Quadrini

Households

$$Max \;\; E_0 \sum_{t=0}^{\infty} \beta^t (1-\sigma)^{-1} c^{1-\sigma}$$

Labor efficiencies:

$$\varepsilon \in \{\varepsilon_1, \varepsilon_2 \ldots \varepsilon_n\}$$

ε follows a first order Markov process with transitions $\Gamma(\varepsilon' | \varepsilon)$.

Entrepreneurial ideas:
$\kappa \in \{0, k_1, k_2 \ldots k_N\};$. 0 means no ideas.

For the household κ is drawn from a probability distribution $P_k(\kappa),$ so that the probability distribution depends on the current $k,$ the current project under implementation

Production

Two sectors, small (non-corporate) and large (corporate). Entrepreneurship is pursued in the small sector. Entrpreneurial risk in small firms is uninsurable and the entrpreneur faces borrowing constraints.

The non-corporate sector:

If a potential entrpeneur draws $k,$ this represents the amount of capital that must be invested-this is a non-divisibility. The entreprenuer can either implement the previous project or the project just drawn.

$$y = g(\eta, k, n) = \eta^{\nu} k^{\nu} n^{1-\nu} \quad 0 < \nu < 1$$

where $n \in \{n_1, n_2 \ldots n_{N_\nu}\}$ are efficiency labor units, and $\eta \in (\eta_1, \ldots \eta_N)$ is a productivity shock following Markov probabilities $Q_k(\eta' | \eta),$ with η_1 a bad persistent shock which makes the entrepreneur quit and become a worker.

The entrepreneur having invested k units in the previous period makes the employment/hiring decision and the project selection after observing serving $\eta.$The entreprenuer can choose to stay with the old project or implement the project associated with the new draw.

Corporate sector

$$Y_c = F(K_c, N_c) = (K_c)^{\theta}(N_c)^{1-\theta}$$

with depreciation δ_c.

Intermediation and Borrowing Constraints

Household deposits are lent to the corporate sector at r_D, at zero intermediation cost cost to the corporate sector and cost φ to households for non-corprate loans for investments at rate $r_L = r_D + \varphi$, up to the maximum secure amount to be defined below.

Let $\eta_{\min}$ be the lowest shock to the entreprenuer implementing project k. Then the minimum resources deployable before debt, after e is observed, are

$$DR_{\min} = Max_n\{(\eta_{\min})^{\nu} k^{\nu} n^{1-\nu} - nw\} + \varepsilon w$$

(Assuming all houshold efficiency labor ε is delivered to market at wage w). Assuming $k > a$, that is that the entrepreneur is a net borrower, so the rate is r_L, debt to bank: $(k-a)(1+r_L) \leq DR_{\min}$.

$$a \geq k - \frac{DR_{\min}}{(1+r_L)}$$

Never optimal to borrow to limit because consumption could then be zero, which would be too costly under the utility function.

Cost of Capital and business profits.

$$\pi(a,k,\eta) = \max_n\{\eta^\nu k^\nu n^{1-\nu} - nw - (1+r)k\}$$

with

$$r = \left\{ \begin{array}{cc} r_D & if\ \ k \le a \\ r_D + \varphi\left(\frac{k-a}{k}\right) & if\ k > a \end{array} \right\}$$

so cost of capital varies with whether it is internal or external. FOC yields

$$w = (1-\nu)\eta^\nu k^\nu n^{-\nu}$$

$$n(\eta,k) = \eta k\left(\frac{1-\nu}{w}\right)^{\frac{1}{\nu}}$$

and profits, substituting for n, are

$$\pi(a,k,\eta) = \nu\eta k\left(\frac{1-\nu}{w}\right)^{\frac{1-\nu}{\nu}} - (1+r)k$$

Household Problem

Beginning of period—If the household runs business, it observes the technology shock η, and given the invested capital k, it decides how much labor to hire. Realizes profits.

End of period—The household observes the entrepreneurial idea κ and the labor ability ε'. Then, knowing the implementable projects (κ, k) and the labor ability ε', it decides, first, whether to invest in the business activity and, second, how much to save.

At the beginning of the period, agents differ over several dimensions or states. The fist state variable, which is not under the control of the agent, is the labor ability ε. The other state variables are given by the net value of assets a, the implemented project k (decided at the end of the previous period), and the technology shock η observed at the beginning of the current period. If $k = 0$, the agent is a worker; in the other cases, the agent is an entrepreneur. Therefore, the full set of individual state variables at the beginning of the period is given by $(\varepsilon, a; k, \eta)$ and the aggregate states of the economy are given by the distribution of agents over individual states represented by the probability measure $\mu(\varepsilon, a; k, \eta)$. It will be assumed that μ is the stationary distribution, prices are taken as constant.

At the end of the period:

$$\tilde{v}(\varepsilon, a, k, \eta, \kappa, \varepsilon') = \max_{a',\, k' \in (k,\kappa)} u(c)$$

$$+ \beta \sum_{\eta'} v(\varepsilon', a', k', \eta') Q_k(\eta' | \eta)$$

ST

$$c = a(1 + r_D) + \pi(a, k, \eta) + \varepsilon w - a'$$

$$a' \geq k' - \frac{v \eta_{\min} k' \left(\frac{1-v}{w}\right)^{\frac{1-v}{v}} + \varepsilon w}{1 + r_L}$$

At the beginning of the period, we have the expected value of end-of period value function

$$v(\varepsilon, a, k, \eta) = \sum_{\kappa, \varepsilon'} \tilde{v}(\varepsilon, a, k, \eta; \kappa, \varepsilon') P_k(\kappa) \Gamma(\varepsilon' | \varepsilon)$$

Steady State Equilibrium

A SS eq are value functions $\tilde{v}(\varepsilon, a, k, \eta, \kappa, \varepsilon')$, $v(\varepsilon, a, k, \eta, \kappa, \varepsilon')$, decision functions $n(k, \eta)$, $a'(\varepsilon, a, k, \eta, \kappa, \varepsilon')$, $k'(\varepsilon, a, k, \eta, \kappa, \varepsilon')$ and interest rates r_D, wages w are equal to the marginal products of capital and labor, $r_L = r_D + \varphi$, capital and labor demand from the corporate sector K_c, L_C, capital and labor demand from the non-corporate sector K_n, L_n, a function $\Psi(\mu)$ mapping the space of the distribution of households μ, ($\mu(\varepsilon, a, k, \eta)$ is a probability measure over the states of the economy) into next period's distribution, and an invariant distribution μ^*, such that decision rules $a'(\varepsilon, a, k, \eta, \kappa, \varepsilon')$, $k'(\varepsilon, a, k, \eta, \kappa, \varepsilon')$ solve the household optimization, $n(k, \eta)$ solves the entrepreneur hiring decision. Since we consider steady state distributions over states, we take prices as constant.

Capital and labor markets clear so that

$$\sum_{\varepsilon,k,\eta}\left\{\int_a k\mu(\varepsilon,a,k,\eta)da\right\}+K_C$$

$$=\sum_{\varepsilon,k,\eta}\left\{\int_a a\mu(\varepsilon,a,k,\eta)da\right\}$$

$$\sum_{\varepsilon,k,\eta}\left\{\int_a n(k,\eta)\mu(\varepsilon,a,k,\eta)da\right\}+N_C$$

$$=\sum_{\varepsilon,k,\eta}\left\{\int_a \varepsilon\mu(\varepsilon,a,k,\eta)da\right\}$$

The distribution μ^* is a fixed point of the map $\Psi(\mu)$.

Calibration: **Labor**:

The labor ability ε is assumed to follow a four-state Markov process with transition probability matrix Γ. To calibrate this process Quadrini makes the following assumptions. Each household is thought of as a sequence of finitely lived generations. In each period, there is a positive probability p that the current generation is replaced by a new generation. This probability is calibrated assuming an average generation duration of 35 years, $p = 1/35$.

The labor ability of each generation follows a two-state Markov process with transition probability matrix Γ_{ε}. However, different generations, are characterized by different mean values of the labor ability ε. More specifically, each generation can be of two types: the labor ability of type 1 takes value in the set $\{\varepsilon_{11}, \varepsilon_{12}\}$, while the labor ability of type 2 takes value in the set

$\{\varepsilon_{21}, \varepsilon_{22}\}$.

When an old generation is replaced by a new one (which, as assumed above, happens with probability p), the earning type of the new generation is determined by a stochastic process that depends on the earning type of the generation from which it descended. The probability that a new generation is of the same earning type of the generation from which it descended is set to 0.75. This implies an intergenerational correlation of earnings of 0.5, which is consistent with the estimates of Behrman and Taubman (1990), Solon (1992), and Zimmerman (1992).

Taking into consideration the probability p that an old generation is replaced by a new one, that mean life is 35 years, and the probability that a new generation is of the same earning type as its descendent, can construct the transition probability across earning types. This probability matrix across generations is denoted by Π and takes the val weues

$$\Pi = \begin{bmatrix} 0.9929 & 0.0071 \\ 0.0071 & 0.9928 \end{bmatrix}$$

so that

$$\Gamma = \Pi \circ \Gamma_{\varepsilon}$$

"To calibrate Γ_ε and $\{\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22}\}$ I assume that, for each generation, the logarithm of the household's labor ability " follows the autoregressive process

$$\ln(\varepsilon_{i,t+1}) = \alpha_i + \rho \ln(\varepsilon_{i,t}) + \nu_{t+1} \qquad \nu_{t+1} \sim N(0, \sigma_\nu^2)$$

where i is the index for the generation type and the parameter α_i is the generation-specific earning parameter characterizing the mean of the earning process. Therefore, the log-earning process of different generation types has the same variance but different means."

Noncorporate Technology

In the noncorporate sector there are three entrepreneurial projects, identified by the capital inputs k_1, k_2, k_3. To calibrate the size (capital requirement)of these projects, Quadrini uses data on the households' distribution of business wealth and gets $\frac{k_2}{k_1} = 10,\ \frac{k_3}{k_1} = 100$.

The probability distribution of the entrepreneurial idea $\kappa = \{0, k_1, k_2, k_3\}$depends only on the project implemented in the current period, and it is denoted by $P_k(\kappa)$. Quadrini assumes that the probabilities of new ideas are positive only for the projects closer to the ones currently being run.This implies that, to run a large-scale project, it is first necessary to run a smaller one.

The assumption is a simple way to formalize the hypothesis of the existence of a learning process through which the ability to run large businesses increases with entrepreneurial tenure. This assumption, together with the assumption that an entrepreneur can always run the project implemented in the previous period, simplifies the calibration of the vectors P_k, for $k \in \{0, k_1, k_2, k_3\}$. What is relevant is only the probability of getting the higher (and closer) idea; therefore, only one component of each vector P_k needs to be calibrated. At the same time, the probability distribution for an entrepreneur running the largest project is irrelevant because a large-scale entrepreneur never chooses to reduce the scale of production, if he or she realizes a good realization of the shock.

Therefore, **only three parameters need to be calibrated**, **and they are determined such that**, **in equilibrium**, **the distribution of entrepreneurs equals the imposed distribution of entrepreneurs among the four projects**—**60**, **30**, **and 10**% **respectively**—**and the total fraction of entrepreneurs equals** 0.12. This is the average fraction of entrepreneurs found in the PSID data for the period 1970–1992 and in the SCF data for the years 1989–1992.

$$P_k(\kappa) = \left\{ \begin{array}{c} 0.024 \\ 0.110 \\ 0.075 \end{array} \right\}$$

TABLE VII
Calibration Values for the Baseline Model

Intertemporal discount rate	β	0.934
Relative risk aversion parameter	σ	2.000
Intermediation cost	ϕ	0.045
Corporate capital income share	θ	0.330
Depreciation rate	δ	0.062
Noncorporate size projects	k	$\left\{1.63 \quad 16.26 \quad 162.61\right\}$
Mean value of the shock	$\bar{\eta}$	2.374
Values of the shock	η	$\left\{\begin{matrix} 1.888 & 2.536 \\ 1.740 & 2.429 \\ 1.842 & 2.390 \end{matrix}\right\}$
Probability distribution of the shock	$Q_k(\eta'/\eta_2)$	$\left\{\begin{matrix} 0.250 & 0.750 \\ 0.080 & 0.920 \\ 0.030 & 0.970 \end{matrix}\right\}$
Arrival probability of a new project	$P_k(\kappa)$	$\left\{\begin{matrix} 0.024 \\ 0.110 \\ 0.075 \end{matrix}\right\}$
Values of labor ability	ε	$\left\{0.334 \quad 0.720 \quad 1.390 \quad 2.989\right\}$
Transition probabilities for the labor ability	$\Gamma = \Pi \otimes \Gamma_\varepsilon$	$\left\{\begin{matrix} 0.743 & 0.250 & 0.005 & 0.002 \\ 0.250 & 0.743 & 0.002 & 0.005 \\ 0.005 & 0.002 & 0.743 & 0.250 \\ 0.002 & 0.005 & 0.250 & 0.743 \end{matrix}\right\}$

TABLE XI
Percentage of Total Wealth and Income Held by Percentile Groups and Gini Indexes

	Top percentiles					Gini index	Zero & neg.
	1%	5%	10%	20%	30%		
Wealth							
Model economy	24.9	45.8	57.1	73.2	84.0	0.74	15.9
PSID data	26.0	47.0	60.6	76.5	86.2	0.76	11.9
Only workers	4.2	15.3	26.2	44.5	58.3	0.55	10.1
Income							
Model economy	7.9	18.2	28.5	46.8	64.0	0.45	0.1
PSID data	7.6	19.9	30.9	47.8	60.9	0.44	0.6
Only workers	3.8	13.4	24.4	45.7	60.2	0.42	0.0

Note. Model economy and average 1984, 1989, and 1994 PSID data.