

The Monetary Transmission Mechanism*

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Abstract

Recent literature on structural vector autoregressions has attempted to identify the effects on the economy of an increase in the stock of money. This work has led to a broad consensus. Initially, an increase in money leads to an increase in economic activity. Output and employment go up, the interest rate declines and prices respond weakly, if at all. Over time, these real effects die out and, in the long run, the only effect of higher money is higher prices. Most writers on the topic have attributed the real effects of money, in the short run, to a barrier of some kind that prevents markets from clearing. We show instead that a competitive market-clearing model in which money enters the production function can reproduce the broad features of data. Our argument exploits the existence of multiple equilibria in a rational-expectations model.

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1 Introduction

In equilibrium business cycle models that are amended to include money it is difficult to set things up in a way that causes simulated time series to mimic real world data. The models display too much price flexibility. Money shocks feed immediately into prices and money is neutral not only in the long run, but also in the short run. In the data, this is not what we observe. Instead, money shocks cause real output responses in the short run and only after a considerable period of time do prices adjust to insulate real quantities from nominal disturbances.¹

There are two popular views of why equilibrium models fail. One view holds that markets, expectations, or both are typically in disequilibrium. According to this view an amended version of the IS-LM model can accurately describe the world and the role of economic theory is to explain why prices do not clear markets. A second view holds that the correlations we observe in the data are examples of reverse causation. Output causes money rather than the other way around and hence there is no puzzle to be explained.

In this paper, we argue that there *is* a puzzle for equilibrium business cycle theory but this puzzle can be resolved within a market-clearing model in which agents have rational expectations. We argue that the world in which we live is one in which the assumption of rational expectations is insufficient to pin down a particular equilibrium since there are infinitely many beliefs that are consistent with rational expectations and market clearing. Agents in the real world have resolved this multiplicity by coordinating on a particular equilibrium that has the property that prices are predetermined one period in advance.

The argument that indeterminacy can be used to explain the observed behavior of prices has been made before.² There have however been few attempts to investigate the *empirical* plausibility of indeterminacy arising from the productive or utility producing role of money.³ For this reason, most macroeconomists have tended to dismiss the idea that indeterminacy of equilibrium can explain the monetary transmission mechanism. In this paper we make the case for the multiple equilibrium approach to the monetary transmission

mechanism by showing that a suitably calibrated model can fit the data well if one is prepared to accept a relatively flexible parameterization of preferences.

2 Setting up a Model Economy

Definitions

In the following discussion we assume that the economy contains a large number of identical representative households. We let Y_t , C_t and L_t represent output, consumption and labor supplied to the market. We allow for two nominal assets money, M_t and bonds B_t and we define i_t to be the interest paid on a nominal bond issued at date t . p_t is the money price of a commodity. We use the timing convention in which the bond B_t is issued at date t and pays $(1+i_t)$ units of money in date $t+1$.

Growth

We assume a structure in which growth is caused by exogenous technical progress and we let the following variables represent ratios of output, consumption etc to a growing trend:

$$y_t \equiv \frac{Y_t}{S_t}, \quad c_t \equiv \frac{C_t}{S_t}, \quad m_t \equiv \frac{M_t}{p_t S_t}, \quad b_t \equiv \frac{B_t}{p_t S_t}. \quad (1)$$

We define the growth factor γ_t as,

$$\gamma_t \equiv \frac{S_t}{S_{t-1}}. \quad (2)$$

In section 5 we will assume that S_t is a geometric random walk with drift although this assumption is not necessary for our results and the main ideas of the paper will also work if productivity growth is mean reverting.

Technology

There are three main ways of introducing money into a representative-agent model. The first is to include the real value of money as an argument in the utility function; the second to include it as an argument in the production function and the third to assume that households

or firms are subject to a cash-in-advance constraint. We choose the first approach. Since the properties of a cash-in-advance model can be replicated by putting money in utility or production, in the following discussion, we concentrate on comparing our assumption with money in the utility function.

We do not offer an explicit microfoundation for a model of money; instead we refer the reader to models in which liquid assets serve a productive role by ameliorating problems that arise from informational asymmetries.⁴ When money enters the production function, a reduction in real balances causes a leftward shift of the labor demand curve. This is in contrast to models in which money enters utility. For these models changes in real balances cause shifts in the labor supply curve. Although we view our model of a spot market as a simplistic abstraction, we are more comfortable with a channel in which a recession is triggered by a contraction of labor demand than by a contraction of labor supply.⁵

We model production with the function, $y_t = f(L_t, m_t)$, where $f(L, m)$ satisfies the assumption of decreasing returns-to-scale in L and m . In our calibrated examples, we use the CES functional form, $y_t^I = (1-a)L_t^{aI} + am_t^I$. We have chosen CES rather than Cobb-Douglas to fit the fact that the interest rate elasticity of the demand-for-money in low frequency data is of the order of -0.5 . In a model with a Cobb-Douglas production function this elasticity is restricted to equal -1 . For our technology, the interest elasticity of the demand for money is equal to $\frac{1}{1-I}$ and for our model to be consistent with the observed interest elasticity we must set I to -1 .⁶

Preferences

Our key idea is to exploit an indeterminate equilibrium to explain price stickiness. In order to generate an indeterminate equilibrium we will need small changes in real balances to have big effects on output and employment. Although this could occur because money is very important as a productive factor, this approach leads to an obvious counterfactual implication. The opportunity cost of holding money is small as a percentage of GDP. Since the

opportunity cost of holding money should equal the elasticity of real balances in production, one can infer that this elasticity must be small and money cannot be important.

Instead of arguing that money is directly important as a productive factor, we assume instead that the economy has large real rigidities in the sense of Ball and Romer (1990). Real rigidity is often defined to mean a flat marginal cost schedule. It can also be shown to imply, in a competitive labor market, that the slopes of labor demand and supply curves must be close.⁷ This can occur either because labor demand curves slope up, as in the literature on increasing returns-to-scale in production, or because labor supply curves slope down.⁸ In our work, we have chosen to model downward sloping labor supply by choosing a non-standard description of preferences. We model period utility with the function, $u = u(C, L, S, \bar{L})$, where C is consumption and L is labor supply. We assume u to be increasing in C , decreasing in L , and jointly concave in these two arguments. The term S represents technical progress and \bar{L} is the average labor supply of other members of society.

The first way, in which our preference specification is non-standard, is the inclusion of technological progress as an argument of utility. This assumption allows our model to be consistent with the existence of a balanced growth path, whilst allowing a flexible representation of income and substitution effects. This is in contrast with more standard models in which income and substitution effects must exactly balance in order to explain the stationarity of U.S. labor supply in the face of growing wages. In our model, consistency with balanced growth implies that $u(C, L, S, \bar{L})$ is homogenous of degree $1-r$ in C and S . We justify our assumption that productivity influences utility by appealing to the work of Benhabib, Rogerson and Wright (1991) on home production. They argue that technical progress has made steel plants more productive but it also has led to the invention of items such as microwave ovens, toasters and washing machines. These are advances that increase the time available to the representative household both for work in market based activities and for the enjoyment of leisure.

The second way, in which our preference specification is non-standard, is our assumption that preferences are interdependent. We assume that the disutility of work is greater when an individual is working more than are other members of society. Since many of the ways in which we enjoy leisure are social activities, the representative individual may be more willing to work when everyone else is working. In our model, individual utility functions are concave, but externalities introduce a non-convexity to the social planner's objective function. An important consequence of the assumption of interdependent utility is that the aggregate labor supply curve may slope down as a function of the real wage.

Maximizing Utility

Our economy consists of a large number of representative families, each of which maximizes the utility function;

$$\text{Max } U = E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[\frac{C_t^{1-r}}{1-r} - \frac{1}{1+c_1} \frac{S_t L_t^{1+c_1}}{\bar{L}^{c+c_1}} \right] \right\}, \quad (3)$$

where r is the rate of time preference, $-r$ is the elasticity of intertemporal substitution, $\frac{1}{1+c_1}$ is the inverse private elasticity of individual labor supply and $\frac{c_1}{1+c_1}$ measures the external effect of other peoples' labor on individual utility. Given our functional form for utility, maximization with respect to $\{C_t\}$ is equivalent to maximizing the utility function

$$\text{Max } U = E_0 \left\{ \sum_{t=0}^{\infty} \mathbf{d} \left[\frac{c_t^{1-r}}{1-r} - \frac{1}{1+c_1} \frac{L_t^{1+c_1}}{\bar{L}^{c+c_1}} \right] \right\},$$

$$\mathbf{d}^0 = 1, \quad \mathbf{d} = \left(\frac{1}{1+r} \right)^t \prod_{v=1}^{t-1} \mathbf{g}^{1-r}, \quad t > 0, \quad (4)$$

with respect to $\{c_t\}$ where $c_t \equiv \frac{C_t}{S_t}$.

The Household Budget Constraint

In the following discussion consumption, output, real balances and the real value of debt are defined as ratios to productivity. Each family chooses how much time to spend in the activity

of production, L_t , how much to consume of the commodities produced by other households, c_t , and how much to save in the form of money m_t and bonds b_t . Households choose sequences that maximize expected utility subject to the budget set defined by the constraints (5)– (8):

$$m_t + b_t = r_t^m m_{t-1} + r_t^b b_{t-1} + f(L_t, r_t^m m_{t-1} + \mathbf{t}_t) - c_t + \mathbf{t}_t, \quad t = 1, \dots, \quad (5)$$

$$r_t^m \equiv \frac{P_{t-1}}{p_t \mathbf{g}}, \quad r_t^b \equiv \frac{P_{t-1}}{p_t \mathbf{g}} (1 + i_{t-1}), \quad (6)$$

$$b_0 = 0, \quad M_0 = m_0, \quad (7)$$

$$\lim_{s \rightarrow \infty} Q_t^s \mathbf{d}(m_s + b_s) \geq 0, \quad Q_t^s = \frac{P_s}{P_t} \prod_{v=1}^s \frac{1}{(1 + i_v)}. \quad (8)$$

Eq.(5) is the household's period budget constraint. We assume that the household receives a lump sum nominal transfer T_t in each period and we define $\mathbf{t}_t \equiv \frac{T_t}{P_t S_t}$ to be the real value of this transfer relative to the growing productivity trend. The production function depends on labor and on $r_t^m m_{t-1} + \mathbf{t}_t$. The term $r_t^m m_{t-1}$ is the real value of cash accumulated in the previous period. The term \mathbf{t}_t is the real value of the transfer received in period t . We allow this transfer to be used in exchange during the period.

We model fiscal policy by assuming that $B_t = 0$, for all t and we define the rate of money creation from the identity,

$$M_t \equiv \mathbf{m}_t M_{t-1}, \quad E_{t-1}[\mathbf{m}_t] = \bar{\mathbf{m}} > 1, \quad (9)$$

where $\bar{\mathbf{m}}$ is the mean money growth factor. We further assume that all output is consumed:

$$y_t = c_t. \quad (10)$$

Finally, the government budget constraint implies,

$$m_t = r_t^m m_{t-1} + t_t. \quad (11)$$

The Equations that Describe Equilibrium

The equations of the model can be separated into those that describe relationships between variables at a single date (intra-temporal equations), and those that describe relationships between variables at consecutive dates (inter-temporal equations). We begin with two intra-temporal equations, the production function and the first order conditions for the choice of labor. In these equations, described below, we have made use of the assumption that utility is separable in consumption and labor supply.

$$y_t = f(L_t, m_t), \quad \text{Production function} \quad (12)$$

$$\frac{-u_L(L_t, \bar{L}_t)}{u_c(c_t)} = f_L(L_t, m_t) \quad \text{Labor demand and supply} \quad (13)$$

Our preference specification is able to capture the feature that aggregate labor supply curves slope down, because labor supply of each individual is shifted by changes in the labor supply of everyone else in society. If we were to decentralize the economy and allow the household to supply labor to a competitive labor market for a real wage of w , the household's labor supply curve would be given by the expression, $\left(\frac{C}{S}\right)^r \frac{SL^{c_1}}{\bar{L}^{c+c_1}} = w$. Log-linearizing this equation leads to the individual labor supply curve,

$$r \log\left(\frac{C}{S}\right) + c_1 \log(L) - (c + c_1) \log(\bar{L}) = \log\left(\frac{w}{S}\right). \quad (14)$$

In equilibrium, individual labor supply must be the same across all individuals and L must equal \bar{L} . It follows that the aggregate labor supply curve will be represented by the equation

$$r \log\left(\frac{C}{S}\right) - c \log(L) = \log\left(\frac{w}{S}\right). \quad (15)$$

In this equation labor supply, controlling for consumption, is a downward sloping as a function of the real wage.

The two intertemporal equations of the model are the Euler equations that arise from the choice of money and bonds. To write the equations in the following way we have made use of the facts that the real returns on money and bonds are related by the identity, $r_t^m (1+i_{t-1}) \equiv r_t^b$, and that the government budget constraint implies that $m_t = m_{t-1}r_{t-1}^m + t_t$.

$$u_c(c_t) = E_t \left[\frac{d_{t+1}}{d} r_{t+1}^m u_c(c_{t+1}) (1 + f_m(L_{t+1}, m_{t+1})) \right], \quad \text{Money} \quad (16)$$

$$u_c(c_t) = E_t \left[\frac{d_{t+1}}{d} r_{t+1}^m u_c(c_{t+1}) (1 + i_t) \right]. \quad \text{Bonds} \quad (17)$$

By combining (16) and (17) and using the definition $X_{t+1} \equiv \frac{d_{t+1} r_{t+1}^m u_c(c_{t+1})}{d}$ one arrives at the following equilibrium condition:

$$E_t [f_m(L_{t+1}, m_{t+1}) X_{t+1}] = E_t [i_t X_{t+1}]. \quad \text{Demand for money} \quad (18)$$

In a non-stochastic model, given the assumption that the production function is CES, Eq(18) would reduce to,

$$i_t = a \left(\frac{y_{t+1}}{m_{t+1}} \right)^{1-I}. \quad \text{Demand for money} \quad (19)$$

Eq(19) is a relatively standard representation of a demand-for-money equation.

We now turn to the dynamic behavior of real balances. First, we rewrite the real return to money in terms of real balances, and the exogenous money growth rate:

$r_{t+1}^m = \frac{p_t}{p_{t+1}\mathbf{g}_{t+1}} = \frac{m_{t+1}}{m_t\mathbf{m}_{t+1}}$. Substituting this expression back into the Euler equation for money,

and noting that $\frac{d_{t+1}}{d_t} = \frac{\mathbf{g}_{t+1}^{1-r}}{1+r}$ leads to the expression,

$$m_t u_c(c_t) = E_t \left[\frac{\mathbf{g}_{t+1}^{1-r}}{\mathbf{m}_{t+1}(1+r)} m_{t+1} u_c(c_{t+1}) (1 + f_m(L_{t+1}, m_{t+1})) \right]. \quad \text{Euler Equation} \quad (20)$$

In the following two sections of the paper we show how to use (12), (13) and (20) to derive an approximate linear difference equation that characterizes an equilibrium. Following this discussion, we calibrate the model and compare time series generated by the model with those from time series data.

3 Money and the Labor Market

In a standard equilibrium model, labor supply is upward sloping and a small shift in labor demand or supply has a small impact on equilibrium labor and equilibrium output. In our model, in contrast, labor supply slopes down and a small shift in labor demand can have a big effect on aggregate supply. The assumption that labor supply slopes down allows us to explain how consumption and employment can be procyclical, in a model driven by demand shocks. It is also consistent with estimates of labor supply in aggregate data, in the work of Mankiw Rotemberg and Summers (1987) and Farmer and Guo (1995).

INSERT FIGURE 1 HERE

Figure 1 illustrates how our equilibrium model is able to generate a large effect of money even though the direct effect of money in production is small. The labor demand curve is the marginal product of labor in production; this is given by the right-hand-side of Eq.(13). Real balances shift the labor demand curve because money is a productive factor. The labor supply curve is the ratio of the marginal disutility of labor to the marginal utility of consumption; this is given by the left-hand-side of Eq.(13). Output shifts the labor supply curve because consumption equals output in equilibrium.

An increase in money has two effects on employment. The first occurs as an increase in real balances causes firms to increase their demand for labor. On Figure 1 the labor demand curve shifts to the right. The second occurs as increased production leads to increased consumption. On Figure 1 the labor supply curve shifts up. The net effect of these shifts in the labor demand and supply curves, for our parameterization of preferences and technology, is to cause an increase in equilibrium output and employment from point A to point B. The effect is big, even though money is relatively unimportant as a productive factor, because the slopes of labor demand and supply are close.

4 Indeterminacy Compared with a More Standard New

Keynesian Approach

In this section we compare our model with a recent literature on the New Keynesian Phillips Curve.⁹ Following Ball and Romer (1990) it is common to distinguish between real and nominal rigidities. New Keynesian economists model nominal rigidities by assuming that some fraction of firms cannot adjust prices in any given period. Ball and Romer (1990) showed that nominal rigidities are insufficient for monetary shocks to have large real effects. It must also be true that there are significant *real rigidities*.

Real rigidity is expressed as a property of an equation relating the real wage to output. In some versions of the New-Keynesian model, this equation is derived by combining a labor supply equation with the production function. In others, Jeanne (1998) for example, it is assumed to derive from union bargaining. A log linear approximation to the wage equation would take the form:

$$w_t = \gamma y_t. \tag{21}$$

In a model with an equilibrium labor market, Eq.(21) would come from linearizing the labor market equilibrium condition,

$$\iota = \frac{-U_L(Y, F^{-1}(Y))}{U_C(Y, F^{-1}(Y))}. \quad (22)$$

If γ is small then the economy displays real rigidities in the sense of Ball and Romer.

In a recent paper, Chari Kehoe and McGrattan (1996) (henceforth CKM) study the ability of New-Keynesian models to explain the persistence of monetary shocks. In a competitive model in which preferences and technologies are relatively standard, the parameter γ must be larger than one. But in order to generate realistic values for the persistence of monetary shocks, γ must be small. For example, in Taylor's (1980) paper on staggered contracts, γ is equal to 0.05 and in this paper, Taylor showed that staggered contracts can do a good job of explaining why nominal shocks have persistent real effects.

INSERT FIGURE 2 HERE

For comparison with Chari Kehoe and McGrattan, Figure 2 compares the predicted impulse response for output, in our model economy, with those of a set of CKM economies for different values of γ . Our model generates more persistence than the CKM economies because the equivalent of the parameter γ in our model is small. Further, our method of making γ small (interdependent preferences) does not have the unpleasant side effect of increasing the impact effect of a money shock. This is in contrast to a related method of lowering γ discussed by Chari et al. They explore the possibility that γ could be made small by studying preferences with small income effects, but they reject this route because it implies that the initial output response is implausibly high.

Our method of solving the persistence problem is not confined to models with indeterminacy. It will work in models with sticky wages or prices just as it works in our model. By allowing labor supply to slope down we can make γ not only small but negative. The parameter γ is equal to the elasticity of output supplied with respect to the real wage that arises from log linearizing (22). For our benchmark parameterization this parameter is equal

to -0.49 . Once one accepts parameterizations in which there are strong real rigidities, indeterminacy follows if real balances enter either the demand or supply of labor in a way that causes a small increase in real balances to have a strong output effect. This effect is absent in much of the New-Keynesian literature mainly because the models are set up in such a way that real balances cannot shift either the demand or the supply of labor.¹⁰

5 Equilibrium in the Model

In this section we study approximate equilibria by approximating the equations of our model around a balanced growth path. The existence of such a path is established in Appendix A.¹¹

Employment, Output and Real Balances

We begin by finding two functions $h(m)$ and $H(m)$ that describe the dependence of employment and output on real balances. These functions arise from solving (12) and (13) simultaneously for L and y as functions of m and for our choice of functional form they are implicitly found by solving the following equations;

$$\frac{Y'}{L} = \frac{Y^{1-\alpha}}{L^{1-\alpha}}, \quad (23)$$

$$Y = (1-\alpha)L^\alpha + \alpha m. \quad (24)$$

Eq.(23) is the first order condition for choice of labor supply and Eq.(24) is the production function. In general, one cannot find closed form expressions for the functions $h(m)$ and $H(m)$, that describe employment and output as functions of real balances. Instead, we linearize the production function and the labor market equation around the steady state to yield the following approximations: details are given in Appendix B. In these expressions, and in our subsequent discussion, we use the symbol $\%$ to mean the log-difference, $\log(x) - \log(x^*)$, of a variable x , $x \in \{y, L, m\}$ from x^* . The equations are represented as follows:

$$\frac{\%}{t} = h \frac{\%}{t}, \quad h \equiv \frac{1}{b - (1 - r)}, \quad (25)$$

$$\frac{\%}{t} = H \frac{\%}{t}, \quad H \equiv \frac{b}{b - (1 - r)}. \quad (26)$$

In Appendix B we show that the parameter b is given by the expression $b \equiv \frac{1 - r}{r - 1 + \dots}$

where $1 - r$ is the degree of homogeneity of the utility function, \dots is labor's share of income

and $\frac{1}{1 - \dots}$ is the interest elasticity of money demand. The parameter \dots is the elasticity of

output with respect to real balances. In our calibrations we set this equal to the average

fraction of GDP lost through holding money rather than interest bearing assets. In the U.S.

this is approximately 1%. Since \dots is small, b must be close to $(1 - \dots)$ for real balances to

have a big effect on employment and output. This condition is equivalent to the assumption

that the labor demand and supply curves have similar slopes.

The Euler Equation

By substituting the expressions $h(m)$ and $H(m)$ into Eq.(20) we arrive at a functional

equation that characterizes equilibrium,

$$G(m_t) = E_t \left\{ \frac{\gamma_{t+1}^{1-r}}{(1 + \dots)_{t+1}} G(m_{t+1}) X(m_{t+1}) \right\}, \quad (27)$$

$$G(m_t) \equiv m_t u_c(H(m_t)), \quad X(m_t) \equiv 1 + f_m(h(m_t), m_t).$$

Linearizing Eq.(27) around the balanced growth path gives the approximate equation,

$$\frac{\%}{t} = E_t \left\{ \frac{1}{b_1} \frac{\%}{t+1} - \frac{b_2}{b_1} \gamma_{t+1} \frac{\%}{t+1} - \frac{b_3}{b_1} \frac{\%}{t+1} \right\}, \quad (28)$$

$$b_1 \equiv \frac{G}{G + X}, \quad b_2 \equiv \frac{(r-1)}{(G + X)}, \quad b_3 \equiv \frac{1}{(G + X)}.$$

An equation like (28) is typical in monetary rational expectations models. Often these models are derived in the context of a two-parameter family of utility functions in which one is able to impose the restriction $|b_1| > 1$. In this case one can show that (28) has a unique rational expectations solution. In contrast, when $|b_1| < 1$, one can find many solutions. In the following section we explore this issue.

A Class of Equilibria

To keep our argument concise we will shut down all real shocks and study the special case in which the money supply is a random walk. Our argument does not depend on these assumptions, which are made to simplify the algebra. An equilibrium must satisfy the approximate Euler equation, Eq.(28). Since our assumptions imply that all the shocks in our model have zero conditional means, this equation can be written as follows:

$$\log(M_t) - \log(P_t) = E_t \left\{ \frac{1}{b_1} [\log(M_{t+1}) - \log(P_{t+1})] \right\}. \quad (29)$$

Since there are multiple equilibria one must supplement the equilibrium equations with an equation that models the process by which agents forecast – we call this a belief function. Some belief functions will implement a rational expectations equilibria and it is on these that we will focus.

Since, when $|b_1| < 1$, the set of rational expectations equilibria is extremely large we will restrict our attention to a subset of equilibria that can be represented as solutions to the following stochastic difference equation,

$$[\log(M_{t+1}) - \log(P_{t+1})] = b_1 [\log(M_t) - \log(P_t)] + e_{t+1}. \quad (30)$$

Providing the conditional expectation of e_{t+1} is zero, sequences of real balances generated by Eq.(30) will satisfy the Euler equation. It follows that for arbitrary stochastic processes with zero conditional mean, one can construct a rational expectations equilibrium by iterating this

equation. We will focus our attention on equilibria for which the sunspot process $\{e_t\}$ is a linear function of the money shock $\{\%_t\}$; that is, on equilibria in the restricted class:

$$\begin{aligned} [\log(M_{t+1}) - \log(P_{t+1})] &= b_1[\log(M_t) - \log(P_t)] + \%_{t+1}, \\ \%_{t+1} &\equiv [\log(M_{t+1}) - \log(M_t)]. \end{aligned} \quad (31)$$

These equilibria are interesting since they are able to explain why nominal shocks have real effects.

Determining Beliefs

Writing down an equilibrium for our model is an important first step, but we must also illustrate how any particular equilibrium comes about. We propose to select an equilibrium by choosing the belief function to be a primitive object of our economy. To see how this works we will allow subjective expectations, determined by the belief function, to differ from rational expectations. In the agent's Euler equation, reproduced below, the left-hand-side is determined by the real money supply.

$$\log(M_t) - \log(P_t) = E_t \left\{ \frac{1}{b_1} [\log(M_{t+1}) - \log(P_{t+1})] \right\}. \quad (32)$$

The right-hand-side is the agents' demand for money and it depends on expectations about the future price level and the future money supply. Suppose that agents use the belief function,

$$\begin{aligned} \log(P_{t+1}^E) &= \alpha_1 \log(M_t) + \alpha_2 \log(M_{t-1}) + \alpha_3 \log(P_{t-1}), \\ \alpha_1 &\equiv (1 - b_1), \quad \alpha_2 \equiv b_1(-b_1), \quad \alpha_3 = b_1^2, \end{aligned} \quad (33)$$

where the parameter b_1 depends on the fundamentals of the economy and parameterizes beliefs. Although this mechanical rule looks arbitrary, we have chosen the parameters α_1 , α_2 and α_3 in such a way that if all agents use the rule, it will support a rational expectations equilibrium. We propose to make b_1 a deep parameter of the model. Using this assumption, our rule will support one and only one rational expectations equilibrium.

To verify that Eq.(33) supports a rational expectations equilibrium, plug it into the right-hand-side of Eq.(20) and apply the expectations operator to the right-hand-side. This leads to the expression¹²

$$\begin{aligned} \log(M_t) - \log(P_t) &= \frac{1}{b_1} \left\{ b_1^2 [\log(M_{t-1}) - \log(P_{t-1})] + b_1 \frac{\%}{\phi} \right\}, \\ \Rightarrow \\ \log(M_t) - \log(P_t) &= b_1 [\log(M_{t-1}) - \log(P_{t-1})] + \frac{\%}{\phi}. \end{aligned} \tag{34}$$

If agents use the function (33) to forecast the price in period $t+1$, then the equilibrium price function in period t will be given by (34). In other words, the belief function supports a rational expectations equilibrium. One can also show, by iterating the right-hand-side of (34) one period, that the belief function (33) is self-fulfilling in the sense that, if agents use this function to predict prices then actual prices will follow the same process.

6 Two Special Cases

Two of the equilibria supported by the belief function (33) are special since they correspond to polar views about price flexibility in the economy. In our first example the parameter γ is equal to 0. We call this case the quantity-theoretic economy (after the Quantity Theory of Money) because nominal shocks feed immediately into prices and money is neutral in both the long run and the short run. In our second example the parameter γ is equal to 1. We call this case the fixed price economy because the price level is predetermined one period in advance, nominal shocks feed immediately into quantities and prices respond only asymptotically.

The Quantity Theoretic Economy

In this example, equilibrium is determined by the difference equation,

$$\log(M_t) - \log(P_t) = b_1 [\log(M_{t-1}) - \log(P_{t-1})], \tag{35}$$

which is the special case of Eq.(34) when $y = 0$. For this case, the log of real balances converges to zero and asymptotically it will be true that $\frac{M_t}{P_t} = 1$. Since real balances converge to one, the price level in equilibrium is equal to the money stock and, since the money supply is a random walk, the expected price level one period ahead must equal the current period's money stock. We can see the same fact by studying the evolution of beliefs which are given by the function,

$$\log(P_{t+1}^E) = \log(M_t) - b_1^2[\log(M_{t-1}) - \log(P_{t-1})]. \quad (36)$$

Since $\log(M_{t-1}) - \log(P_{t-1})$ converges to zero, in the steady state, the expected price in period $t+1$ is equal to the money stock in period t .

The Predetermined Price Equilibrium

A second interesting case occurs when $y = 1$. In this case equilibrium is determined by the equation:

$$\log(M_t) - \log(P_t) = b_1[\log(M_{t-1}) - \log(P_{t-1})] + \% \phi. \quad (37)$$

Using the fact that the money shock $\% \phi$ is equal to $\log(M_t) - \log(M_{t-1})$ we can derive an equation that describes how the price level will be determined in equilibrium,

$$\log(P_t) = \log(M_{t-1}) - b_1[\log(M_{t-1}) - \log(P_{t-1})]. \quad (38)$$

The right-hand-side of Eq.(38) contains only variables dated at time $t-1$. It follows that the price level must be predetermined at date t . It is this sense in which our model leads to a description of an economy in which prices may be sticky. There are no barriers to prevent prices from responding to new information. Instead, it is the way that individuals use that information to adapt their beliefs about *future* inflation that causes prices to respond slowly to nominal shocks.

7 Calibrating the Model

In this section we calibrate our model and investigate its implications for the moments of simulated data. In Table 1 we summarize the information on our calibrated parameters and the evidence that we used to choose them.

INSERT TABLE 1 HERE

The calibration of lsh , and hence of \mathbf{a} is relatively standard. We chose the parameter α to equal the average interest rate in U.S. data divided by M1 velocity. This gives a measure of the resource cost of using liquid assets. The parameter β comes from studies that estimate the cointegrating relationship between M1 velocity and the interest rate, for example, the cross country study of Hoffman, Rasche and Tieslau (1995). The remaining parameters r and \mathbf{c} are chosen to cause the slopes of labor demand and supply to be close and to therefore allow the model to display an indeterminate equilibrium.

Table 2 gives the values of the derived parameters of the model. The parameter b is related to the slope of the labor demand and supply curves. Our explanation of labor market

INSERT TABLE 2 HERE

dynamics is very sensitive to b and to make our explanation work we must choose the slope of the labor supply curve, by choosing r and \mathbf{c} , such that b is close to $-\beta$. This is because when the direct effect of money is small, the slopes of the labor demand and supply curves must be very close, if the equilibrium of the economy is to be indeterminate.

The parameters η , and η_H determine the responsiveness of employment and output to money shocks and η_X measures the sensitivity of one plus the marginal product of money to changes in real balances. Since the marginal product of money is equated in equilibrium to the interest rate, this parameter also determines the elasticity of interest rate fluctuations with respect to money shocks. Finally, b_1 is the slope of the characteristic equation. It is this

parameter that determines whether the equilibrium is indeterminate. Indeterminacy requires $|b_1| < 1$.

8 Evidence From Simulated Data

In this section we illustrate the idea that prices may be “sticky” in equilibrium by simulating data from our model economy. In our simulations we chose the parameter b_1 to equal 1; in other words, we simulated a predetermined price equilibrium.¹³

How We Simulated Our Data

To facilitate comparison with actual data, we fed shocks into our model, recovered from actual U.S. data. For the sequence $\{\% \}$ we used the log growth rate of U.S. M1 and for the productivity shock $\{S_t\}$ we used the Solow residual, computed as $\log(S_t) = \log(Y_t) - 0.67 \log(L_t) - 0.33 \log(K_t)$.¹⁴

Our simulated data was constructed by first generating a sequence of 59 values for $\log(m_t)$ by iterating the equation,

$$\log(m_t) = b_1 \log(m_{t-1}) + \% , \quad m_0 = 0 , \quad (39)$$

where $b_1 = 0.3$ and $\{\% \}$ was the sequence of actual log money supply growth rates. Next,

we generated the stationary series $\{d \log(\bar{m}_t)\}_{t=1930}^{1988}$ from the equation,

$$d \log(\bar{m}_t) = \log(m_t) - \log(m_{t-1}) + v_t . \quad (40)$$

The notation $d \log(\bar{m}_t)$ stands for the first difference of the log of real balances and v_t is the first difference of the log of the productivity shock. Eq.(40) comes from differencing the

identity, $\log\left(\frac{M_t}{P_t}\right) \equiv \log(m_t) + \log(S_t)$. For the series $\{v_t\}_{t=1930}^{1988}$ we fed in the actual

values of the log difference of the Solow residual taken from the U.S. data. Then we

constructed the series $\{d \log(y_t)\}_{t=1930}^{1988}$ by taking the difference of Eq.(26);

$$d \log(y_t) = \mathbf{H} [\log(m_t) - \log(m_{t-1})] + v_t. \quad (41)$$

Interest Rate Volatility

The interest rate in our model is found from linearizing Eq.(18),

$$E_t [f_m(L_{t+1}, m_{t+1}) X_{t+1}] = E_t [i_t X_{t+1}], \quad (42)$$

$$d \log(i_t) = (\mathbf{I} - 1)(1 - \mathbf{e}_H) [\log(m_{t+1}) - \log(m_t)] + w_t.$$

The first line of this expression is the asset market equilibrium equation. By including the interest rate inside the expectation operator on the left side of (42) we are implicitly assuming that bonds are not perfectly safe assets. Since the period of our model is a year, and since portfolios are rebalanced daily, this does not seem an unreasonable assumption. It has the advantage of allowing us to capture observed interest rate volatility. The second line of Eq.(42) is the log linearization of this condition. This is the equation we used to simulate the series $d \log(i_t)$.

To capture the fact that the interest rate in real data is relatively volatile, we added the sequence of random variables $\{w_t\}_{t=1930}^{1988}$ to our simulated interest rate series. To generate $\{w_t\}$ we took a sequence of mean zero normal random variables with a variance of .065, a number chosen to replicate the observed standard deviation of interest rate fluctuations in the data.

Characteristics of the Simulated Data

Figures 3 and 4 graph the actual series for the log differences of GDP, real balances and the interest rate against simulated data for a single simulation and Table 3 compares the volatility of the data with the volatility of the simulated series.

INSERT TABLE 3 HERE

INSERT FIGURES 3 AND 4 HERE

INSERT TABLES 4 AND 5 HERE

Tables 4 and 5 present the correlation matrix of the simulated and actual series. It is apparent from these tables that, in the simulations, real balances move a little too closely with GDP. The interest rate also has the wrong correlation with GDP. However, the broad features of actual and simulated series are similar.

To get a better feel for the dynamics of the model, compared with data, we estimated a three variable vector autoregression on actual and simulated data series. In each case we included two lags of the log difference of money growth, the log difference of GDP growth and the log difference of the interest rate. We used actual data on GDP and the interest rate in one case and data simulated from a single run of the model in the other. Since the model was driven by the actual log difference of nominal money growth, we used the actual money growth series in both cases.

In actual data we also looked at a four variable autoregression, including real balances in the system, with similar results. We could not run a four variable system on the simulated data as the four variable simulated system is singular: there are only three independent shocks. To check that this did not affect the qualitative features of the results we experimented with two different three variable VAR's; one with GDP, the interest rate and nominal money and one with real balances, the interest rate and nominal money. We also ran the four variable VAR on the actual data and compared the impulse responses with each of the three variable systems to make sure that the qualitative features of the systems did not change. The main findings from the comparison of these two sets of figures is the broad

INSERT FIGURE 5 HERE

similarity in the qualitative and quantitative nature of the responses of the model economy with that of the data. Notice in particular, the response of the real economy to a monetary shock depicted in the second panel of Figure 5.

The Labor Market

It is perhaps worth drawing attention to one aspect of our model that is a common failing of equilibrium models in which demand shocks play an important role. Since output movements are generated by movements along a concave neoclassical production function, productivity is predicted to be counter-cyclical in response to monetary shocks. In the data productivity is procyclical. However, since our model is a two shock model, it performs relatively well in this dimension.

INSERT TABLE 6 HERE

Table 6 compares the standard deviations of output and employment series, .048 for employment and .065 for GDP in the actual data: .055 for employment and .064 for GDP in simulated data. A second aspect of the labor market behavior of the model that is worth pointing out is its ability to capture the covariation of real wages with employment. It has been pointed out by a number of authors that the actual covariance of real wages and employment is low. In a one shock model, the covariance should be high. The resolution is to add demand side shocks as in the work by Christiano and Eichenbaum (1992). Since our model is driven by both demand and supply shocks it is perhaps unsurprising that we are also able to replicate this feature of the actual data.

9 Conclusions

The idea that general equilibrium models can generate indeterminate equilibria has been understood for some time although it is only recently that such models have been calibrated to fit existing data. Two criticisms are frequently leveled at economic models with indeterminacy. The first is that the degree of increasing returns required to generate indeterminacy is implausible. The second is that models with a multiplicity of equilibria cannot be used to make concrete predictions. In this paper we have addressed both of these criticisms by supplementing a monetary model with a model of how agents form beliefs. In

our model, indeterminacy arises for parameter values that we argue are plausible, even when the technology satisfies constant returns-to-scale.

Perhaps the most unsatisfactory element of our explanation of the monetary transmission mechanism is our reliance on voluntary fluctuations in labor supply to explain employment variation. In this regard, we are following in the tradition of real business cycle models. We believe that the equilibrium approach to the labor market is the right one, although we would prefer a more sophisticated model, perhaps based on search theory, with a role for unemployment. Developments of this kind may add realism to the model, an important consideration if we wish our explanation to have an impact on the monetary policy debate. But it is unlikely to alter our main conclusions.

We have argued that models of multiple equilibria are not devoid of predictive content. In fact, each of the equilibria that might arise has a very different concrete prediction for the behavior of data.¹⁵ Provided one imposes the discipline that agents form expectations in a stable way, the existence of indeterminacy should provide no more of a problem for econometricians than the assumption that utility functions are stable over time. In recent literature, a number of authors have exploited the idea that equilibria may be indeterminate to generate explanations of business cycles that are driven and propagated by “animal spirits”.¹⁶ In this paper we have argued that equilibrium models in which there may be an indeterminate set of equilibria may also be used to explain why monetary policy has real effects.

10 Appendix A

In this appendix we establish the existence of a solution to (12), (13), (16) and (17) evaluated, along a balanced growth path for specific functional forms. We seek values $\{y^*, c^*, L^*, m^*, i^*\}$ that solve the following steady state equations. Eqns.(43) and (44) come from evaluating (12) and (13) along the balanced growth path. Eq.(45) is the steady state money demand equation, derived from combining (16) and (17). Eq. (46) is the Euler equation for money, (20).

$$c^* = y^* = f(L^*, m^*) \quad \Rightarrow \quad y^{*I} = (1-a)L^{*aI} + am^{*I}, \quad (43)$$

$$\frac{-u_L(y^*, L^*)}{u_c(y^*, L^*)} = f_L(L^*, m^*), \quad \Rightarrow \quad \frac{(y^{*1-r} + A)}{(r-1)y^{*-r}(L^* - B)} = (1-a) L^{*-1} y^{*1-}, \quad (44)$$

$$i^* = f_m(L^*, m^*) \quad \Rightarrow \quad i^* = a \left(\frac{y^*}{m^*} \right)^{1-}, \quad (45)$$

$$(1+r)(1+m) = [1 + f_m(L^*, m^*)] \quad \Rightarrow \quad (1+r)(1+m) = 1 + a \left(\frac{y^*}{m^*} \right)^{1-I}. \quad (46)$$

To solve these equations, first let $v^* \equiv \frac{y^*}{m^*}$ be the steady state value of real balances. From

(46) it follows that $v^* = \left[\frac{(1+r)(1+m) - 1}{a} \right]^{\frac{1}{1-I}}$ and from (45) that $i^* = av^{*1-}$. Now express

(43) and (44) as follows,

$$L^* = \left[\left(1 - \frac{a}{v^{*I}} \right) \left(\frac{1}{1-a} \right) \right]^{\frac{1}{aI}} y^{*\frac{1}{a}}, \quad (47)$$

$$L^* = \left(\frac{1}{1-a} \right)^{\frac{1}{c+aI-1}} y^{*\frac{r-1+I}{c+aI-1}}. \quad (48)$$

and solve these two simultaneous equations to find y^* and L^* as functions of the parameters of the model.

11 Appendix B

We begin by log linearizing the production function. The parameter d is the elasticity of output with respect to real balances evaluated along a balanced growth path:

$$\%y = (1 - a)\%L_t + \%m, \quad (49)$$

$$\text{where } a \equiv \frac{am^*}{(1-a)L^* + m^*}$$

The labor market equation takes the form,

$$\frac{Y^r}{L} = \frac{Y^{1-r}}{L^{1-r}}. \quad (50)$$

Log-linearizing (50) leads to the expression:

$$\%y = b\%L_t, \quad (51)$$

$$\text{where } b \equiv \frac{+}{b - (1 -)} - 1.$$

Putting together (49) and (51) one can solve for $\%y$ and $\%L_t$ as functions of $\%m$,

$$\%L_t = e_h \%m, \quad e_h \equiv \frac{d}{b - a(1 - d)}, \quad (52)$$

$$\%y = {}_H \%m, \quad H \equiv \frac{b}{b - (1 -)}. \quad (53)$$

These are linearized versions of the functions $h(m)$ and $H(m)$ described in the text.

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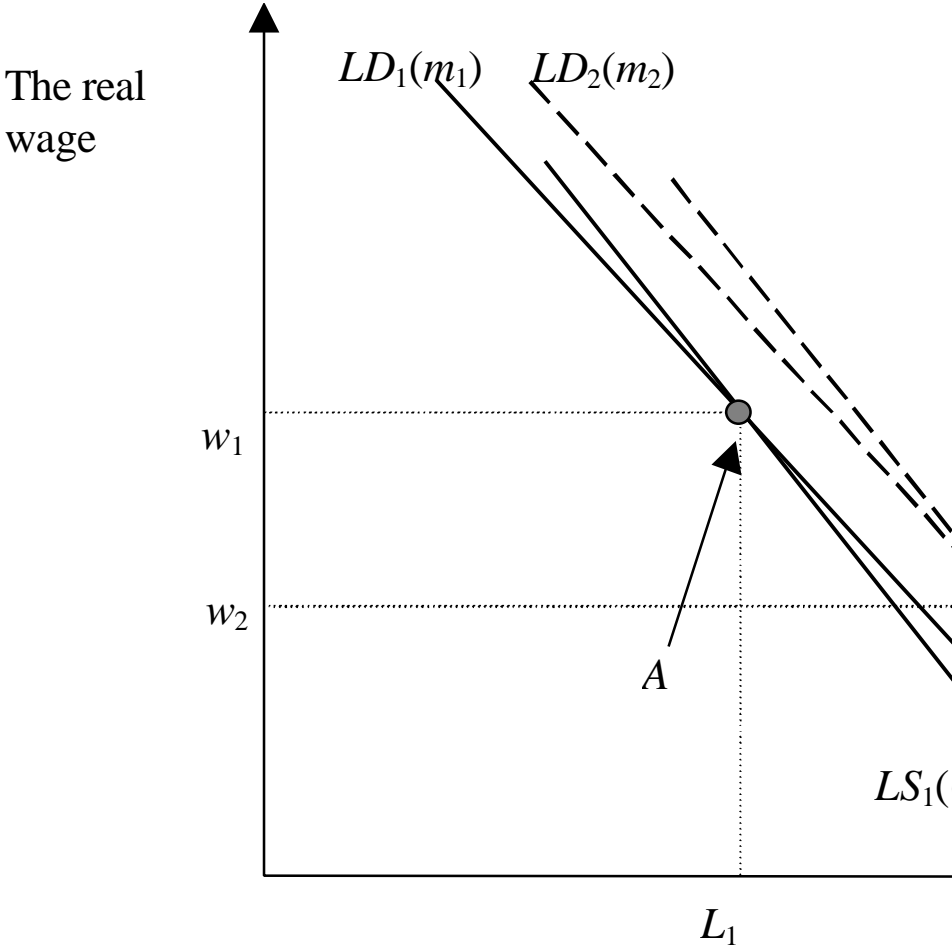
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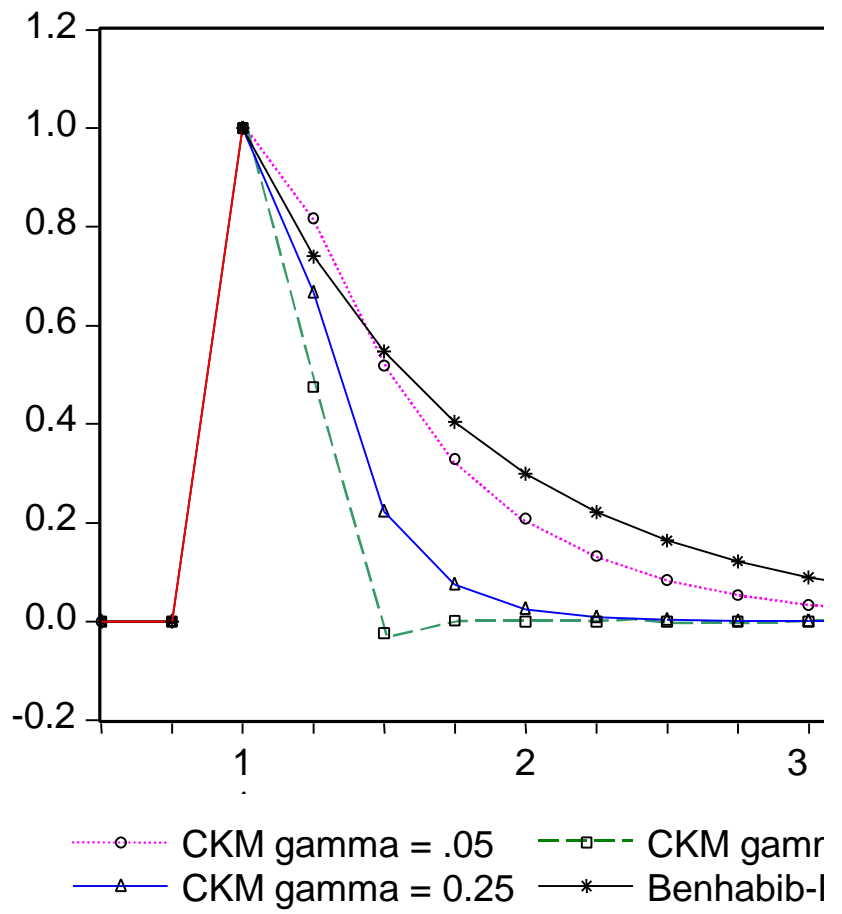
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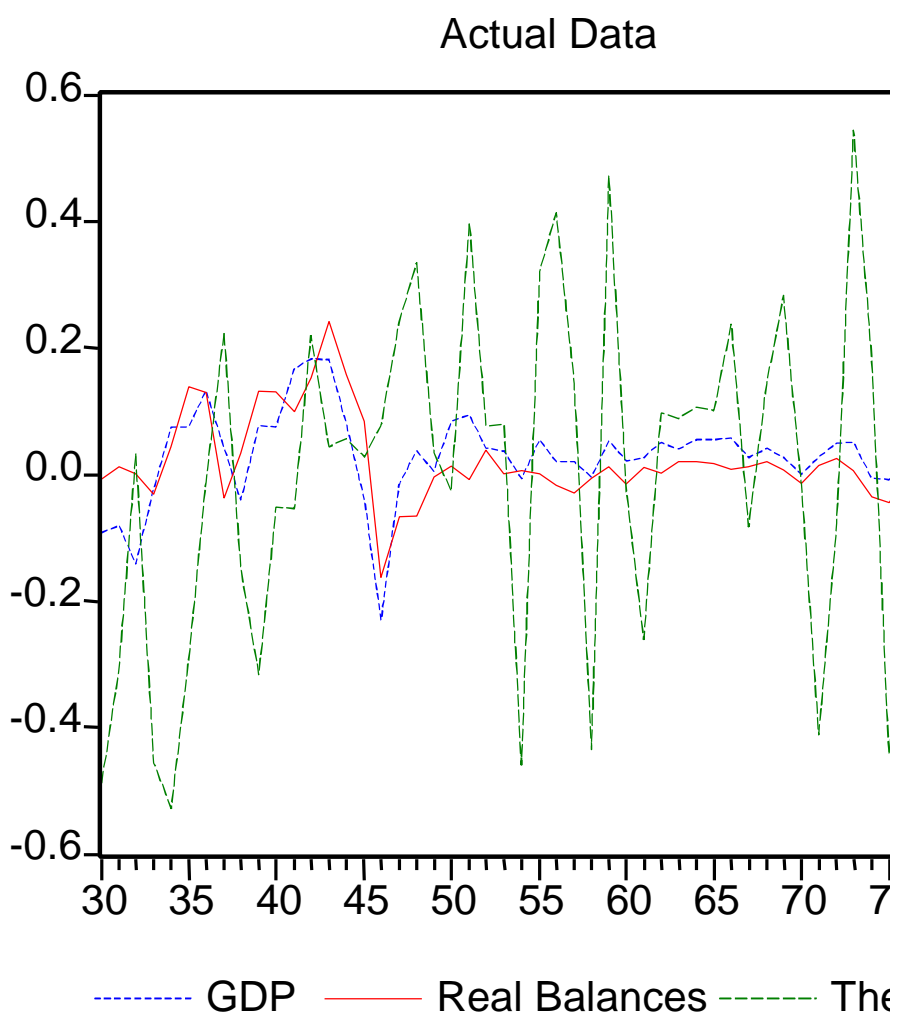
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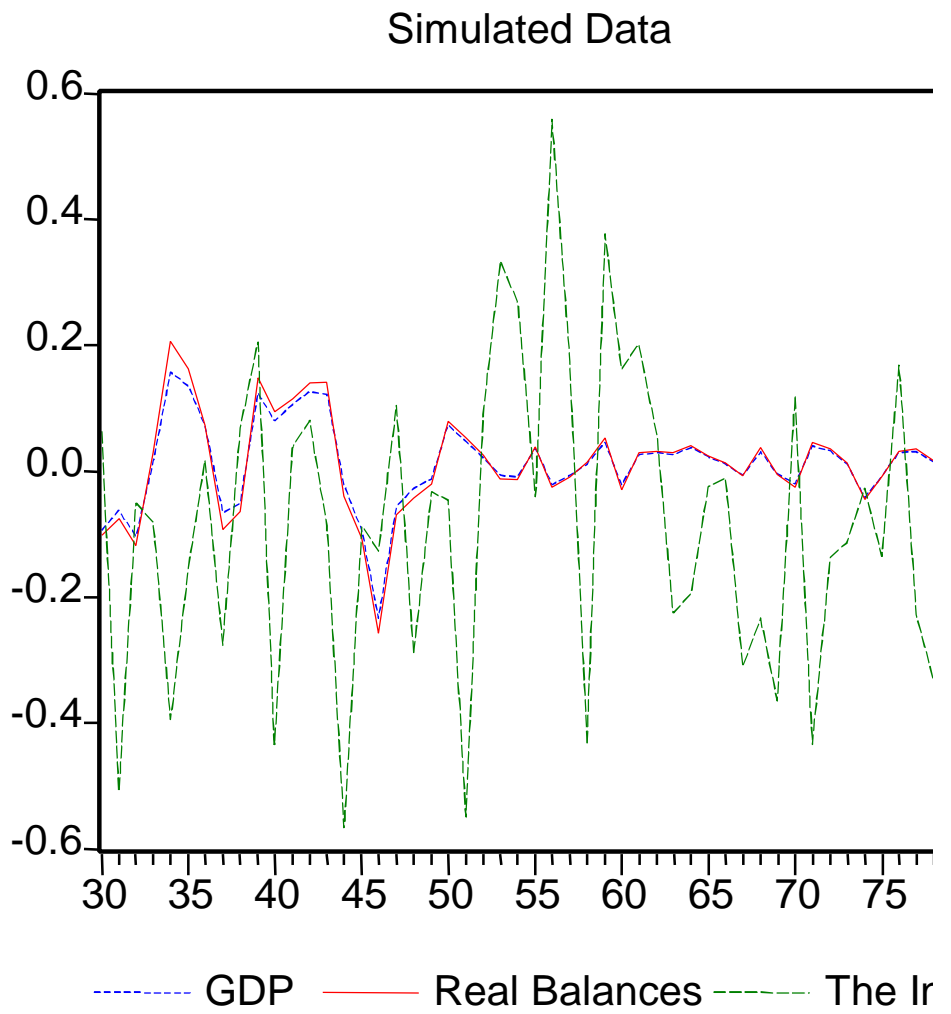
Review of Economic
Dynamics
Benhabib and Farmer
Figure 1

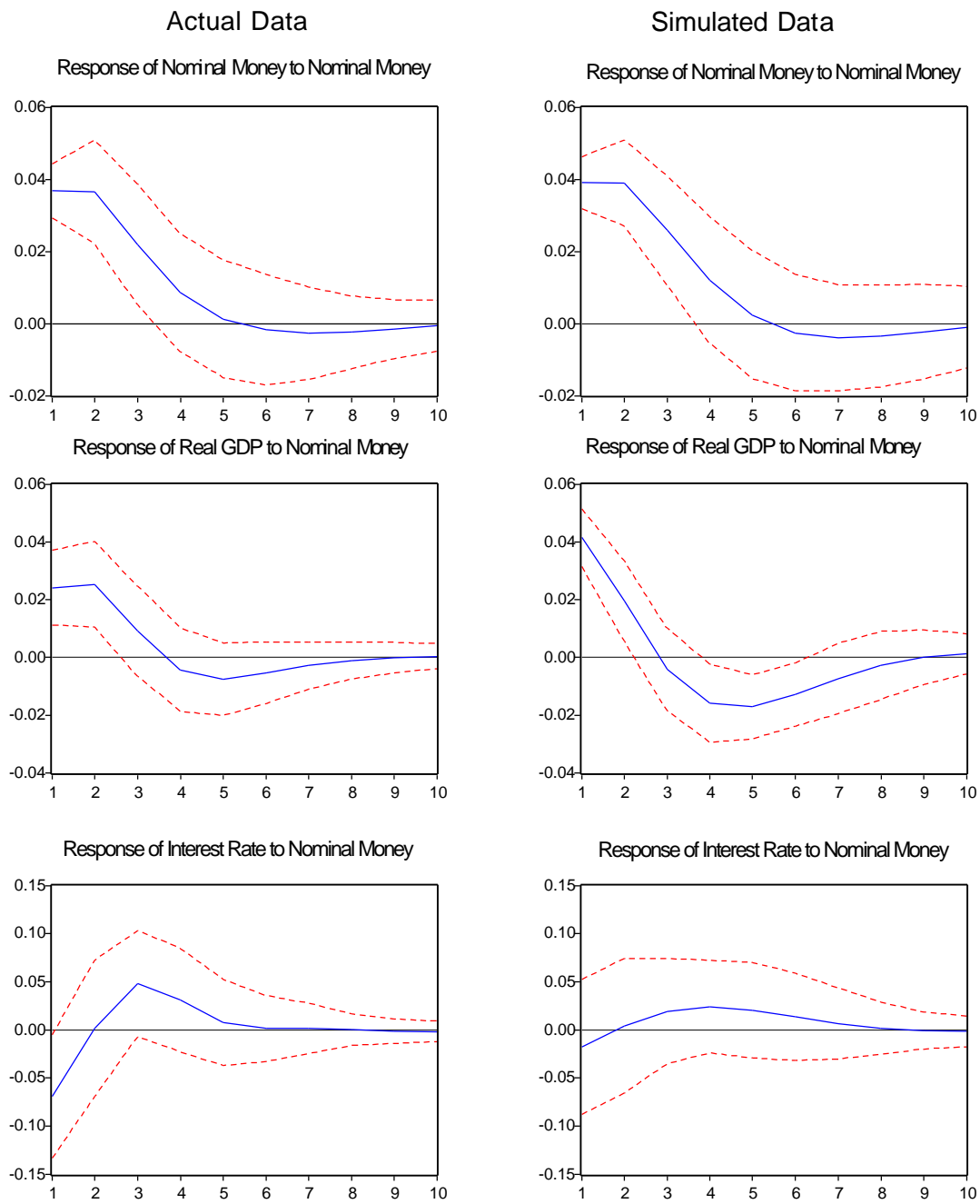


Review of Economic
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Figure 2









Legends for Figures

FIGURE 1: The Labor Market in Our Model

FIGURE 2: Persistence of Monetary Shocks in Different Models

FIGURE 3: GDP, Real Balances and the Interest Rate in US Time Series from 1930 through 1988

FIGURE 4: GDP, Real Balances and the Interest Rate in Simulated Data Using Actual Money growth and the Actual Solow Residual as Shocks

FIGURE 5: Impulse Responses to a Money Growth Shock

<i>Table 1:</i>		
<i>Parameter</i>	<i>Magnitude</i>	<i>Evidence</i>
<i>lsh</i>	0.67	Labor's share of Income
$\equiv msh$	0.01	Money's Share of Income
	0.68	$\equiv \frac{lsh}{1 - msh}$
	-1	Cointegrating relationship of velocity and the interest rate
<i>r</i>	1.35	Asset market studies (1 is log preferences)
	1.23	Makes H equal to 0.75

<i>Table 2</i>		
<i>Derived parameters</i>	<i>Magnitude</i>	<i>Interpretation</i>
b	0.67	Reduced form elasticity of y w.r.t. L
h	1.10	Reduced form elasticity of L w.r.t. m
H	0.75	Reduced form elasticity of y w.r.t. m
x	-0.024	Reduced form elasticity of $(1+i)$ w.r.t. m
G	-0.010	Elasticity of $mu_c(H(m))$ w.r.t. m
b_1	0.3	Slope coefficient of characteristic equation

<i>Table 3: Standard Deviations</i>	<i>GDP</i>	<i>Real Balances</i>	<i>Interest Rate</i>
<i>Std. Dev. (Simulation)</i>	0.068	0.080	0.23
<i>Std. Dev. (Actual Data)</i>	0.066	0.065	0.26

<i>Table 4: Correlation Matrix of Simulated Data</i>	<i>GDP</i>	<i>Real Balances</i>	<i>Interest Rate</i>
<i>GDP</i>	1.000000	0.997161	-0.021490
<i>Real Balances</i>	0.997161	1.000000	-0.010626
<i>Interest Rate</i>	-0.021490	-0.010626	1.000000

<i>Table 5: Correlation Matrix of Actual Data</i>	<i>GDP</i>	<i>Real Balances</i>	<i>Interest Rate</i>
<i>GDP</i>	1.000000	0.656839	0.192373
<i>Real Balances</i>	0.656839	1.000000	-0.180435
<i>Interest Rate</i>	0.192373	-0.180435	1.000000

<i>Table 6: Volatility Data</i>	<i>Actual Employment</i>	<i>Actual GDP</i>	<i>Simulated Employment</i>	<i>Simulated GDP</i>
<i>Std. Dev.</i>	0.048231	0.065772	0.055441	0.064056

Endnotes

¹ Recent summaries of the evidence include Bernanke and Gertler (1995), Christiano, Eichenbaum and Evans (1997) and Christiano and Eichenbaum (1999).

² The fact that monetary models might display indeterminate equilibria has been known at least since the work of Brock (1974). Other authors who have studied this issue include Calvo (1979), Beaudry and Devereux (1993), Benhabib and Bull (1983), Benhabib and Farmer (1991), Farmer (1991:a), (1991:b), (1992), Farmer and Woodford (1997), Gray (1983), Lee (1993), Matsuyama (1991), Matheny (1992), (1998), Obstfeld and Rogoff (1983) and Woodford (1987), (1994).

³ One such attempt is given by Benhabib and Farmer (1991), who rely on aggregate monetary externalities; another is by Beaudry and Devereux (1993), who rely on increasing returns to scale.

⁴ Farmer (1987) models the effects of money on employment in model in which there are informational asymmetries and this offers one possible route through which money may appear as a productive asset.

⁵ In the limited participation model of Christiano and Eichenbaum (1997), a contraction in credit operates through the demand for labor. It is likely that a version of this model, supplemented with a labor market rigidity of the kind that we study in this paper, will also lead to indeterminacy.

⁶ For estimates of the interest elasticity of money demand using low frequency movements see the paper by Hoffman Rasche and Tieslau (1995).

⁷ Farmer (2000) compares models of indeterminacy with New Keynesian models that assume price stickiness due to dynamic menu costs.

⁸ There is a version of the model presented in this paper, in which real balances have large effects because labor supply slopes up as in the work of Benhabib and Farmer (1994). We have chosen not to follow this approach because the degree of increasing returns required to explain indeterminacy appears to be empirically implausible.

⁹ A sample (by no means comprehensive) list of recent papers in this literature includes work by Ascari, (1997), Ascari and Garcia, (1999), Fuhrer and Moore (1995), Gali and Gertler (1998), Jeanne (1998), Kimball (1995), and Rotemberg and Woodford (1998:a), (1998:b).

¹⁰ See Farmer (2000) which compares the sticky price approach with the model laid out in this paper.

¹¹ To derive this path we set $v_t = 0$ and $\theta_t = \theta$ for all t and we let y^* , L^* , c^* and m^* represent the stationary values of y_t , c_t , and m_t that satisfy equations (11)–(12), (15), and (16).

¹² To derive (33) we use the fact that $\mu_t \equiv \log(M_t) - \log(M_{t-1})$ and that $E_{t-1}[\mu_t] = 0$.

¹³ A copy of the Gauss code that we used to simulate our data is available at

<http://www.iue.it/Personal/Farmer/Pdf%20Files/DataAppendixfor%20MonTran.pdf>.

¹⁴ Y_t is GDP, L_t is full and part time equivalent employees and K_t was constructed from the U.S. investment data using a perpetual inventory method. Details can be found in Farmer and Ohanian (1998).

¹⁵ For an elaboration this point see the paper by Farmer and Guo (1995) and the discussion by Aiyagari (1995).

¹⁶ See the collection of papers on this issue in the *Journal of Economic Theory*, Vol. 63 no. 1, 1994.