Bubbles and Credit Constraints

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ECON 101

Bubbles

- Bubble Growth $\frac{\dot{B}}{B} \leq g$ or eventually they get bigger than the economy, where g is the overall growth rate of the economy.
- Without liquidity or dividend yield arbitrage $\frac{\dot{B}}{B} = r$ where r is the return on other assets or discount rate.
- With liquidity or dividend yield q on B, $\frac{\dot{B}}{B} - y = r$, so $\lim_{t\to\infty} e^{-rt} B(t) = \lim_{t\to\infty} e^{-rt} B(0)\, e^{(r-q)t} = 0$

Household Utility

- Representative enterpreneur owns all unit measure of $j \in (0, 1)$ firms:

$$Max \int_0^\infty e^{-rt} C(t)\, dt$$

$$C_t + \int_j V_t^j \dot{\psi}_j^t dj = \int_j \Pi_t^j \psi_t^j dj + w_t N_t$$

where V_t^j denotes firm j's stock price, ψ_t^j denotes holdings of firm j's stocks, Π_t^j denotes firm j's profits, w_t is the wage, N_t is labor supply.

- From linear utility, set $\psi_t^j \equiv 1$. Linear utility gives

$$rV_t^j = \Pi_t^j + \acute{V}_t^j$$

Firms

Production

- Continuum $j \in (0,1)$

$$
\begin{aligned}
Y_t^j &= (K_t^j)^\alpha (N_t^j)^{1-\alpha} \\
R_t K_t^j &= Max_{N_t^j}\ (K_t^j)^\alpha (N_t^j)^{1-\alpha} - w_t N_t^j \\
w_t &= (1-\alpha)\left(\frac{K_t^j}{N_t^j}\right)^\alpha \\
R_t &= \alpha\left(\frac{K_t^j}{N_t^j}\right)^{\alpha-1} = \alpha\left(\frac{w_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \\
D_j^t &= R_t K_t^j - Q_t(\dot{K}_t^j + \delta K_t^j) \qquad \text{dividend}
\end{aligned}
$$

- Investment Opportunities

$$K_{t+dt}^{j} = \begin{array}{ll} (1-\delta dt) K_t^j + I_t & \textit{with prob } \pi dt \\ (1-\delta dt) K_t^j & \textit{with prob } (1-\pi) dt \end{array}$$

- Investment

$$0 \leq I_t \leq L_t^j \quad \textit{where } \; L_t^j \textit{ is the loan}$$

- Loans L_t^j are fully repaid after the realization of investment returns $Q_t I_t^j$ when $Q_t I_t^j \geq L_t^j$:

Collateral Constraint

$$L_t^j \leq V_t \left(\zeta K_t^j \right)$$

The value of the firm, subject to above constraints, is

$$rV_t\left(K_t^j\right) = Max_{I_t^j}\ D_t^j + \dot{V}_t\left(K_t^j\right) - \pi\left(\left(L_t^j + Q_t I_t^j\right) - \left(I_t^j + L_t^j\right)\right) \quad (1)$$

and

$$D_j^t = R_t K_t^j - Q_t(\dot{K}_t^j + \delta K_t^j) \quad \text{dividend} \quad (2)$$

- Here δ represents the depreciation rate of capital and D_t^t represents dividends excluding net investment returns. Firm j receives internal funds $R_t K_t^j$and purchases (or sells) capital K_t^j at price Q_t.
- When an investment opportunity arrives at the Poisson rate π, the firm borrows L_j^t from other firms that do not have investment opportunities, and makes investment I_t^j before receiving investment returns $Q_t I_t^j$.
- The change in firm value is equal to $\left(L_t^j + Q_t I_t^j\right) - \left(I_t^j + L_t^j\right)$.

Equilibrium

- Firms choose $\left\{K_t^j,\ I_t^j, N_t^j, L_t^j\right\}$
- Households choose $\{C_t\}$
- Markets clear:
  $K_t = \int_0^1 K_t^j dj,\ I_t = \int_0^1 I_t^j dj,\ N_t = \int_0^1 N_t^j dj = 1,\ Y_t = (K_t)^\alpha$ at prices $\{w_t, R_t\}$

Firm Value

$$V\left(K_t^j\right) = v_t K_t^j + b_t$$

Tobin's Q_t :

$$Q_t = e^{-rt}\frac{\partial V_{t+dt}}{\partial K_t^j} = e^{-rt}v_{t+dt} \neq \frac{e^{-rt}V_{t+dt}}{K_{t+dt}} = average\ \ Q_t$$

Bubble:

$$B_t = e^{-rt}b_{t+dt}$$

Let$dt \to 0$. For $Q > 1$ invest maximally (constraint binds):

$$I_t^j = L_t^j = V\left(\zeta K_t^j\right) = Q_t \zeta K_t^j + B_t \tag{3}$$

Then dynamics are

$$\begin{aligned} r &= \frac{\dot{B}}{B} + \pi(Q_t - 1) \\ \dot{Q}_t &= (r+\delta)\, Q_t - R_t - \pi\,(R_t + \zeta Q_t)\,(Q_t - 1) \\ \dot{K}_t &= -\delta K_t + \pi\,(R_t K_t + \zeta Q_t K_t + B_t) \end{aligned}$$

with $R_t = \alpha K_t^{\alpha-1}$

- Transversality: $\lim_{t\to\infty} e^{-rt} Q_t K_t = e^{-rt} B_t = 0$;
- $V_t(K_t^j) = Q_t K_t^j + B_t,\ Q_t = v_t,\ B_t = b_t$

- To see this simplify Bellman eq. (1), using (2) and (3) :

$$\begin{aligned} rV_t(K_t^j) &= \\ rQ_tK_t^j + rB_t &= Max_{I_t^j} \left[R_tK_t^j - Q_t(\dot{K}_t^j + \delta K_t^j)_t^j \right] \\ & -\pi\left(Q_t - 1\right)\left(Q_t\zeta K_t^j + B_t\right) \\ & + \left[\dot{Q}_tK_t^j + Q_t\dot{K}_t^j + \dot{B}_t\right] \end{aligned}$$

and equate coefficients to get equations for Dynamics.

- Note: Under Kiyotaki-Moore constraint, for $Q > 1,\ I_t^j = L_t^j = Q_t\zeta K_t^j$. Substituting for I_t^j the dynamic equation for the bubble, without the liquidity or collateral yield, has to grow faster, $\dot{B} = rB$, which violates transversality. So, $\lim_{t\to\infty} e^{-rt}B_0e^{rt} = B_0 = 0$, no bubble: $B_t \equiv 0$.

Bubbleless:

$B_t = 0$

- At steady state, if constraints are slack, which occurs if $\zeta \geq \frac{\pi}{\delta}$

$$Q_t = Q^* = 1, \quad K = K^* \textit{ where } R^* = \alpha (K^*)^{\alpha - 1} = (r + \delta)$$

- This is the planners problem: with linear utility you jump to the steady state immediately.
- At steady state, if constraints are not slack and firms are credit constrained because ζ is low, which occurs if $\zeta \leq \frac{\pi}{\delta}$,

$$\tilde{Q} = \frac{\delta}{\zeta\pi}, \qquad \alpha \left(\tilde{K}\right)^{\alpha-1} = \frac{r\delta}{\zeta\pi} + \delta, \qquad \tilde{K} < K^*$$

- In both bubbless equilibria, the steady states are saddles in the $(K_t, \ Q_t)$ dynamics (no B_t).

Bubble Equilibrium

- In this case Bubble provides a liquidity yield or dividend: $\pi(Q_t - 1)$.
- Bubble Steady State:

$$\begin{aligned}\frac{B}{K_b} &= \frac{\delta}{\pi} - \zeta\left(\frac{r}{\pi} + 1\right) \\ Q_b &= \frac{r}{\pi} + 1 \\ \alpha K_b^{\alpha-1} &= [(1-\zeta)\, r + \delta]\left(\frac{r}{\pi} + 1\right)\end{aligned}$$

iff (when ζ is low)

$$0 < \zeta < \frac{\delta}{r + \pi}$$

- Note that condition $\zeta < \frac{\delta}{r+\pi}$ implies condition $\zeta \leq \frac{\pi}{\delta}$. Thus, if condition $\zeta < \frac{\delta}{r+\pi}$ holds, then there exist two steady state equilibria: one bubbleless and the other bubbly.

Results

- Bubble effects: Two steady states, $\left(B, \tilde{Q}_b, K_b\right)$ and $\left(0, \tilde{Q}, \tilde{K}\right)$. B and $\tilde{Q}$ are jump vaiables
- i) $K^* > K_b > \tilde{K}$ (Raise steady state K), ii) $Q_b < \tilde{Q}$ (Lower shadow value because $K_b > \tilde{K}$, lower MPK), iii) $C^* > C_b > \tilde{C}$.
- Bubbless steady states are saddles with two roots, one (+) and one (-). Bubbly steady state indeterminate with one (+) root, two (-) roots.
- Global Dynamics?

Stochastic Bubbles (in brief)

- After Collapse economy follows bubbless equilibrium, with Value V^*, by jumping to the saddle path of bubbless equilibrium.
- Before Collapse: Probability of collapse is θ. Adjust firm values V and bubbly equilibrium path dynamics to allow jump with probability θ to bubbless saddle path. Solve similarly. (Not MIT shock: jump probability incorporated.)