The distribution of wealth and fiscal policy in economies with finitely lived agents

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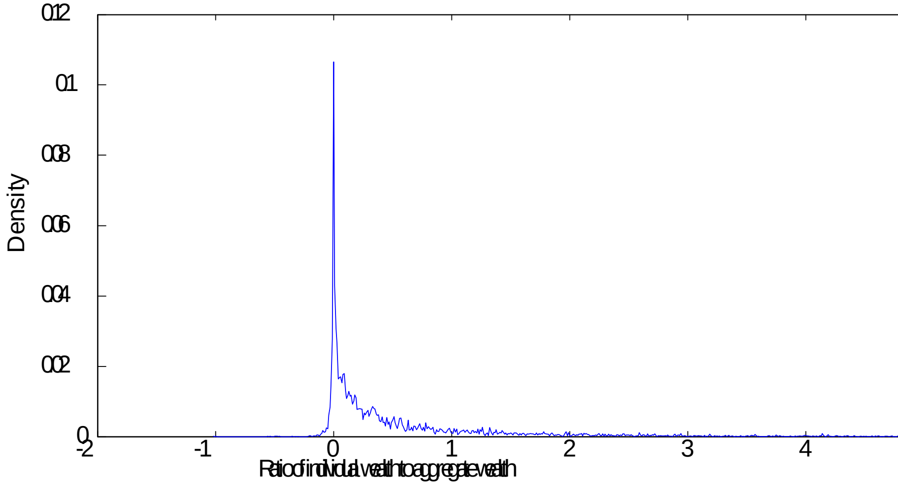
U.S. Wealth Distribution

- Survey of Consumer Finances, 2004

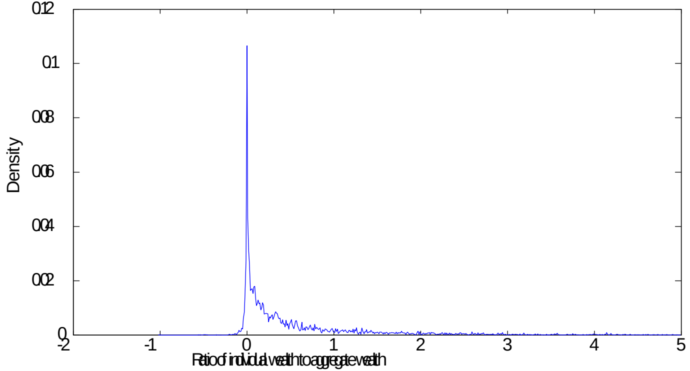


Motivations

- We set up a finite OLG model with stochactic and capital labor income across finitely-lived generations. We solve it analytically to characterize the tail of wealth distribution which exhibits:
  - Right skewness
  - Heavy upper tail
- We study the persistence of talent across generations.
- We explore the effects of capital and estate taxes on wealth distribution.

Mechanism to produce a Pareto distribution

- Pareto distribution: $f(x) \sim x^{-\beta} \quad x \geq x^*$



- A standard old mechanism to generate Pareto distribution is to construct a stochastic process with negative drift and a lower reflecting barrier, as in Champernowne (1953), or using a birth and death process as in Wold and Whittle (1957).

History

- After Pareto (1897), Cantelli (1921), Kalecki (1945), Champernowne (1953), Rutherford (1955), Wold and Whittle (1957), among others, explored the economics of wealth dynamics through stochastic processes that converged to skewed distributions.
- Mincer (1958) pleaded for microfoundations and more explicit determinants of income and wealth distributions, like human capital:

"From the economist's point of view, perhaps the most unsatisfactory feature of the stochastic models, which they share with most other models of personal income distribution, is that they shed no light on the economics of the distribution process. Non-economic factors undoubtedly play an important role in the distribution of incomes. Yet, unless one denies the relevance of rational optimizing behavior to economic activity in general, it is difficult to see how the factor of individual choice can be disregarded in analyzing personal income distribution, which can scarcely be independent of economic activity."
Jacob Mincer, JPE, 1958

History (cont'd)

- With the advent of simulation techniques, stochastic features like luck in labor endowments, together with optimizing agents, were reintroduced to analyze income and wealth distributions.
- Calibrated models aimed at reproducing the wealth distribution in the data through consumption-saving models which 'filter' the observed distribution of earnings.
- -Precautionary savings for uninsurable earnings risk; Huggett (1996), Castaneda-Gimenez-Rios-Rull(2003)...
- -Heterogeneous discount factors, Krusell-Smith (1998)
- - Preferences for bequest (non homogeneous in wealth); Laitner(2001), De Nardi(2004)
- -Heterogeneity in entrepreneurial ability and borrowing constraints; Quadrini(2000), Cagetti-De Nardi (2005)
- See also Econophysics

TABLE 1
DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN SELECTED MODEL ECONOMIES

	Gini	Bottom 40%	Top 5%	Top 1%
		A. U.S. Economy		
Earnings	.63	3.2	31.2	14.8
Wealth	.78	1.7	54.0	29.6
		B. Aiyagari (1994)		
Earnings	.10	32.5	7.5	6.8
Wealth	.38	14.9	13.1	3.2
		C. Castañeda et al. (1998)		
Earnings	.30	20.6	10.1	2.0
Wealth	.13	32.0	7.9	1.7
		D. Quadrini (1998)		
Earnings	...	...	...	...
Wealth	.74	...	45.8	24.9
		E. Krusell and Smith (1998)		
Earnings	...	...	...	...
Wealth	.82	...	55.0	24.0
		F. Huggett (1996)		
Earnings	.42	9.8	22.6	13.6
Wealth	.74	.0	33.8	11.1
		G. De Nardi (1999)		
Earnings	...	...	...	...
Wealth	.61	1.0	38.0	15.0

TABLE 7

Distributions of Earnings and of Wealth in the United States and in the Benchmark Model Economies (%)

Economy	Gini	Quintile: First	Second	Third	Fourth	Fifth	Top Groups (Percentile): 90th–95th	95th–99th	99th–100th
				A. Distributions of Earnings					
United States	.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
				B. Distributions of Wealth					
United States	.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85

Idiosyncratic investment risk

- Bertaut and Starr-McCluer (2002) (BSM), using the Survey of Consumer Finances (SCF) in 1998, document that **19.2% of households have direct investment in stocks** as a financial asset, and 89.9% of households have nonfinancial assets most of which with highly nondiversified risk.
- The 2004 SCF shows that about 49.77% **of all households have assets in quasi-liquid pension funds**, **constituting** 13.45% **of household wealth.**
- According to BSM, **the share of household sector financial assets invested in corporate equities, either directly or through a mutual fund, retirement account, other managed asset or DC pension fund was** 35% **in 1998, involving** 49% **of households.**

Idiosyncratic investment risk-cont'd

- The catalogue of nonfinancial assets consists of primary residence, investment real estate, business equity, and other nonfinancial assets. 66.3% **of households choose to own rather than rent their primary residence.** 18.6% **of households have investments in real estate and** 11.5% **of households have business equity.**
- BSM also document that the **sum of primary residence (home), private business equity and investment real estate account for more than** 50% **of the total assets of U.S. households in 1998.**
- **These financial and non-financial assets are subject to considerable idiosyncratic risk**.
- Using SCF, we can compute the distribution of average returns on wealth household weath, which seems highly variable.

Benhabib-Zhu Model

- There is a continuum of agents in the economy. Constant death rate p. When the agent dies, one child is born.
- Investment opportunity
  - riskless asset
  $$dQ(t) = Q(t)rdt$$
  - risky asset
  $$dS(t) = S(t)\alpha dt + S(t)\sigma dB(t)$$
  where $r < \alpha$.
- Joy-of-giving' bequest motive
  $Z(s,t)$ denotes the bequest that the agent born at time s leaves at time t if the agent dies. Life insurance purchase is $P(s,t)$. Annuities are negative life insurance.
  $$Z(s,t) = W(s,t) + \frac{P(s,t)}{\mu}$$
- In a fair market $\mu = p$ where p is the probability of death.

A portfolio selection problem with uncertain lifespan-Richard (1975)

- The agent's utility maximization problem is

$$\max_{C,\omega,P} E_t \int_t^{+\infty} e^{-(\theta+p)(v-t)}\left[\frac{C^{1-\gamma}(s,v)}{1-\gamma} + p\chi\frac{((1-\zeta)Z(s,v))^{1-\gamma}}{1-\gamma}\right]dv \tag{1}$$

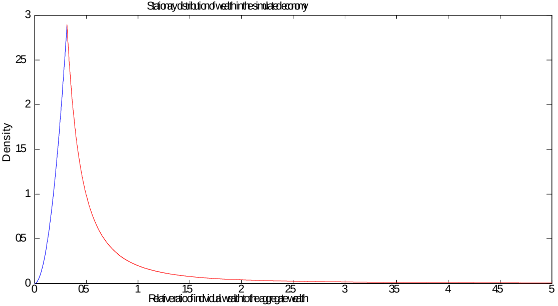
subject to

$$\begin{aligned} dW(s,t) &= [(r-\tau)W(s,t)+(\alpha-r)\omega(s,t)W(s,t) \\ &\quad -C(s,t)-P(s,t)]dt+\sigma\omega(s,t)W(s,t)dB(s,t) \end{aligned} \tag{2}$$

where θ is the time discount rate, χ is bequest motive parameter, τ is capital income tax, ζ is estate tax and $\omega(s,t)$ is the share of wealth the agent invests in risky asset.

Problems

- No stochastic labor income.
- Perpetual Youth (buys stationarity).
- Agents born with too low inheritance maybe because parents had bad luck, are topped up. Alternatively everyone receives a transfer at birth–which may be reinterpreted as discounted value of non-stochastic lifetime earning stream.
- Results: an analytic "Double-Pareto" solution plus comparative statics on inequality and welfare



This Model

- Agents:

$$U = \int_{t=s}^{s+T} \; e^{-\rho t} \frac{c\left(s,t\right)^{1-\sigma}}{1-\sigma} + \chi e^{-\rho T} \frac{\left[\left(1-b\right) w\left(s, s+T\right)\right]^{1-\sigma}}{1-\sigma} dt$$

subject to

$$\dot{w}\left(s,t\right) = r\left(t\right) w\left(s,t\right) + y\left(t\right) - c\left(s,t\right)$$

Simplification

- Idiosyncratic rates of returns and labor income are drawn from a distribution at birth, possibly correlated with those of the parent. Deterministic within life.

Let an individual's age be denoted $\tau = t - s$. Let human capital of an agent born at s at time t, $h(s,t)$, be defined as:

$$h(s,t) = \int_t^{s+T} y(\tau) e^{-\ (r)\tau} d\tau$$

Proposition

The optimal consumption path satisfies

$$c(s,t) = m(\tau)\left(w(s,t) + h(s,t)\right),$$

where the propensity to consume out of financial and human wealth, $m(\tau)$, is independent of $w(s,t)$ and $h(s,t)$. Furthermore, $m(\tau)$ is i) decreasing in age τ; ii) decreasing in the estate tax b and in capital income tax ζ, iii) independent of b for $\sigma = 1$.

Solution to Agent's Problem

Substituting the optimal consumption path into the budget constraint, we can write the dynamics of individual wealth as a function of age τ, to obtain the following linear differential equation with variable coefficients:

$$\dot{w}(\tau) = \tilde{r}(\tau)\, w(\tau) + yq(\tau)$$

The dynamics of individual wealth therefore satisfies the indefinite integral:

$$w(\tau) = e^{\int \tilde{r}(\tau) d\tau} \left(Z + y \int q(\tau)\, e^{-\int \tilde{r}(\tau) d\tau} d\tau \right)$$

where Z is a constant to be determined by initial conditions. In fact, we can solve for the dynamics of individual wealth in exact closed form.

Proposition

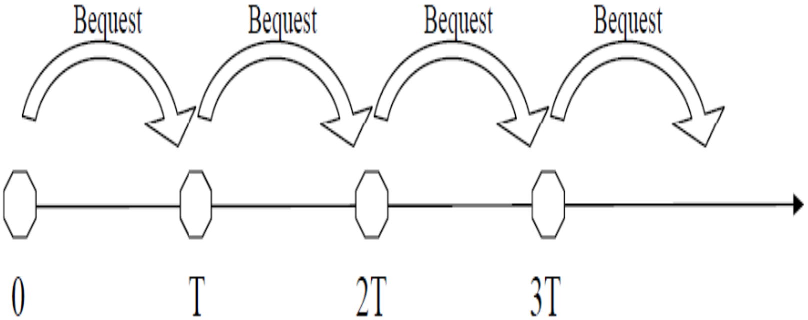
The wealth of an individual of age τ is:

$$
\begin{aligned}
w(\tau) &= \frac{\left(e^{A(r)T} + (A(r)B(b)-1)\, e^{A(r)\tau}\right)}{(A(r)B(b)-1)\, e^{A(r)T}} e^{A(r)(T-\tau)+r\tau} \times \\
&\times \left(\frac{(A(r)B(b)-1)}{\left(e^{A(r)T} + A(r)B(b)-1\right)} w(0) + y(Q^T(r) - Q^0(r)) \right)
\end{aligned}
$$

where

$$
\begin{aligned}
Q^T(r) &= \{Q(r,\tau)\}_{\tau=T}\,,\ \ Q^0(r) = \{Q(r,\tau)\}_{\tau=0}\,,\ \ \textit{and} \\
Q(r,\tau) &= \int q(\tau) \frac{(A(r)B(b)-1)\, e^{A(r)T}}{e^{A(r)T} + (A(r)B(b)-1)\, e^{A(r)\tau}} e^{-A(r)(T-\tau)-r\tau} d\tau
\end{aligned}
$$

Exploiting the solution for $w(T)$ in terms of $w(0)$, we can construct a discrete time map for each dynasty equating post-tax bequests from parents with initial wealth of children. Let $w_n = w(0, nT)$ be the initial wealth of the n'th dynasty. A generic individual of the n'th dynasty faces constant rate of return of wealth and initial earnings over his/her lifetime. On the other hand, the rate of return of wealth and earnings are stochastic across individuals and generations; we let $(r_n)_n$ and $\left(y_n e^{g'n}\right)_n$ denote, respectively the stochastic process for the rate of return of wealth and initial earnings, over dynasties n.



We can then construct a stochastic difference equation for the initial wealth of dynasties, mapping w_{n-1} into w_n. It is in fact convenient to work with discounted variables:

$$z_n = \left(e^{-g'}\right)^n w_n, \quad z_{n-1} = \left(e^{-g'}\right)^{n-1} w_{n-1}$$

We thus obtain the following stochastic difference equation:

$$z_{n+1} = \alpha_n z_n + \beta_n$$

where

$$\begin{aligned}
\alpha_n &= (1-b)\frac{A(r_n;\rho,\sigma)B(b;\chi,\sigma)e^{r_n T}}{(A(r_n;\rho,\sigma)B(b;\chi,\sigma)-1)+e^{A(r_n)T}}e^{-g'} \\
\beta_n &= (1-b)\,e^{r_n T} y_n e^{-g'}\left(Q^T(r_n) - Q^0(r_n)\right)
\end{aligned}$$

After repeated substitutions

$$z_{n+N} = \left(\prod_{l=0}^{N-1}\alpha_{n+l}\right) z_n + \sum_{l=0}^{N-1}\beta_{n+l}\prod_{m=l+1}^{N-1}\alpha_{n+m}$$

Assumptions

Let $\bar{r} = \{\bar{r}_1, ..., \bar{r}_m\}$ denote the state space of $(r_n)_n$, and P its transition matrix. Similarly, let $\bar{y} = \{\bar{y}_1, ..., \bar{y}_l\}$ denote the state space of $(y_n)_n$. and $\alpha(\bar{r})$ denote the state space of $(\alpha_n)_n$ as induced by the map $\alpha(r_n)$.

Assumption

$\bar{r} = \{\bar{r}_1, ..., \bar{r}_m\}$, $\bar{y} = \{\bar{y}_1, ..., \bar{y}_l\}$ and P are such that: (i) $\bar{r}_i$, $\bar{y}_j > 0$ and bounded , for $i = 1, ...m$ and $j = 1, ...l$, (ii) $P\alpha(\bar{r}) < \mathbf{1}$, (iii) $\exists \bar{r}_i$ such that $\alpha(\bar{r}_i) > 1$, (iv) the elements of the trace of the transition matrix P are positive, that is $P_{ii} > 0$, for any i.

Remark

(ii) can be weakened so $E\alpha(\bar{r}) < 1$ where expectation is taken wrt stationary distribution of $(r_n)_n$.We can discuss later.

Some technical assumptions to avoid degeneracies

Assumption

The process $(\alpha_s, \beta_s)_{n=s}^{\infty}$ *is independent of the realization of* β_{s-1}, *all* s.

The assumption would be satisfied, for instance, if a single Markov process $(x_n)_n$ corresponding e.g., to say productivity shocks, drove parameters of the distributions of returns on capital $(r_n)_n$, as well as labor income $(y_n)_n$.

Assumption

$\Pr(\alpha_0 x + \beta_0 = x) \neq 1$ *for any* $x \in \mathbb{R}_+$

Assumption

The elements of the vector $\bar{\alpha} = \{\ln \overline{\alpha}_1 \ldots \ln \overline{\alpha}_m\} \subset \mathbb{R}_+^m$ *are not integral multiples of the same number.*

Recall the stochastic difference equation

$$z_n = \alpha_n z_{n-1} + \beta_n$$

where z_n is the discounted initial wealth of generation n. The multiplicative component α_n can be interpreted as the effective lifetime rate of return on discounted initial wealth (after subtracting consumption spending and incorporating the bequest motive for accumulation). The additive component β_n can in turn be interpreted as a measure of effective lifetime labor income (again after subtracting the affine part of consumption). Intuitively then, to induce a limit stationary distribution of $(z_n)_n$ it is required that the contractive and expansive components of the effective rate of return tend to balance, i.e., that the distribution of α_n display enough mass on $\alpha_n < 1$ as well some as on $\alpha_n > 1$; and that effective earnings β_n be positive and bounded, hence acting as a reflecting barrier. This in effect what a *reflective* process $(\alpha_n, \beta_n)_n$ guarantees.

Let the shorthand $\bar{\alpha} = \{\alpha(\bar{r}_1), ..., \alpha(\bar{r}_m)\} = \{\bar{\alpha}_1, ...\bar{\alpha}_m\}$ denote induced state space of $(\alpha_n)_n$ and $\bar{\beta} = \{\bar{\beta}_1, ...\bar{\beta}_{lm}\}$ the state space of $(\beta_n)_n$. Let A be the diagonal matrix with diagonal elements $\bar{\alpha}_i$.

Theorem

(Saporta (2005), Thm 1). Consider

$$z_n = \alpha_{n-1} z_{n-1} + \beta_{n-1}, \qquad z_0 > 0$$

Let $(\alpha_n, \beta_n)_n$ satisfy above Assumptions. Then the tail of the stationary distribution of z_n, $P_{.>}(z_n > z)$, is asymptotic to a Pareto law

$$\Pr{}_{>}(z_n > z) \sim cz^{-\mu}$$

where $\mu > 1$ satisfies

$$\lambda\left(A^{\mu} P'\right) = 1$$

and where $\lambda\left(A^{\mu} P'\right)$ is the dominant root of $A^{\mu} P'$

Remark

The analysis of this section holds more generally, when $(\alpha_n, \beta_n)_n$ *is not restricted to be a finite Markov chain. In fact for Markov Chains, the power law exponent* $\mu > 1$ *satisfies*

$$\lambda \left(A^{\mu} P'\right) = \lim_{N \to \infty} \left(E \prod_{n=0}^{N-1} (\alpha_{-n})^{\mu}\right)^{\frac{1}{N}} = 1.$$

Note also that, when $(\alpha_n)_n$ *is i.i.d., condition* $\lim_{N\to\infty} \left(E \prod_{n=0}^{N-1} (\alpha_{-n})^{\mu}\right)^{\frac{1}{N}} = 1$ *reduces to* $E(\alpha)^{\mu} = 1$.

Remark

The term $\prod_{n=0}^{N-1} \alpha_{-n}$ *arises from using repeated substitions for* z_n. *Later I'll try to give a heuristic explanation for it.*

Aggregation

The wealth at age τ of an agent born with wealth $z(0)$, return r, and income y is

$$z(\tau) = \sigma_w(r,\tau)z(0) + \sigma_y(r)y$$

Let $f_0(z)$ denote the density of the stationary distribution of initial wealth z. Then a simple change of variable gives the distribution $f_\tau(z)$ at age τ, with the same tail exponent. The aggregate distribution then is:

$$f(z) = \int_0^T f_\tau(z)d\tau$$

Given the linear map above of initial wealth to wealth at age τ, since the asymptotic power law property with the same power μ for each age is preserved under integration, it is now straightforward to show the following:

Proposition

Suppose the tail of the limiting distribution of initial wealth z^0 is asymptotic to a power law, $P_{.>}(z_n > z) \sim cz^{-\mu}$, then the tail of the distribution of wealth in the population is also asymptotic to a power law, with the same exponent μ.

The Theorem above characterizes the tail of the wealth distribution when the process $(\alpha_n, \beta_n)_n$ is *reflective*. It follows from the Theorem that, as long as $(\alpha_n, \beta_n)_n$ is *reflective,* the stochastic properties of labor income risk, $(\beta_n)_n$, have no effect on the tail stationary distribution of wealth. In fact heavy tails in the stationary distribution require that the economy has sufficient capital income risk, with some $a_i > 1$. Consider instead an economy with limited capital income risk, where $\alpha_i \leq \tilde{\alpha} < 1$ for all i and where $\tilde{\beta}$ is the upper bound of β_n. In this case it is straightforward to show that the stationary distribution of wealth would be bounded above by $\frac{\tilde{\beta}}{1-\tilde{\alpha}}$.

More generally, we can also show that wealth inequality increases with the capital income risk agents face in the economy, as measured by a "mean preserving spread" on the distribution of the stochastic process of "effective return on wealth" $(\alpha_n)_n$.

Let G be the Gini coefficient. Then

$$G = \frac{1}{2\mu - 1}$$

Definition

Define a *mean preserving spread* on the distribution of the stochastic process of $(\alpha_n)_n$ as any change of the state space $\bar{\alpha}$ and/or the transition matrix P which increases the variance of $\lim_{N\to\infty}\left(\prod_{n=0}^{N-1}\alpha_{-n}\right)^{\frac{1}{N}}$ while keeping the mean constant.

Note that, for a finite Markov chain, the mean of the random variable $\lim_{N\to\infty}\left(\prod_{n=0}^{N-1}\alpha_{-n}\right)^{\frac{1}{N}}$ is equal to the dominant root of AP', $\lambda\left(AP'\right)$. In the $i.i.d.$ case, $E\left(\alpha\right) = \lambda\left(AP'\right)$.

Proposition

Wealth inequality, as measured by the Gini coefficient of the tail G, increases with a mean preserving spread on the distribution of the stochastic process of $(\alpha_n)_n$.

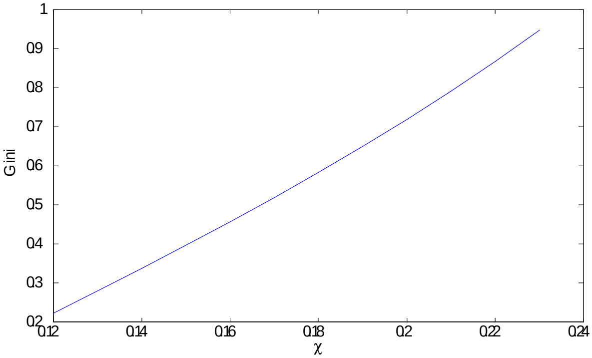
Effect of Bequest Motive on Inequality

Wealth inequality depends on the bequest motive, as measured by the preference parameter χ. An agent with a higher preference for bequests will save more and accumulate wealth faster with a higher α_n, which in turn will lead to higher wealth inequality.

Proposition

Wealth inequality, as measured by the Gini coefficient of the tail G, increases with the bequest motive χ.

Suppose $(\alpha_n, \beta_n)_n$ is *iid*; there is no transmission of talent across generations. Calibrate the stationary return r_n with a discrete uniform distribution, taking values 0.01, 0.11 and 0.21, with a mean return $\bar{r} = 0.11$. Set $\sigma = 1$, $g' = 0.01$, $\rho = 0.04$, and the estate tax rate $b = 0.20$. Finally, set working life span $T = 45$. Under this calibration (with perfect social mobility) we obtain for $\chi = .2$ a Gini coefficient of 0.72 (see the Figure).As χ is increased to .25, the Gini coefficient rapidly grows over .95.



Effects of Fiscal Policies on Inequality

Fiscal policies in our economy are captured by the parameters b and ζ, representing, respectively, the estate tax and the capital income tax.

Proposition

Wealth inequality, as measured by the Gini coefficient of the tail G, is decreasing in the estate tax b and in the capital tax ζ.

Furthermore, let $\zeta(r_n)$ denote a non-linear tax on capital, such that the net rate of return of physical wealth for generation n becomes $r_n\left(1-\zeta(r_n)\right)$. Since $\frac{\partial \alpha_n}{\partial r} > 0$, the Corollary below follows immediately from the argument used in the proof of Proposition 5.

Corollary

Wealth inequality, as measured by the Gini coefficient of the tail G, is decreasing if a non-linear tax on capital $\zeta(r_n)$ is applied which induces a relative shift of the state space.to the left.

The Table below illustrates instead the effects of taxes on the Gini coefficient G of the tail of the distribution of wealth. We calibrate the economy as before, under perfect social mobility. We also fix $\chi = 0.2$. The table shows G as we vary b and ζ.

$b \backslash \zeta$	0.15	0.175	0.20
0.10	$G = 0.91$	$G = 0.57$	$G = 0.34$
0.15	$G = 0.81$	$G = 0.50$	$G = 0.29$
0.20	$G = 0.72$	$G = 0.44$	$G = 0.24$

Social Mobility:

MA(1)

$$\ln \alpha_n = \eta_n + \theta \eta_{n-1}$$

AR(1)

$$\ln \alpha_n = \theta \ln \alpha_{n-1} + \eta_n$$

where $\theta < 1$ and $(\eta_n)_n$ is an $i.i.d.$ process with bounded support.

Proposition

Suppose that $\ln \alpha_n$ *satisfies MA(1). The tail of the limiting distribution of initial wealth* w_n *is then asymptotic to a Pareto law with tail exponent* μ_{MA} *which satisfies*

$$Ee^{\mu_{MA}(1+\theta)\eta_n} = 1.$$

If instead $\ln \alpha_n$ *satisfies AR(1), the tail exponent* μ_{AR} *satisfies*

$$Ee^{\frac{\mu_{AR}}{1-\theta}\eta_n} = 1.$$

In either the MA(1) or the AR(1) case, the higher is θ, the lower is the tail exponent. That is, the more persistent is the process for the rate of return on wealth (the higher are frictions to social mobility), the thicker is the tail of the wealth distribution.

Calibration

Our benchmark calibration postulates a rate of return r_n which is $i.i.d.$ across generations (perfect social mobility). We set the before-tax rate of return to a discrete distribution, with mean 1.0921.The distribution of labor earnings is also discrete, with mean 1.675.These values are chosen so that the labor income share be about twice the capital income share: 1.94 in fact at the stationary distribution. We set the life span to the average working span, $T = 45$, and we set $\sigma = 2$, $\rho = 0.04$, $g = .01$, and $\chi = 0.25$. We also set the estate tax rate $b = 0.20$ (which is the average tax rate on bequests), and the capital income tax $\zeta = 0.15$. Finally, we introduce the possibility of frictions to social mobility: starting from our benchmark $i.i.d$ case for r_n, expressed as a Markov transition matrix with identical rows, we move a mass ε of probability from the off-diagonal terms in each row to the diagonal term of that row in order to introduce persistence across generations.

Economy	Percentiles		
	$90th - 95th$	$95th - 99th$	$99th - 100th$
$U.S.$	0.113	0.231	0.347
$Model\ \ with\ \varepsilon = 0$	0.099	0.207	0.385
$\varepsilon = 0.001$	0.093	0.212	0.418
$\varepsilon = 0.004$	0.088	0.230	0.461

Table 5.1: Percentiles of the top tail

Economy	Quintiles				
	First	*Second*	*Third*	*Fourth*	*Fifth*
U.S.	-0.003	0.013	0.05	0.122	0.817
Model with $\varepsilon = 0$	0.022	0.040	0.057	0.091	0.789
$\varepsilon = 0.001$	0.020	0.035	0.050	0.082	0.813
$\varepsilon = 0.004$	0.015	0.027	0.038	0.064	0.856

Economy	*Tail index* μ	*Gini*
U.S.	1.49	0.803
Model with $\varepsilon = 0$	1.795	0.740
$\varepsilon = 0.001$	1.693	0.767
$\varepsilon = 0.004$	1.489	0.814

$b \backslash \zeta$	0	0.05	0.1	0.15
0	1.155	1.301	1.483	1.718
0.1	1.174	1.323	1.510	1.753
0.2	1.195	1.348	1.542	1.795
0.3	1.219	1.378	1.579	1.844

Table 5.3: Tax experiments - Tail index μ

$b\backslash\zeta$	0	0.05	0.1	0.15
0	.838	.790	.772	.734
0.1	.842	.787	.760	.741
0.2	.821	.785	.757	.740
0.3	.832	.783	.761	.735

Table 5.4: Tax experiments - Gini Coefficient

The results above indicate that taxes have a dampening effect on the tail of wealth distribution. This is the case despite the presence of bequest motives. Becker and Tomes (1979), on the contrary, find that tax increases not only have a negligible influence on the stationary wealth distribution, but may in fact increase inequality even if the tax receipts were redistributed lump sum in an egalitarian fashion. In their model the utility of parents depends on the expected income of children, and parents can anticipate and essentially offset any fiscal policy, dampening any wealth equalizing aspects of these policies. They say:

"One of the surprising implications is that even a progressive tax and public expenditure system may widen inequality in disposable income...Consequently our analysis offers no comfort to the prevailing view that a redistribution within a progressive tax-subsidy scheme reduces relative inequality in disposable income."

"Finally, as far as the policy experiment of abolishing estate taxation is concerned, we find that the steady-state implications of this policy change are to increase output by 0.35 percent and the stock of capital by 0.87 percent, and that its distributional implications are very small."
Castaneda, Jimenez and Rios-Rull (2003), JPE
"We find that abolishing estate taxation would not generate large increases in inequality, and would, in some cases, generate increases in aggregate output and capital accumulation. If, however, the resulting revenue shortfall were financed through increased income or consumption taxation, the immensely rich, and the old among those in particular, would experience a welfare gain, at the cost of welfare losses for the vast majority of the population."
"Estate Taxation, Entrpreuneurship and Taxes," Cagetti and DeNardi (2009), AER

In our model fiscal policy is used for government consumption and does not have any direct redistributive and wealth equalizing effect. Nonetheless, both capital income and estate taxes dampen the effect of luck on the tail acting through the stochastic returns on capital. The effect of a streak of luck acting multiplicatively on wealth can be powerful and in fact generates the skewness and fat tails of the wealth distribution. Any dampening either of returns to wealth through capital taxes or of the transmission of wealth through estate taxes will tend to flatten the heavy tails of the wealth distribution. Capital income risk, inducing a stochastic return on capital, therefore, is the main reason why our results onfiscal policies differ substantially from those of Becker and Tomes (1979) and others.

Intuition

For simplicity take $\beta_n = \beta$ constant (but constant support will do as well). Then, letting $P(z)$ denote the stationary distribution of $(z_n)_n$,

$$P(z) = \int P\left(\frac{z-\beta}{\alpha}\right) \pi(\alpha)\, d\alpha$$

where $\pi(\alpha_n)$ is the density of α_n. For large z,in the tail, we can approximate the solution by the solution to

$$P(z) = \int P\left(\frac{z}{\alpha}\right) \pi(\alpha)\, d\alpha$$

by $P(z) = C(z)^{-\mu}$:

$$Cz^{-\mu} = \int Cz^{-\mu} (\alpha)^{\mu} \pi(\alpha)\, d\alpha = Cz^{-\mu} \int (\alpha)^{\mu} \pi(\alpha)\, d\alpha$$

Therefore, μ solves

$$\int (\alpha)^{\mu} \pi(\alpha)\, d\alpha = 1 = E\alpha^{\mu}$$

Conclusions

- The main conclusion of this paper is that capital income risk, that is, idiosyncratic returns on wealth, have a fundamental role in understanding the distribution of wealth.
- Capital income risk appears crucial in generating the thick tails observed in wealth distributions across a large cross-section of countries and time periods.
- Furthermore, when the wealth distribution is shaped by capital income risk, wealth inequality in the tail is very sensitive to fiscal policies.
- Higher taxes in effect dampen the multiplicative stochastic return on wealth, which is critical to generate the thick tails through lucky runs.
- Interestingly, this role of capital income risk as a determinant of the distribution of wealth seems to have been lost also on Vilfredo Pareto.

- Pareto explicitly noted that an identical stochastic process for wealth across agents will not induce the skewed wealth distribution (*Cours d.Economie Politique*, 1897, Notes to #962, p. 416). :

  C'est à dire, le hasard seul ne suffit pas pour expliquer la répartition des revenues, il faut, en outre, admettre nécessairement l'hétérogénéité sociale.

- He therefore introduced (Pareto) skewness into the distribution of "talents" or the endowments of agents.
- Left with the distribution of talents and endowments as a determinant of the wealth distribution, he was perhaps lead to his "Pareto's Law," enunciated e.g., by Samuelson (1965) as follows:

  In all places and all times, the distribution of income remains the same. Neither institutional change nor egalitarian taxation can alter this fundamental constant of social sciences.

- But maybe Pareto thought that fical policies are endogenous, and that the rich have their way.