Immigration and Median Voter Policy

Individuals are indexed by the units of capital that they own, $k$. The density of individuals is given by the continuous density function $N(k)$, defined on $[0, \infty)$. The initial capital stock, $K(0) = K_0$, is given by

$$K_0 = \int_0^\infty N(k)\,dk$$

and the initial population size, $L(0) = L_0$, is

$$L_0 = \int_0^\infty N(k)\,dk$$

The median type, $k_m$, is the solution to the equation

$$\frac{\int_0^{k_m} N(k)\,dk}{L_0} = 0.5$$

The density function of potential (one shot) immigrants is $I(k)$. 
An immigration policy $P(s, q)$ restricts the types of potential immigrants to those with $s \leq k \leq q$, so that the total immigration flow is $\int_s^q I(k)dk$. Of course, $s, q \in [0, \infty)$. The post-immigration capital-labor ratio is given by

$$R(s, q) = \frac{K_0 + \int_s^q I(k)kdk}{L_0 + \int_s^q I(k)dk}$$

Let $R_0 = R(s, s)$.

We assume a constant-returns neoclassical production function $F(K, L) = Lf(K/L)$. The competitive wage rate, as usual, is $w = f(K/L) - f'(K/L)K/L$. The interest rate is $r = f'(K/L)$. 
Let \( k_i \) be the type indifferent to immigration policy \( P(s, q) \), so that her pre-immigration and post-immigration incomes are identical. Then
\[
\begin{align*}
f(R_0) - f'(R_0)R_0 &+ f'(R_0)k_i \\
&= f(R^s_q) - f'(R^s_q)R^s_q + f'(R^s_q)k_i
\end{align*}
\]
Adding and subtracting \( f'(R^s_q)R_0 \) to the right of 5 and rearranging, we obtain
\[
\begin{align*}
f(R_0) - f(R^s_q) - f'(R_0)(R_0 - R^s_q) \\
&= -f'(R^s_q)R^s_q + f'(R^s_q)k_i - f'(R_0)k_i + f'(R_0)(R^s_q) \\
&= (f'(R^s_q) - f'(R_0))(k_i - R^s_q)
\end{align*}
\]
(6i)
Proposition 1:
a) The average person of type $k = R_0$ obtains a higher income than his pre-immigration income under any immigration policy $P(s, q)$ if $R_0 \neq R^s_q$.

b) If $R^s_q < R_0$, then $R^s_q < k_i < R_0$ and all natives of type $k > k_i$ have a higher post-immigration income under policy $P(s, q)$;

c) If $R^s_q > R_0$ then $R^s_q > k_i > R_0$ and all natives of type $k < k_i$ have a higher post-immigration income under policy $P(s, q)$;

d) If we assume that natives vote against an immigration policy that reduces their income, then a policy $P(s, q)$ will be defeated in a referendum if

(i) $k_m < k_i$ when $R^s_q < R_0$

(ii) $k_m > k_i$ when $R^s_q > R_0$

e) Any immigration policy $P(s, q)$ such that $R_0 \neq R^s_q$ will be approved if the median
wealth, $k_m$, is sufficiently close to the average wealth, $R_0$. 
Proof: (b) and (c) follow since the left-hand side of (6.i) is negative and the left-hand side of (6.ii) is positive under the strict concavity of $f$. (a) follows from (5) if we replace $k_i$ with $R_0$ and note that the strict concavity of $f$ implies that the left-hand side is smaller than the right-hand side. (d) is obvious and immediate. (e) is obvious from inspecting (d.i), (d.ii) in conjunction with (b) and (c).
Median Voter Choice

Next we would like to investigate which immigration policy would defeat any other policy in a pairwise contest under majority voting. We first characterize the policies for which the highest and the lowest post immigration capital-labor ratios are obtained.

Some elementary algebra shows that the maximum $R(s, q)$ is attained by $R(s_-, \infty)$ where $s$ is the highest $s_-$ such that $R(s, \infty) = s$. That is we can start by permitting the immigration from the highest end of the distribution $I(k)$ and reduce $s$ until the average capital-labor ratio $R(s, \infty)$ attained is equal to the marginal capital-labor ratio $s$. Any $s > s_-$ obviously raises the capital-labor ratio and must be allowed to immigrate in order to attain the maximum $R(s, q)$. Similarly to minimize the capital-labor ratio we must start at the bottom to allow the immigration of all types.
$[0, q^-]$, where $q^-$ is the lowest $q$ such that $R(0, q) = q$. Of course if $R_0 = 0$, $q^- = 0$. 
Consider now the preferred immigration policy of an arbitrary type $k_p$.

We define the maximum attainable $R$ as $\hat{R} = R(s, \infty)$ and the minimum attainable $\tilde{R}$ as $R = R(0, \infty)$.

The preferred capital-labor ratio $R$ of type $k_p$ is the solution to:

$$\max_{R \in [\tilde{R}, \hat{R}]} f(R) - f'(R)R + f'(R)k_p$$

The objective function is convex with a minimum at $k_p = \hat{R}$. Thus type $k_p$ either prefers $\tilde{R}$ or $\hat{R}$. The population will be polarized between those who prefer $\hat{R}$ and therefore immigration policy $P(s, \hat{R})$ to all other policies, and those who prefer $\tilde{R}$, and therefore immigration policy $P(0, \tilde{R})$ to all other policies.
To sort out what the majority prefers consider the type $k_I$ who is indifferent between $\tilde{R}$ and $\hat{R}$. We have

$$f(R_0) - f'(R_0)R_0 + f'(R_0)k_i$$

$$= f(R_q^s) - f'(R_q^s)R_q^s + f'(R_q^s)k_I$$

Constructing the analogues of equations (6) with $\tilde{R}$ and $\hat{R}$ replacing $R_0$ and $R_q^s$, it is easy to show that $\tilde{R} < k_I < \hat{R}$. It follows that all types $k < k_I$ prefer $P(s_\cdot, \infty)$ and types $k > k_I$ prefer $P(0, q^-)$. We can summarize these points in Proposition 2.

Proposition 2: If $k_m < k_I (k_m > k_I)$, and voters care only about their income, $P(s_\cdot, \infty) (P(0, q^-))$ defeats all other immigration policies under majority voting with pairwise alternatives.