"Is it reasonable to assume that the ongoing transmission of a seemingly useless stock of knowledge such as Aesop’s fables, Plato’s dialogues, the Bible, Mozart’s music, Euclid’s geometry, and so forth somehow enhances productivity growth? " 
Lifetime utility is $U(c, c_0)$

$s =$ saving

$\tau =$ fraction of time spent learning = ‘years of schooling’

$w =$ wage

$p =$ price of teachers per unit of $\tau$

$x =$ labor productivity

Agents are price takers

$$\max_{s \geq 0, \tau \in [0, 1]} U(c, c_0)$$

subject to

$$c = wx(1 - \tau) - s - \tau p$$

$$c' = (1 + r)s + \tau p'$$

Note that $\tau$ is unproductive, its sole return to the worker is its resale value $p'$. The effective price of the asset $\tau$ is $p + wx$. You pay a teacher and it takes time to acquire culture.
The FOCs

\[ U_1 = (1 + r)U_2 \]

\[ U_1 = \frac{p'}{p + wx} U_2 \quad (1) \]

imply if the economy is to accumulate both capital and culture, the rates of return to the two kinds of savings must be equal, which means that

\[ \frac{p'}{p + wx} = 1 + r \quad (2) \]

Production function is CRS

\[ Y = F(K,xL), \]

or letting \( y = \frac{Y}{xL}, \quad k = \frac{K}{xL} \)

\[ y = F(k, 1) \equiv f(k) \]
Wages and rentals equal marginal products:

\[ r = f'(k) \equiv r(k) \]

\[ w = f(k) - kf'(k) \equiv w(k) \]

Normalize population to 1. Labor mkt equilibrium:

\[ L = 1 - \tau \]

Moreover, capital next period equals savings out of output

\[ s = k' \quad (3) \]

Growth of productivity.—There is an external effect from aggregate schooling today to next-period productivity \( x' \) as follows:

\[ x' = [1 + \gamma(\tau)]x \quad (4) \]

where \( \gamma' > 0 \).
Equilibrium.–We will consider only equilibria in which $\tau$ and $k$ are fixed. In that case the ratio in (2) is fixed which means that $p/wx$ is constant, which means that $p$ must grow at the same rate as $x$. But there is a continuum of such equilibria including the one in which $\tau = 0$ and $x$ does not grow.
The labor input is \((1 - \tau)x\) and grows at the same rate as \(x\). Since \(K\) also grows at the same rate, \(Y\) grows at that rate as well and therefore the bubble does not overtake output.

With all these facts, the steady state reduces to just 2 equations: First, (2) implies the first equality in

\[
\frac{1 + \gamma(\tau)}{1 + \frac{w(k)x}{p}} = 1 + r(k) = \frac{U_1}{U_2}
\]

(5)

where \(r(k)\) and \(w(k)\) are given above. The second equality is implied by the first FOC.
Suppose $U$ is homothetic. Then \( \frac{U_1}{U_2} \) depends only on the ratio
\[
\frac{c'}{c} = \frac{wx(1 - \tau) - s - \tau p}{(1 + r)s + \tau p'}
\]
\[
= \frac{w(k)x}{p} \left( (1 - \tau) - (1 + \gamma(k)) \frac{k}{p} - \tau \right)
\]
\[
= \frac{w(k)x}{p(1+\gamma(k))} \left( (1 - \tau) - \frac{k}{p} - \frac{\tau}{(1+\gamma(k))} \right)
\]
\[
= (1 + r) \frac{k}{p} + \tau
\]
where the second equality uses (3) and the fact that $k$ grows at the rate $\gamma(k)$, and the third the fact that $p$ also grows at the rate $\gamma$. 

Therefore (5) has 2 equations in 3 unknowns: \((\tau, p, k)\) and all other equations hold where \(\gamma(\tau) = \gamma(k)\).

\[
\frac{1 + \gamma(\tau)}{1 + \frac{w(k)x}{p}} = 1 + r(k) = \frac{w(k)x}{p(1+\gamma(\tau))} (1 - \tau) - \frac{k}{p} - \frac{\tau}{(1+\gamma(\tau))} (1 + r) \frac{k}{p} + \tau
\]

Therefore a continuum of constant-growth paths exist, being indexed by \(\tau\). From the equations above

\[
1 + \gamma > 1 + r
\]

i.e., the rate of growth exceeds the rate of interest.

- Culture is useless at the individual level but raises growth via the external effect.
- The whole thing could have been done with exogenous per capita growth instead of the mechanism in (4).