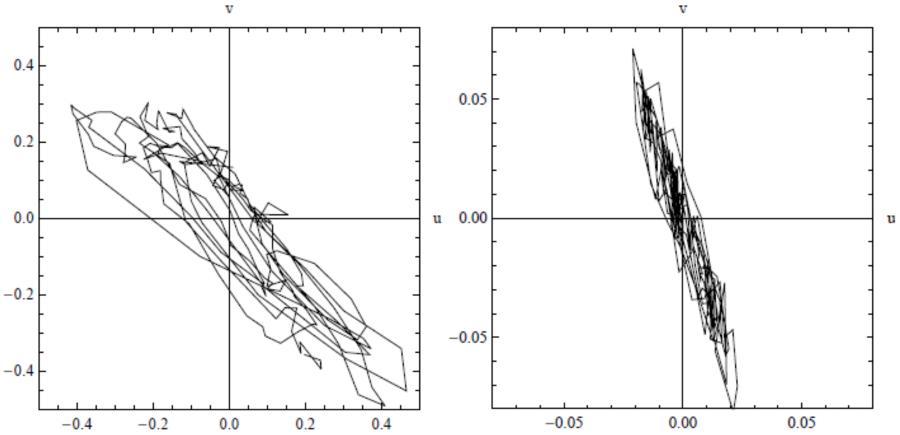
Sniekers

ECON 101

- Workers can be either employed or unemployed.
- The labor force is normalized to one, so that the total number of employed workers, n, is equal to one minus the unemployment rate u.
- If a worker is unemployed, he can search for a job with a certain intensity s.
- The discouraged worker effect - unemployed workers stop looking for a job if the prospect of finding one is very bad - is modeled by s, because the labor force is fixed while search intensity will increase with labor market tightness.
- The vacancy rate is v (scaled by the labor force), and aggregate search effort given by the unemployment rate times the search intensity of the unemployed.



(a) United States, 1951-2003.

(b) Simulation of Shimer (2005).

Figure 1: Cyclical component of the Beveridge curve.

Notes: Data are quarterly averages of a monthly series, taken from Shimer (2005). Vacancies are help-wanted advertisements as constructed by the Conference Board. Unemployment data are the seasonally adjusted series from the CPS. The trend is an HP-filter with smoothing parameter 10^5. The simulation is a representative realization with productivity shocks only.

- Matching Function:

$$m(v; su) = m_0 v^\eta (su)^{1-\eta}, \qquad 0 < \eta < 1, \quad m_0 > 0.$$

- Labor market tightness is $\theta = \frac{v}{su}$. An individual unemployed worker finds work at the Poisson rate

$$\frac{m_0 v^\eta (su)^{1-\eta}}{u} = s m_0 \theta^\eta = s\lambda(\theta).$$

- Similarly,individual vacancies are filled at a rate

$$\frac{m_0 v^\eta (su)^{1-\eta}}{v} = \frac{\lambda(\theta)}{\theta}.$$

- $\lambda(\theta)$ is increasing and concave.
- Jobs are destroyed at an exogenous rate $\delta \in (0; 1]$. These assumptions give the change in unemployment in terms of unemployment and labor market tightness

$$\dot{u} = \delta (1 - u) - s\lambda(\theta) u \qquad (1)$$

- Employers face a constant cost of opening a vacancy per unit of time, denoted by $k > 0$
- Unemployed workers encounter periodical search costs $c(s)$, increasing and strictly convex in the intensity of search, with $c(0) = c'(0) = 0$.
- In addition, they receive a gross value of leisure $z > 0$, so that the net flow benefit of the unemployed is $z - c(s)$.
- The parameter z is assumed to be independent of labor market conditions and captures the combination of the unemployment benefit, the stigma of unemployment, the value of home production and the pure value of leisure that comes with unemployment.
- Note that the gross value of leisure does not affect the search costs, which seems a reasonable assumption for risk-neutral agents. This assumption highlights the effect that the value of leisure has on vacancy creation rather than labor supply.

- All agents have utility that is linear in consumption, and discount the future with the same constant and exogenous rate r.
- These assumptions result in the following asset price equations for the value of holding a vacancy V for an employer,

$$rV = -k + \frac{\lambda(\theta)}{\theta}(J^e - V) + \dot{V}, \tag{2}$$

and of being unemployed U for a worker,

$$rU = z - c(s) + s\lambda(\theta)(W^e - U) + \dot{U}, \tag{3}$$

where J^e and W^e are the expected values of a match to an employer and worker respectively.

- The stock of vacancies follows from a free entry condition. The competitive behavior of the employers ensures that for interior solutions the value of a vacancy in equilibrium is always zero.
- However, the value of a vacancy can be negative if employers cannot reduce vacancies because they did not open any at all, so that $V \leq 0$ with $V = 0$ if $\theta > 0$.
- Using the asset price equation in 2 for all $\theta > 0$, with now $V \equiv 0$, the expected value of a filled vacancy can then be expressed in terms of the costs and benefits of a vacancy, where the latter depends on labor market tightness,

$$J^e = \frac{k\theta}{\lambda(\theta)} \qquad \textit{for} \quad \lambda > 0 \tag{4}$$

- At $\theta = 0$, J^e may be negative, but $\frac{k\theta}{\lambda(\theta)}$ is always bigger than or equal to zero.

Demand Externalities

- The demand externalities are modeled by a production function of a single worker-employer match that is increasing in the aggregate level of employment. The expected periodical flow benefit of a match can thus be denoted by:

$$\phi\left(1-u\right), \quad with \quad \phi'\left(1-u\right) > 0$$

- The per period flows to an employer are then the expected value of output $\theta\left(1-u\right)$ minus the wage w. This wage is the per period flow income to the worker.
- The asset price equations of a job J and W, to an employer and a worker respectively, are then

$$rJ = \phi\left(1-u\right) - w - \delta\left(J - V\right) + \dot{J} \tag{5}$$

and

$$rW = w - \delta(W - U) + \dot{W}$$

- Without aggregate shocks the representative worker and employer face no uncertainty, so that rational expectations and perfect foresight are equivalent.
- Employers are therefore correct in their expectations of the value of a filled vacancy, and thus the number of vacancies opened is actually maximizing the expected utility of employers.
- Using the free entry condition 4 correspondingly so $J^e = J$, and acknowledging that $V = 0$ for all interior equilibria $\theta > 0$, 5 can be rearranged to a law of motion of the value of a job in w, u and θ only:

$$\dot{J} = (r + \delta)\frac{k\theta}{\lambda(\theta)} - \phi(1 - u) + w \tag{6}$$

Bargaining, Wages and Search Intensity

- The wage is determined by Nash bargaining over the surplus of a match $p = J + W - V - U$, with worker's bargaining power equal to $\beta \in (0, 1)$ and separation (U, V) as threat point.
- This assumption about the distribution implies that the worker's rent is equal to his share of the surplus and thus that

$$W - U = \frac{\beta}{1-\beta}(J - V), \quad \dot{W} - \dot{U} = \frac{\beta}{1-\beta}(\dot{J} - \dot{V})$$

where the latter follows because wages are continuously renegotiated.

- With rational expectations and Nash bargaining, and the condition that for interior equilibria $V = 0$ and thus also $\dot{V} = 0$, the asset price equations can be simplified to yield the wage w :

$$w = \beta\phi(1-u) + \beta sk\theta + (1-\beta)[z - c(s)] \tag{7}$$

Derivation of 7

$$rW = w - \delta (W - U) + \dot{W}$$
$$rU = z - c(s) + s\lambda(\theta)(W - U) + \dot{U}$$

$$r(W-U) = w - z + c(s) - s\lambda(\theta)(W-U) - \delta(W-U) + \dot{W} - \dot{U}$$
$$w = (r + \delta + s\lambda(\theta))(W-U) + z - c(s) - \frac{\beta}{1-\beta}(\dot{J} - \dot{V}); \quad no$$
$$w = (r + \delta + s\lambda(\theta))\frac{\beta}{1-\beta}J + z - c(s) - \frac{\beta}{1-\beta}\left((r+\delta)\frac{k\theta}{\lambda(\theta)}\right.$$
$$w = (r + \delta + s\lambda(\theta))\frac{\beta}{(1-\beta)}\frac{k\theta}{\lambda(\theta)} + (z - c(s)) - \frac{\beta}{1-\beta}\left((r+\right.$$
$$w\left(\frac{1}{1-\beta}\right) = (r + \delta + s\lambda(\theta))\frac{\beta}{(1-\beta)}\frac{k\theta}{\lambda(\theta)} + (z - c(s)) - \frac{\beta}{1-\beta}\left((r+\right.$$
$$w = (r + \delta + s\lambda(\theta))\beta\frac{k\theta}{\lambda(\theta)} + (1-\beta)(z - c(s)) - \beta\left((r+\delta)\right.$$
$$w = s\beta k\theta + (1-\beta)(z - c(s)) + \beta(\phi(1-u))$$

- The search intensity s in this wage equation is optimally chosen by the unemployed worker, knowing that the surplus of a match will be shared by Nash bargaining.
- From 3, the worker's net expected income from search activity g is given by

$$s(\theta)(W^e - U) - c(s) = s(\theta)\left(\frac{\beta}{1-\beta}\right) J - c(s)$$

. With the expression for J^e in 4, net expected income from search activity can be expressed in terms of labor market tightness:

$$g(\theta) = max_s \left[\frac{\beta}{1-\beta} sk\theta - c(s)\right] \qquad (8)$$

- So the wage in 7 can be written as:

$$w = g(\theta) + z + \beta[\phi(1-u) - g(\theta) - z] \qquad (9)$$

- As usual, the wage is a linear combination of the net expected income from search and the gross value of leisure - together the outside option of the worker - and the expected value of output of the match.

- In 8 the worker balances the benefits of search with the costs, such that

$$\frac{\beta}{1-\beta} k\theta = c'(s) \tag{10}$$

- Given an increasing and strictly convex search cost function with $c\prime(0) = 0$, an optimal intensity exists and is unique. Moreover, since the benefits of search increase in labor market tightness, while the cost function is independent of it, the optimal intensity increases in tightness. It Is zero if tightness is zero, since without vacancies there is no payoff to search.
- If we choose a constant elasticity specification for the cost function, so that $c(s) = \frac{c_0}{\gamma} s^{\gamma}$ with $\gamma > 1$ this functional form results in the following closed-form solution

$$s(\theta) = \left(\frac{\beta}{1-\beta} \frac{k}{c_0} \theta \right)^{\frac{1}{\gamma-1}}$$

Dynamics of Tightness

- The derivative of 4 with respect to time yields a differential equation for the expected value of a job which holds for all interior solutions:

$$\dot{J}^e = \frac{k\dot{\theta}\lambda(\theta) - k\theta\lambda'(\theta)\dot{\theta}}{(\lambda(\theta))^2} = \frac{k\dot{\theta}}{\lambda(\theta)}\left(1 - \frac{\lambda'(\theta)}{\lambda(\theta)}\right) \equiv \frac{k\dot{\theta}}{\lambda(\theta)}(1-\eta) \qquad (11)$$

- Therefore, equation 11 implies the following law of motion for labor market tightness

$$\dot{\theta} = \dot{J}^e \frac{\lambda(\theta)}{k(1-\eta)}$$

- By opening or closing vacancies, employers translate changes in expectations about the surplus of a filled vacancy into changes in labor market tightness.

- Substituting the actual law of motion of J in 6 for the expected law of motion, and using the wage from 9, yields the following second differential equation in θ and u, valid for all interior solutions:

$$\dot{\theta} = (r + \theta)\frac{\theta}{1 - \eta} + (1 - \beta)(r + \theta)\frac{\lambda(\theta)}{k(1 - \eta)} + (g(\theta) + z - \phi(1 - u)) \tag{12}$$

- Together with the law of motion for unemployment in 1,

$$\dot{u} = \delta(1 - u) - s\lambda(\theta) u$$

12 describes rational expectations equilibria. High employment expectations make employers open vacancies immediately, because revenue per worker is expected to be higher in the future, but then hiring will be more costly too. Since more vacancies bring about higher employment, expectations are self-fulfilling.

- The next section shows that these self-fulfilling expectations may result in multiple equilibria. For an equilibrium, however, the laws of motion are restricted to a certain space. Unemployment can never be negative or exceed one, and labor market tightness can never be negative either.
- An equilibrium path starts at the given initial unemployment rate, and satisfies the transversality condition that tightness goes to zero if time goes to infinity, since then no surplus of a filled vacancy can be expected anymore. ($J_t^e - \frac{k\theta}{\lambda(\theta)} \to 0$ as $t \to \infty$; no bubble)

- In the presence of search frictions, an equilibrium is not necessarily efficient.
- Positive externalities of search and recruiting activity occur for the trading partners, for whom matching is more likely because of the availability of more (effective) trading partners.
- Negative externalities of search and recruiting activity occur for searchers of the same type, for whom matching is less likely because of increased congestion for trading partners.
- These externalities only cancel once the net private returns from search and recruiting activity equal the net social returns.
- This happens if the familiar Hosios (1990) condition is satisfied. That is, if and only if the bargaining power of employers $1 - \beta$ is equal to the elasticity of the matching function η, then search intensity s and labor market tightness θ (by employers' adjustment of vacancies) is efficient. We'll see its effect on dynamics too.
- Note that the Hosios condition only concerns the search externalities, not the demand externalities.

Dynamics

- This section presents the steady states of the dynamical system in labor market tightness θ and unemployment u, given by the differential equations in 1 and 12, and studies their stability.
- If there exists a steady state with a positive employment level and $z > 0$, then there are generically a multiple of them. Knowledge of the stability of these steady states helps to understand the Beveridge cycle that ultimately explains the data.

Steady State Equilibria

- Setting 1 equal to zero results in steady state unemployment in terms of labor market tightness $\theta = \frac{\nu}{su}$, the $\dot{u} = 0$ locus or unemployment nullcline:

$$u = \frac{\delta}{\delta + s(\theta)\lambda(\theta)} = \frac{\delta}{\delta + \left(\frac{\beta}{1-\beta}\frac{k}{c_0}\right)^{\frac{1}{1-\gamma}} m_0 \theta^{\eta + \frac{1}{1-\gamma}}} \tag{13}$$

- All workers are unemployed $(u = 1)$ if tightness is zero, and unemployment decreases in tightness via the job finding rate.
- Choosing a constant elasticity production per match will be given by $\phi_0 (1-u)^{\alpha}$ where $1 > \alpha > 0$ is the elasticity of the external effects.
- For all interior equilibria, the tightness nullcline for $\dot{\theta} = 0$ derived from 12 can then be written as

$$u = 1 - \left[(r+\delta)\frac{k\theta}{\phi_0(1-\beta)\lambda(\theta)} + \frac{g(\theta)+z}{\phi_0}\right]^{\frac{1}{\alpha}} \tag{14}$$

- 14 uses the Cobb-Douglas matching function and the optimal search

- Again unemployment decreases in tightness: at a lower level of unemployment, the expected value of output will be higher due to demand externalities, and therefore in equilibrium employers open more vacancies, and unemployment drops.
- At some level of unemployment, the nullcline crosses the $\theta = 0$ axis, and employers do not want open any vacancies.
- From 14 with $\dot{\theta} = 0$ we can see that, with $g(0) = 0$, this happens at 15

$$u_0 = 1 - \left(\frac{z}{\phi_0}\right)^{\frac{1}{\alpha}} \tag{15}$$

- This u_0 is the upper limit on the unemployment level, and thus the lower limit on the expected value of output, to have any vacancy creation at all.
- Also, for smaller values of leisure utility, z,wages are lower. As a result, there is more surplus of a match, and employers open more vacancies.

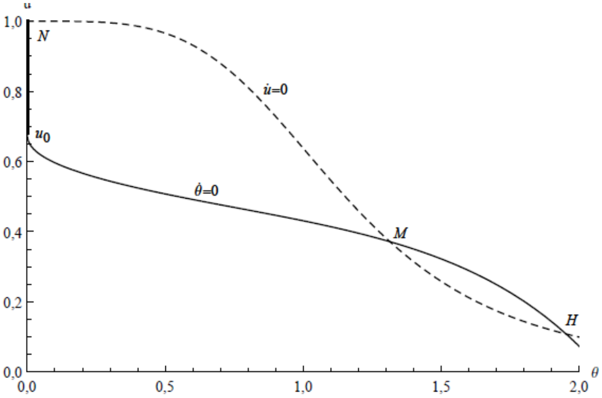


Figure 2: Nullclines of unemployment (dashed) and labor market tightness, resulting in the three steady states N, M, and H.

- **Notes**: Except for $z > 0$, parameters are as much as possible from Mortensen (1999). They are $k = 0.3$, $\delta = 0.15$, $r = 0.01$, $\eta = 0.6$, $\gamma = 1.29$, $c_0 = 0.3$, $\phi_0 = 0.12$, $\alpha = 0.3$, $\beta = 0.29$, $m_0 = 4.3$ and $z = 0.71\phi_0$.

- For $z = 0$ it can be seen that 14 and 13 intersect at $\theta = 0$ and $u = 1$, so that a no-trade steady state exists for $z = 0$, as in Mortensen (1999).
- For $z > 0$, u_0 will be smaller than 1, but the no-trade equilibrium will still exist, due to the linear section of the tightness nullcline on the $\theta = 0$ axis that intersects with 13 at $\theta = 0$ and $u = 1$.
- The example of the Figure results in three steady states: the no-trade steady state N at zero vacancies and zero employment, a steady state M with a relatively low but positive employment and labor market tightness, and a steady state H with high employment and tightness.

Theorem

Suppose ϕ_0 is large enough to guarantee the existence of a steady state strictly in the positive quadrant. Then if $\alpha \leq 1$, a sufficient condition for the existence of exactly two positive steady states is
$z > \phi_0 \left(1 - \frac{\delta}{\delta+(r+\delta)(\eta-1)\eta(\gamma-1)}\right)^{\alpha}$.

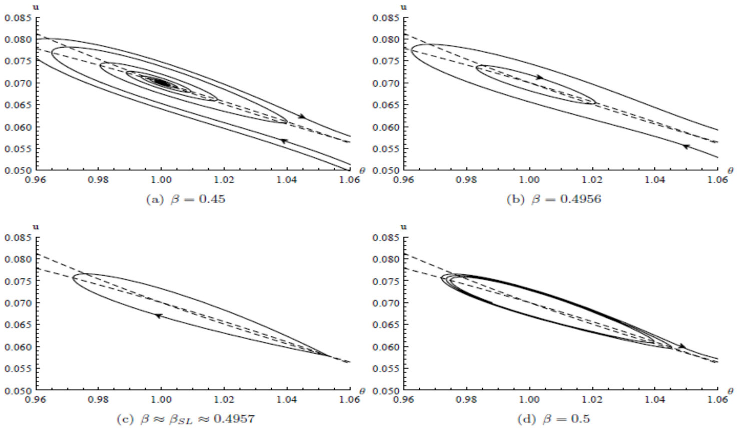
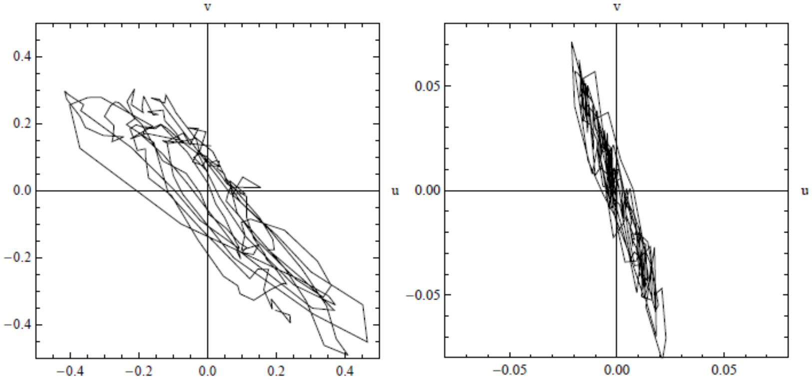


Figure 3: Representative phase diagrams for $\alpha = 0.185$, varying β: (a) Stable steady state; (b) Stable limit cycle; (c) Homoclinic orbit of the saddle-loop bifurcation; (d) No closed orbits at efficient bargaining.

Notes: Nullclines are dashed, intersecting twice. k and c_0 used to normalize steady state M tightness to 1. Parameter values other than β from final calibration as presented in Subsection 5.2. $\beta_{Hopf} \approx 0.49557$.



(a) United States, 1951-2003. (b) Simulation of Shimer (2005).

Figure 1: Cyclical component of the Beveridge curve.

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Y. Kuznetsov, Elements of Applied Bifurcation Theory, Springer 1998.

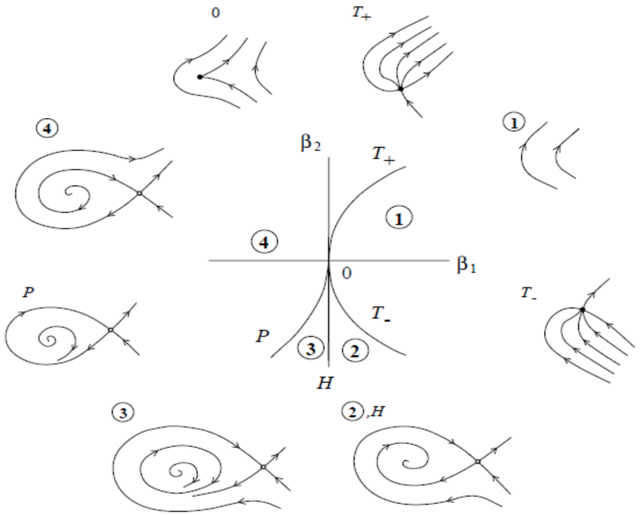

FIGURE 8.8. Bogdanov-Takens bifurcation.

ADDENDUM: Deriving Equation 7

Steady States:

$$S = J + W - V - U$$

$$\begin{aligned} rJ &= \theta\left(1-u\right) - w - \delta\left(J - V\right) \\ rW &= w - \delta(W - U) \end{aligned}$$

$$\begin{aligned} r(J+W) &= \theta\left(1-u\right) - w - \delta\left(J - V\right) + w - \delta(W - U) \\ r(J+W) &= \theta\left(1-u\right) - \delta S \end{aligned}$$

$$rU = z - c\left(s\right) + s\lambda\left(\theta\right)\left(W - U\right)$$

Nash Bargaining

$$W - U = \frac{\beta}{1-\beta}\left(J - V\right)$$

Free entry→ $V = 0$

$$r(J + W - U) = rS = \theta\left(1-u\right) - \delta S - z + c\left(s\right) - s\lambda\left(\theta\right)\left(W - U\right)$$

$$r(J + W - U) = rS = \theta\left(1-u\right) - \delta S - z + c\left(s\right) - s\lambda\left(\theta\right)\left(\frac{\beta}{1-\beta}J\right)$$

Now from Nash Bargaining

$$\begin{aligned}(1-\beta)(W-U) &= \beta(J-V)\\ (1-\beta)(W-U)+(1-\beta)(J-V) &= (1-\beta)S = J-V = J\end{aligned}$$

So

$$\begin{aligned}rS &= \theta(1-u)-\delta S - z + c(s) - s\lambda(\theta)\left(\frac{\beta}{1-\beta}J\right)\\ (r+\delta)S &= \theta(1-u) - z + c(s) - \frac{\beta s\lambda(\theta)}{1-\beta}\frac{k\theta}{\lambda(\theta)}\end{aligned}$$

But, with $V=0$, from

$$\begin{aligned}rV &= -k+\frac{\lambda(\theta)}{\theta}(J-V)\\ J &= \frac{k\theta}{\lambda(\theta)} = (1-\beta)S\\ S &= \frac{k\theta}{\lambda(\theta)(1-\beta)}\end{aligned}$$

So

$$\frac{k\theta}{\lambda(\theta)(1-\beta)} = \frac{\theta(1-u) - z + c(s)}{(r+\delta)} - \frac{\beta s}{(1-\beta)(r+\delta)} k\theta$$

$$\frac{k\theta}{\lambda(\theta)} = \frac{(1-\beta)\theta(1-u) - (1-\beta)(z - c(s))}{(r+\delta)} - \frac{\beta s k\theta}{(r+\delta)}$$

But

$$\begin{aligned}
(r+\delta) J &= \theta(1-u) - w \\
w &= \theta(1-u) - (r+\delta) J \\
w &= \theta(1-u) - (r+\delta)\frac{k\theta}{\lambda(\theta)} \\
w &= \theta(1-u) \\
&\quad - (r+\delta)\left[\frac{(1-\beta)\theta(1-u) - (1-\beta)(z-c(s))}{(r+\delta)} - \frac{\beta s k\theta}{(r+\delta)}\right] \\
w &= \beta\theta(1-u) + (1-\beta)(z - c(s)) + \beta s k\theta
\end{aligned}$$