Collateral Constraints and Multiplicity

Jess Benhabib   Pengfei Wang

New York University

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Introduction

- Firms and businesses face borrowing costs to finance their working capital that depends on the collateral value of their assets and output. Any positive shock that appreciates the value of a firm's collateral will decrease the cost of external finance, increase profitability and amplify the effect of the initial shock.
- This mechanism suggests the possibility of self-fulfilling multiple equilibria: Optimistic expectations of higher output may well lead increased lending to financially constrained firms.

Mechanism

- Even though our model has no increasing returns in production, the relaxation of the borrowing constraint implies that unit marginal costs can increase with output as firms compete hire more labor and capital. In such a case markups can become countercyclical and factor returns can increase sufficiently so that the expectation of higher output can become self-fulfilling.
- The purpose of this paper is to show that multiple equilibria and indeterminacies can easily arise in a simple financial accelerator model for realistic parameter calibrations, and that it can reasonably match some of the quantitative features of economic data.

- We introduce borrowing constraints into an otherwise standard Dixit-Stiglitz monopoly competition model. Firms rent capital and hire labor in the competitive markets to produce differentiated intermediate goods.
- The firms however may default on their promise or contract to repay their debt. We assume therefore that firms face borrowing constraints to finance their working capital, determined by the fraction of firm revenue and assets that the creditors can recover, minus some fixed collection costs. This constrains the output as well as the unit marginal costs of firms.

- Given the fixed collection costs however, if households expect a higher equilibrium output, they will be willing to increase their lending to firms, even if the marginal costs of firms increase and their markups decline as they compete for additional labor and capital.
- At the new equilibrium both output and factor returns will be higher. Despite the income effects on labor supply, the increase in wages associated with lower markups will allow employment and output to increase, so the optimistic expectations of higher output will be fulfilled.

Final good

- $$Y_t = \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i)di \right]^{\frac{\sigma}{\sigma-1}},$$

- The final goods producer solves

$$\max_{y_t(i)} \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i)di \right]^{\frac{\sigma}{\sigma-1}} - \int P_t(i) Y_t(i) di.$$

  where $P_t(i)$ the price of the i'th intermediate good.

- The first-order conditions lead to the following inverse demand functions for intermediate goods:

$$P_t(i) = Y_t^{-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}},$$

  where the aggregate price index is

$$1 = \left[\int P_t^{1-\sigma}(i)di \right]^{\frac{1}{1-\sigma}}.$$

The Financial Constraint.

- We assume that in the beginning of period, the ith intermediate goods firm decides to rent capital $K_t(i)$ from the households and hire labor $N_t(i)$. The firm promises to pay $w_t N_t(i) + r_t K_t(i) \equiv b_t(i)$ to the households.
- However the firm may default on their contract or promise. We assume that if the firm does not pay its debt $b_t(i)$, the households can recover a fraction $\xi < 1$ of the firm's revenue $P_t(i) Y_t(i)$ by incurring a liquidation cost f.
- One possibility is that the firm must pay the wages to labor as production takes place , and that creditors can always redeem the physical capital, but that the interest on borrowing may not be fully recoverable.

- So if the household can recover $\xi P_t(i)Y_t(i) - f$, they will lend to the firm only if $\xi P_t(i)Y_t(i) - f$ will at least cover the wage bill plus principal and interest.
- Knowing that the household can not recover more than $\xi P_t(i)Y_t(i) - f$, the firm will also has no incentive to repay more than $\xi P_t(i)Y_t(i) - f$. The incentive-compatiblity constraint for the firm then is:

$$\begin{aligned} & P_t(i)Y_t(i) - [w_t N_t(i) + r_t K_t(i)] \\ \geq \quad & P_t(i)Y_t(i) - [\xi P_t(i)Y_t(i) - f], \end{aligned}$$

or

$$w_t N_t(i) + r_t K_t(i) \leq \xi P_t(i)Y_t(i) - f.$$

Cost Minimization

$$Min\ r_t K_t + w_t N_t \qquad S.T.\ \ A K_t^a N_t^{1-\alpha} \geq Y_t$$

$$Max\ L = \ r_t K_t + w_t N_t - \phi_t \left(Y_t - A\ K_t^a N_t^{1-\alpha} \right)$$

FOC:

$$r_t = \alpha \phi_t A\ K_t^{a-1} N_t^{1-\alpha}; \qquad w_t = (1-\alpha)\, \phi_t A\ K_t^a N_t^{-\alpha}$$

where ϕ_t is the Lagrange multiplier interpreted as marginal cost, the marginal effect of relaxing the income constraint on marginal costs. Then

$$\left(\frac{r_t}{\alpha}\right)^{\alpha} = \left(\phi_t A\ K_t^{a-1} N_t^{1-\alpha}\right)^{\alpha}; \qquad \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} = \left(\phi_t A\ K_t^a N_t^{-\alpha}\right)^{1-\alpha}$$

Mulltiplying:

$$\begin{aligned}
\left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} &= \phi_t \left(A\ K_t^{\alpha(\alpha-1)+\alpha(1-\alpha)} N_t^{\alpha(1-\alpha)-\alpha(1-\alpha)} \right) = A\phi_t \\
\phi_t &= A^{-1} \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}
\end{aligned}$$

- After substituting $P_t(i)$, the profit maximization for the $i'th$ firm becomes

$$\max_{Y_t(i)} Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i),$$

subject to

$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}.$$

- Given w_t, r_t, final output Y_t, and the borrowing constraint, the feasible choice of $Y_t(i)$ can be illustrated by shaded area of Figure 1.

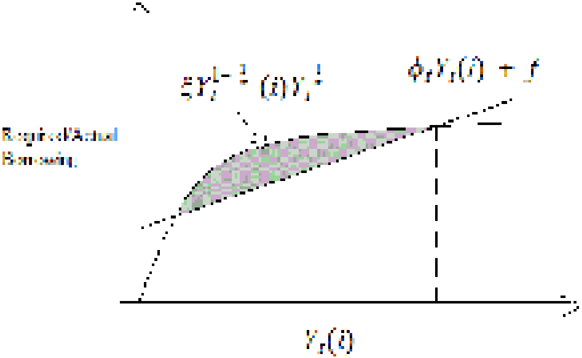


Figure 1. The Credit Constraints and Feasible Ouptut Choice.

Households

- $$\int_0^\infty [\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}] e^{-\rho t} dt,$$

  subject to

  $$\dot{K}_t = r_t e_t K_t - \delta(e_t) K_t + w_t N_t - C_t + \Pi_t,$$

- For simplicity we assume that the households choose the capacity utilization rate e_t. A higher e_t implies that the capital is more intensively utilized, at the cost of faster depreciation, so that $\delta(e_t)$ is convex increasing function.

- The first-order conditions for the consumer's optimization problem are given by equations:

$$\frac{\dot{C}_t}{C_t} = r_t e_t - \rho - \delta(e_t),$$

$$r_t = \delta'(e_t),$$

and

$$\psi N_t^{\chi} = \frac{1}{C_t} w_t.$$

- The equilibrium in the economy is a collection of price processes $\{w_t, r_t, P_t(i)\}$ and quantities $\{K_t(i), N_t(i), Y_t(i), Y_t, K_t, N_t, e_t, \Pi_t\}$, such that given the prices and the aggregate Π_t, the households choose K_t and N_t to maximize their utility;
- Given $P_t(i)$, the final goods firm chooses $\{Y_t(i)\}$ to maximize its profits
- Given w_t, r_t, and the financial constraint (the intermediate goods maximizes its profit by choosing $K_t(i)$ and $N_t(i)$; and all markets clear.
- Since firms are symmetric, we have $K_t(i) = K_t$, $N_t(i) = N_t$, $P_t(i) = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t K_t$.

- The budget constraint becomes

$$\dot{K}_t = Y_t - C_t - \delta K_t.$$

The wage w_t and the interest rate r_t are

$$w_t = (1-\alpha)\phi_t \frac{Y_t}{N_t},$$

and

$$r_t = \alpha\phi_t \frac{Y_t}{K_t}.$$

- Lemma: If $\xi(1-\frac{1}{\sigma}) < \phi_t < 1-\frac{1}{\sigma}$, then the final constraint binds; that is
$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}.$$
- Why? Marginal revenue $= 1-\frac{1}{\sigma} >$ Marginal cost $= \phi_t >$ Marginal Revenue in case of default $= \xi\left(1-\frac{1}{\sigma}\right)$
- First inequality says firms want to borrow more to expand output, second inequality says they cannot because what the household can recover in case of default does not cover marginal cost.
- Using the fact $Y_t(i) = Y_t$, the constraint implies
$$\phi_t = \xi - \frac{f}{Y_t}$$

- To summarize, the following system of equations fully characterize the equilibrium

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta(e_t), \\
\dot{K}_t &= Y_t - \delta(e_t) K_t - C_t, \\
\psi N_t^{\chi} &= \frac{1}{C_t} \phi_t \frac{(1-\alpha)\, Y_t}{N_t}, \\
Y_t &= A(e_t K_t)^{\alpha} N_t^{1-\alpha}, \\
\phi_t \frac{\alpha Y_t}{e_t K_t} &= \delta'(e_t) \\
\phi_t &= \xi - \frac{f}{Y_t},
\end{aligned}$$

subject to the constraint $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$.

- Let the depreciation function be given by $\delta(e_t) = \delta_0 \frac{e_t^{1+\mu}}{1+\mu}$.We then have

$$\phi \frac{\alpha Y}{eK} = \delta'(e) = \delta_0 e^{\mu}$$

We first solve deterministic steady state. We normalize δ_0 such that $e = 1$.

$$\delta(e) = \frac{1}{1+\mu}\phi\frac{\alpha Y}{K}$$

$$K = \frac{\mu}{1+\mu}\frac{\phi\alpha}{\rho}Y$$

$$e = \left[\frac{(1+\mu)\rho}{\mu\delta_0}\right]^{\frac{1}{1+\mu}}$$

$$\begin{aligned}
\frac{C}{Y} &= 1 - \delta(e)\frac{K}{Y} \\
&= 1 - \frac{\rho}{\mu}\frac{\mu}{1+\mu}\frac{\phi\alpha}{\rho} \\
&= 1 - \frac{\phi\alpha}{1+\mu}
\end{aligned}$$

$$N = \left[\phi\frac{(1-\alpha)}{\frac{C}{Y}}\frac{1}{\psi}\right]^{\frac{1}{1+\chi}}$$

$$Y = \left(\frac{\mu}{1+\mu} \frac{\phi\alpha}{\rho} \right)^{\frac{\alpha}{1-\alpha}} \left[\phi \frac{(1-\alpha)}{1 - \frac{\phi\alpha}{1+\mu}} \frac{1}{\psi} \right]^{\frac{1}{1+\chi}} \equiv Y(\phi)$$

Finally from the definition of $\phi = \xi - \frac{f}{Y}$, we have

$$f = (\xi - \phi) Y(\phi) \equiv \Psi(\phi)$$

to determine the steady-state value of ϕ. For the existence of a steady state however we will need to assume $\xi(1 - \frac{1}{\sigma}) < \phi < 1 - \frac{1}{\sigma}$. See diagram: f cannot be too high, or alternatively constrain ϕ.

$$
\begin{aligned}
\dot{c}_t &= \rho[\hat{Y}_t - \hat{K}_t + \hat{\phi}_t] \\
\dot{k}_t &= \frac{(1+\mu)\delta}{\alpha\phi}(\hat{Y}_t - \hat{K}_t) - \left(\frac{(1+\mu)\delta}{\alpha\phi} - \delta\right)(\hat{C}_t - \hat{K}_t) \\
& \quad -\delta(\hat{Y}_t + \hat{\phi}_t - \hat{K}_t) \\
\chi\hat{N}_t &= \hat{\phi}_t + \hat{Y}_t - \hat{N}_t - \hat{C}_t \\
\hat{Y}_t &= \alpha(\hat{K}_t + \hat{e}_t) + (1-\alpha)\hat{N}_t \\
\hat{e}_t &= \frac{1}{1+\mu}(\hat{\phi}_t + \hat{Y}_t - \hat{K}_t) \\
\hat{\phi}_t &= \frac{f/Y}{\xi - f/Y}\hat{Y}_t \equiv \gamma\hat{Y}_t
\end{aligned}
$$

where we use $\delta = \frac{\rho}{\mu}$.

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} \tag{1}$$

Using the factor $\rho = \delta\mu$, we have

$$J = \delta \begin{bmatrix} \frac{(1+\mu)}{\alpha\phi}\lambda_1 - (1+\gamma)\lambda_1 & \frac{(1+\mu)}{\alpha\phi}(\lambda_2 - 1) + 1 - (1+\gamma)\lambda_2 \\ \mu\left[(1+\gamma)\lambda_1 - 1\right] & \mu(1+\gamma)\lambda_2 \end{bmatrix}$$

- Proposition: Let γ and ϕ satisfy the following two constraints

$$(1+\gamma) > \frac{(1+\mu)(1+\chi)}{\alpha(1+\chi)+(1+\mu)(1-\alpha)}$$

and

$$1+\gamma \quad < \quad \min\Big(\frac{1+\mu}{\alpha}, \frac{\left(\frac{1+\mu}{\phi}\right)(1+\chi)}{\alpha(1+\chi)+(1+\mu)(1-\alpha)}, \frac{(1-\alpha)(1+\chi)}{(1+\mu)(1-\alpha)\frac{1}{1+\mu-\alpha\phi}+(1+\chi)\alpha}+1\Big)$$

Then

$$\mathit{Trace}\,(J) < 0, \det(J) > 0$$

- To gain intuition for self-fulfilling expectations of higher output and higher factor rewards, we first focus on labor demand and supply curves incorporating the equilibrium effects of the borrowing constraint on marginal costs and markups.
- The labor demand curve is given by

$$\hat{w}_t = (1+\gamma)\hat{Y}_t - \hat{N}_t \tag{2}$$

$$\hat{w}_t = \frac{\alpha\nu}{1+\nu-(1+\gamma)\alpha}\hat{K}_t + \left[\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1\right]\hat{N}_t \tag{3}$$

and the labor supply curve in the economy is

$$\hat{w}_t = \hat{C}_t + \chi\hat{N}_t \tag{4}$$

- This slope of the labor market demand curve is positive and steeper than the labor supply curve under the condition

$$(1+\gamma) > \frac{(1+\mu)(1+\chi)}{\alpha(1+\chi)+(1+\mu)(1-\alpha)}$$

.

- Unlike earlier works, our model has no increasing returns in the production technology. Instead indeterminacy arises from the borrowing constraints and their indirect effects on marginal costs through wages and the rental rate on capital. If households expect a higher equilibrium output, they will be willing to increase their lending to firms.

- Given the positive fixed collection costs f, an expected increase in output levels relaxes the borrowing constraint so that unit marginal costs of firms, $\phi_t = \xi - \frac{f}{Y_t}$, can rise and markups can decline.
- This implies that as firms compete for inputs, factor rewards will also with increase with Y_t. The labor demand curve incorporating these general equilibrium effects on marginal costs can now be positively sloped and steeper than the labor supply curve.
- Normally, higher output levels tend to increase the demand for leisure, so barring inferiorities in preferences, the higher demand for labor will be contained by the income effect on labor supply.

- However if the labor demand slopes up more steeply than labor supply, employment will increase robustly as the labor supply curve shifts to the left with income effects.
- The rise in labor hours as well as the accumulation of capital will raise output, so that optimistic output expectations of households will be self-fulfilling.
- We can show that these indeterminacy results can hold even in the absence of variable capacity utilization, but we include it into our model to improve calibration results in the next section.

- Figure 2 illustrates the combinations of f and ξ which may yield indeterminacy with the other parameters set at $\mu = 0.3$, $\alpha = \frac{1}{3}$, $\rho = 0.01$. The shaded areas are the feasible ξ and f that support a steady state equilibrium with $\xi^{\frac{\sigma-1}{\sigma}} < \phi < \frac{\sigma-1}{\sigma}$.

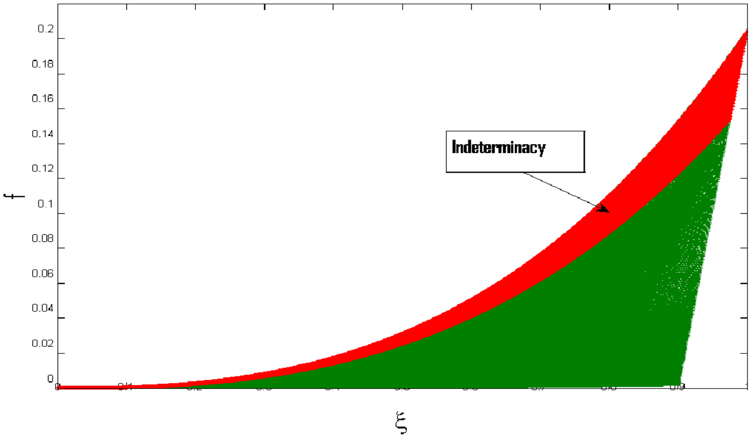


Figure 2. Parameter Spaces for Indeterminacy.

Calibrations

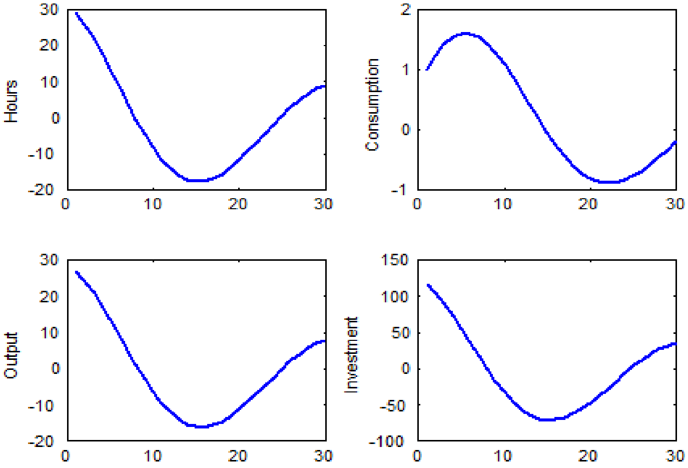
We write the model in discrete time and solve it by log-linearizing the equations that characterize the equilibrium around the steady state. We adopt standard parameterization: $\beta = \frac{1}{1+\rho} = 0.99$, $\alpha = 1/3$, $\delta = 0.033, \sigma = 10$ and $\mu = 0.3$. We set $\xi = 0.9768$, $f = 0.1908$ and fix the productivity level at $A = 1$. These parameter values imply steady state values $\phi = 0.88$ and $\gamma = \frac{f/Y}{\xi - f/Y} = 0.11$.

We begin without fundamental shocks. In the case of indeterminacy, the model's solution takes the form

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = M \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_{t+1} \end{pmatrix}$$

where M is a two by two matrix and $\varepsilon_{t+1} = \hat{C}_{t+1} - E_t \hat{C}_{t+1}$ is the sunspot shock. The remaining variables can be written as functions of $\hat{K}_t$ and $\hat{C}_t$:

$$\begin{pmatrix} \hat{Y}_t \\ \hat{I}_t \\ \hat{N}_t \\ \hat{e}_t \end{pmatrix} = H \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix}$$



Figre 3. Impulse Responses to a Consumption Shock.

- From Figure 3 we see that output, investment, consumption and hours comove.
- The impulse responses also demonstrate that labor is slightly more volatile than output, an important feature of the data that the standard RBC model has difficulty explaining with a T.F.P shock.
- The impulse responses also show cycles in output, investment, consumption and hours, so the model has the potential to explain the boom-bust patterns observed in many episodes in data.

- However, as in the models with increasing returns to scale, the extremely large impact of autonomous consumption on output and investment seems empirically unjustified.
- On the impact period, one percentage increase consumption leads to 27 percent increase in output and 116 percent increase in investment..

This volatile response of output and investment can be understood by studying the effect of consumption on labor. Equating the labor demand (2) and labor supply (4) we have

$$\hat{N}_t = \frac{1}{\frac{(1+\gamma)(1+\mu)(1-\alpha)}{1+\mu-(1+\gamma)\alpha} - 1 - \chi}\hat{C}_t. \tag{5}$$

where $\frac{(1+\gamma)(1+\mu)(1-\alpha)}{1+\mu-(1+\gamma)\alpha} - 1$ is the slope of the labor demand curve and χ is the slope of labor supply curve. When these two slopes are close, a one percentage point of autonomous consumption increase can lead to a huge increase in labor and hence output. Denote s as the steady state investment to income ratio. Then from the resource constraint,

$$s\hat{I}_t + (1-s)\hat{C}_t = \hat{Y}_t, \tag{6}$$

so it is clear that smooth consumption plus volatile income will make investment even more volatile as $s << 1$. In the current calibration $s = 0.23$. So it implies that response of investment at impact will be about 4.4 times that of output.

Table 1: Sample and Model Moments

	US Sample			Model		
var	σ_X/σ_Y	$corr_{XY}$	$corr_{X_t X_{t-1}}$	σ_X/σ_Y	$corr_{XY}$	$corr_{X_t X_{t-1}}$
Y	1.00	1.00	0.87	1.00	1.00	0.91
N	1.01	0.88	0.92	1.08	0.99	0.91
C	0.52	0.83	0.90	0.07	0.48	0.97
I	3.33	0.92	0.92	4.31	0.99	0.91
ϕ	0.32	0.16	0.70	0.11	1.00	0.91

Note: Variables (Y, N, C, I, ϕ) stands for output, labor (hours), consumption, investment and marginal cost respectively. The marginal cost in the data can be computed via $\phi = \frac{labor\ share}{1-\alpha}$. σ_X/σ_Y is the relatively standard deviation of variable X to output, $corr(X, Y)$ computes the correlation between X and output and $corr(X_t, X_{t-1})$ compute the first-order autocorrelation of X_t.

To better match the relative volatility of consumption and output we now introduce a TFP shock into the model. We assume that the technology level in the economy follows an AR(1) process

$$\hat{A}_{t+1} = \rho_a \hat{A}_t + \sigma_a \varepsilon_{at+1} \tag{7}$$

Following Benhabib and Wen (2004), we assume the sunspots shocks and technology shocks are correlated. Following King and Rebelo (1999) , we assume $\rho_a = 0.98$. We assume the technology shock ε_{at} and sunspots shocks ε_t are perfectly correlated and the relative volatility of sunspots and technology shocks is set to $\sigma_s / \sigma_\varepsilon = 1.5$. These bring the relative volatility of consumption closer to data. The moments with correlated TFP shocks and sunspots shocks are in Table 2.

Table 2: Moments with correlated TFP and Sunspot Shocks

	Model with Correlated Shocks			The RBC Model		
var	σ_X/σ_Y	$corr_{XY}$	$corr_{X_t X_{t-1}}$	σ_X/σ_Y	$corr_{XY}$	$corr_{X_t X_{t-1}}$
Y	1.00	1.00	0.98	1.00	1.00	0.95
N	0.96	0.95	0.99	0.53	0.73	0.90
C	0.37	0.55	0.99	0.62	0.86	0.99
I	3.38	0.96	0.98	2.65	0.88	0.91
ϕ	0.11	1.00	0.98	0	N.A	N.A

The RBC model refers to $f = 0$, *so* $\gamma = 0$, *in which we have* $\phi = \xi$ *is a constant. We select the parameter values in a way such that these two models have the same steady state. For the RBC model, we use TFP shocks with* $\rho_a = 0.98$ *as the only driving forces.*

Hump-Shaped output Dynamics

- We illustrate how our indeterminacy model can also predict some of the aspects of actual fluctuations that standard RBC models cannot explain, such as the hump-shaped, trend reverting impulse response of output to transitory demand shocks and the substantial serial correlation in the output growth rate in data (see Cogley and Nason(1995)).
- Since there is significant empirical evidence favoring demand shocks as a main source of business cycle, (e.g., see Blanchard and Quah(1989), Waston (1993), Cogley and Nason (1995) and Benhabib and Wen (2004), it is important to examine whether demand shocks can generate persistent business cycles.

- We consider two types of demand shocks as in Benhabib and Wen (2004): government spending shocks and preference shocks. With preference shocks the period-by-period utility function changes to $U = \exp(\Delta_t) \log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}$. We assume that the preference shocks Δ_t follows an AR(1) process, namely $\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_{\Delta t}$. With government spending, G_t, in period t, the resource constraint changes to $\dot{K}_t = Y_t - \delta(e_t) K_t - C_t - G_t$. We assume $\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_{gt}$. We choose $\rho_g = \rho_\Delta = 0.90$ as in Benhabib and Wen (2004).

- To highlight the effect of indeterminacy on the propagation mechanism of RBC models, we graph the impulse responses to a persistent government spending shock with and without indeterminacy in Figure 4. Figure 5 graphs the impulse response of the model to a persistent preference shock. For the model without indeterminacy we set $f = 0$ and reset $\xi = 0.88$ such that the model with and without indeterminacy have the same steady state.
- Several features of Figure 4 deserves particular mention.
- First, in the case $f = 0$, we have marginal cost $\phi_t = \xi$ is a constant. Hence the impulse responses of our model with financial constraints resemble these of a standard RBC model. Figure 4 and Figure 5 show that some difficulties of the standard RBC model in generating business cycle fluctuations. Figure 4 shows that consumption and investment move against each other after a positive government spending shock. An increase in government spending generates a negative wealth effect, which reduces both consumption and leisure. The decrease in leisure leads to an increase in output. An increase in output together with a decrease in consumption imply that investment has to increase.

- Second, even though the model generates comovement without indeterminacy under persistent preference shocks, the responses of output to such demand shocks is monotonic. Neither government spending shocks nor preference shocks can generate the hump-shaped output dynamics observed in the data. And this monotonic and persistent output responses to demand shocks mostly come from the persistence of shocks, not from an inner propagation mechanism. If one reduces the persistence of the shocks, the persistence of output responses will reduced accordingly.

- Third, when the model is indeterminate, the responses of output to both the government spending shocks and the preference shocks are dramatically changed.
- Figure 4 and Figure 5 clearly show persistent and hump-shaped responses of output to both shocks. In addition, this persistent response of output is not due to the persistence in shocks. As Figure 3 has already demonstrated, the model with indeterminacy can generate persistent fluctuations even under i.i.d shocks.
- Figure 4 and Figure 5 again highlight the similarity of our indeterminacy model with those based on increasing return to scale, so it has a similar ability to explain other puzzles. For example, Benhabib and Wen (2004) demonstrated that their indeterminacy model based on increasing returns to scale can explain the forcecastable-movement puzzle as pointed out by Rotemberg and Woodford (1996), namely that they are highly forecastable and comove.

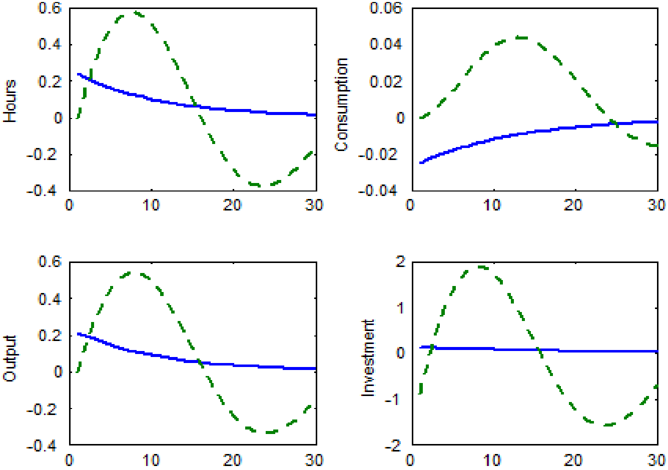


Figure 4. Impulse response to a government shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

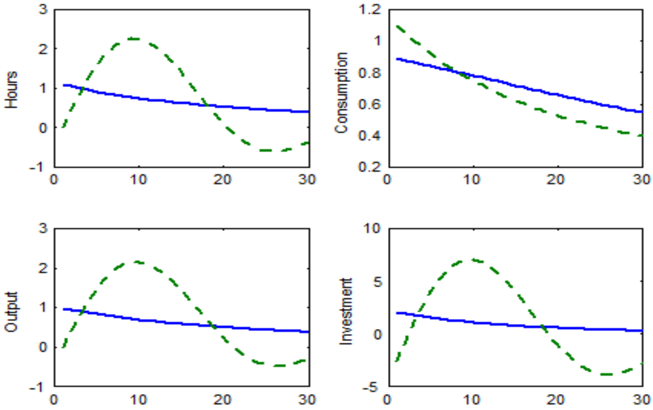


Figure 5. Impulse responses to a preference shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

We conclude that borrowing or collateral constraints can be a source of self-fulfilling fluctuations in economies that have no increasing returns to scale in production. Expectations of higher output can relax borrowing constraints, and firms can expand their output by bidding up factor prices and eliciting a labor supply response that allows the initial expectation to be fulfilled. The parameter ranges and markups where self-fulfilling expectations can occur are within realistic ranges and compatible with the data. Simulating our data we obtain moments and impulse responses that can reasonably match the US macroeconomic data.