Age, Luck, and Inheritance

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Motivations

- We set up a parsimonious model and solve it analytically to match the wealth distribution in terms of
  - Gini coefficient
  - Right skewness
  - Heavy upper tail
  - Pareto distribution

- We disentangle the contribution of luck, age and inheritance for the wealth inequality.
Mechanism to produce a Pareto distribution

- Pareto distribution
  \[ f(x) \sim x^{-\beta} \quad x \geq x^* \]

- A standard mechanism to generate Pareto distribution is to construct a stochastic process with negative drift and a lower reflecting barrier. Champernowne (1953). Because of luck, we’ll loose the reflecting barrier and get a "Double-Pareto".
A Pareto Distribution looks like this:

\[ y = 1.6x^{-2.6}, \quad x \geq 1 \]
An equivalent mechanism to generate Pareto distribution with a reflecting barrier:

- Positive growth rate and death. At death new agent is reinjected at a lower wealth \( x^* \).

  \[
  w = x^* e^{gT}, \quad T \sim p e^{-pT} \implies w \sim w^{-\left(\frac{p}{g} + 1\right)}, \quad w \geq x^*
  \]

- More realistically, at death the individual wealth process can jump to some place other than the lower bound \( x^* \) if there is inheritance, taxes, annuities or life insurance. Benhabib and Bisin (2006).
What is new of this paper?

- We introduce the stochastic rate of capital return (Geometric Brownian motion) to Benhabib and Bisin (2006) and generate a "Double Pareto" distribution.
- We produce the wealth inequality of Gini as high as 0.64 with a parsimonious model.
- We disentangle the contributions of luck, age and inheritance for the wealth inequality. And we find the two opposing roles of inheritance for the wealth inequality.
- The mechanism is similar to Reed (2001). But Reed (2001) does not discuss inheritance.
Building blocks

- Yarri (1965) RES, Blanchard (1985) JPE
- Merton (1971) JET
- Richard (1975) JFE
- Wold and Whittle (1957) *Econometrica*
There is a continuum of agents in the economy. Constant death rate $p$. When the agent dies, one child is born.

**Investment opportunity**

- **riskless asset**
  \[ dQ(t) = Q(t)rdt \]

- **risky asset**
  \[ dS(t) = S(t)\alpha dt + S(t)\sigma dB(t) \]
  where $r < \alpha$.

**‘Joy-of-giving’ bequest motive**

$Z(s, t)$ denotes the bequest that the agent born at time $s$ leaves at time $t$ if the agent dies. Life insurance purchase is $P(s, t)$. The price of the life insurance is $\mu$. Annuities are negative life insurance. In a fair market $\mu = p$.

\[ Z(s, t) = W(s, t) + \frac{P(s, t)}{\mu} \]
The agent’s utility maximization problem is

$$\max_{C, \omega, P} E_t \int_t^{+\infty} e^{-(\theta+p)(v-t)} \left[ \frac{C^{1-\gamma}(s, v)}{1-\gamma} + p\chi \frac{((1-\zeta)Z(s, v))^{1-\gamma}}{1-\gamma} \right] dv$$

subject to

$$dW(s, t) = [(r-\tau)W(s, t) + (\alpha-r)\omega(s, t)W(s, t)$$
$$-C(s, t) - P(s, t)]dt + \sigma\omega(s, t)W(s, t)dB(s, t)$$

where $\theta$ is the time discount rate, $\chi$ is bequest motive parameter, $\tau$ is capital income tax, $\zeta$ is estate tax and $\omega(s, t)$ is the share of wealth the agent invests in risky asset.
The agent’s optimal policies are characterized by

\[ C(s, t) = A^{-\frac{1}{\gamma}} W(s, t), \omega(s, t) = \frac{\alpha - r}{\gamma \sigma^2}, \]

\[ Z(s, t) = \left( \frac{p\chi}{\lambda \mu} \right)^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1 - \gamma}{\gamma}} W(s, t) \]

with \( A = \left( \frac{\theta + p - (1 - \gamma)(r - \tau + \mu + \frac{(\alpha - r)^2}{2\gamma \sigma^2})}{\gamma(1 + (p\chi)^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma} \mu \frac{1 - \gamma}{\gamma} (1 - \zeta)^{\frac{1 - \gamma}{\gamma}})} \right)^{-\gamma} \) and

\[ dW(s, t) = gW(s, t)dt + \kappa W(s, t)dB(s, t). \]

with \( g = \frac{r - \tau + \mu - \theta - p}{\gamma} + \frac{1 + \gamma (\alpha - r)^2}{2\gamma \gamma \sigma^2} \) and \( \kappa = \frac{\alpha - r}{\gamma \sigma}. \)
Inheritance and government redistributive policy

- heterogenous bequest motives
  - A fraction of $\frac{q}{p}$ people leave bequest. ($\chi > 0$)
  - A fraction of $\frac{p-q}{p}$ people do not leave bequest. ($\chi = 0$)

- Government subsidy
  - If the newborn’s inheritance is lower than a threshold level that is proportional to the aggregate wealth, the government gives the newborn a subsidy that brings their starting wealth to the threshold level. This is financed by estate and capital income taxes. (For example expenditures per pupil in public elementary and secondary education, approximately $9000 per annum.)
  - If the newborn’s inheritance is higher than the threshold, the newborn does not receive a government wealth subsidy.
  - The newborn whose parent does not have a bequest motive obtains a wealth subsidy financed by estate and capital income taxes and starts life at the threshold level of wealth.
  - Alternatively, wealth at birth could be the discounted lifetime labor income: then our solution approximates this case.

- Government expenditure $\eta W(t)$. Government budget is balanced.
The connection of wealth between two consecutive generations

Let $x^* W(s)$ be the threshold level of wealth below which newborns qualify for the government wealth subsidy. Let $\rho = \left( \frac{p\chi(1-\zeta)}{A\mu} \right)^{\frac{1}{\gamma}}$.

- If $W(e, s) \geq x^* \frac{W(s)}{\rho}$,
  
  $$W(s, s) = (1 - \zeta) \left( \frac{p\chi}{A\mu} \right)^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1-\gamma}{\gamma}} W(e, s) = \left( \frac{p\chi(1-\zeta)}{A\mu} \right)^{\frac{1}{\gamma}} W(e, s)$$

  where $W(e, s)$ is wealth level of dying father who is born at period $e$, and $W(s, s)$ is wealth level of son who is born at period $s$.

  $$W(s, s) = \rho W(e, s)$$

- If the parents have a bequest motive but their wealth level $W(e, s) < x^* \frac{W(s)}{\rho}$, or if the parents do not have a bequest motive, then the government subsidizes their children.

  $$W(s, s) = x^* W(s)$$
Wealth distribution and inequality

To investigate the cross-sectional distribution of the wealth, we discount the individual wealth level by the aggregate economy growth rate $\tilde{\gamma}$. Since aggregate wealth level is growing, we investigate the distribution of the ratio of individual wealth to aggregate wealth. Let $X(s, t)$ be the ratio of the individual wealth to the aggregate wealth.

$$X(s, t) = e^{-\tilde{\gamma} t} W(s, t)$$

We normalize

$$W(0) = 1$$

Note that $X(s, t)$ is also a Geometric Brownian Motion.

$$dX(s, t) = (g - \tilde{\gamma}) X(s, t) dt + \kappa X(s, t) dB(s, t)$$

where we assume that $g - \tilde{\gamma} - \frac{1}{2} \kappa^2 \geq 0$. 

PDE for the evolution of wealth distribution

\[
\frac{\partial f(x, t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} (\kappa^2 x^2 f(x, t)) - \frac{\partial}{\partial x} ((g - \tilde{g}) x f(x, t)) \\
- pf(x, t) + q f \left( \frac{x}{\rho}, t \right) \frac{1}{\rho}, \quad x > x^* 
\]  

(10)

\[
\frac{\partial f(x, t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} (\kappa^2 x^2 f(x, t)) - \frac{\partial}{\partial x} ((g - \tilde{g}) x f(x, t)) \\
- pf(x, t), \quad x < x^* 
\]  

(11)

The endogenous value \( x^* \) is determined by government’s subsidy policy:

\[
(p - q)x^* + q \int_0^{x^*} (x^* - \rho x) f(x) dx \\
= q\left( \frac{p\chi}{A\mu} \right)^{\frac{1}{\gamma}} (1 - \zeta)^{\frac{1-\gamma}{\gamma}} \zeta + \tau - \eta
\]  

(13)
Theorem

The stationary distribution has the following kernel

\[
f(x) = \begin{cases} 
  C_1 x^{-\beta_1} & \text{when } x < x^* \\
  C_2 x^{-\beta_2} & \text{when } x > x^* 
\end{cases}
\]

where \( \beta_1 \) is the smaller root of the characteristic function

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \bar{g}) \right) \beta + \kappa^2 - p - (g - \bar{g}) = 0 \tag{14}
\]

and \( \beta_2 \) is the larger solution of the characteristic function

\[
\frac{\kappa^2}{2} \beta^2 - \left( \frac{3}{2} \kappa^2 - (g - \bar{g}) \right) \beta + \kappa^2 - p - (g - \bar{g}) + q \rho^{\beta-1} = 0. \tag{15}
\]

where \( \beta_1 < 1 \) and \( \beta_2 > 2 \).
The calibrated economy

- The parameters: $\theta = 0.03$, $\chi = 15$, $\sigma = 0.26$, $\gamma = 3$, $r = 1.8\%$, $\alpha = 8.8\%$, $p = 0.016$, $q = 0.75p = 0.012$, $\zeta = 0.19$ and $\tau = 0.004$
Distributions of wealth in the United States and in the benchmark model economies (Qintile)

<table>
<thead>
<tr>
<th>Economy</th>
<th>Gini</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.78</td>
<td>-0.39</td>
<td>1.74</td>
<td>5.72</td>
<td>13.43</td>
<td>79.49</td>
</tr>
<tr>
<td>Model</td>
<td>0.64</td>
<td>4.07</td>
<td>6.21</td>
<td>8.16</td>
<td>12.24</td>
<td>69.32</td>
</tr>
</tbody>
</table>

Top groups (Percentile)

<table>
<thead>
<tr>
<th>Economy</th>
<th>90th – 95th</th>
<th>95th – 99th</th>
<th>99th – 100th</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>12.62</td>
<td>23.95</td>
<td>29.55</td>
</tr>
<tr>
<td>Model</td>
<td>8.84</td>
<td>15.75</td>
<td>34.33</td>
</tr>
</tbody>
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Wealth distribution conditional on age

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Gini coefficient on age

Gini coefficient conditional on age

Age
Gini coefficient
Gini coefficient conditional on age

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Disentangle

- Contribution of luck
  The Gini coefficient in the economy without risky asset is 0.439633. Comparing this Gini coefficient with that of the economy with risky asset, 0.636731, we find that the Gini coefficient would decrease by about 31% if we close the investment opportunity of agents for the risky asset.

- Intergenerational transmission and age effect
  - With inheritance the wealth process is more persistent across generations.
  - On the other hand, if people leave bequests, the aggregate economy growth rate increases because people cut back on consumption to leave bequests. Since the economy grows faster if there is inheritance the lucky who earn high returns relative to the economy will not break out and leave others behind by as much. (Life cycle savings cannot account for US capital stock. Bequests plus inter-vivos transfer account for more than 50%. Kotlikoff and Summers (1981), Gale and Scholtz (1994).)

- Inheritance can decrease the wealth inequality in the economy.
Contribution of age effect
We pick $\tau = 0.0107$ so that the relative growth rate $g - \tilde{g} \approx \frac{1}{2} \kappa^2$. In this economy, the Gini coefficient is 0.402667. In the general case of our economy, the Gini coefficient is 0.636731. After we close the age effect, the Gini coefficient decreases by about 37%.
Redistributive Policies

- **Tax and wealth inequality**
  - The minimum of Gini coefficient is 0.4020, which is obtained when $\zeta = 0.95$ and $\tau = 0.0047$.
  - The maximum of Gini coefficient is 0.7143, which is obtained by $\zeta = 0.83$ and $\tau = 0$.

- **Taxes and welfare**
  The aggregate welfare is maximized by $\zeta = 0.18$ and $\tau = 0.0063$.
Conclusions

- We set up a parsimonious model and solve it analytically to replicate the wealth distribution which displays
  - Right skewness
  - Heavy upper tail
  - Pareto distribution
- The Gini coefficient is as high as 0.64. This number is lower than that of the data. But we have fewer parameters (5) than the other models using simulations (35). Castaneda, Diaz-Gimenez and Rios-Rull (2003).
- Luck contributes about 31% to wealth inequality as measured by the Gini coefficient. Age effect contributes about 37% to wealth inequality as measured by the Gini coefficient.
- Surprisingly, inheritance can decrease the wealth inequality in the economy.