Indeterminacy Under Constant Returns to Scale in Multisector Economies*

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Abstract

The purpose of this paper is to characterize the possibility of indeterminacy in multisector growth models that exhibit constant marginal returns to scale at the social level, with empirically realistic small external effects. Our results demonstrate that indeterminacy does not require increasing returns to scale, large external effects, or close to linear utility functions. A small divergence between the social and private returns is sufficient for multiple equilibria.

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1 Introduction

Recently there has been a renewed interest in indeterminacy, or alternatively put, in the existence of a continuum of equilibria in dynamic economies that exhibit some market imperfections[1]. One of the primary concerns of this literature has been the empirical plausibility of indeterminacy, which arises in markets with external effects or with monopolistic competition, often coupled with some degree of increasing returns. While the early results on indeterminacy relied on relatively large increasing returns and high markups, more recently Benhabib and Farmer ([3]) showed that indeterminacy can also occur in two-sector models with small sector-specific external effects and very mild increasing returns. Nevertheless, a number of empirical researchers, refining the earlier findings of Hall ([9]) on disaggregated US data, have concluded that returns to scale seem to be roughly constant, if not decreasing[2]. While one can argue whether the degree of increasing returns required for indeterminacy in Benhabib and Farmer ([3]) falls within the standard errors of the recent empirical estimates, one may also inquire whether increasing returns are at all needed for indeterminacy to arise in a plausible manner.

In this paper we argue that in multisector models indeterminacy can arise as a type of coordination problem, even without increasing returns, if there is a small wedge between private and social returns. In simple one-sector models increasing returns, sustained in a market context by external effects or monopolistic competition, can create a coordination problem. In such a setting, if all agents were to simultaneously increase their investment in an asset above the level associated with the initial equilibrium, the rate of return on that asset would tend to increase, justifying the higher level of investment. In a multisector model however, the rates of return and marginal products depend not only on the stocks of assets, but also on the composition of output across sectors. The rate of return on an asset can increase with the stock of the asset even in the absence of increasing returns. For example, consider a two-sector

[1]See for example Benhabib and Farmer ([2]), Benhabib and Perli ([5]), Benhabib, Perli and Xie ([6]), Boldrin and Rustichini ([7]), Bond, Wang and Yip ([8]), Schmitt-Grohé ([11]), or Xie ([12]).

[2]See Basu and Fernald ([1]).

model with a pure consumption and a pure capital good. Increasing the relative price and hence the output of the capital good by moving along the production possibility frontier will increase the marginal product of the capital good if it is relatively more capital intensive. When combined with market distortions and external effects, the consequent rise in the stock of capital may not be enough to offset the initial increase of its marginal product. Both the stock and the marginal product of the capital good would rise simultaneously, mimicking the effect of increasing returns in the one-sector model. It is therefore possible to have constant aggregate returns in all sectors at the social level, and still to obtain indeterminacy with minor or even negligible external effects in some of the sectors. We illustrate this in the next section in the context of a two-sector endogenous growth model and discuss some extensions.

2 An endogenous growth model with non-linear utility

We consider an economy without fixed factors that exhibits unbounded growth. A representative agent optimizes an additively separable utility function with discount rate $(r-g) > 0$ and g is the depreciation rate. We have

$$Max \int_0^\infty U(c)\, e^{-(r-g)t} dt \tag{1}$$

subject to:

$$y_j = e_j \prod_{i=1}^{2} (x_{ij})^{\beta_{ij}}, \qquad j = 1, 2 \tag{2}$$

$$\frac{dx_1}{dt} = y_1 - g x_1 - c \tag{3}$$

$$\frac{dx_2}{dt} = y_2 - g x_2 \tag{4}$$

$$\sum_{j=1}^{2} x_{ij} = x_i \qquad\qquad i = 1,2 \tag{5}$$

where we assume that $\beta_{ij} > 0$.[3] We specify the utility function as $U(c) = (1-\sigma)^{-1} c^{1-\sigma}$, $\sigma \geq 0$. Note that as in one sector growth models, there is no pure consumption good in this model: the first good is both a factor of production and a consumption good. Production is subject to an external effect e_j, treated as a constant by the agent:

$$e_j = \prod_{i=0}^{2} x_{ij}{}^{b_{ij}}, \quad j = 1,2 \tag{6}$$

Therefore the true production functions are

$$y_j = \prod_{i=1}^{2} (x_{ij})^{\beta_{ij}+b_{ij}}, \qquad i = 1,2; \;\; j = 1,2 \tag{7}$$

where under constant social returns $\sum_{s=1}^{2} (\beta_{sj} + b_{sj}) = 1$.

We can write the Hamiltonian associated with the problem given by (1) as:

$$\begin{aligned} H &= U(c) + \bar{p}_1 \left(e_1 \prod_{i=1}^{2} (x_{i1})^{\beta_{i1}} - g x_1 - c \right) \\ &\quad + \bar{p}_2 \left(e_2 \prod_{i=1}^{2} (x_{i2})^{\beta_{i2}} - g x_2 \right) \\ &\quad + \sum_{i=1}^{2} \bar{w}_i \left(x_i - \sum_{j=1}^{2} x_{ij} \right), \end{aligned}$$

Here $\bar{p}_j$ and $\bar{w}_i$ are Lagrange multipliers, representing utility prices of the capital goods and their rentals, respectively. The static first order conditions for this problem are given by:

$$U'(c) = c^{-\sigma} = \bar{p}_1 \tag{8}$$

$$\overline{w}_s = \bar{p}_j \left(\beta_{sj} \prod_{i=1}^{2} (x_{ij})^{\beta_{ij}+b_{ij}} \right) (x_{sj})^{-1} \tag{9}$$

[3] We assume $\beta_{ij} > 0$, which assures that all inputs are used in the production of all goods, for computational and analytical simplicity. It is not difficult to relax this assumption but the notation becomes cumbersome.

for $j = 1, 2$ and $s = 1, 2$. Note that to derive equation (9), we use the fact that in equilibrium the inputs x_{ij} generating external effects e_j are identical to the inputs chosen by the firm.

Under constant returns the unit cost functions are independent of output levels and are invertible. Therefore factor rentals $\overline{w} = (\overline{w}_1, \overline{w}_2)$ are uniquely determined by output prices $\overline{p} = (\overline{p}_1, \overline{p}_2)$.

The laws of motion for problem (1) are given by (3), (4), where $y_i = y_i(x, \bar{p})$, and by

$$\left(\frac{d\bar{p}_i}{dt}\right) = r\bar{p}_i - \bar{w}_i(\bar{p}) \qquad i = 1, 2 \tag{10}$$

Constant social returns coupled with small external effects imply that some sectors must have a small degree of decreasing returns at the private level. This is in contrast to models of indeterminacy with social increasing, but private constant returns to scale. An implication of decreasing private returns is positive profits. Unless the number of firms is fixed, we must assume that there is a fixed entry cost to determine the number of firms along the equilibrium trajectories. As is clear from Proposition 1 below, the external effects and the degree of decreasing returns required for indeterminacy may be arbitrarily small, and generate only a small amount of profits. If the discounted value of profits along equilibrium trajectories that converge to the balanced growth path is small, a small fixed cost of entry will be sufficient to deter new entrants[4].

Let the growth rate of c and x_i along the balanced growth path be μ. It follows from equation (8) that prices must then decline at the rate $\sigma\mu$. We define discounted variables as

$$\chi_i = e^{-\mu t}x_i, \quad \pi_i = e^{\sigma\mu t}\bar{p}_i, \quad \psi_i = e^{-\mu t}y_i, \quad \omega_i = e^{\sigma\mu t}\overline{w}_i \tag{11}$$

Note that $e^{-\mu t}c = e^{-\mu t}(\bar{p}_1)^{-\frac{1}{\sigma}} = (\pi_1)^{-\frac{1}{\sigma}}$. Since there are no fixed factors, outputs y are homogenous of degree one in the stocks x, and homogenous of degree zero in

[4]The existence of a balanced growth path is easily proved, with small modifications to allow for external effects, along the lines of the proof in Bond, Wang and Yip ([8]) . To assure positive prices and quantities, a lower bound to the discount rate is required, as shown in footnote 5 below.

prices p, and the factor prices $\overline{w}$ are homogenous of degree one in prices . Then the equations (3), (4) and (10) can be written as:

$$\frac{d\chi_1}{dt} = \psi_1(\chi, \pi) - (g+\mu)\chi_1 - (\pi_1)^{-\frac{1}{\sigma}} \tag{12}$$

$$\frac{d\chi_2}{dt} = \psi_2(\chi, \pi) - (g+\mu)\chi_2 \tag{13}$$

$$\frac{d\pi_i}{dt} = (r+\sigma\mu)\pi_i - \omega_i(\pi) \quad , i = 1, 2 \tag{14}$$

The balanced growth path corresponds to the stationary point $(\chi_1^*, \chi_2^*, \pi_1^*, \pi_2^*)$ of the above system. The Jacobian of this system is given by:

$$J = \begin{bmatrix} \left[\frac{\partial\psi}{\partial\chi}\right] - (g+\mu) I & \left[\frac{\partial\psi}{\partial\pi}\right] - Z \\ 0 & (r+\sigma\mu) I - \left[\frac{\partial\omega}{\partial\pi}\right] \end{bmatrix}$$

where Z is a matrix with zeros except for the element of the first row and first column, which is $\left(\frac{1}{\sigma}\right)\pi_1^{-\frac{1}{\sigma}-1}$.

Using (9) and the social constant returns restriction $\sum_{t=1}^{2}(\beta_{sj}+b_{sj}) = 1$, we find that output prices satisfy

$$\bar{p}_j = \prod_{s=1}^{2}\left(\frac{\overline{w}_s}{\beta_{sj}}\right)^{\beta_{sj}+b_{sj}}, \qquad j = 1, 2 \tag{15}$$

or

$$\pi_j = \prod_{s=1}^{2}\left(\frac{\omega_s}{\beta_{sj}}\right)^{\beta_{sj}+b_{sj}}, \qquad j = 1, 2 \tag{16}$$

and unit input coefficients are:

$$a_{ij} = \frac{\beta_{ij} p_j}{\overline{w}_i} \tag{17}$$

$$= \frac{\beta_{ij}}{\overline{w}_i}\prod_{k=1}^{2}\left(\frac{\overline{w}_k}{\beta_{kj}}\right)^{\beta_{kj}+b_{kj}} \tag{18}$$

$$= \frac{\beta_{ij}}{\omega_i}\prod_{k=1}^{2}\left(\frac{\omega_k}{\beta_{kj}}\right)^{\beta_{kj}+b_{kj}}, \qquad i, j = 1, 2 \tag{19}$$

Let A be the input coefficient matrix with elements a_{ij}. Full employment of factors implies $A\psi = \chi$. Differentiating, we have:

$$Ad\psi + S\psi = d\chi \tag{20}$$

where the elements of the matrix S are $\left[\sum_{s=1}^{2} \frac{\partial a_{ij}}{\partial \omega_s} d\omega_s\right]$. In order to obtain the dual relationship of price and output in the context of externalities, we express the price function in terms of input coefficients and Cobb-Douglas exponents. Let

$$\hat{a}_{ij} = a_{ij}(\beta_{ij} + b_{ij})/\beta_{ij} \tag{21}$$

and define $\hat{A} = [\hat{a}_{ij}]$. Prices satisfy (16). Differentiating (16) and using (19) we obtain:

$$d\pi = \left[\hat{A}\right]' d\omega \tag{22}$$

Using equations (20) and (22) the Jacobian J becomes:

$$J = \begin{bmatrix} [A]^{-1} - (g+\mu) I & \left[\frac{\partial \psi}{\partial \omega}\right] - Z \\ 0 & (r + \sigma\mu) I - \left[\hat{A}'\right]^{-1} \end{bmatrix} \tag{23}$$

On a balanced growth path the full employment of factors, $A\psi = \chi$, implies:

$$[I - A(g+\mu)]\chi^* = z \tag{24}$$

where z is an 2 vector given by $(a_{11}\pi_1^*, a_{21}\pi_1^*)$. The price equations, $\pi = \left[\hat{A}\right]' \omega$, imply that on a balanced growth path:

$$\left[\left[\hat{A}\right]' (r + \sigma\mu) - I\right] \pi^* = 0 \tag{25}$$

Note that the above relation implies that the matrix $\left[\left[\hat{A}\right]' (r + \sigma\mu) - I\right]$ must be singular and it corresponds to the lower right submatrix of the Jacobian J. As expected this always yields root for J that is identically zero. The vector π will be determined up to a multiplicative constant, while equation (24) will determine χ. The vector χ pins down not the level of stocks x, but their discounted values, as is clear from equations (11). The same is true for π which does not pin down the prices $\bar{p}$, but their upcounted values. Thus on a growth balanced growth path quantities x, y, c grow at the rate μ, while prices $\bar{p}$ decline at the rate $\sigma\mu$.

To determine the growth rate μ, we note from equation (25) that the quantity $(r+\sigma\mu)$ corresponds to the inverse of the Frobenius root of the nonnegative matrix $\hat{A}'$. This is the only root that is associated with the positive eigenvector π^*.[5]

The signs of the roots of J are the same as those of the roots of $\left[[A]^{-1}-(g+\mu)I\right]$ and $\left[(r+\sigma\mu)I-\left[\hat{A}'\right]^{-1}\right]$. The system will be locally determinate if J has one negative root and two roots with positive real parts. Then we can choose the initial prices so that initial conditions lie on the two dimensional manifold spanned by the one dimensional center manifold corresponding to the balanced growth path, and the one dimensional stable manifold corresponding to the negative root[6]. If J has two roots with negative real parts however, the system will be indeterminate. In this case, depending on initial stocks, the initial prices can be chosen on the three dimensional space spanned by the one dimensional center manifold corresponding to the balanced growth path, and the two dimensional stable manifold corresponding to the two negative roots. For example, if $[A]^{-1}$ has one root with negative real part and $\hat{A}'$ has at least two real positive roots, the system will be indeterminate. In a multisector version of the model, the system will be locally indeterminate if $[A]^{-1}$ has $(n-1)$ roots with negative real parts and $\hat{A}'$ has at least two real positive roots (see Proposition 2

[5]Since the matrix $\hat{A}'$ is indecomposable, π^* is the unique nonnegative eigenvector of $\hat{A}'$ and the associated positive Frobenius root $\hat{\lambda}=(r+\sigma\mu)^{-1}$ is the largest root in absolute value. Since with positive externalities the elements of $\hat{A}$ are at least as large as those of A, the Frobenius root of $\hat{A}$ will be at least as large as that of A. From these observations it also follows that if $\hat{\lambda}^{-1}>g+\mu$, the inverse of $[I-A(g+\mu)]$ will be a positive matrix and assure, from equation (24) that $\chi>0$. The restriction on the discount rate then is $(1-\sigma)\left(\hat{\lambda}^{-1}-g\right)<r-g$. If some externalities are negative, the elements of $\hat{A}$ may be smaller than as those of A,but if the Frobenius root of A is less than μ^{-1} we can choose g less than r so that $[I-(g+\mu)A]^{-1}$ is a positive matrix.

[6]A standard alternative method to working with a system that has an identically zero root is to reduce the dimension of the system using ratios of stocks, $\frac{x_i}{x_1}$, as in Mulligan and Sala-i-Martin [10] or Benhabib and Perli [5].

below).

Let $B = [\beta_{si}]$ and $\hat{B} = [\beta_{si} + b_{si}]$. We assume:

Assumption 1: The matrices B and $\hat{B}$ are strictly positive and non-singular.

Let Ω denote the 2×2 diagonal matrix with diagonal elements $\omega_i,\ i = 1, 2$ and zero off-diagonal elements. Similarly let Π denote the 2×2 diagonal matrix with diagonal elements $\pi_i,\ i = 1, 2$ and zero off-diagonal elements. Note from (9) that $a_{ij} = \frac{p_j \beta_{ij}}{w_i}$, and therefore that $\hat{a}_{ij} = \frac{p_j(\beta_{ij} + b_{ij})}{w_i}$. It follows that $A = \Omega^{-1} B \Pi$ and $\hat{A} = \Omega^{-1} \hat{B} \Pi$, and from Assumption 1 that A and $\hat{A}$ will be non-singular.

Lemma 1 *: Along the balanced growth path the sign pattern of roots of B is the same as that of $A^{-1} = \Pi^{-1} B^{-1} \Omega$, and the sign pattern of roots of $\widehat{B}$ is the same as that of $\widehat{A}^{-1} = \Omega \left[\widehat{B}'\right]^{-1} \Pi^{-1}$.*

Proof. Along the balanced growth path where $\omega_i^* = (r + \sigma\mu)\, \pi_i^*$. The lemma follows from noting that $\mid A^{-1} \mid = \mid \Pi^{-1} B^{-1} \Omega \mid = (r + \sigma\mu)^2 \left|B^{-1}\right|$ and that every principle minor of $[\Pi^{-1} B^{-1} \Omega]$ of order i will be given by the corresponding principle minor of B^{-1} multiplied by $(r + \sigma\mu)^i$.If the characteristic equation of B^{-1} is $f(\lambda) = (-\lambda)^2 + b_1(-\lambda) + b_0 = 0$, the coefficients b_{n-i} will be the sum of principal minors of order i. Therefore, the characteristic polynomial of $[\Pi^{-1} B^{-1} \Omega]$ will have coefficients $(r + \sigma\mu)^i\, b_{2-i}$. If the characteristic equation of $[\Pi^{-1} B^{-1} \Omega]$ is given by $g(\nu) = 0$, then

$$\begin{aligned} (r + \sigma\mu)^{-2}\, g(\nu) &= (r + \sigma\mu)^{-2} (-\nu)^2 + (r + \sigma\mu)^{-1} b_1 (-\nu) + b_0 \\ &= f\left(v / (r + \sigma\mu)\right) \end{aligned}$$

Therefore if λ is a root of B^{-1}, then $\lambda / (r + \sigma\mu)$ is a root of $[\Pi^{-1} B^{-1} \Omega]$ and the sign pattern of the roots of B and B^{-1} is the same as that of $[\Pi^{-1} B^{-1} \Omega]$. The proof that the inertia of $\hat{B}'$ is the same as that of $\left[\Omega \left[\widehat{B}'\right]^{-1} \Pi^{-1}\right]$ is identical. ■

Proposition 1 below gives conditions for indeterminacy that are independent of the utility function. From lemma 1 the factor intensity difference $a_{22}/a_{12} - a_{21}/a_{11}$

is directly related to $\beta_{22}/\beta_{12} - \beta_{21}/\beta_{11}$. Therefore we may say that the consumable capital good (first good) is intensive in the pure capital good (second good) **from the private perspective** if $(\beta_{22}\beta_{11} - \beta_{21}\beta_{12} < 0)$, but that it is intensive in itself **from the social perspective** if $(\beta_{22} + b_{22})(\beta_{11} + b_{11}) - (\beta_{21} + b_{21})(\beta_{12} + b_{12}) > 0$. The proposition follows from noting the signs of the determinant and trace of the matrices B and $\hat{B}$.

Proposition 1 *In the two-sector endogenous growth model, if the consumable capital good is intensive in the pure capital good* **from the private perspective**, *but it is intensive in itself* **from the social perspective** , *then the balanced growth path is indeterminate.*

Proof. If the consumable capital good is intensive in the pure capital good from the private perspective, B has negative determinant. This implies that B^{-1} has negative determinant and one negative root. In this case $A^{-1} - (g + \mu) I$ has at least one negative root.

If the consumable capital good is intensive in itself from the social perspective, $\widehat{B}$ has positive trace and positive determinant. In this case $\widehat{B}$, and hence $\hat{A}'$ have two positive roots. One of the positive roots of $\hat{A}'$ is the Frobenius root $(r + \sigma\mu)^{-1}$ which has to be real, and the other one is smaller in modulus. Therefore the positive real root of $\left[\hat{A}'\right]^{-1}$ other than the inverse of the Frobenius root of $\hat{A}'$ will dominate $(r + \sigma\mu)$. On the other hand as the inverse of the Frobenius root $(r + \sigma\mu)$ is the root of $\left[\hat{A}'\right]^{-1}$, $\left[\hat{A}'\right]^{-1} - (r + \sigma\mu) I$ has one zero root and one negative root. Therefore J has one zero root and at least two negative roots. ■

The analysis of the model above can easily be recast in an n-sector framework. In a multisector model, the matrices B and $\hat{B}$,composed of the Cobb-Douglas exponents, will be of higher dimension than two, with $i,\ j = 1, ...n$. Indeterminacy will now follow if 2n-dimensional matrix J has less than n roots with positive real parts. Then the proposition above generalizes as follows:

Proposition 2 *In the multisector endogenous growth model, if the matrix B has $(n-1)$ roots with negative real parts and the matrix $\hat{B}$ has at least two roots with positive real parts, the system will be indeterminate.*

Proof. From Lemma 1, at the steady state the root structure of the n-dimensional input matrices A and $\hat{A}$ corresponds to the root structure of B and $\hat{B}$. The system will be indeterminate if the now 2n-dimensional matrix J has less than n positive roots. This will happen if $[A]^{-1}$ has $(n-1)$ roots with negative real parts and $\left[\hat{A}'\right]$ has at least two real positive roots. Since one of the positive real roots of $\hat{A}'$ is the Frobenius root $(r+\sigma\mu)^{-1}$, all the other roots are smaller in modulus. Therefore the real positive roots of $\left[\hat{A}'\right]^{-1}$ other than the Frobenius root will dominate $(r+\sigma\mu)$. Therefore J will have at least $n+1$ roots with negative real parts. ■

The result of Proposition 1 holds exactly in a two-sector non-endogenous growth model with fixed labor supply if the first good is a pure consumption good, the second good is a pure capital good, and if utility is linear (see Benhabib and Nishimura [4]). The reason that we need to resort to linear utility in a two-sector model with fixed labor and a pure consumption good is simple. The existence of multiple equilibrium paths implies that for a given level of the capital stock, there is a continuum of ratios of initial investment to consumption that are consistent with equilibrium. However some curvature in the utility function may destroy the possibility of multiple equilibria if the cost of forgoing current consumption is large relative to future benefits that come from higher initial investment. We can regain some flexibility if the first good is both a consumption good and a capital good, as in the one sector model. In that case increasing the investment level in the second (pure capital) good does not solely come at the expense of consumption. However if we stay with our previous setup, we would end with two goods and three factors, one of which is labor. This would significantly complicate the analysis. Switching to an endogenous growth model without a fixed factor avoids this difficulty by making the number of goods and factors equal.

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