Cozzi-Culture as a Bubble JPE-1998

"Is it reasonable to assume that the ongoing transmission of a seemingly useless stock of knowledge such as Aesop’s fables, Plato’s dialogues, the Bible, Mozart’s music, Euclid’s geometry, and so forth somehow enhances productivity growth? "

Lifetime utility is $U(c, c_0)$

s= saving

τ= fraction of time spent learning = 'years of schooling'

w= wage

p = price of teachers per unit of τ

x= labor productivity

Agents are price takers

$$max_{s \geq 0, \tau \in [0,1]} U(c, c_0)$$

subject to

$$c = wx(1 - \tau) - s - \tau p$$

$$c' = (1 + r)s + \tau p'$$

Note that τ is unproductive, its sole return to the worker is its resale value p'. The effective price of the asset τ is $p + wx$. You pay a teacher and it takes time to acquire culture.

The FOCs

$$U_1 = (1+r)U_2$$

$$U_1 = \frac{p'}{p+wx}U_2 \qquad (1)$$

imply if the economy is to accumulate both capital and culture, the rates of return to the two kinds of savings must be equal, which means that

$$\frac{p'}{p+wx} = 1+r \qquad (2)$$

Production function is CRS

$$Y = F(K, xL),$$

or letting $y = \frac{Y}{xL}, \quad k = \frac{K}{xL}$

$$y = F(k, 1) \equiv f(k)$$

Wages and rentals equal marginal products:

$$r = f'(k) \equiv r(k)$$

$$w = f(k) - kf'(k) \equiv w(k)$$

Normalize population to 1. Labor mkt equilibrium:

$$L = 1 - \tau$$

Moreover, capital next period equals savings out of output

$$s = k' \qquad (3)$$

Growth of productivity.–There is an external effect from aggregate schooling today to next-period productivity x' as follows:

$$x' = [1 + \gamma(\tau)]x \qquad (4)$$

where $\gamma' > 0$.

Equilibrium.–We will consider only equilibria in which τ and k are fixed. In that case the ratio in (2) is fixed which means that p/wx is constant, which means that p must grow at the same rate as x. But there is a continuum of such equilibria including the one in which $\tau = 0$ and x does not grow.

The labor input is $(1-\tau)x$ and grows at the same rate as x. Since K also grows at the same rate, Y grows at that rate as well and therefore the bubble does not overtake output.

With all these facts, the steady state reduces to just 2 equations: First, (2) implies the first equality in

$$\frac{1+\gamma(\tau)}{1+\frac{w(k)x}{p}} = 1+r(k) = \frac{U_1}{U_2} \tag{5}$$

where $r(k)$ and $w(k)$ are given above. The second equality is implied by the first FOC.

Suppose U is homothetic. Then $\frac{U_1}{U_2}$ depends only on the ratio

$$\begin{aligned}
\frac{c'}{c} &= \frac{wx(1-\tau) - s - \tau p}{(1+r)s + \tau p'} \\
&= \frac{\frac{w(k)x}{p}(1-\tau) - (1+\gamma(k))\frac{k}{p} - \tau}{(1+r)(1+\gamma(k))\frac{k}{p} + \tau\frac{p'}{p}} \\
&= \frac{\frac{w(k)x}{p(1+\gamma(k))}(1-\tau) - \frac{k}{p} - \frac{\tau}{(1+\gamma(k))}}{(1+r)\frac{k}{p} + \tau}
\end{aligned}$$

where the second equality uses (3) and the fact that k grows at the rate $\gamma(k)$, and the third the fact that p also grows at the rate γ.

Therefore (5) has 2 equations in 3 unknowns: (τ, p, k) and all other equations hold$\Big($ where $\gamma(\tau) = \gamma(k)\Big)$.

$$\frac{1+\gamma(\tau)}{1+\frac{w(k)x}{p}} = 1 + r(k) = \frac{\frac{w(k)x}{p(1+\gamma(\tau))}(1-\tau) - \frac{k}{p} - \frac{\tau}{(1+\gamma(\tau)}}{(1+r)\frac{k}{p} + \tau}$$

Therefore a continuum of constant-growth paths exist, being indexed by τ. From the equations above

$$1 + \gamma > 1 + r$$

i.e., the rate of growth exceeds the rate of interest .

• Culture is useless at the individual level but raises growth via the external effect.

• The whole thing could have been done with exogenous per capita growth instead of the mechanism in (4) ?.