# Redistribution, Taxes, and the Median Voter*

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**Abstract**

We study a simple model of production, accumulation, and redistribution, where agents are heterogeneous in their initial wealth, and a sequence of redistributive tax rates is voted upon. Though the policy is infinite-dimensional, we prove that a median voter theorem holds if households have identical, Gorman aggregable preferences; furthermore, the tax policy preferred by the median voter has the "bang-bang" property.

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## 1 Introduction

We study a simple model of production, accumulation, and redistribution, where agents are heterogeneous in their initial wealth, and where a sequence of redistributive tax rates are chosen through voting. Decisions to save are endogenous, which means that they depend on the time profile of future tax rates.

Since the tax rate need not be constant, we encounter all the richness and all the difficulty of the optimal tax literature (see for example Chamley (1985), (1986), Judd (1985), Benhabib and Rustichini (1997), Chari and Kehoe (1999), Atkeson, Chari and Kehoe (1999)). Furthermore, since agents vote over a sequence of tax rates, the usual single-peakedness assumption required for the median voter theorem cannot be used. While poorer households will tend to favor more redistribution through capital-income taxes than richer households, the specific path preferred by each household may vary in complicated ways that depend on the distribution of local elasticities of intertemporal substitution.

The contribution of our paper is twofold. First, we prove that, when preferences are identical and Gorman aggregable, a Condorcet winner exists. This is achieved by showing that preferences satisfy an order restriction, as discussed in Rothstein (1990,1991) and in Gans and Smart (1996). This technique can have a wider application in environments where voting occurs over sequences of policies but heterogeneity among voters is restricted to a single dimension. Second, we prove that the policy chosen by majority voting has the "bang-bang" property: capital income taxes remain at the upper bound until they drop to 0, with at most one period in between. This generalizes to heterogeneous households a result first proven by Chamley (1986) in the context of a representative agent. When redistributive considerations are strong and the government ability to tax capital is limited, it is possible that the median voter prefers to keep the tax rate at its maximum allowed value in all periods, as we show through an example.

The "bang-bang" property collapses a multidimensional policy vector into a single choice over the optimal stopping time, with poorer households favoring maximal capital-income taxes for a longer period than rich ones. Gorman aggregation, however, delivers a stronger result, as a median voter result applies to all pairwise comparisons of policy sequences.

## 2 The Model

We have a production economy where output $y_t$ at time $t$ is produced by competitive firms using capital $k_t$ and labor $l_t$ according to the production function

$$y_t = F(k_t, l_t), \tag{1}$$

with $F$ linearly homogeneous. Without loss of generality, capital depreciates fully in each period.[1]

There is a continuum of agents of unit measure, with agents indexed by $i$. In the initial period 0, they each own a wealth level $W_0^i$ (made of capital and government debt). In each period, the government levies nonnegative proportional taxes on labor income $\nu_t$ and capital income $\tau_t$, subject to an exogenous upper bound $\bar{\tau}$.[2] The government uses tax receipts and one-period debt to pay for spending on a public good at an exogenous rate $\{g_t\}_{t=0}^{\infty}$, and to finance a lump-sum transfer $T_t$ that is used for redistribution.[3]

Household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right)$$

where $c_t^i$ is period $t$ consumption by agent $i$. We will assume that household utilities are restricted to satisfy Gorman aggregation, that is to have linear Engel curves. These preferences are exhausted by the exponential and power classes (see Pollak, 1971) which coincide with the HARA class of utility functions.

**Assumption 1** *Household preferences are given by*[4]

$$u(c) = \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c + B\right)^{1-\sigma}. \tag{2}$$

[1]Any undepreciated capital can be accounted for by adding $(1-\delta)k_t$ to the definition of $F$.

[2]The upper bound could be justified by the presence of a “black market technology" that allows households to shield their income from observation by the tax collector at a proportional cost $\bar{\tau}$. The zero lower bound will not be binding for the distribution of wealth that we will consider. Also, notice that we assumed the tax rate to hit *both* principal and income from capital. This can be changed with no effect on the results.

[3]Bassetto (2005) proves that the full specification of a government strategy is needed to ensure that a given policy choice does not give rise to multiple equilibria. We assume here that the government will adjust government spending in response to unanticipated shortfalls/excesses in tax or borrowing revenues in any given period. This ensures a unique equilibrium, given tax rates. While government spending is allowed to vary, voters are restricted to choose policies such that the exogenous target $\{g_t\}_{t=0}^{\infty}$ is achieved at the equilibrium, which would be justified if all households receive a sufficiently large disutility from deviations from this target. Our analysis could be extended to account for endogenous spending.

[4]The HARA class reduces to logarithmic utility for $\sigma = 1$, to quadratic utility for $\sigma = -1$, to CRRA utility for $\sigma > 0$ and $B = 0$, to linear utility for $\sigma = 0$, and to exponential utility for $\sigma \to \infty$ and $B > 0$.

Each household is endowed with 1 unit of labor, which it supplies inelastically. $W_t^i$ is the wealth of household $i$ at time $t$, consisting of capital and maturing bonds. The period-by-period household budget constraint is

$$y_t^i = (1-\tau_t)\, r_t W_t^i + (1-\nu_t)\, w_t + T_t \geq c_t^i + W_{t+1}^i, \tag{3}$$

where $r_t$ is the gross return to capital and $w_t$ is the wage at time $t$. We have imposed the no arbitrage condition by stipulating that the net return on bonds and capital are equal. We also assume that households cannot run Ponzi schemes:

$$\lim_{t\to\infty}(\prod_{s=1}^{t+1}(r_s(1-\tau_s))^{-1})W_{t+1}^i \geq 0 \tag{4}$$

The household budget constraint in present value form is:

$$\sum_{t=0}^{\infty} c_t^i \prod_{s=0}^{t} \left(r_s(1-\tau_s)\right)^{-1} = W_0^i + \sum_{t=0}^{\infty} \left(T_t + w_t\,(1-\nu_t)\right) \prod_{s=0}^{t} \left(r_s(1-\tau_s)\right)^{-1} \tag{5}$$

For convenience we define prices $q_t$ :

$$q_t = \beta^{-t} \prod_{s=0}^{t} (r_s)^{-1} \tag{6}$$

Rather than working with the sequence $\{\tau_t\}_{t=0}^{\infty}$, it is more convenient to work with the following transformation, which is one to one over the relevant domain:

$$1+\theta_t := \prod_{s=0}^{t}(1-\tau_s)^{-1} \tag{7}$$

We can write (5) as:

$$\sum_{t=0}^{\infty} \beta^t c_t^i q_t\,(1+\theta_t) = W_0^i + \sum_{t=0}^{\infty} \left(T_t + w_t\,(1-\nu_t)\right) \beta^t q_t\,(1+\theta_t) \tag{8}$$

Let $c_s$ and $b_s$ represent the aggregate levels of consumption and bonds at time $s$.

**Definition 1** *A competitive equilibrium is*

$$\left\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \left\{c_s^i, W_s^i\right\}_{i\in(0,1)}\right\}_{s=0}^{\infty}$$

*that satisfies*

1. $$\left\{\left\{c_s^i, W_s^i\right\}_{i\in(0,1)}\right\}_{s=0}^{\infty} \tag{9}$$

   *maximize household utilities subject to (3) and (4);*

2. *Factor prices equal their marginal products:*

$$r_t = F_k\left(k_t, 1\right), \;\; w_t = F_l\left(k_t, 1\right);$$

3. *Markets clear*

$$\int c_t^i di = c_t, \;\; \int W_t^i di = k_t + b_t;$$

4. *The government budget satisfies:*

$$a) \;\; \tau_t r_t\left(k_t + b_t\right) + \nu_t w_t + b_{t+1} = r_t b_t + g_t + T_t \tag{10}$$

$$b) \;\; \lim_{t\to\infty}\left(\prod_{s=1}^{t+1}(r_s(1-\tau_s))^{-1}\right)b_{t+1} = 0$$

Let the sequence $\{g_s\}_{s=0}^{\infty}$ represent the exogenous government expenditures. With no assumptions on the sequence, it would of course be possible that some $g_s > f^s\left(f^{s-1}\left(...f^0(k_0)\right)\right)$ where $f^n\left(k_n\right) = F\left(k_n, 1\right)$, so that the feasibility condition (11) in the Theorem below cannot be satisified; however, a competitive equilibrium will exist if we assume that $\{g_s\}_{s=0}^{\infty}$ is not too high. We assume further that this sequence is sufficiently small that no household faces a budget constraint with negative resources in any of the competitive equilibria analyzed below.[5] First, we characterize properties of the competitive equilibria that will be used in proving the subsequent theorems.

**Theorem 1** *For any sequence $\{c_s, k_s\}_{s=0}^{\infty}$, there exists a competitive equilibrium*

$$\left\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \left\{c_s^i, W_s^i\right\}_{i\in(0,1)}\right\}_{s=0}^{\infty}$$

*if and only if the sequence satisfies*

[5]The analysis could be extended to the case in which some households are insolvent, but this would require assumptions about the consequences of insolvency. Notice also that we abstract from the nonnegativity constraint on consumption; if $\sigma > 0$ and $B \leq 0$, the constraint will never be binding.

*1.*

$$k_{t+1} + c_t + g_t = F(k_t, 1) \tag{11}$$

*2.*

$$\left(F_k(k_{t+1}, 1)(1 - \bar{\tau})\right)^{-1} \leq \frac{\beta^t u'(c_{t+1})}{u'(c_t)} \leq \left(F_k(k_{t+1}, 1)\right)^{-1} \tag{12}$$

**Proof.** First, we prove that conditions 1 and 2 are sufficient. We set $r_t = F_k(k_t, 1)$, $w_t = F_l(k_t, 1)$. Taxes on capital satisfy

$$(1 - \tau_{t+1})^{-1} = \frac{\beta^t r_{t+1} u'(c_{t+1})}{u'(c_t)} = \frac{1 + \theta_{t+1}}{1 + \theta_t}. \tag{13}$$

In present value form the government budget constraint is:

$$\sum_{t=0}^{\infty} (w_t \nu_t - T_t) \beta^t q_t (1 + \theta_t) = \sum_{t=0}^{\infty} (g_t - \tau_t r_t k_t) \beta^t q_t (1 + \theta_t) + b_0$$

Pick any sequence $\{\nu_t, T_t, b_{t+1}\}_0^{\infty}$ that satisfies the above, as well as (10).[6] This sequence is not unique because of Ricardian equivalence.

The household first order condition, iterating (13), is

$$\frac{A\left(\frac{A}{\sigma} c_t^i + B\right)^{-\sigma}}{A\left(\frac{A}{\sigma} c_0^i + B\right)^{-\sigma}} = \frac{q_t(1 + \theta_t)}{1 + \theta_0} = \frac{A\left(\frac{A}{\sigma} c_t + B\right)^{-\sigma}}{A\left(\frac{A}{\sigma} c_0 + B\right)^{-\sigma}} \tag{14}$$

This implies

$$\frac{A}{\sigma} c_t^i + B = \alpha^i \left(\frac{A}{\sigma} c_t + B\right) \tag{15}$$

If $\alpha^i$ is set such that the present value budget constraint (5) holds at the given prices and taxes, the resulting sequence solves the household's optimization problem.

We now have to show that $\int c_t^i di = c_t$, which will hold if $\int \alpha^i di = 1$. Integrating the present value budget constraints over households we have:

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t q_t (1 + \theta_t) \int \left(\alpha^i c_t + \frac{\sigma B(\alpha^i - 1)}{A}\right) di = \\ &W_0 + \sum_{t=0}^{\infty} (T_t + w_t(1 - \nu_t)) \beta^t q_t (1 + \theta_t) \end{aligned} \tag{16}$$

[6] It is possible that for a given $\{g_s\}_{s=0}^{\infty}$ with large elements some of the lump-sum transfers $T_t$ will be negative, since we have not explicitly constrained them to be non-negative. However, as is clear from Theorem 3 below, a median voter with wealth below the mean will not prefer a tax sequence that relies solely on lump-sum taxes because of distribution considerations.

From the feasibility constraint we have $c_t + g_t = F(k_t, 1) - k_{t+1}$, which implies

$$\sum_{t=0}^{\infty} (c_t + g_t) \beta^t q_t (1 + \theta_t) = \sum_{t=0}^{\infty} F(k_t, 1) \beta^t q_t (1 + \theta_t) - \sum_{t=0}^{\infty} k_{t+1} \beta^t q_t (1 + \theta_t)$$

Using the zero-profit condition of the firms we have:

$$\sum_{t=0}^{\infty} (c_t + g_t) \beta^t q_t (1 + \theta_t) = k_0 + \sum_{t=0}^{\infty} (w_t + \tau_t r_t k_t) \beta^t q_t (1 + \theta_t)$$

Using (10) this is equivalent to:

$$\sum_{t=0}^{\infty} c_t \beta^t q_t (1 + \theta_t) = W_0 + \sum_{t=0}^{\infty} (T_t + w_t (1 - \nu_t)) \beta^t q_t (1 + \theta_t) \qquad (17)$$

For (17) and (16) to hold simultaneously we must have $\int \alpha^i di = 1$, which implies market clearing.

To prove necessity, the feasibility condition 1 is implied by market clearing and the household and government budget constraints; condition 2 is implied by the first-order conditions of the household problem (14) and the bounds on capital-income tax rates. ■

## 3 The Median Voter Theorem

In this section we prove that the median voter theorem will hold in our setting. We assume that the sequence of taxes and transfers is set by voting at time 0. Since agents vote over an infinite sequence of tax rates and transfers, the usual single-peakedness assumption required for the median voter theorem cannot be used. Our proof will instead rely on the following two observations:

1. Households differ from each other along a single dimension, i.e., their initial capital holdings.

2. Any change in tax rates affects households in two ways: by the redistribution of wealth that it implies, and by the distortion in after-tax prices that it generates. Gorman aggregation and the fact that all households have the same discount factor imply that the distortion in after-tax prices has a proportional effect on all households, independent of wealth. Thus, all households trade off a single, common measure of distortions against the degree of redistribution engineered by the distortion. Households of different wealth will disagree on the optimal point along this trade-off, but their disagreement will naturally be ordered according to their initial wealth level.

To prove the theorem we first establish the following Lemma:

**Lemma 1** *For each household $i$ there exists a function $G : R^4 \to R$ such that the utility of the household in a competitive equilibrium is $G\left(V, c_0, \tau_0, W_0^i - W_0\right)$ where $V$, the utility of the agent with average wealth, is $\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$. Also,*

$$sign\left(\frac{\partial G\left(V, c_0, \tau_0, W_0^i - W_0\right)}{\partial c_0}\right) = sign\left(\frac{\partial G\left(V, c_0, \tau_0, W_0^i - W_0\right)}{\partial \tau_0}\right) = sign\left(W_0 - W_0^i\right).$$

**Proof.** Subtracting the average budget constraint from the budget constraint of household $i$ and substituting (14) we obtain:

$$\sum \beta^t \left(c_t^i - c_t\right)\left(\frac{A}{\sigma}c_t + B\right)^{-\sigma} = r_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}$$

Using (15) we obtain:

$$\begin{aligned}\left(\alpha^i - 1\right)\frac{\sigma}{A}\sum \beta^t \left(\frac{A}{\sigma}c_t + B\right)^{1-\sigma} &= r_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma} \\ \left(\alpha^i - 1\right)\left(\frac{1-\sigma}{A}\right)V &= r_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}\end{aligned}$$

$$\alpha^i = 1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}$$

Therefore the utility attained by household $i$ is:

$$\begin{aligned}G\left(V, c_0, \tau_0, W_0^i - W_0\right) = \left(\alpha^i\right)^{1-\sigma} V = \\ \left[1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V\end{aligned} \tag{18}$$

The partial derivatives follow trivially. ■

**Theorem 2** *The tax sequence preferred by the household with median wealth is a Condorcet winner.*[7]

[7] An alternative proof of this result which only works for CRRA preferences and linear technology is in Benhabib and Przeworski (2006).

**Proof.** We will use an order restriction to prove the theorem. Consider two competitive equilibrium sequences $\{c_t\}_{t=0}^{\infty}$ and $\{\hat{c}_t\}_{t=0}^{\infty}$ , and two initial tax rates $\tau_0$ and $\hat{\tau}_0$. Define $V := \sum_{t=0}^{\infty} \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t + B\right)^{1-\sigma}$ and $\hat{V} := \sum_{t=0}^{\infty} \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} \hat{c}_t + B\right)^{1-\sigma}$.

Construct the set of households that (weakly) prefer the competitive equilibrium associated with $(\{c_t\}_{t=0}^{\infty}, \tau_0)$ to the one associated with $(\{\hat{c}_t\}_{t=0}^{\infty}, \hat{\tau}_0)$:

$$
\begin{aligned}
H :=\Bigg\{ W_0^i : & \left[1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V \geq \\
& \left[1 + \frac{Ar_0\left(1-\hat{\tau}_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}\hat{c}_0 + B\right)^{-\sigma}}{\hat{V}\left(1-\sigma\right)}\right]^{1-\sigma} \hat{V} \Bigg\}
\end{aligned} \tag{19}
$$

Conversely, let $\hat{H}$ be the set of households that (weakly) prefer the other equilibrium:

$$
\begin{aligned}
\hat{H} :=\Bigg\{ W_0^i : & \left[1 + \frac{Ar_0\left(1-\tau_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}}{V\left(1-\sigma\right)}\right]^{1-\sigma} V \leq \\
& \left[1 + \frac{Ar_0\left(1-\hat{\tau}_0\right)\left(W_0^i - W_0\right)\left(\frac{A}{\sigma}\hat{c}_0 + B\right)^{-\sigma}}{\hat{V}\left(1-\sigma\right)}\right]^{1-\sigma} \hat{V} \Bigg\}
\end{aligned} \tag{20}
$$

We need to prove that both $H$ and $\hat{H}$ are convex, independently of the choice of sequences. To do so, we consider the ratio of the utility for a household with initial capital $W_0^i$ in the two equilibria, and we take the derivative of its logarithm, which is

$$
(1-\sigma)\frac{\frac{Ar_0(1-\tau_0)\left(\frac{A}{\sigma}c_0+B\right)^{-\sigma}}{V(1-\sigma)} - \frac{Ar_0(1-\hat{\tau}_0)\left(\frac{A}{\sigma}\hat{c}_0+B\right)^{-\sigma}}{\hat{V}(1-\sigma)}}{\left[1+\frac{Ar_0(1-\tau_0)\left(W_0^i-W_0\right)\left(\frac{A}{\sigma}c_0+B\right)^{-\sigma}}{V(1-\sigma)}\right]^{-1}\left[1+\frac{Ar_0(1-\hat{\tau}_0)\left(W_0^i-W_0\right)\left(\frac{A}{\sigma}\hat{c}_0+B\right)^{-\sigma}}{\hat{V}(1-\sigma)}\right]^{-1}}
$$

The sign of the derivative is independent of $W_0^i$. This proves the convexity of $H$ and $\hat{H}$, and the theorem. ■

# 4 The Taxes Preferred by the Median Voter

We assume that the median voter's wealth, $W_0^m$, is below the mean.[8] We show in the theorem below that the capital-income taxes preferred by the

[8] If the median voter has wealth above the mean, then using arguments similar to those below, it can be shown that he will prefer zero capital taxes forever.

median voter are decreasing over time and have have the "bang-bang" property: they are either at their maximum level or at 0 in all periods except at most one. The median voter will always prefer maximal taxes on capital in period 0 and strictly positive taxes in period 1, which rules out no taxes as the voting equilibrium. We provide an example where the preferred tax rate remains at the upper bound for ever.[9]

**Theorem 3** *The capital tax sequence $\{\tau_t\}_0^\infty$ preferred by the median voter has the bang-bang property: if $\tau_t < \bar{\tau}$, then $\tau_s = 0$ for $s > t$.*

**Proof.** From Lemma 1, the initial tax will be $\bar{\tau}$ if the wealth of the median voter is below the mean.

**Case 1**: At the allocation preferred by the median voter $G\left(V, c_0, \tau_0, W_0^m - W_0\right)$ is increasing in $V$.

To derive implications for the entire sequence of taxes, consider first the consumption and capital sequence that maximizes $V$. This will solve

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t + B\right)^{1-\sigma}$$

subject to (11) and (12). The solution involves the Euler equation $u'\left(c_{t-1}\right) = (\beta F_k\left(k_t, 1\right))u'\left(c_t\right)$ and hence setting $\tau_t = 0$ for all $t \geq 1$: since $V$ is the utility of the household having mean wealth, this household does not have any incentive to distort the economy. Since we assumed the government is constrained to nonnegative capital income taxes, this solution is at a corner; in terms of the allocation, it is at the maximum growth compatible with the constraint (12), though the constraint is not binding.

Let $\{c_t^*, k_t^*\}_{t=0}^{\infty}$ be the preferred sequences of aggregate consumption and capital by a household with median wealth within those that satisfy (11) and (12). This allocation will be implemented by a sequence of capital income taxes $\{\tau_t^*\}_1^\infty$.

An equivalent statement for the theorem is, if $u'\left(c_t^*\right) > [\beta F_k\left(k_{t+1}, 1\right)(1-\bar{\tau})]u'\left(c_{t+1}^*\right)$, then $u'\left(c_s^*\right) = \beta F_k\left(k_{s+1}, 1\right)u'\left(c_{s+1}^*\right)$ for all $s > t$. Suppose this were not true. Then $\{c^{*i}\}_{s=t+1}^{\infty}$ does not satisfy the first-order conditions for solving

$$\max_{\{c_s\}_{s=t+1}^{\infty}} \sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t + B\right)^{1-\sigma} \tag{21}$$

[9] This example is not inconsistent with Judd (1985): Judd proves that the taxes preferred by any agent in the economy converge to 0 if the economy is at a steady state *and the equilibrium is interior.*

subject to (11) and (12) from period $t+1$ on, given $k_{t+1}$. It is thus possible to find an alternative sequence $\{c_s^{**}\}_{s=t+1}^{\infty}$ such that $(\{c^*\}_{s=0}^{t}, \{c_s^{**}\}_{s=t+1}^{\infty})$ satisfies (11) and (12), but such that, for sufficiently small $\varepsilon > 0$,

$$\begin{aligned}
&\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t^{**}+B\right)^{1-\sigma} = \sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t^{*}+B\right)^{1-\sigma} + \varepsilon \Longrightarrow \\
&\sum_{s=0}^{t} \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t^{*}+B\right)^{1-\sigma} + \sum_{s=t+1}^{\infty} \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t^{**}+B\right)^{1-\sigma} \\
&= \sum_{s=0}^{\infty} \beta^s \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c_t^{*}+B\right)^{1-\sigma} + \varepsilon
\end{aligned} \tag{22}$$

The new sequence has the same initial consumption, but a higher value for the utility aggregate $V$. By hypothesis we have $G(V, c_0, \tau_0, W_0^m - W_0)$ strictly increasing in $V$. As a consequence, the new sequence would be preferred by the median household, which is a contradiction.

**Case 2:** At the allocation preferred by the median voter $G(V, c_0, \tau_0, W_0^m - W_0)$ is decreasing in $V$. Then we can prove by contradiction that capital income taxes must be at their upper bound at all periods. Suppose this were not true. Then pick the first period in which the growth rate of marginal utility is below its lower bound:

$$[\beta(1-\bar{\tau})F_k(k_N,1)] < \frac{u'(c_{N-1})}{u'(c_N)}$$

Then change $u'(c_{N-1})$ proportionately by a factor $d\Psi$ (this entails raising the capital tax rate in period $N$) and change $u'(c_N)\,...u'(c_M)$ by a corresponding factor $d\Phi$ so feasibility remains satisfied, where $M$ is the first period after $N$ where:

$$[\beta F_k(k_{M+1},1)] > \frac{u'(c_M)}{u'(c_{M+1})}$$

The required adjustment in $c_t$ is:

$$\frac{dc_{N-1}}{d\Psi} = \frac{u'(c_{N-1})}{u''(c_{N-1})} = -\left(\sigma^{-1}c_{N-1} + A^{-1}B\right)$$

$$\frac{dc_t}{d\Phi} = \frac{u'(c_t)}{u''(c_t)} = -\left(\sigma^{-1}c_t + A^{-1}B\right) \quad for \quad t = N, ...M$$

The implied change in capital is:

$$dk_N = -dc_{N-1} = d\Psi\left(\sigma^{-1}c_{N-1} + A^{-1}B\right)$$

$$dk_t = d\Psi\left(\prod_{j=N}^{t-1} F_k(k_j,1)\right)\left(\sigma^{-1}c_{N-1}+A^{-1}B\right)+d\Phi\sum_{s=N}^{t-1}\prod_{j=s+1}^{t-1}F_k(k_j,1)\left(\sigma^{-1}c_s+A^{-1}B\right)$$

for $t = N+1, ...M+1$. Since $dk_{M+1} = 0$,[10]

$$0 = d\Psi\left(A\sigma^{-1}c_{N-1}+B\right)+d\Phi\sum_{s=N}^{M}\prod_{j=N}^{s}\left(F_k(k_j,1)\right)^{-1}\left(A\sigma^{-1}c_s+B\right)$$

From the Euler equation, and the non-negativity of capital income taxes:

$$\beta F_k(k_t,1)\left(A\sigma^{-1}c_t+B\right)^{-\sigma} \geq \left(A\sigma^{-1}c_{t-1}+B\right)^{-\sigma}$$

If $d\Psi < 0$ this implies that

$$-d\Psi\left(A\sigma^{-1}c_{N-1}+B\right)^{1-\sigma} \leq d\Phi\sum_{s=N}^{M}\beta^{s-N+1}\left(A\sigma^{-1}c_s+B\right)^{1-\sigma}$$

The effect that this change has on the utility index is:

$$\begin{aligned} dV &= \sum_{s=N-1}^{M}\beta^s u'(c_s)dc_s \\ &= -\beta^{N-1}\left(A\sigma^{-1}c_{N-1}+B\right)^{1-\sigma}d\Psi - \sum_{s=N}^{M}\beta^s\left(A\sigma^{-1}c_s+B\right)^{1-\sigma}d\Phi \leq 0 \end{aligned}$$

The change will be strictly negative, unless $u'(c_{N-1})/u'(c_N) = \beta F_k(k_N,1)$, i.e., unless $\tau_N = 0$. If instead $\tau_N = 0$, then the unperturbed allocation $(c_{N-1},\ldots,c_M)$ maximizes $\sum_{t=N-1}^{M}\beta^t u(c_t)$ subject to (11) and to the initial and terminal values for capital, $k_{N-1}$ and $k_{M+1}$. By strict concavity of utility, the perturbed allocation has a negative (second-order) effect on $V$.

Finally we can show that the perturbation constructed above to satisfy (11) also satisfies (12). The perturbation raises $c_{N-1}$ and decreases $c_N$ through $c_M$, in a way that leaves the marginal rate of substitution between $c_j$ and $c_{j+1}$ unaffected when $N < j < M$. So between $N-1$ and $N$ the perturbation raises the capital income tax, which was not at the maximum. Between $j$ and $j+1$, with $N < j < M$, the marginal rate of substitution is unaffected, but the marginal productivity of capital is higher due to the higher consumption and lesser accumulation in period $N-1$. Therefore for the competitive equilibrium to hold, we need to

[10] If $M = \infty$, we can take $\lim_{M\to\infty}\left(\prod_{j=N}^{M}F_k(k_j,1)\right)^{-1}dk_{M+1} = 0$.

raise the capital income tax from 0 to a positive number, which is feasible. Between periods $M$ and $M+1$, we need to lower the capital income tax and we can do that since it is positive.

**Case 3:** At the allocation preferred by the median voter $G\left(V, c_0, \tau_0, W_0^m - W_0\right)$ is stationary with respect to $V$. Then we can prove again that capital income taxes must be at their upper bound at all periods. Suppose this were not true. Then pick the first period in which the growth rate of marginal utility is below its lower bound:

$$\left[\beta\left(1-\bar{\tau}\right)F_k\left(k_N, 1\right)\right] < \frac{u'\left(c_{N-1}\right)}{u'\left(c_N\right)}$$

Then we can increase the consumption in periods $0, \ldots, N-1$ and decrease consumption in periods $N, \ldots, M$, where $M$ is the first period after $N$ where:

$$\left[\beta F_k\left(k_{M+1}, 1\right)\right] > \frac{u'\left(c_M\right)}{u'\left(c_{M+1}\right)}$$

This perturbation can be achieved by raising capital income taxes in period $N$ and lowering them in period $M+1$, while keeping them at $\bar{\tau}$ for $t < N$ and at 0 for periods $N+1 < t < M+1$. This perturbation yields a first-order increase in initial consumption $c_0$, which brings about a first-order benefit on the median voter; if $G$ is stationary with respect to $V$, any cost from the distortions imposed on $V$ would only have a higher-order impact. Hence, the perturbation would be beneficial. ■

**Theorem 4** *In period 0, the capital tax preferred by the median voter is $\bar{\tau}$. In period 1, it is strictly positive.*

**Proof.** Equation (18) is strictly increasing in $\tau_0$ for $W_0^i < W_0$, hence setting $\tau_0 = \bar{\tau}$ is optimal for the median voter. Intuitively, the initial capital-income tax is lump-sum: it achieves redistribution at no cost in terms of distortions.

To prove that the preferred level of $\tau_1$ is strictly positive, notice that $V$ is maximized by the sequence $\tau_t = 0, t \geq 1$, and that the constraint $\tau_t \geq 0$ is not binding. Hence, any increase in $\tau_1$ locally has only second-order effects on $V$, but feasibility and (14) imply that $c_0$ has a first-order increase. Equation (18) will thus increase as $\tau_1$ is perturbed from 0 to a strictly positive number (holding all future taxes at 0). ■

While the theorem above rules out no taxes for ever as a possible outcome, it is instead possible that the median voter will find maximal taxes forever to be its preferred choice, if the redistribution concern is sufficiently strong. Let the ratio of the initial median wealth to mean wealth be $R = \frac{W_0^m}{W_0}$.

**Corollary 1** *If preferences are CRRA ($B = 0$ and $\sigma > 0$ in (2)) and production is linear ($y = rk$ in (1)), the capital tax preferred by the median voter is $\bar{\tau}$ forever if* $1 + \frac{\sigma r(1-\bar{\tau})(R-1)}{\left(1-\beta^{\frac{1}{\sigma}} r^{\frac{1-\sigma}{\sigma}} (1-\tau)^{\frac{1}{\sigma}}\right) r \left(1-\beta^{\frac{1}{\sigma}} (r(1-\tau))^{\frac{1-\sigma}{\sigma}}\right)^{-1}} \leq 0$, *which can only happen if $\sigma > 1$.*[11]

**Proof.** Under the CRRA preferences and linear technology $G(V, c_0, \tau_0, W_0^m - W_0)$ is (weakly) decreasing in $V$ if

$$\left[1 + \sigma \frac{Ar_0 (1 - \tau_0)(W_0^i - W_0)\left(\frac{A}{\sigma} c_0 + B\right)^{-\sigma}}{(1 - \sigma) V}\right] \leq 0 \tag{23}$$

Furthermore, it is straightforward to prove that, when $\sigma > 1$, $\partial^2 G / \partial V^2 < 0$, $\partial^2 G / \partial c_0 \partial V > 0$, and $\partial^2 G / \partial V \partial \tau_0 > 0$. The competitive equilibrium with $\tau_t = \bar{\tau}$ for all periods $t \geq 0$ has both the lowest value of $V$ and the highest values of $c_0$ and $\tau_0$ among all competitive equilibria. Hence, it is sufficient to check that $\partial G / \partial V$ is strictly negative at this equilibrium to ensure that it is negative at all possible equilibria. Theorem 3 then implies the desired result. For taxes set at $\tau_t = \bar{\tau}, t \geq 0$, the discounted utility and initial consumption of the agent with average wealth $W_0$ are $V = \sigma (1 - \sigma)^{-1} (c_0)^{1-\sigma} \left(1 - \beta^{\frac{1}{\sigma}} (r_0 (1 - \bar{\tau}))^{\frac{1-\sigma}{\sigma}}\right)^{-1}$ and $c_0 = \left(\left(1 - \beta^{\frac{1}{\sigma}} (r (1 - \bar{\tau}))^{\frac{1-\sigma}{\sigma}}\right)(1 - \bar{\tau}) + \bar{\tau}\right) rW_0$. Substituting these into (23) we obtain the Corollary. ■

As an example of the Corollary, consider the case $r = 1/\beta$, $\sigma = 2$, $B = 0$ and $\beta = .96$, and no government spending.[12]. If more than 50% of the population has no capital, maximal taxes for ever will be the political outcome whenever $\bar{\tau} < 2.63\%$.[13]

# 5 Conclusion

In this paper, we established a median voter result for a class of economies in which an entire sequence of tax rates is chosen once and for all, and we characterized the solution preferred by the median voter. We proved that each household finds it optimal for the economy to converge to a steady state, so that Judd's (1985) result of no capital income taxes in

[11] It can also be proven that the condition in the corollary is necessary; for linear preferences and CRRA utility, taxes will necessarily converge to 0 if (18) is strictly increasing in $V$.

[12] $A$ can be normalized to any positive value.

[13] If the tax rate applies only to capital income net of depreciation, rather than to both capital and its income, i.e., only to $(r-1)k$ rather than to $rk$, the corresponding tax rate is 66%.

the limit applies, with the exception of cases in which an upper bound on capital income taxation binds for ever.

Our results can be useful even in environments where government policy is not set once and for all. First, establishing that the household with median wealth is pivotal in all pairwise comparisons of policy sequences is a likely useful step in proving existence of a median voter for dynamic political-economic equilibria where policy choices are made more frequently. Secondly, it is often the case that governments choose infrequently sequences of tax policies that will remain in effect for a *finite* period of time. Our method, based on Rothstein's (1990,1991) original insight, can be adapted to prove the existence of a median voter for these multidimensional choices as well.[14]

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[14]For a related application of Rothstein's method, see Jack and Lagunoff (2006).

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