Persistence of Business Cycles in Multisector RBC Models*

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August 24, 2005

Abstract

In this paper we explore whether the changing composition of output in response to technology shocks can play a significant role in the propagation of shocks over time. For this purpose we study two multisector RBC models, with two and three sectors. We find that, while the two sectors model requires a high intertemporal elasticity of substitution of consumption to match the various dynamic properties of U.S. macroeconomic data, the three sector model has a strong propagation mechanism under conventional parameterizations, as long as the factor intensities in the three sectors are different enough.

Key Words: Real Business Cycles, Persistence
JEL Classification: E00, E3, O40.

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1 Introduction

Over the past several years, attention has been drawn to the fact that standard one sector, stochastic optimal growth models—the paradigm of RBC theory—do not have a strong enough endogenous mechanism to propagate shocks over time. As a consequence, these models are not capable of generating persistent business cycles.[1] The purpose of this paper is to show that multisector models have a potentially strong propagation mechanism which does not rely upon any extra features.

We identify the presence of a strong propagation mechanism with a hump–shaped impulse response of output growth, i.e., with the presence of a strong trend–reverting component; with an autocorrelation function of output growth which is significantly positive for at least three lags, as it seems to be in the U.S. data; and with a power spectrum of output growth which has a peak at business cycle frequencies, again as it is the case in the U.S. data.[2] Cogley and Nason, [11], have shown that the standard RBC model has a monotonically decreasing impulse response function of output to a non–permanent shock, an autocorrelation function of output growth which is always very close to zero and possibly negative, and a flat spectrum of output growth.

Several modeling strategies are known to mitigate the problem: quadratic adjustment costs to capital and labor, as in Cogley and Nason, [11]; variable factor utilization rates, as in Burnside and Eichenbaum, [10]; variable factor utilization rates within an efficiency wages framework, as in Beaudry and Devereaux, [5]; the embedding of a search–theoretic approach to the labor market within an otherwise standard RBC model, as in Andolfatto, [1]; habit formation in leisure coupled with increasing returns to scale, as in Wen, [18]; home production with enough productive externalities to generate multiple equilibria, as in Perli, [15]; sector-specific externalities, as in Benhabib and Farmer, [8]; the addition of a human capital sector with low elasticity of

[1] This fact was pointed out, for example, by Cogley and Nason, [11], and Rotemberg and Woodford, [17].

[2] We follow the same definition of propagation mechanism given in Cogley and Nason, [11]; as consequence we feed strongly autocorrelated shocks to our models. Alternatively, we could have looked at the autocorrelation and spectra of levels, rather than growth rates, and have fed white noise shocks to the models. This is the approach taken by Beaudry and Devereaux, [5].

substitution between raw labor and human capital, as in Perli and Sakellaris, [16]; and human capital with variable factor utilization rates, as in DeJong *et al.*, [12].

All the previous papers essentially try to break the post-impact inverse relationship between consumption and labor of a non–permanent shock.[3] While in the period of impact consumption, labor and output may all move in the same direction, after the impact the intratemporal marginal efficiency condition, which has to be satisfied in any one sector RBC model, forces consumption and labor to move in opposite directions. Since consumption typically continues to increase after a positive shock, labor decreases forcing output to decrease as well, unless capital responds extremely strongly.[4] The papers mentioned above that rely on adjustment costs or variable factor utilization rates or search, [11], [10], [5], [1], [12], introduce a delay in the response of labor to the technology shock, whereas the other papers, [18], [15], [16], [8], try to modify the working of the intratemporal marginal efficiency condition, to allow consumption and labor to move in the same direction for a few periods after impact.

In a more recent paper, Gali [13] decomposes the shocks into a technology and non–technology component. Using VARs he demonstates that, in response to a technology shock, hours may decline on impact and in fact induce a negative correlation between productivity and hours. Indeed, in response to a technology shock, "the initial increase in labor productivity and the (smaller) increase in output is reflected in a short-lived, though persistent (and significant), decline in hours."[5] Gali suggests that this is compatible with a model of sticky prices, so that an increase in productivity does not lead to a sufficient rise in output and sales, and therefore must be reflected in a curtailment of hours. Basu, Fernald and Kimball [4], in a model of firms with quasi-fixed inputs and variable capacity utilization, also find that in response to a technology shock input use and the output of non-residential investment decline, and argue that this provides evidence in favor of sticky price models.

In this paper we pursue a different strategy: instead of adding extra fea-

[3]We assume in what follows that the shocks are always strongly autocorrelated, but do not contain a unit root.

[4]For a detailed discussion of this problem see Perli, [15], and Perli and Sakellaris, [16], who, in turn, build on the early paper by Barro and King, [2].

[5]See Gali [13], p. 259, and Figure 3, p. 262.

tures to a one sector model, we explore the propagation behavior of two multi–sector models. Our idea is that looking at the composition of output may also help understanding the way shocks are propagated in the economy. While in one sector models output is defined simply as the output of a single production function, in multi-sector models output is the result of the composition of consumption and investment goods, i.e., $Y_t = C_t + \sum_{i=1}^{n} p_{i_t} X_{i_t}$. Thinking in terms of impulse response functions, it is clear that output can display a hump–shaped pattern in response to a shock even if neither the consumption good nor the investment goods have a hump–shaped response, or if only one of them has it. In the case of a two sector model, for example, output could have the appropriate impulse response if, after the impact of a positive shock, consumption increases more than what investment decreases. In particular, we confine our attention to a one–capital–good, two–sector model and a two–capital–goods, three–sector model. We show that the latter has a strong endogenous propagation mechanism for a wide range of empirically plausible parameters, as long as the three sectors have sufficiently different factor intensities. The one–capital–good, two–sector model, on the other hand, requires the strong additional assumption of a very low intertemporal elasticity of substitution of consumption to generate artificial data with the same dynamic properties as the real U.S. data. It is nonetheless useful to examine the behavior of this model since it is easier to grasp the intuition behind the persistence results within its simpler structure. In particular, we show that composition effects not only can produce the hump-shaped impulse responses observed in the data, but also can explain the impulse responses generated by Gali, [13], and "the fact that the bulk of the joint variation in employment and productivity arising from a technology shock takes place on impact, with both variables moving in opposite directions."[6]

The paper is organized as follows: the next section discusses the one–capital–good, two–sector model; section 3 discusses the two–capital–goods, three–sector model; and section 4 concludes.

[6] See Gali, [13], p. 259.

2 A Two–Sector Model

In this section we consider a standard model with a single capital good and two sectors, consumption and investment. The representative agent chooses how to allocate capital and labor across the two sectors in order to maximize the discounted sum of each period utilities, subject to the production constraints in the two sectors and the law of motion of capital and of the technologies. Formally:

$$\max_{K_{C_t}, L_{C_t}, L_{I_t}} \mathrm{E}_0 \sum_t \rho^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \theta \frac{(1 - L_{C_t} - L_{I_t})^{1-\gamma}}{1-\gamma} \right]$$

subject to:

$$\begin{aligned} C_t &= q_C z_{C_t} K_{C_t}^{\alpha} L_{C_t}^{1-\alpha} \\ X_t &= q_X z_{X_t} K_{X_t}^{\beta} L_{X_t}^{1-\beta} \\ K_{t+1} &= (1-\delta) K_t + X_t \\ z_{C_{t+1}} &= z_{C_t}^{\xi} e_t \\ z_{X_{t+1}} &= z_{X_t}^{\zeta} u_t \\ K_t &= K_{C_t} + K_{X_t} \end{aligned}$$

To solve this problem, we substitute the produciton function for C_t into the utility function; the first order conditions with respect to K_C, L_C, and L_I are, respectively:

$$MUC_t \cdot MPK_{C_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} \cdot MPK_{X_t} \tag{1}$$

$$MUC_t \cdot MPL_{C_t} = MUL_{C_t} \tag{2}$$

$$MUL_{X_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} \cdot MPL_{X_t} \tag{3}$$

where MPw and MUw denote the marginal product and marginal utility, respectively, of variable w and v_{t+1} is the value function at time $t+1$. To find an expression for $\mathrm{E}(\partial v_{t+1}/\partial K_{t+1})$ consider that:

$$\frac{\partial v_t}{\partial K_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} (1 - \delta + MPK_{X_t}) \tag{4}$$

From (1):

$$\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} = \frac{MUC_t \cdot MPK_{C_t}}{MPK_{X_t}}$$

and therefore, substituting into (4) and updating one period :

$$\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} = \frac{MUC_{t+1} \cdot MPK_{C_{t+1}}}{MPK_{X_{t+1}}} \cdot \left(1 - \delta + MPK_{X_{t+1}}\right) \tag{5}$$

The model has to be solved numerically; to this end, we use a log–linearization around the steady state. We can thus write the system as follows:

$$\begin{pmatrix} \widehat{K}_{t+1} \\ \widehat{L}_{C_{t+1}} \\ \widehat{z}_{C_{t+1}} \\ \widehat{z}_{X_{t+1}} \end{pmatrix} = J \cdot \begin{pmatrix} \widehat{K}_{t} \\ \widehat{L}_{C_{t}} \\ \widehat{z}_{C_{t}} \\ \widehat{z}_{X_{t}} \end{pmatrix} + Q \cdot \begin{pmatrix} e_t \\ u_t \end{pmatrix} \tag{6}$$

where J is the Jacobian matrix and $\widehat{w}$ indicates the percentage deviation of variable w from its steady state. All other variables can be expressed as (approximately) linear functions of total capital, labor in the consumption sector, and the two shocks. System (6) can be simulated numerically; of course, an appropriate value of $\widehat{L}_C$ must be chosen at any point in time as a function of $\widehat{K}$, $\widehat{z}_C$ and $\widehat{z}_X$ since the model has a unique equilibrium.

Once we simulate artificial time series for all the variables we must compute output. One way to do it is to write:

$$Y_t = C_t + p_t X_t$$

where p_t is the price of the investment good in terms of the consumption good. We can obtain p_t from the static first order conditions of the firms with respect to capital, $MPK_{C_t} = r_t$, and $p_t MPK_{X_t} = r_t$. Dividing one by the other we get:

$$p_t = \frac{MPK_{C_t}}{MPK_{X_t}}$$

i.e., the price of the investment good is equal to the ratio of the marginal product of capital in the two sectors. The same relation could have been obtained using the first order conditions with respect to labor. Output computed in this way would correspond to "nominal" output, since current prices are used. Alternatively, one could use $Y_t = C_t + p * X_t$, where $p*$ is the steady state value of the price level and also the starting point in our simulation. This would correspond to "real output", expressed in terms of prices at time zero.

2.1 Calibration and Results

We calibrate some of the parameters in a standard way, and we choose the remaining ones so that the model exhibits a degree of persistence of the shocks compatible with what we observe in the U.S. data. We then ask the question of whether the latter values are plausible or not in view of the available empirical evidence.

We set the depreciation of capital, δ, to 0.025 and the discount factor, ρ, to 0.9898, since we want to simulate quarterly data. Moreover, as it is standard in the RBC literature, we assume that the utility function is linear in leisure, i.e., we set $\gamma = 0$. We then set $\theta = 1.51$ so that, in the steady state, the number of hours worked is 1/3 of the total time available. Finally, we assume that the shocks to both sectors are highly persistent; in particular we set $\xi = \zeta = 0.95$.

We choose the remaining parameters, α, β, and σ, in order for the model to have a significant endogenous propagation mechanism. We provide an intuition for two cases under which endogenous persistence arises. As noted in the introduction, persistence can arise in this model due to changes in the composition of output. Note that we say that we have persistence when the impulse response of output is "hump shaped", i..e., when output increases when a positive, non permanent shock first hits the economy, and continues to increase for a few periods after that. Since output is composed of consumption and investment, this can happen if both variables continue to increase after the impact of the shock; but it can also happen if one of the two variables increases more than the other decreases. Below we present a case in which the latter possibility occurs.

Suppose that both sectors are subject to the same technology shocks, i.e., that e_t and u_t are identical.[7] Assume that a positive, persistent shock hits

[7] The same results that we report below hold also for different innovations, as long as they are strongly positively correlated.

both sectors. If households like to smooth consumption, we will typically observe that both consumption and investment increase at impact, and then decrease monotonically; this clearly is not going to generate any persistence in output. Things could be different, however, if households are not too concerned about consumption smoothing, i.e., if the utility of consumption is linear or close to linear. If that is the case, the higher interest rate induced by the positive shock provides a strong incentive for households to substitute future consumption for present consumption; indeed, with very little curvature of the utility function, households reduce consumption at date zero, the period of impact. Conversely, investment strongly increases—more so than consumption declines—so output increases at date zero. In the periods after the shocks have hit the economy, however, households will start consuming more (the consumption they deferred at date zero), and investment will start declining. Given its low intertemporal elasticity of substitution, and given the high elasticity of the labor supply that we assume ($\gamma = 0$), consumption actually increases faster than investment decreases, and therefore output keeps increasing. Eventually, both consumption and investment will go back to their steady states, and so output has to start decreasing and go back to its steady state after a few periods. But, since at least initially C increases faster than pX decreases, output has a hump–shaped impulse response and therefore the model has a strong endogenous propagation mechanism.

The first restriction that we have to impose to our remaining parameters, therefore, is that σ has to be low; we choose $\sigma = 0.07$, the higher value that gives the desired persistence result. Note, moreover, that we can not have a low labor supply elasticity: if γ is high, total labor is practically constant, and the only movements in labor that we see are between the two sectors. These intersectoral movements are simply not sufficient to push up consumption enough to generate noticeable persistence; we need some extra labor going to the consumption sector from leisure after the impact of the shock. A linear utility of leisure is standard in RBC models. We view an almost linear utility of consumption however as a significant difficulty for the model above, and will address and correct it in the three–sector model in the next section.

Those on σ and γ are the only restrictions that we need in order to achieve persistence. We want, however, to calibrate the model so that not only persistence, but also other statistics are in line with what we observe in the data. For example, if the factor intensities are identical in the two sectors, consumption remains below the steady state for several periods after impact; this has the unpleasant implication of making consumption weakly correlated with output, or even countercyclical, depending on parameter values.[8] The problem can be easily corrected assuming that the investment sector is more capital intensive than the consumption sector, i.e., that $\beta > \alpha$. In particular we choose $\alpha = 0.2$ and $\beta = 0.4$, although the same results below would be obtained with many other different combinations; what matters is the ratio of the two capital shares. In this way, after impact, labor will tend to move back faster to the consumption sector, which is labor intensive, and consumption output will respond more strongly and rapidly to the inflow of labor. With this calibration consumption is below its steady state only for the period of impact, which implies a much higher correlation with output.

	Y	C	X	L	K
Standard Deviation	1.00	0.94	4.52	0.93	0.41
	(1.00)	(0.49)	(2.82)	(0.86)	(0.34)
Correlation with Output	1.00	0.72	0.66	0.99	0.44
	(1.00)	(0.76)	(0.96)	(0.86)	(0.14)
AR(1) Coefficient	0.75	0.76	0.66	0.75	0.97
	(0.90)	(0.84)	(0.76)	(0.90)	(0.96)

Table 2.1 (U.S. Data in parenthesis)

The standard RBC set of statistics for several variables is in Table 2.1. We see that the model performs more or less like other RBC models, i.e., it captures several "static" aspects of the U.S. business cycle quite well. The biggest problem, however, is that consumption is almost twice as volatile as in the U.S. data; this was obviously to be expected, given the extremely high intertemporal elasticity of substitution of consumption that we assumed.

The performance of the model in terms of persistence is shown in figures 1–3. The impulse response function of output is shown in figure 1; with respect to the impulse response of U.S. output, we see that Y takes too long

[8] If the factor intensities are equal in the two sectors, and the shocks are the same, the model is equivalent to a one–sector model. The one–sector model, therefore, can exhibit persistence if the utility of consumption is very flat. The problem is not only that this assumption is quite unrealistic; standard RBC statistics are also quite off the mark in that case.

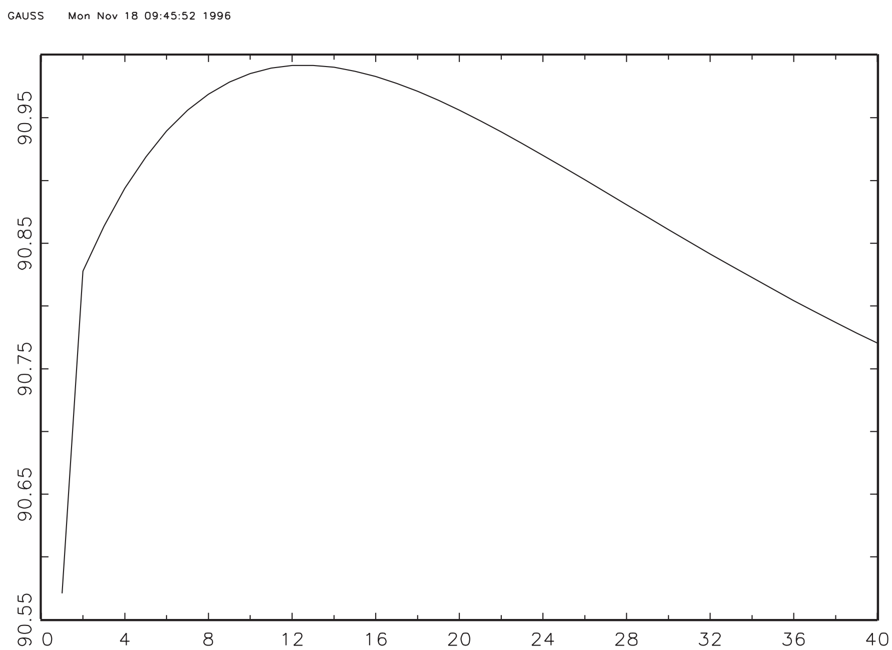

Figure 1: Impulse Response

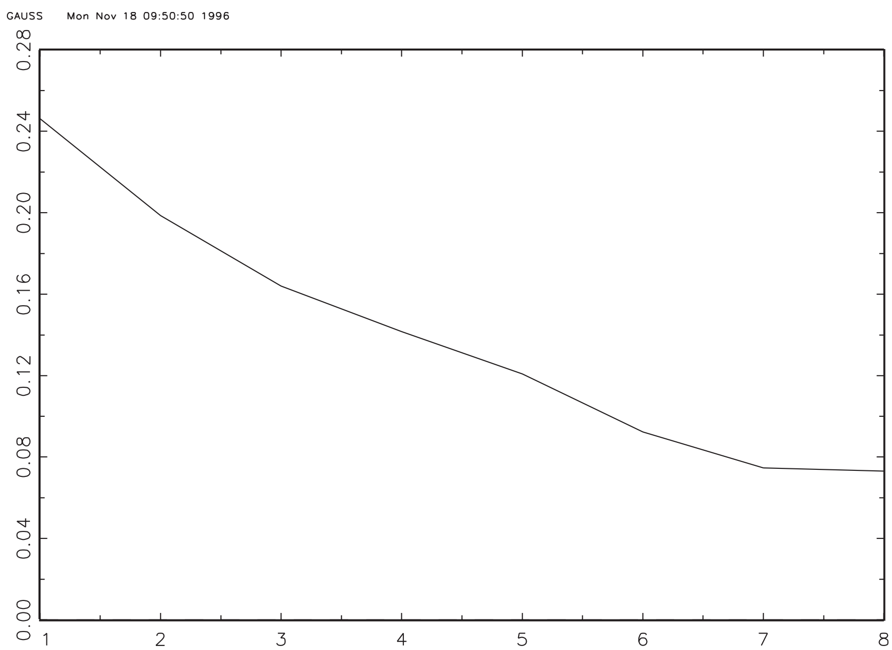

Figure 2: Autocorrelation

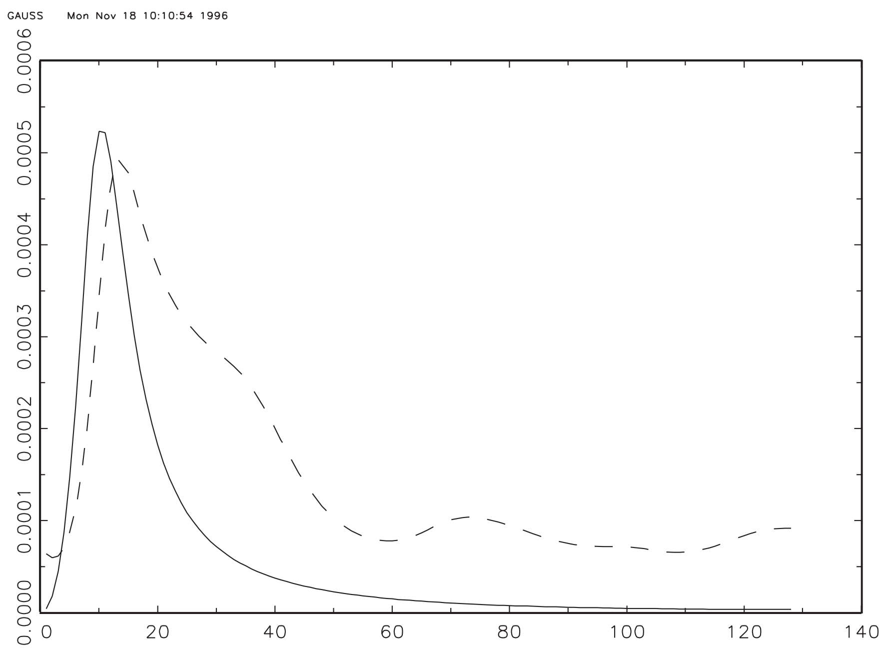

Figure 3: Power spectrum of output growth: two–sector model

to go back to its steady state, a sign that our model has too much, rather than too little persistence. Figure 2 shows the first ten lags of the autocorrelation function of output growth; they are all positive, which is another sign of too much persistence relative to the U.S. data. Finally, figure 3 shows the power spectrum of output growth; the presence of a peak is clear, although it occurs at frequencies slightly lower than the typical business cycle frequencies; this again says that the model has too much persistence.

While with the parameterization above we are able to obtain a degree of persistence comparable to, and actually higher than, what is observed in the U.S. data, we have the problem that the intertemporal elasticity of substitution of consumption has to be too high. In the next section we show that a three–sector model can produce the same level of persistence with a more conventional logarithmic utility of consumption.

3 A Three–Sector Model

In the previous section we showed that a two sector model can generate persistence of technology shocks comparable to what we observe in the U.S. data if consumption increases faster than investment decreases for a few periods after the impact of a positive technology shock. This required, however, a very high intertemporal elasticity of substitution of consumption. The idea of this section is to have a second investment good to absorb some of the role played by consumption in the two sector model. Here we assume a conventional intertemporal elasticity of substitution of consumption equal to one, and we introduce a second investment sector, which we denominate W. The model therefore has one consumption good and two capital goods. Even if here consumption does not react very strongly to technology shocks, as it appears to be the case in the U.S. data, the two investment goods are generating persistence according to a mechanism similar to that described in section 2.

The working of the three–sector model is standard: as in the previous section, the representative agent chooses how to allocate capital and labor across the three sectors in order to maximize the discounted sum of each period's utilities, subject to the production constraints in the three sectors, the laws of motion of the capital stocks, and the laws of motion of the technologies. Formally:

$$\max_{K_{C_t}, L_{C_t}, L_{I_t}} \mathrm{E}_0 \sum_t \rho^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \theta \frac{(1 - L_{C_t} - L_{I_t})^{1-\gamma}}{1-\gamma} \right]$$

subject to:

$$\begin{aligned}
C_t &= q_C z_{C_t} L_{C_t}^{\alpha_0} K_{XC_t}^{\alpha_1} K_{WC_t}^{1-\alpha_0-\alpha_1} \\
X_t &= q_X z_{X_t} L_{X_t}^{\beta_0} K_{XX_t}^{\beta_1} K_{WX_t}^{1-\beta_0-\beta_1} \\
W_t &= q_W z_{W_t} L_{W_t}^{\gamma_0} K_{XW_t}^{\gamma_1} K_{WW_t}^{1-\gamma_0-\gamma_1} \\
K_{X_{t+1}} &= (1-\delta) K_{X_t} + X_t \\
K_{W_{t+1}} &= (1-\delta) K_{W_t} + W_t \\
z_{C_{t+1}} &= z_{C_t}^{\xi} e_t \\
z_{X_{t+1}} &= z_{X_t}^{\zeta} u_t \\
z_{W_{t+1}} &= z_{W_t}^{\eta} v_t \\
K_{X_t} &= K_{XC_t} + K_{XX_t} + K_{XY_t} \\
K_{W_t} &= K_{WC_t} + K_{WX_t} + K_{WW_t}
\end{aligned}$$

Total real output is again defined as $Y_t = C_t + p_{X_t} X_t + p_{W_t} W_t$, where p_X and p_W are the prices of the two capital goods in terms of the consumption good, and are computed in the same way as with the two–sector model; total investment is $I_t = p_{X_t} X_t + p_{W_t} W_t$. The representative agent chooses how much to work and how much capital to use in the three sectors; total capital is just the sum of the capital stocks used in the individual sectors. The first order conditions of this problem can be written as:

$$
\begin{aligned}
MUC_t \cdot MPL_{C_t} &= MUL_{C_t} \\
MUL_{X_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPL_{x_t} \\
MUL_{W_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPL_{W_t} \\
MUC_t \cdot MPK_{XC_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{XW_t} \\
\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPK_{XX_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{XW_t} \\
MUC_t \cdot MPK_{WC_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{WW_t} \\
\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPK_{WX_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{WW_t}
\end{aligned}
$$

where the two derivatives of the value function next period are given by:

$$
\begin{aligned}
\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} &= \frac{MUL_{X_{t+1}}}{MPL_{X_{t+1}}} \cdot \left(1 - \delta + MPK_{XX_{t+1}}\right) \\
\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} &= \frac{MUC_{t+1} \cdot MPK_{WC_{t+1}}}{MPK_{WW_{t+1}}} \cdot \left(1 - \delta + MPK_{WW_{t+1}}\right)
\end{aligned}
$$

Like the two–sector model, this model cannot be solved analytically. We solve it numerically using the same technique as above, i.e., we linearize the Euler equations around the steady state. Since we have now two state variables (the two capital stocks) plus three shocks, the resulting linearized dynamical system consists of seven difference equations in seven variables (the two states, two controls, and the three shocks):

$$
\begin{pmatrix} \widehat{K}_{X_{t+1}} \\ \widehat{K}_{W_{t+1}} \\ \widehat{L}_{X_{t+1}} \\ \widehat{L}_{W_{t+1}} \\ \widehat{z}_{C_{t+1}} \\ \widehat{z}_{X_{t+1}} \\ \widehat{z}_{W_{t+1}} \end{pmatrix} = J \cdot \begin{pmatrix} \widehat{K}_{X_t} \\ \widehat{K}_{W_t} \\ \widehat{L}_{X_t} \\ \widehat{L}_{W_t} \\ \widehat{z}_{C_t} \\ \widehat{z}_{X_t} \\ \widehat{z}_{W_t} \end{pmatrix} + Q \cdot \begin{pmatrix} e_t \\ u_t \\ v_t \end{pmatrix}
$$

All the other choice variables can be expressed in terms of the above variables using the first order conditions. Appropriate values of $\widehat{L}_{X_t}$ and $\widehat{L}_{W_t}$ have to be chosen in every period to make sure that the transversality conditions are satisfied, i.e., to make sure that the dynamics takes always place on the stable branch of the saddle point.

3.1 Calibration and Results

The model is calibrated using the same strategy as in the previous section, In particular, we set the depreciation of capital, δ, to 0.025, the discount factor, ρ, to 0.9898, and the inverse of the labor supply elasticity, γ, to zero. We set $\theta = 1.73$, so that, again, total hours worked in the steady state are 1/3 of the total time available. Unlike the previous case we also set $\sigma = 1$, which implies that the utility is logarithmic in consumption, a standard feature of RBC models. We further assume that the shocks to all three sector are highly persistent, $\xi = \zeta = \eta = 0.95$, and identical.

The factor shares are set in order to get the desired persistence and acceptable results in terms of other types of statistics. As it turns out, there are several combinations of parameter values that yield relatively good results. One such case involves assuming that the consumption sector is relatively labor intensive, and that the investment sector W is relatively more capital–W intensive than the investment sector X. For example figures 4, 5 and 6 and Table 3.1 below were obtained with the following parameters: $\alpha_0 = 0.58$, $\alpha_1 = 0.32$; $\beta_0 = 0.56$, $\beta_1 = 0.24$; $\gamma_0 = 0.39$, $\gamma_1 = 0.12$.

	Y	C	I	L	K
Standard Deviation	1.00	0.70	3.23	1.22	0.65
	(1.00)	(0.49)	(2.82)	(0.86)	(0.34)
Correlation with Output	1.00	0.34	0.88	0.74	0.29
	(1.00)	(0.76)	(0.96)	(0.86)	(0.14)
AR(1) Coefficient	0.93	0.93	0.85	0.82	0.75
	(0.90)	(0.84)	(0.76)	(0.90)	(0.96)

Table 3.1 (U.S. Data in parenthesis)

As one can see in figure 4, the impulse response function to a single aggregate technology shock hitting all three sectors simultaneously is clearly

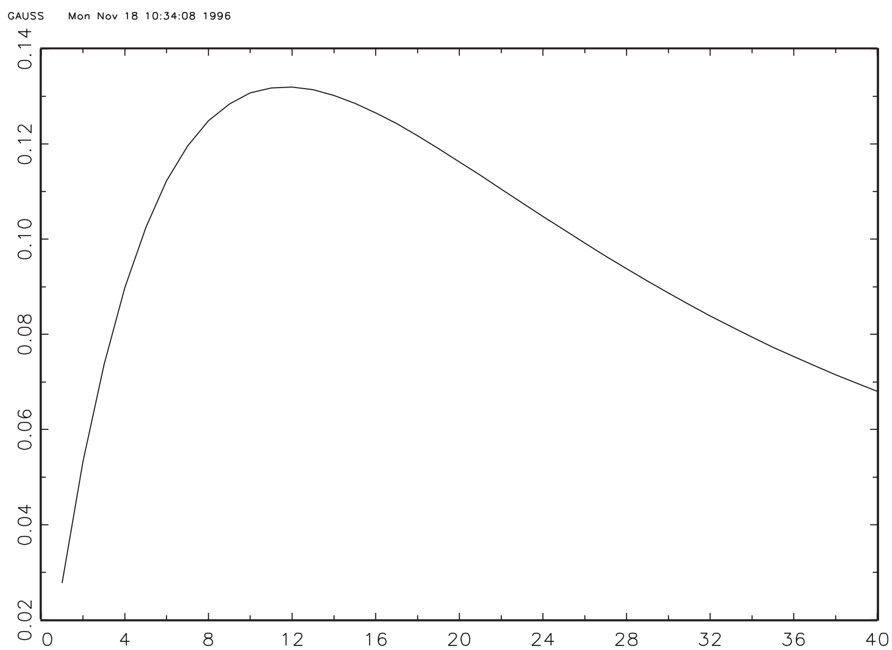

Figure 4: Impulse Response

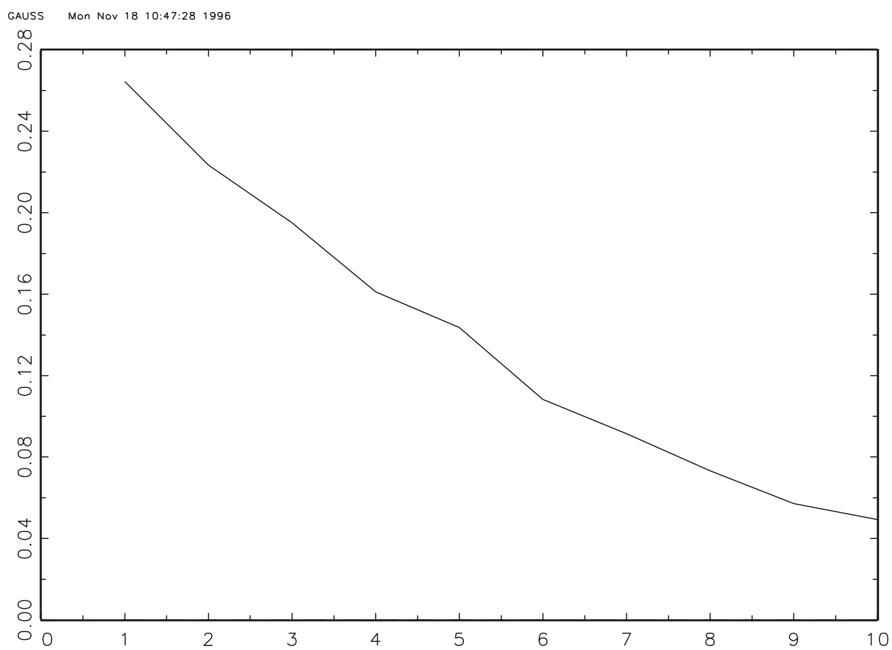

Figure 5: Autocorrelation

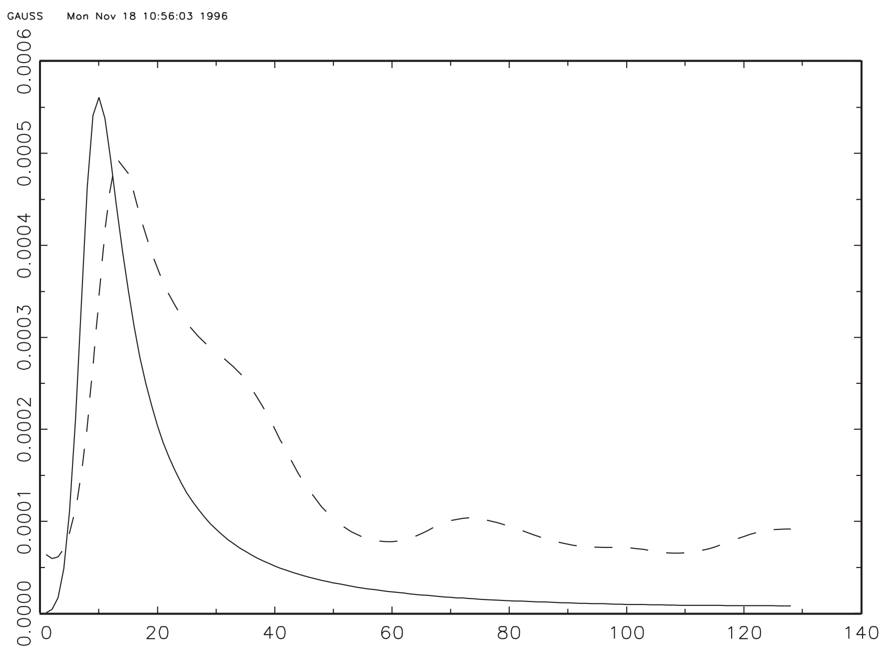

Figure 6: Power spectrum of output growth: three– sector model

hump–shaped and the autocorrelation function of output growth in figure 5 is always positive for the first 10 lags, a sign that, like in the previous section, we have too much, rather than too little, persistence. The results of Table 3.1 are similar to those in Table 2.1 for the two–sector model, with some differences in the properties of consumption. As was to be expected, C is now less volatile than before, since σ is smaller; the reduction in consumption volatility, however, came at the expense of an increased labor volatility. In addition, consumption is now only mildly procyclical.

The reason why we have persistence for the plausible parameter values we chose has again to do with the changing composition of output. When a positive shock hits all three sectors with the same intensity at date zero, we see again that consumption still slightly decreases, but very little in percentage terms. The two investment goods, however, respond much more strongly; in particular X increases and W decreases. Since X increases more than what the combined decrease in C and W, the impulse response of output to the shock is positive at date zero. Note that here consumption decreases at impact even if its intertemporal elasticity of substitution is not high as in the two–sectors model; this happens because the response of the investment goods to the shock is very pronounced; the resulting higher real interest rate provides an incentive to postpone consumption which is so strong that it overcomes households' natural predisposition for sonsumption smoothing. After the impact of the shock, we see that consumption and the second investment good, W, start increasing, while X starts decreasing. Again, the key here is that W and C increase more than X decreases, and therefore total output continues to increase. This lasts for several periods, and consequently output has the pronounced hump that is the characteristic of persistence. Note that we are exploiting the same effects that yielded persistence in the two–sector model when the investment sector was relatively capital intensive. Only, here we do not need a high intertemporal elasticity of substitution of consumption, thanks to the extra reallocation of resources across the two investment sectors. In other words, here one of the two investment sectors, W in particular, plays part of the role that was played by consumption in the two–sector model.

For this mechanism to work, investment good W must be countercyclical (or a lagging, rather than a leading, sector), and very volatile. Empirically it seems more plausible that the output of an investment sector, rather than the output of the consumption sector, is more volatile than output and countercyclical .[9] The fact that investment good W is countercyclical (its correlation with output is -0.41) does not of course mean that total investment is also countercyclical. Since the other investment good, X, is strongly procyclical, total investment is indeed procyclical, as shown in table 3.1.

The particular parameter values that we chose are not the only ones that give persistence. Simulations show that there is a wide region in the parameter space that yields results equivalent to those shown above. This region is characterized by the fact that the factor intensities in the three sectors have to be sufficiently different. If the they are too close to each other, consumption will be countercyclical consumption. Unlike the two sector model, it is also possible that some parametrizations lead to a negative response of output to the shocks; this happens when the decline in the production of good W is too strong.

Another interesting feature of this model is its ability to magnify extremely small shocks. To generate artificial time series for output that have the same variance of the real U.S. output we need shocks in each sector with a standard deviation of only 0.00023. Altogether, the three shocks have a standard deviation which is about 10 times smaller than what is required by other standard one–sector RBC models. Although we do not pursue this argument here any further, multisector models seem therefore to be interesting not only for their intrinsic propagation mechanism, but also for their amplification mechanism.

We now turn to a slightly different calibration of our three-sector model to explore the results on the negative impact response of labor hours to technology shocks, obtained by Gali, [13] and by Basu, Fernald and Kimball [4]. The instantenous utility function is specified as above, but now we set $\sigma = \gamma = 1$, implying that the labor supply elasticity is also equal to 1. We set the quarterly discount factor to $\rho = 0.99$ and the quaterly depreciation rate to $\delta = 0.025$. Technology is specified as before, with the persistence parameters for the shocks set as $\xi = \zeta = \eta = 0.95$ and the standard deviation of the innovations to the normally distributed, zero mean sectoral technology shocks to 0.0039. The Cobb-Douglas exponents for the production functions are $\alpha_0 = 0.10$, $\alpha_1 = 0.80$; $\beta_0 = 0.97$, $\beta_1 = 0.02$; $\gamma_0 = 0.74$, $\gamma_1 = 0.06$. This

[9] One could interpret the output of the W sector as human capital, making its countercyclicality more natural.

specification for technology represents a strong curvature in the production possibility frontier, reflected in the significant differences of the Cobb-Douglas exponents across sectors[10] and implies that it is quite costly to reallocate factors across sectors.

The calibration results for the model are in Table 3.2. While the match is reasonable, consumption is too smooth relative to output, and aggregate investment (aggregated through relative prices) is less volatile than in the data, although the individual sectoral components of investment and output could be more volatile.

	Y	C	I	L
Standard Deviation	1.00	0.50	2.20	0.79
	(1.00)	(0.49)	(2.82)	(0.86)
Correlation with Output	1.00	0.56	0.96	0.84
	(1.00)	(0.76)	(0.96)	(0.86)
AR(1) Coefficient	0.96	0.99	0.95	0.93
	(0.90)	(0.84)	(0.76)	(0.90)

Table 3.2 (U.S. Data in parenthesis)

We now turn to the impulse response functions, again with an aggregative technology shock hitting all three sectors simultaneously, and depicted in figure 7. As suggested by Gali [13] labor hours exhibit a pronounced decline at the impact of the technological shock, while output, consumption and investment exhibit the standard hump-shaped response. Note that, as in Gali [13] and Basu, Fernald and Kimball [4], hours initially fall below the steady state level, but overshoot the steady state and then converge back to the steady state level from above. Of course output and investment are values aggregated by contemporaneous relative prices, so sectoral "physical" output levels may not all be procyclical. Nevertheless the multisector model generates the impulse responses in the data, with a negative response of labor hours to technology shocks through compositional effects, but without invoking sticky prices.

Finally, we should note that our multisector approach has a close connection to the recent works of Beaudry and Portier, [6] and [7], who propose to study the impulse responses and the procyclicality of output variables in

[10] A specification similar in spirit would postulate significant adjustment costs in the reallocation of factors accross sectors. Such a specification prevents the excessive volatility of factor movements and therefore of sectoral outputs. For an adjustment cost treatment see Huffman and Wynne [14].

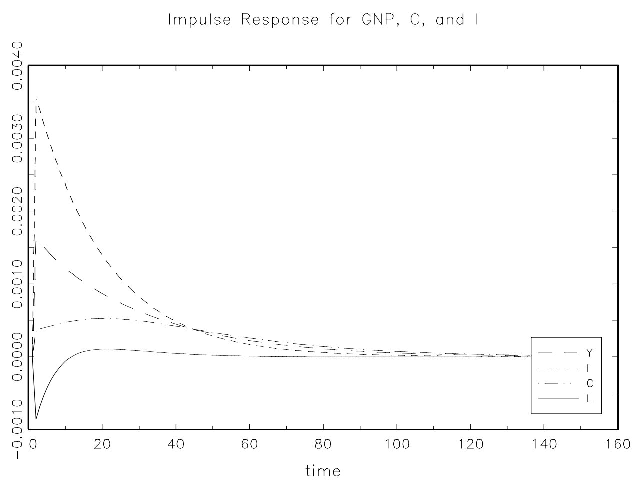

Figure 7: Impulse Responses: Output, Consumption, Investment and Labor

response to news about future but not contemporaneous shocks. In such a setup, the output expansion cannot rely on the increased capacity provided by a contemporaneous technology shock, so it must either be driven by a strong and positive labor supply response, or a countercyclical sector like home production, or complementarities and externalities in production. The latter has been explored by Benhabib and Nishimura ([9]) in a three sector model, where the positive news are represented by sunspots and therefore operate under full rational expectations[11]. The precise conditions on disaggregated technology that will permit business cycle fluctuations consistent with macroeconomic data in response to non-technology shocks will require further work and analysis.

4 Conclusion

In this paper we presented two multisector real business cycle models and explored their implications for the propagation of shocks over time. We found that both models, and the three–sector, two–capital–goods model in particular, are able to generate artificial data with the same degree of persistence observed in the U.S. data. The models are also able to match several other key business cycle statistics to a degree similar to other standard one sector models. The message that we think is coming out clearly from our analysis is that compositional effects play a significant role in the dynamic properties of aggregate economic variables. In particular, sectoral reallocations of productive factors in response to productivity shocks may be one of the fundamental reasons why we see shocks that have persistent effects on U.S. output.

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