Age, Luck, and Inheritance

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Motivations

- We set up a parsimonious model and solve it analytically to match the wealth distribution in terms of
  - Gini coefficient
  - Right skewness
  - Heavy upper tail
  - Double Pareto distribution

- We disentangle the contribution of luck, age and inheritance for the wealth inequality.
U.S. Wealth Distribution

- Survey of Consumer Finances, 2004
Mechanism to produce a Pareto distribution

- Pareto distribution
  \[ f(x) \sim x^{-\beta} \quad x \geq x^* \]

- A standard mechanism to generate Pareto distribution is to construct a stochastic process with negative drift and a lower reflecting barrier, as in Cantelli (1921), or Champernowne (1953), or using a birth and death process as in Wold and Whittle (1957).
A Pareto Distribution looks like this:

\[ y = 1.6x^{-2.6}, \quad x \geq 1 \]
An equivalent mechanism to generate Pareto distribution with a reflecting barrier:

- Positive growth rate and death. At death new agent is reinjected at a lower wealth $x^*$.

$$w = x^* e^{gT}, \quad N \sim p e^{-pT} \implies w \sim w^{-(\frac{p}{g}+1)}, \quad w \geq x^*$$

- More realistically, at death the individual wealth process can jump to some place other than the lower bound $x^*$ if there is inheritance, taxes, annuities or life insurance. Benhabib and Bisin (2006).

- Adding luck (good and bad) in the stochastic rate of return, we’ll loose the reflecting barrier and get a "Double-Pareto".
After Pareto (1897), Cantelli (1921), Kalecki (1945), Champernowne (1953), Rutherford (1955), Wold and Whittle (1957), among others, explored the economics of wealth dynamics through stochastic processes that converged to skewed distributions.

Mincer (1958) pleaded for microfoundations and more explicit determinants of income and wealth distributions, like human capital:

"From the economist’s point of view, perhaps the most unsatisfactory feature of the stochastic models, which they share with most other models of personal income distribution, is that they shed no light on the economics of the distribution process. Non-economic factors undoubtedly play an important role in the distribution of incomes. Yet, unless one denies the relevance of rational optimizing behavior to economic activity in general, it is difficult to see how the factor of individual choice can be disregarded in analyzing personal income distribution, which can scarcely be independent of economic activity."

Jacob Mincer, JPE, 1958
With the advent of simulation techniques, stochastic features like luck in labor endowments, together with optimizing agents, were reintroduced to analyze income and wealth distributions.

Calibrated models aimed at reproducing the wealth distribution in the data through consumption-saving models which ‘filter’ the observed distribution of earnings.

- Precautionary savings for uninsurable risk; Huggett (1996), Castaneda-Gimenez-Rios-Rull (2003)...
- Heterogeneous discount factors, Krusell-Smith (1998)
- Preferences for bequest (non homogeneous in wealth); Laitner (2001), De Nardi (2004)
- Heterogeneity in entrepreneurial ability and borrowing constraints; Quadrini (2000), Cagetti-De Nardi (2005)
What is new of this paper?

- We introduce a stochastic rate of capital return (Geometric Brownian Motion) to Benhabib and Bisin (2006) and solve analytically in closed form to obtain a "Double Pareto" distribution.
- We produce a Gini as high as 0.64 with a parsimonious model.
- We disentangle the contributions of luck, age and inheritance for the wealth inequality. And we find two opposing effects of inheritance on wealth inequality.
Building blocks

- Yaari (1965) RES, Blanchard (1985) JPE
- Merton (1971) JET
- Richard (1975) JFE
- Wold and Whittle (1957) *Econometrica*
Model Simplifications

- Perpetual Youth
- Benchmark Model has no Labor Income
- Returns not Persistent
- No Non-Homogeneity in Bequest Motive, Savings Rates etc.
Model

- There is a continuum of agents in the economy, and a constant death rate $p$. When the agent dies, one child is born.
- Investment opportunities:
  - riskless asset
    \[ dQ(t) = Q(t)rdt \]
  - risky asset
    \[ dS(t) = S(t)\alpha dt + S(t)\sigma dB(t) \]
    where $r < \alpha$.
- ‘Joy-of-giving’ bequest motive
  $Z(s, t)$ denotes the bequest that the agent born at time $s$ leaves at time $t$ if the agent dies. Life insurance payment is $P(s, t)$. The price of the life insurance is $\mu$. Annuities are negative life insurance. In a fair market $\mu = p$.
  \[ Z(s, t) = W(s, t) + \frac{P(s, t)}{\mu} \]
The agent’s utility maximization problem is

$$\max_{C, \omega, P} E_t \int_t^{+\infty} e^{-\alpha t} [C^{1-\gamma}(s, v) + \beta \chi (1 - \zeta) Z(s, v)]^\gamma dv$$

subject to

$$dW(s, t) = [(r - \tau) W(s, t) + (\alpha - r) \omega(s, t) W(s, t)$$

$$- C(s, t) - P(s, t)] dt + \sigma \omega(s, t) W(s, t) dB(s, t)$$

where $\theta$ is the time discount rate, $\chi$ is bequest motive parameter, $\tau$ is capital income tax, $\zeta$ is estate tax and $\omega(s, t)$ is the share of wealth the agent invests in the risky asset.
Theorem

The agent’s optimal policies are characterized by

\[ C(s, t) = A^{-\frac{1}{\gamma}} W(s, t), \quad \omega(s, t) = \frac{\alpha - r}{\gamma \sigma^2}, \]

\[ Z(s, t) = \left( \frac{p\chi}{A\mu} \right)^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1-\gamma}{\gamma}} W(s, t) \]

with \( A = \left( \frac{\theta + p - (1 - \gamma)(r - \tau + \mu + \frac{(\alpha - r)^2}{2\gamma \sigma^2})}{\gamma (1 + (p\chi)^{\frac{1}{\gamma}} \mu^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1-\gamma}{\gamma}})} \right)^{-\gamma} \) and

\[ dW(s, t) = gW(s, t)dt + \kappa W(s, t)dB(s, t). \]

with \( g = \frac{r - \tau + \mu - \theta - p}{\gamma} + \frac{1 + \gamma (\alpha - r)^2}{2\gamma \gamma \sigma^2} \) and \( \kappa = \frac{\alpha - r}{\gamma \sigma}. \)
Heterogenous bequest motives

- A fraction of $\frac{q}{p}$ people leave bequest. ($\chi > 0$)
- A fraction of $\frac{p-q}{p}$ people do not leave bequest. ($\chi = 0$)
Government redistributive policy

- Government subsidy
  - If the newborn’s inheritance is lower than a threshold level that is proportional to the aggregate wealth, the government gives the newborn a subsidy that brings their starting wealth to the threshold level. This is financed by estate and capital income taxes. (At about 9 – 10% of GDP in the U.S., transfers amount to about $9,000 per household per annum.)
  - If the newborn’s inheritance is higher than the threshold, the newborn does not receive a government wealth subsidy.
  - The newborn whose parent does not have a bequest motive obtains a wealth subsidy financed by estate and capital income taxes and starts life at the threshold level of wealth.
  - Alternatively, wealth at birth could be the discounted lifetime labor income: then our solution approximates this case.

- Government expenditure $\eta W(t)$. Government budget is balanced at any time.
The connection of wealth between two consecutive generations

- Let $x^*W(s)$ be the threshold level of wealth below which newborns qualify for the government wealth subsidy. Let $\rho = \left( \frac{p\chi(1-\zeta)}{A\mu} \right)^{\frac{1}{\gamma}}$.

- If $W(e, s) \geq \frac{x^*}{\rho}W(s)$,

$$W(s, s) = (1 - \zeta) \left( \frac{p\chi}{A\mu} \right)^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1-\gamma}{\gamma}} W(e, s) = \left( \frac{p\chi(1-\zeta)}{A\mu} \right)^{\frac{1}{\gamma}} W(e, s)$$

where $W(e, s)$ is wealth level of dying father who is born at period $e$, and $W(s, s)$ is wealth level of son who is born at period $s$.

$$W(s, s) = \rho W(e, s)$$

- If the parents have a bequest motive but their wealth level $W(e, s) < \frac{x^*}{\rho}W(s)$, or if the parents do not have a bequest motive, then the government subsidizes their children.

$$W(s, s) = x^*W(s)$$
Wealth distribution and inequality

- To investigate the cross-sectional distribution of the wealth, we discount the individual wealth level by the aggregate economy growth rate $\tilde{g}$. Since aggregate wealth level is growing, we investigate the distribution of the ratio of individual wealth to aggregate wealth. Let $X(s, t)$ be the ratio of the individual wealth to the aggregate wealth.

$$X(s, t) = e^{-\tilde{g} t} W(s, t)$$

We normalize initial mean wealth:

$$W(0) = 1$$

Note that $X(s, t)$ is also a Geometric Brownian Motion.

$$dX(s, t) = (g - \tilde{g})X(s, t) dt + \kappa X(s, t) dB(s, t)$$

where we assume that $g - \tilde{g} - \frac{1}{2} \kappa^2 \geq 0$. 
PDE for the evolution of wealth distribution

\[
\frac{\partial f(x, t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} \left( \kappa^2 x^2 f(x, t) \right) - \frac{\partial}{\partial x} \left( (g - \tilde{g}) x f(x, t) \right)
\]
\[
- p f(x, t) + q f\left( \frac{x}{\rho}, t \right) \frac{1}{\rho}, \quad x > x^* \tag{10}
\]

\[
\frac{\partial f(x, t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} \left( \kappa^2 x^2 f(x, t) \right) - \frac{\partial}{\partial x} \left( (g - \tilde{g}) x f(x, t) \right)
\]
\[
- p f(x, t), \quad x < x^* \tag{11}
\]

The endogenous value \( x^* \) is determined by government’s subsidy policy:

\[
(p - q)x^* + q \int_0^{x^*} (x^* - \rho x) f(x) \, dx = q\left( \frac{p \chi}{A \mu} \right)^{\frac{1}{\gamma}} \left( 1 - \zeta \right)^{\frac{1}{\gamma}} \zeta + \tau - \eta \tag{13}
\]
The stationary distribution is ergodic and has the following kernel

\[ f(x) = \begin{cases} C_1 x^{-\beta_1} & \text{when } x < x^* \\ C_2 x^{-\beta_2} & \text{when } x > x^* \end{cases} \]

where \( \beta_1 \) is the smaller root of the characteristic equation

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \bar{g}) \right) \beta + \kappa^2 - p - (g - \bar{g}) = 0 \quad (14)
\]

and \( \beta_2 \) is the larger solution of the characteristic equation

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \bar{g}) \right) \beta + \kappa^2 - p - (g - \bar{g}) + q \rho^{\beta-1} = 0. \quad (15)
\]

where \( \beta_1 < 1 \) and \( \beta_2 > 2 \).
The calibrated economy

- The parameters: $\theta = 0.03$, $\chi = 15$, $\sigma = 0.26$, $\gamma = 3$, $r = 1.8\%$, $\alpha = 8.8\%$, $p = 0.016$, $q = 0.75p = 0.012$, $\zeta = 0.19$ and $\tau = 0.004$
The U.S. economy

The image shows a graph with the title "Ratio of individual wealth to aggregate wealth". The y-axis represents the density, ranging from 0.0 to 0.12, while the x-axis represents the ratio of individual wealth to aggregate wealth, ranging from -2 to 5.
Distributions of wealth in the United States and in the benchmark model economies (Quintile)

<table>
<thead>
<tr>
<th>Economy</th>
<th>Gini</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
</tr>
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<tbody>
<tr>
<td>United States</td>
<td>0.78</td>
<td>-0.39</td>
<td>1.74</td>
<td>5.72</td>
<td>13.43</td>
<td>79.49</td>
</tr>
<tr>
<td>Model</td>
<td>0.64</td>
<td>4.07</td>
<td>6.21</td>
<td>8.16</td>
<td>12.24</td>
<td>69.32</td>
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</tbody>
</table>

Top groups (Percentile)

<table>
<thead>
<tr>
<th>Economy</th>
<th>90th — 95th</th>
<th>95th — 99th</th>
<th>99th — 100th</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>12.62</td>
<td>23.95</td>
<td>29.55</td>
</tr>
<tr>
<td>Model</td>
<td>8.84</td>
<td>15.75</td>
<td>34.33</td>
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Wealth distribution conditional on age

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Gini coefficient on age

Gini coefficient conditional on age

Age

Gini coefficient

0 10 20 30 40 50 60 70 80
0.55
0.6
0.65
0.7
0.75
0.8

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January 11, 2008  25 / 29
Contribution of luck
The Gini coefficient in the economy without risky asset is 0.439633. Comparing this Gini coefficient with that of the economy with the risky asset, 0.636731, we find that the Gini coefficient would decrease by about 31% if we close the market for the risky asset.

Contribution of age effect
We pick $\tau = 0.0107$ so that the relative growth rate $g - \tilde{g} \approx \frac{1}{2} \kappa^2$. In this economy, the Gini coefficient is 0.402667. Initially, the Gini coefficient is 0.636731. After we close the age effect, the Gini coefficient decreases by about 37%.

Intergenerational transmission

- With inheritance the wealth process is more persistent across generations.
- On the other hand, if people leave bequests, the aggregate economy growth rate increases because people cut back on consumption to leave bequests. Since the economy grows faster if there is inheritance the lucky who earn high returns relative to the economy will not break out and leave others behind by as much.
- Inheritance can decrease the wealth inequality in the economy. In the experiment above eliminating the risky asset decreases the Gini coefficient from 0.636731 to 0.0.439633 but eliminating the bequest motive by setting $\chi = 0$ increases it back to 0.6.
Tax and wealth inequality

- The minimum of Gini coefficient is 0.4020, which is obtained when \( \zeta = 0.95 \) and \( \tau = 0.0047 \).
- The maximum of Gini coefficient is 0.7143, which is obtained by \( \zeta = 0.83 \) and \( \tau = 0 \).

Taxes and welfare

The aggregate welfare is maximized by \( \zeta = 0.18 \) and \( \tau = 0.0063 \) with the Gini coefficient of 0.5260.
We set up a parsimonious model and solve it analytically to replicate the wealth distribution which displays

- Right skewness
- Heavy upper tail
- Pareto distribution

The Gini coefficient is as high as 0.64. This number is lower than that in the data. But we have fewer calibrated parameters (11) than the other models using simulations.

Luck contributes about 31% to wealth inequality as measured by the Gini coefficient. Age effect contributes about 37% to wealth inequality as measured by the Gini coefficient.

Surprisingly, inheritance can decrease the wealth inequality in the economy.