$x^* = (1 - m\theta)^{1/m}$$

$$\bar{x} = \left[\frac{1}{2}\left(2+(m^2-m)\theta-\frac{m^2 r}{cB}-\text{sign}(m)\left(\left(2+(m^2-m)\theta-\frac{m^2 r}{cB}\right)^2-4(1-m\theta)\right)^{\frac{1}{2}}\right)\right]^{\frac{1}{m}}$$

Note that:

- The innovator threshold only depends on the ratio of marginal productivities
- The equilibrium imitator productivity depends on c separately
- $\frac{\partial D'(x;c)}{\partial c} = \frac{\partial^2 D(x;c)}{\partial x \partial c} < 0$, so as $c \uparrow$, the fall-back incentive $sxD'(x) \uparrow$

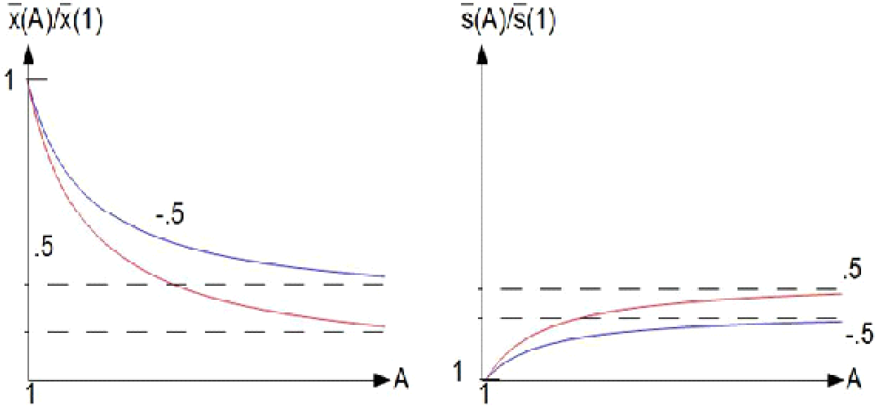
Comparative Statics with "Hicks-Nutral" Technology Change

Uniformly change the productivity of growth technologies

- Fix $\bar{\theta} = \frac{\bar{\sigma}}{\bar{c}}$
- Multiply both by a common A: $\sigma = A\bar{\sigma}$ and $c = A\bar{c}$
- This seemingly Hicks-neutral change distorts the equilibrium

1. $x^*(A) = x^*(1) = \left(1 - m\bar{\theta}\right)^{1/m}$, no distortion
2. $\frac{d\bar{x}(A)}{dA} < 0 \implies \bar{x}(A)$ is decreasing in A
3. $\frac{d\bar{s}(A)}{dA} > 0 \implies \bar{s}(A)$ is increasing in A
4. $\lim_{A\to\infty} \bar{x}(A) > 0$

Equilibrium with "Hicks Neutral" A



- Increasing Hicks-neutral progress causes fall back
- $A \uparrow \implies g = \bar{\sigma} AB - r \uparrow$, so the imitators have to invest more, $s \uparrow$, to keep up with the frontier.

Conclusion:Key Results

- The technology frontier, and the BGP productivity growth rate, depend only on the innovation technology
- Agents endogenously segment into innovators and imitators:
  - The proportion of innovators depends on the relative productivity of diffusion vs. innovation
- Agents may endogenously choose to "fall-back" to a lower productivity relative to the frontier
- As diffusion becomes less efficient imitators fall further behind the frontier on the BGP
- A seemingly Hicks-neutral technology increase:
  - Has no effect on the proportion of innovators
  - Decreases the equilibrium relative productivity of imitators

Motivation Revisited

- Rapid, sustained growth of some developing countries
  - "Catch-up" not surprising—we've seen it before
  - Historically, growth rates depend on distance to frontier
- European convergence to GDP ratio below unity and lower innovation
  - —Technology diffusion efficiency incentives
- "Fall-back "
  - Not neccesarily sub-optimal—

The Lucas-Moll Model-Baseline Model Described: Poisson arrivals

What happens in a meeting?
— Agent (a draw z from $F(z,t)$) meets another, z'.
— Not symmetric: z is active, learns from meeting, but z' is passive.
— If $z' \geq z$, nothing happens; if $z' \leq z$ agent z adopts z'
— Poisson Idea Arrivals: over time interval $(t, t+\Delta)$ meet another with probability $\alpha\Delta$
— More than one meeting with with probability $o(\Delta)$ (where the function $o(\Delta)$ satisfies $lim_{\rightarrow\infty} \frac{o(\Delta)}{\Delta} = 0$)

Now motivate differential equation for right cdf
$F(z,t) = \Pr\{\tilde{z} \geq z \text{ at date } t\}$

$$F(z, t+\Delta) = F(z,t) \times Pr\{\text{no lower cost idea darrives in } (t, t+\Delta)$$

$$\begin{aligned} Pr\{\text{no lower cost arrives in}(t, t+\Delta) &= Pr\{\text{no ideas arrive in } (t, t \\ &+Pr\{\text{one idea} > z \text{ arrives from } F(z,t) \text{ in } (\\ +Pr\{\text{more than one idea} &> z \text{ arrives in } (t, t+\Delta)\} \\ &= 1 - \alpha + \alpha\Delta F(t,z) + o(\Delta) \end{aligned}$$

Combine to get

$$F(z, t+\Delta) = F(z,t)\left[1 - \alpha\Delta + \alpha\Delta F(t,z) + o(\Delta)\right]$$

$$\frac{F(z,t+\Delta) - F(z,t)}{\Delta} = -F(z,t)\left[\alpha - \alpha F(z,t) - \frac{o(\Delta)}{\Delta}\right]$$

$$\frac{F(z,t+\Delta)-F(z,t)}{\Delta} = -F(z,t)\left[\alpha - \alpha F(z,t) - \frac{o(\Delta)}{\Delta}\right]$$

Let $h \longrightarrow 0$ to get

$$\frac{\partial F(z,t)}{\partial t} = -\alpha F(z,t)\left[1 - F(z,t)\right]$$

Now fix z, treat as ODE, solve given initial $F(z,0) = G(z,)$ and characterize.
For fixed z solution is:

$$F(z,t) = \frac{G(z)}{G(z) + e^{\alpha t}(1 - G(z))}$$

and if the initial distribution is bounded below at $\bar{z}$ so $G(\bar{z}) = 1$, then all mass converges down to $\bar{z}$.