Earnings Inequality and Other Determinants of Wealth Inequality

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I. Introduction

Increasing income and wealth inequality has led to renewed interest in understanding and explaining wealth and income distributions, and in particular the recent growth in their top shares (Piketty, 2014). The literature has largely emphasized the role of earnings inequality in explaining wealth inequality. Indeed, Bewley-Aiyagari economies, which focus on precautionary savings as an optimal response to stochastic earnings, represent the most popular approach of introducing heterogeneity into a representative consumer framework to study the distribution of wealth (see Benhabib and Bisin, 2017, for a survey).

However, models of earnings inequality and precautionary savings find it generally difficult to reproduce the thick right-tail of the wealth distribution observed in the data. In particular, these models cannot reproduce wealth distributions with substantially thicker right-tails (larger top shares) than earnings distributions. But, while comparable estimates of the statistical properties of wealth and earnings distributions are available only for a few countries, they invariably show that thicker wealth tails are a critical and robust feature of data. Consider to-date estimates of the tail index, a measure of the rate of decay of the right-tail of a distribution and hence a measure which is inversely related to its thickness:[1] Wealth and earnings indices are, respectively, $1.48 - 1.55$ and 2 in the U.S.; $1.63 - 1.85$ and 3 in Sweden; and $1.33 - 1.54$ and 2 in Canada.[2]

More specifically, in the context of Bewley-Aiyagari models, simulations tend to produce tail indices of wealth close to those of the distribution of labor earnings which has been fed into the the model. This is explicitly noted, for instance by Carroll, Slacalek and Tokuoka (2013). Similar results are obtained by De Nardi et al (2016), which argues that adopting the exceptional recently available earnings data from Guvenen, Karahan, Ozkan, and Song (2015) allows for a much better fit of the wealth distribution relative to the bottom 60% of agents, but generates too little wealth concentration at the top of the wealth distribution; and most recently by Hubmer, Krusell, and Smith (2017), which aptly concludes: "the wealth distribution inherits not only the Pareto tail of the earnings distribution but also its Pareto coefficient. Because earnings are considerably less concentrated than wealth, the resulting tail in wealth is too thin to match the data [...]."

Most importantly, in economic environments in which wealth accumulation is mainly driven by stochastic earnings, it is natural to expect a positive relationship between earnings and wealth inequality: higher earning risk tends to increase wealth accumulation via precautionary savings, thereby spreading the distribution which in turn, under borrowing constraints, tends to increase wealth inequality (Aiyagari, 1994). Interestingly, on the other hand, the cross-country data does not display a statistically significant correlation between inequality in earnings and wealth, indicating a significant role for other factors to drive the distribution of wealth. Consider Gini coefficients, the standard inequality measure (which can also be consid-

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[1] In the standard and simplest case of a Pareto distribution, whose cumulative is $F(x) = 1 - \left(\frac{x_m}{x}\right)^a$ for $x \in [x_m, \infty)$ and $x_m, \alpha > 0$, the tail index coincides with the exponent α. For a survey of power laws in economics, see Gabaix (2016).

[2] Wealth estimates: Vermeulen (2015), Table 8, for U.S.; Cowell (2011) for Canada and Sweden. Earnings estimates: Badel, Dayl, Huggett and Nybom (2016).

ered a proxy for the inverse of tail indices), reported in see Figure 1.

Indeed, the slope coefficient from a linear regression of wealth Gini on earnings Gini is 0.258, not statistically significant with a standard error of 0.296. Though only suggestive due to the paucity of data, we consider this as additional evidence that earnings inequality does not adequately explain wealth inequality.

II. A Theoretical Explanation

A simple but deep theoretical result is useful to understand why it is difficult to reproduce important statistical properties of the wealth distribution which are observed in the data with earnings inequality and precautionary savings alone.

Consider a linear individual wealth accumulation equation,

$$w_{t+1} = r_t w_t + y_t - c_t; \tag{1}$$

where w_t, y_t, c_t and r_t are wealth, earnings, consumption and rate of return at time t. We may assume $\{y_t, r_t\}$ are stationary stochastic processes.

Consider also a linear consumption function,

$$c_t = \psi w_t + \chi_t.$$

We can then write the wealth accumulation equation as

$$w_{t+1} = (r_t - \psi)\, w_t + (y_t - \chi_t)\,. \tag{2}$$

Suppose that r_t and $y_t > 0$ are both random variables, independent and *i.i.d* over time and independent of w_t. Suppose also that $\chi_t \geq 0$,[3] $0 < E(r_t) - \psi < 1$, and *prob* $(r_t - \psi > 1) > 0$ for any $t \geq 0$.[4]

The stationary distribution for w_t can be characterized by applying a theorem due to Grey (1994), extending results of Kesten (1973), to (2).

Theorem 1. *Suppose $y_t - \chi_t$ has a thick right-tail, with tail index $\beta > 0$. If $E\left((r_t - \psi)^\beta\right) < 1$, and $E\left((r_t - \psi)^\gamma\right) < \infty$ for some $\gamma > \beta > 0$, then the right-tail index of the stationary distribution of wealth will be β. If instead $E\left((r_t - \psi)^\gamma\right) = 1$ for $\gamma < \beta$, then the right-tail index of the stationary distribution of wealth will be γ.*

The Theorem makes clear that the right-tail index of the wealth distribution induced by the accumulation equation, (2), is either γ, which depends on the stochastic properties of returns, or β, the right-tail of $y_t - \chi_t$. With $\chi_t \geq 0$ the right tail of $y_t - \chi_t$ will be no thicker than that of than that of y_t.[5] In other words, it is either stochastic returns via the the accumulation process or skewed earnings which determine the thickness of the right-tail of the wealth distribution, not both.

Theorem 1 is of course obtained under very specific assumptions and, furthermore, pertains literally only to economies with linear consumption rules, that is, to very special microfoundations. Indeed infinitely-lived agent models with stochastic earnings and precautionary savings as in Aiyagari-Bewley economies generally display concave consumption functions. But assumptions can be substantially relaxed to allow for persistent (Markov-dependent) earning processes $y_t - \chi_t$, as well as for earnings and returns r_t which are correlated (see Ghosh et al., 2010 and Roitershtein, 2007). Also, with Constant Relative Risk Aversion preferences, the consumption function in this class of models becomes linear at high wealth levels and the Theorem applies asymptotically (Benhabib, Bisin and Zhu, 2016, for a rigorous exposition and proofs). Moreover, linearity obtains in a larger class of Overlapping Generations Economies (Benhabib, Bisin and Zhu, 2011).

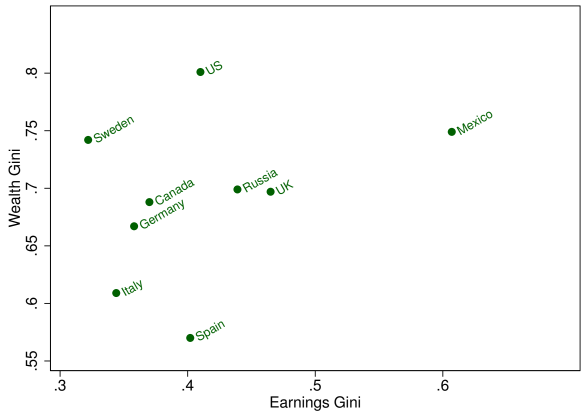
Even when holding as an approximation, the result does clearly point to the potential difficulty of matching the right-tail of wealth distribution by relying solely on earnings. First of all, since the distribution of wealth has a thicker tail than the dis-

[3] Note that χ_t will depend on the stochastic properties (i.e. the persistence and variance of its innovations) of the earnings process.

[4] Some additional regularity conditions are required; see Benhabib, Bisin, Zhu (2011) for details.

[5] This is because $y_t - \chi_t$ is a left shift of the earnings density y_t and if indeed it has a thick power tail, it must by its definition be decreasing in the right tail.

Figure 1. Earnings and Wealth Gini



Note: Wealth: Davies, Sandström, Shorrocks, and Wolff (2011). Earnings: special issue of the *Review of Economic Dynamics* (2010), titled "Cross-sectional facts for macroeconomists," edited by Krueger, Perri, Pistaferri, and Violante.

tribution of earnings, Theorem 1 directly suggests that the distribution of earnings cannot by itself explain the thick tail of the wealth distribution. In fact, the implications of the Theorem are even more striking: the distribution of earnings cannot even partially contribute to explain the thickness of the tail of the wealth distribution; the burden for explaining the thick tails of wealth distribution will have to rely on other factors, like stochastic idiosyncratic returns on wealth r_t. Second, if indeed it is other factors which are driving wealth inequality, then the lack of a a positive relationship between earnings and wealth inequality which we observe in the available cross-country data, Figure 1, is in fact not surprising at all.

III. Further Empirical Considerations

Theorem 1 also suggests an explanation why several studies which postulate extraordinarily high earnings states, originally to account for top-coding in earnings data, do in fact match the wealth distribution even if relying solely on earnings and precautionary savings as a determinant wealth accumulation. In fact, Theorem 1 suggests that, working with models in which earnings and precautionary savings are the main determinant of wealth accumulation, a much thicker distribution of earnings than the observed distribution is required to fit wealth data. This is exactly what the *awesome state* estimates, introduced with great success by Castañeda et al. (2003), effectively achieve.

More precisely, an *awesome state* is a state added to the observed stochastic process for earnings whose properties are estimated in order to better match the wealth distribution. Castañeda et al. (2003), in a rich overlapping-generation model with various demographic and life-cycle features, obtain estimates of the *awesome state* which requires the top 0.039% earners to have about $1,000$ times the average labor endowment of the bottom 61%. With the recent availability of earnings data which have not been top coded we can assess the reliability of this estimate. In fact, the ratio between even the top .01% and the median is at most of the order of 200 in the World Wealth and Income Database (WWID) by Alvaredo, Atkinson, Piketty, Saez, and Zucman (since 2011).[6] Similarly,

[6] We use WWID earnings data for 2014. The argument is not much changed even when considering average income, excluding capital gains.

Díaz et al. (2003) estimate a top 6% of the population to earn 46 times the labor earnings of the median, while the top 5% in WWID earns about 5 times the median.

To better account for wealth inequal-ity, and especially top wealth shares, we conclude, it is necessary to rely on other factors. Remaining close to the Bewley-Aiyagari environment, for instance, several papers exploit heterogeneous life-spans, adding death rates independent of age ("perpetual youth") to amplify wealth inequality (see Benhabib and Bisin, 2017, for a survey). In such a framework, however, standard calibrations of demo-graphics imply that a significant fraction of agents enjoy counter-factually long life-spans. With stochastic but realistic finite life-spans, these models fail to match the top shares of the wealth distribution (De Nardi et al, 2016, p. 44).

Theorem 1 suggests instead a role for stochastic idiosyncratic returns to wealth. Available evidence suggests that the idiosyncratic rate of return on wealth (cap-ital income) is composed in large part of returns to entrepreneurship (returns to pri-vate business equity).[7] Since a good mea-sure of these returns is generally hard to find, Benhabib, Bisin, Luo (2016) explicitly estimates the stochastic properties of the Markov process for returns to match the distribution of wealth.[8] Its conclusions are that stochastic idiosyncratic returns are essential for explaining the thickness of the wealth distribution.

Finally, other promising factors which possibly help explain the thick tail of the wealth distribution include non-homogeneous bequests (see De Nardi, 2004) and savings rates (increasing in wealth) as well as returns to wealth which are increasing in wealth (see Fagereng, Guiso, Malacrino and Pistaferri, 2015). Benhabib, Bisin, Luo (2016) find that all these are statistically significant in a model which includes also stochastic earnings as well as stochastic returns to wealth.

[7]See Quadrini (2000) and Cagetti and De Nardi (2006), and equivalently for stochastic discount factors, see Krusell and Smith (1998).

[8]Interestingly, the mean and standard deviation of estimated returns, 2.76% and 2.54%, respectively, closely match those estimated by Fagereng, Guiso, Malacrino and Pistaferri (2015) for the idiosyncratic component of the lifetime rate of return on wealth.

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