Inequality, redistribution and cultural integration in the Welfare State

Alberto Bisin∗and Thierry Verdier†

September 2016

Abstract

This paper constructs a simple theoretical political economy model to analyze the dynamic interactions between redistribution, public good provision and cultural integration of minority groups. Cultural differentiation erodes the support for general public good provision and vertical redistribution, reducing in turn the attractiveness of adoption of the mainstream culture by the minority groups. Our model shows the possibility for multiple politico-cultural steady state trajectories depending strongly on the initial degree of cultural differentiation in the society. An exogenous increase in income inequality is shown to increase the likelihood of multiple steady state trajectories. In a context with multiple minority groups, cultural fragmentation favors integration into the mainstream culture.

JEL Classification Numbers: J13, J15, Z10.
Keywords: political economy, cultural integration, inequality, redistribution

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1 Introduction

Two important features currently impact western societies: increased income inequalities and structural immigration flows emanating from low income countries. The first element leads to enhanced political pressures for vertical redistribution. The second brings cultural diversity and its horizontal redistributive demands at the forefront of the political agenda. The interactions between these two motivations for redistribution is at the heart of the debate on the dynamics of welfare state systems in culturally diverse societies. Related to this, social scientists have dedicated significant attention to how immigrants’ integration patterns alter the design and the political economy of public policies in an host society. An important issue concerns the sustainability of welfare state institutions in the context of multicultural societies (Banting, 1998; Banting and Kymlicka, 2003). In particular, it has been argued that cultural diversity may erode the sense of social solidarity that constitutes the founding pillar of democratic welfare state systems, and that it may lead to diminishing political support for universal social programs.

Several political economy mechanisms have been highlighted in this respect. Cultural diversity may affect the sentiment towards national community that underlies the social consensus for redistribution. It may also divide coalitions rooted in socio-economic classes that traditionally sustained the welfare state, and therefore change the pattern of political alliances for social policies (Esping-Andersen 1990, Stephens 1979). Cultural minorities may prefer private or communal provision of public services that better fit their cultural preferences. The focus on such group specific public goods then creates divisions among pro-welfare coalitions. Support for affirmative action, group rights, or greater autonomy for the expression of cultural differences may weaken the links with majority community members, and undermine their support for welfare policies. Furthermore, divisions among different minority groups may as well hurt coalition formation processes. As well, in political environments in which minorities challenge the mainstream culture, majorities might also tend to oppose programs that channel resources to such communities they do not recognize as their own. This effect may be magnified when socio-economic differences and cultural differences are highly correlated (i.e., when the poor are mostly minorities and the minorities are mostly poor).

Importantly, most of the previous political economy arguments are articulated in a static way, as if the degree of cultural diversity in the society is taken as given and constant over time. An important dimension of the question however resides in the evolution of cultural diversity and its political economy consequences in a multicultural context. The purpose of
this paper is to build up a simple framework to explore such issue and to analyze the dynamic relationships between the political economy of public good provision and redistribution, and the cultural integration process of minority groups inside the welfare state.

For this, we construct a simple theoretical political economy model of public good provision in which a majority "mainstream" group interacts with a minority group whose members do not necessarily share the same preferences as the "mainstream" group. Members of the minority group may progressively adopt the "mainstream" preferences through a process of cultural socialization while others on the contrary demonstrate cultural resistance. We investigate how the degree of cultural diversity (i.e. fraction of "culturally integrated" minority individuals) interacts with income inequality to affect the political equilibrium of provision of some public good valued by the mainstream society. We also consider how the resulting political outcomes feedbacks on the cultural evolution of preferences inside the minority group.

Our analysis highlights a dynamic complementarity between the political economy of redistribution, public good provision and the process of cultural integration of the minority group. Cultural diversity shifts political coalitions, and may erode the support for large scale general public good provision and vertical redistribution from the rich to the poor. Dynamically, the attractiveness of adopting the mainstream culture is reduced and the minority group more likely to remain culturally non integrated. This in turn has negative feedbacks effects on the demand for redistribution and general public good provision over the next generations.

Specifically, our model indicates that, the political long run sustainability of vertical redistribution and provision of the mainstream public goods depends crucially on the initial size of the minority group. When the minority is small enough, the society converges towards a state of long run cultural integration in which the mainstream preferences get diffused to most minority members. Such situation supports therefore a social redistributive system providing a large amount of mainstream public goods. Conversely, a large enough initial minority group will generate the opposite politico-cultural dynamics, leading to little cultural integration and little vertical redistribution. Interestingly, for intermediate sizes of the minority group, one obtains multiple politico-cultural steady state trajectories depending strongly on the initial degree of cultural differentiation in the society.

We then investigate the impact of changes in income inequality on the comparative dynamics of the politico-cultural trajectory. An exogenous increase in income inequality affects the current demand for vertical redistribution, but also the induced dynamics of cultural integration. This in turn has consequences for the future demand of redistribution
and public good provision over generations. In such a context, we show that an increase in income inequality is likely to make the equilibrium politico-economic trajectory more sensitive to the initial conditions of the minority group, increasing therefore the range of situations in which one may obtain multiple steady state trajectories. Similarly, an increase in the poverty rate of the minority community makes it more likely to obtain a low integration equilibrium, especially so when the minority group is large and more culturally differentiated in terms of its preferences for public policies.

Extending the model to multiple minority groups introduces the possibility of positive dynamic externalities across these groups and alter significantly the process of cultural integration. Specifically, the capacity for cultural resilience of one group may now crucially depend on the degree of cultural resilience of other minority groups in the society. Even when individuals from different groups do not directly influence each other culturally, they may still impose on each other a political economy externality through the channel of provision of public goods. Indeed, culturally non-integrated minority members from different groups may essentially politically ”team-up” against the provision of ”mainstream” public goods. This will shift the position of the median voter and affect the political equilibrium of public good provision and redistribution, feeding back on the incentives to culturally assimilate to the mainstream preferences. This in turn may increase the likelihood of a low cultural integration equilibrium.

As said, our paper is motivated by a large sociological literature discussing the viability and sustainability of multicultural welfare states in western societies (Banting, 1998; Banting and Kymlicka, 2003). Among economists, part of the discussion has turned around the comparison between the degree of redistribution in the American and the European political systems, where it has been argued that the lower redistributive character of the American political system is partly related to the fact that the American society is more culturally fragmented that the European ones (Alesina, Glaeser and Sacerdote, 2001, Alesina and Glaeser 2004).

Consistent with our model, there is also substantial empirical evidence for the US that cultural diversity (measured by ethnic or racial diversity) is associated to a reduced provision of public goods or general redistribution, at the regional, city, or district level (Alesina, Baqir and Easterly (1999, 2000), Poterba (1997), Vigdor (2004)). Using a broad cross-section of countries, Desmet, Weber and Ortuño-Ortín (2009) also finds that linguistic diversity has a negative impact on redistribution.

Related to the persistence of minority cultural traits, an interesting recent literature has addressed the specific issue of the preferences for redistribution of immigrants. Using
the separation and reunification of Germany as a natural experiment, Alesina and Fuchs-Schündeln (2007) find that those who lived in the former East Germany more strongly prefer redistribution after reunification. Similarly, Guiso, Sapienza, and Zingales (2006) find that country-of-ancestry fixed effects are significant determinants of preferences for redistribution in the General Social Survey in the U.S. Luttmer and Singhal (2011) use the three waves 2002/2003, 2004/2005, and 2006/2007 of the European Social Survey (ESS) and show that the average preference for redistribution in an immigrant’s country of birth has a large and significant effect on his own preference for redistribution. All these analyses indicate that migrant communities “bring” with themselves the preferences for redistribution formed in the origin country. Passed on to second generations through cultural transmission, these inherited cultural values tend to shape the political support for redistribution in the host countries, at least as long as they are effectively activated through civic and political participation.\footnote{For an overview of the experiences of different countries in terms of cultural integration in Europe, see also the books by Kahanec and Zimmermann (2011) and Algan et al. (2012).}

On the theoretical side, our work connects to the literature on immigration and income redistribution that considers the political economy implications of migration on host economies and the sustainability of welfare state institutions (Razin, Sadka and Swagel (2002). Roemer and Van der Straeten (2006), Dolmas and Huffman (2004), Ortega (2009), Jain, Majumdar and Mukand 2011). These approaches do not consider though the issue of the dynamics of cultural integration and how that interacts with the size of governments.

Our paper is also related to Fernandez and Levy (2008) which investigates the political economy interaction of income and preference heterogeneity. It shows in particular that as taste diversity increases in society within a specific range, the set of equilibrium policies becomes more and more tilted towards specific groups and against general redistribution. We also connect to Corneo and Neher (2015) which highlights how the possibility for democracies to implement a minority-backed amount of redistribution that can be explained by the electoral bundling of redistribution with values and rights issues. Our framework complements these analyses by allowing preference heterogeneity to be endogenous and partly determined by the political equilibrium on general redistribution.

Closer to us, Shayo (2009) provides an interesting formal model of the endogenous interaction between social class or national identity formation and redistributive policies. Building on the social psychology insight that an individual is more likely to identify with a group the more similar he is to that group and the higher is the relative status of that group, the paper highlights a relationship between national identification and income (the poor are more
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Extending the model to multiple minority groups introduces the possibility of positive dynamic externalities across these groups and alter significantly the process of cultural integration. Specifically, the capacity for cultural resilience of one group may now crucially depend on the degree of cultural resilience of other minority groups in the society. Even when individuals from different groups do not directly influence each other culturally, they may still impose on each other a political economy externality through the channel of provision of public goods. Indeed, culturally non integrated minority members from different groups may essentially politically "team-up" against the provision of "mainstream" public goods. This will shift the position of the median voter and affect the political equilibrium of public good provision and redistribution, feeding back on the incentives to culturally assimilate to the mainstream preferences. This in turn may increase the likelihood of a low cultural integration equilibrium.

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nationalist), and a link between preferences for redistribution and national identification (nationalists are for less redistribution, at a given income level). We depart from Shayo’s paper on two dimensions. First, instead of a static social identity choice model a la Akerlof-Kranton (2000), our framework involves preferences dynamics, based on a micro-founded intergenerational cultural transmission process a la Bisin and Verdier (2001). Second, the focus of our analysis is different. We are concerned on how political outcomes on general redistribution or public good provision interacts with the dynamics of cultural integration and we highlight the comparative dynamics implications of income inequality changes on this interaction.

The paper is organized in the following way. Section 2 reviews the literature connected the paper. Section 3 presents the basic model, describing the political economy block and the cultural transmission framework. Section 4 discusses the nature of the equilibrium politico-cultural steady states. In section 5 we provide some comparative dynamics discussion with respect to income inequality, poverty rates and other parameters. Section 6 briefly discusses the extensions to direct cultural influence by the mainstream group, while section 7 presents the extension with multiple minority groups. Finally section 8 concludes. Proofs are relegated to an appendix.

2 A model of public good provision and cultural integration

We consider a simple economy composed of two social groups $A$ and $B$. Group $A$ of size $L_A$ normalized to 1 is the majority “mainstream” group. It is composed of two types of individuals differentiated by their income levels: rich $R$, and $P$ with respective incomes $\omega^R > \omega^P$ and fractions $\lambda^R_A$ of rich and $\lambda^P_A$ of poor (with $\lambda^R_A + \lambda^P_A = 1$). Group $B$ is a minority group of size $\mu < 1$, composed also of rich and poor individuals with income $\omega^R$ and $\omega^P$. As it is often the case empirically we assume the minority group to be poorer on average than the mainstream group, with fractions of poor and rich $\lambda^R_B$, and $\lambda^P_B$ (with $\lambda^R_B + \lambda^P_B = 1$) such that $1/2 < \lambda^P_A < \lambda^P_B$. This ensures that both groups have a majority of poor and that the minority group $B$ is on average poorer than group $A$.

- Preferences:

Individuals in group $A$ are concerned with the provision of a so-called “mainstream” public good $G^a$. We view $G^a$ as a public good in the large sense, such as public and secular
education, public secular health services provision, security and rule of law under a secular modern justice system, equal rights and access to goods and markets, protection against gender discrimination. Essentially we think of the type of general public good for which a broad consensus exists among "mainstream" individuals who belong to a typical socio-liberal modern democracy. The preferences take the following simple quasi-linear form over private consumption and the mainstream public good:

$$u(c, G) = c + V^a(G)$$

with $V^a(.)$ an increasing concave function such that $V^a(0) = 0$, $V^a_G(0) = +\infty$ and $V^b_G(\infty) = 0$.

Individuals in group $B$ constitute a minority community and are differentiated along their public good preferences: a fraction $1 - q$ of these individuals are "culturally integrated" to the mainstream society, and as such share the same preferences as the mainstream group $A$. The residual fraction $q$ of individuals in group $B$ are "culturally non integrated" and do not value the "mainstream" public good $G$. They have preferences over some other community specific public good $G^b$ that can only be produced inside community $B$. This public good $G^b$ can be associated to the provision or support for group specific activities and services, such as ethnic or religious education, marriage and social matching making, informal dispute settlement or arbitration services.

The preferences of the "non integrated" individuals in group $B$ write as:

$$u^b(c, G^b) = c + V^b(G^b)$$

with $V^b(.)$ an increasing strictly concave function such that $V^b(0) = 0$, and the derivatives satisfy $V^b_G(0) = +\infty$, $V^b_G(\infty) = 0$.

The mainstream good $G^a$ is provided by majority voting of the whole society through proportional taxation on income (at a flat tax rate $\tau$). The government budget constraint writes simply as $\tau \bar{\omega} = G^a$ with total income $\bar{\omega} = (\lambda_A^R + \mu \lambda_B^R)\omega^R + (\lambda_A^P + \mu \lambda_B^P)\omega^P$.

The amount of specific community $B$ good $G^b$ is decided by some community organization or leader, out of contributions obtained inside group $B$. Consistent with Prummer and Siedlarek (2016), and Verdier and Zenou (2015, 2016), we recognize here the role of community leaders as important socializing agents and public good providers in immigrant or minority

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Prummer and Siedlarek (2016) highlight the importance of social connections inside the minority group as an important factor of cultural resilience. Verdier and Zenou (2015, 2016) consider the strategic and forward looking implications of cultural leaders for the process of cultural integration of minority groups. These papers do not consider the political economy issues analyzed in this paper.
communities. These leaders are typically incarnated by immigrant religious organizations and their agents (Imams, Rabbis or Priests)\(^3\) or foreign-language media \(^4\).

Moreover we assume that all individuals in group \(B\) follow the prescription of the community leader and contribute to the group specific public good \(G^b\), even if they have no taste for that good (ie. are "culturally integrated"). The idea here is the fact that all individuals of group \(B\) are somewhat subject to some informal community mechanism that enforces the payment of the contributions for good \(G^b\). In particular, "culturally integrated" individuals may feel compelled to follow the community leader’s suggestion, although they do not value privately good \(G^b\). This captures the fact that individuals are socially embedded into community networks through which provision is publicly observed by others. Default of doing this could trigger some penalty cost associated to a loss of reputation, social pressure or ostracization. Another possibility is the fact that "culturally integrated" individuals have internalized a psychological cost to deviate from the obligation to contribute to the community public good even when they do not have individual preferences over such goods.\(^5\)

The important feature from a political economy perspective is the fact that a "culturally integrated" individual may still express his "mainstream" preferences in the secret of the ballot when voting over the provision of the general public good \(G^a\).\(^6\)

Substituting the public budget constraint, one obtains the utility of each type of agents:

- for group \(A\):

\[
w^A(\tau, \omega, \bar{w}) = (1 - \tau)\omega + V^a(\tau \bar{w})
\]

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\(^3\)For instance in Turkish communities in Germany the leadership role falls to Imams and in particular the “DITIB”, an institution of the Turkish government (Yasar (2012)). Imams part of such institution influence the norms and values of their communities in Germany. Through their sermons and teachings they provide group specific services (religious education, community services and counseling), that affect the assimilation of Turkish immigrants. To finance such services, monetary contributions are asked in the community (Ceylan (2010)).

\(^4\)For instance in Hispanic communities in the US, Spanish-language media provide spanish communication services in the community and have an influence on the attitude towards integration (Portes and Sensenbrenner (1993)), Subervi-Velez (1986)).

\(^5\)Hence implicitly we assume that there is a latent characteristic that still attaches the "culturally integrated" individuals to their minority community, even when they have shifted to mainstream public good preferences.

\(^6\)The model could be extended to allow "culturally integrated" individuals to choose whether or not to pay the contribution \(x^b\), at a psychological or social cost \(C\) if they do not pay. In such a case, when \(C\) is large enough, the community leader will always choose an equilibrium contribution \(x^b\) that satisfies the incentive compatibility constraint \(x^b < C\) inducing the "culturally integrated" individual to contribute to the group specific public good.
- for group B: the "integrated" individuals have the same policy preference but contribute to the group specific public good $G^b$.

$$w^A_i(\tau, \omega^i, \overline{\omega}, x^b) = [(1 - \tau)\omega^i - x^b] + V^a(\tau\overline{\omega})$$

where $x^b$ is the individual contribution inside community $B$ to produce good $G^b$.

Given $G^b = \sum_{i \in B} x^{bi} = \mu x^b$. The "non integrated" individuals in group $B$ have the following policy preferences

$$w^B_i(\tau, \omega^i, \overline{\omega}, x^b) = [(1 - \tau)\omega^i - x^b] + V^b(\mu x^b)$$

**Provision of the community specific public good**

We assume that the cultural leader inside group $B$ decides about the contributions of all individuals, weighting the "integrated" and "non integrated" types with respective weights $\theta$ and $1 - \theta$. Typically $\theta$ reflects the degree of openness of the community leader towards the "mainstream" culture. Alternatively, $\theta$ could also reflect the internal political economy representation of progressive (pro integration) versus conservative (pro non integration) factions inside the community institution. We then have the following maximization problem

$$\max_{x^b} \theta [(1 - q) \cdot w^A_i(\tau, \omega^i, \overline{\omega}, x^b)] + (1 - \theta) [q \cdot w^B_i(\tau, \omega^i, \overline{\omega}, x^b)]$$

or equivalently

$$\max_{x^b} -[\theta(1 - q) + (1 - \theta) q] x^b + (1 - \theta) q V^b(\mu x^b)$$  \hspace{1cm} (1)

Denote $\Phi(q, \theta) = 1 + \frac{\theta(1 - q)}{(1 - \theta) q}$. Using the first order condition of (1), we then obtain the optimal level of contribution $\hat{x}(q, \mu, \theta) = \frac{V^b_{G^{-1}}}{\mu} (\Phi(q, \theta))$ and level of community specific public good $G_{A}(q, \theta, \mu) = V^b_{G^{-1}}(\Phi(q, \theta))$.

**Provision of the "mainstream" public good $G^a$**

In each period $t$, the whole society decides about the provision of the "mainstream" or general public good $G^a$ by majority voting. To determine the political economy equilibrium it is therefore useful to consider the preferred provision of the mainstream public good for the different groups (members of the majority group $A$, integrated and non integrated individuals of the minority group $B$). For this denote $\hat{g}_{j,k}$ the level of general public good $G^a$ preferred by an individual with income $\omega^i i \in \{R, P\}$ (ie. $R$ for rich, $P$ for poor) , in group $j \in \{A, B\}$
(ie. $A$ for mainstream, $B$ for minority) and cultural type $k \in \{I, N\}$ (ie. $I$ for "integrated", $N$ for "non integrated") if he belongs to group $B$.

Consider then an individual with income $\omega^i$, $i \in \{R, P\}$ from the mainstream group $A$, or from the minority group $B$ and culturally integrated (ie. $k = I$). Substituting the public budget constraint into the indirect utility of such agent, it is straighforward to see that his preferred level of $G^a$ is given by:

$$\arg \max_g \left\{ \omega^i - \frac{\omega^i}{\bar{\omega}} g + V^a(g) \right\}$$  \hspace{1cm} (2)

the solution of which is given by $\tilde{g}_i^R = \tilde{g}_i^P = \tilde{g}_i$ for $i \in \{R, P\}$.

Conversely for a "non integrated" individual (type $N$) that belongs to group $B$, we obviously get $\tilde{g}_{B,N} = 0$ whatever his income level $i \in \{R, P\}$. From this, we can easily rank the preferred levels of general public goods $\tilde{g}_{i,k}^j$ of the different types of agents as:

$$\tilde{g}_{B,N}^P = \tilde{g}_{B,N}^R = 0 < \tilde{g}_i^R = \tilde{g}_i^P < \tilde{g}_i = \tilde{g}_i^P = \tilde{g}_i^P$$

Note that aggregate income $\bar{\omega} = \bar{\omega}(\mu) = (\lambda_A^R + \mu \lambda_B^R)\omega^R + (\lambda_A^P + \mu \lambda_B^P)\omega^P$ is increasing in $\mu$. Consequently the optimal policy levels $\tilde{g}_i^R = \tilde{g}_i^R(\mu)$ and $\tilde{g}_i^P = \tilde{g}_i^P(\mu)$ are also increasing in $\mu$, reflecting a standard size effect of public good provision.

Given that the preferences of the agents satisfy the single crossing property, the equilibrium is therefore obtained by the preferred policy of the median voter. To characterize this median voter, denote $\overline{\tau}(\mu)$ the threshold such that

$$\lambda_A^P + \lambda_B^P \mu(1 - q) = 1 - \lambda_A^P + (1 - \lambda_B^P)\mu(1 - q) + \mu q$$

The left hand side of this equation reflects the total number of individuals in favor of the policy platform $\tilde{g}_i^P$ rather than $\tilde{g}_i^R$, while the right hand side reflects the total number of individuals that support $\tilde{g}_i^R$ rather than $\tilde{g}_i^P$. Then $\overline{\tau}(\mu)$ is given by

$$\overline{\tau}(\mu) = 1 + \frac{2\lambda_B^P - 1 - \mu}{2\lambda_B^P} > 0$$

It is just the fraction of integrated individuals in group $B$ that ensures that the two political platforms $\tilde{g}_i^R$ and $\tilde{g}_i^P$ get the same amount of voting support. Note that $\overline{\tau}(\mu)$ is decreasing in $\mu$ as we assumed that $2\lambda_B^P - 1 > 0$.

When $q < \overline{\tau}(\mu)$, the median voter is easily characterized as a mainstream/ or minority "integrated" poor and the equilibrium provision of good $G^a$ is given by $g^*_A(q, \mu) = \tilde{g}_i(\mu)$. 


Conversely, when \( q > \tau(\mu) \), the median voter is a mainstream rich individual and the provision of good \( G^n \) is now given by \( g_\lambda(q, \mu) = \hat{g}_R^R(\mu) < \hat{g}_P^P(\mu) \). Typically, when the number of ”non integrated” minority individuals is sufficiently large, there is less equilibrium provision of the mainstream public good \( G^n \), and there is as well less redistribution (through the financing of that public good across income class groups).

- Cultural evolution inside the minority community \( B \).

Consider now the pattern of cultural integration of the minority group \( B \). For this we embed our previous model of political economy of public goods provision into a framework of cultural transmission a la Bisin and Verdier (2001) in which individuals in the minority group can acquire either the mainstream trait \( A \) (and be ”culturally integrated”) or the minority trait \( B \) (and be culturally ”non integrated”) by an intergenerational process of cultural transmission. Parents play an active socialization role in such a process.

Specifically, at each point of time \( t \), the timing is the following: 1) well socialized individuals vote, contribute and decide which public good to consume (mainstream or group specific) depending on their preferences; 2) then each minority individual before dying has one child and there is cultural transmission of the two traits (integrated \( I \) or non integrated \( N \)) inside the minority group \( B \). Cultural transmission is the result of direct vertical (parental) socialization and horizontal/oblique socialization inside the minority group. More precisely,

i) direct vertical socialization to the parent’s trait, say \( i \in \{I, N\} \), occurs with probability \( d^i \);

ii) if a child from a family with trait \( i \) is not directly socialized, which occurs with probability \( 1 - d^i \), he/she is horizontally/obliquely socialized by picking the trait of a role model chosen randomly in the minority population (i.e., he/she picks the ”non integrated” trait \( N \) with probability \( q \) and the ”integrated” trait \( I \) with probability \( 1 - q \)).

\(^7\)See also Lazear (1999) and Kónya (2005) for models of cultural assimilation of migrants respectively in a static and a dynamic context, but without endogenous political economy considerations.

\(^8\)For an economic approach see a serie of papers by Bisin and Verdier (1998, 2000a,b), 2001) which build upon the work of Cavalli-Sforza and Feldman (1981) in evolutionary biology and Boyd and Richerson (1985) in anthropology; see also Bisin and Verdier (2010) for a survey of the literature.

\(^9\)We assume here that socialization to the integrated trait (ie. value the general public good) can only be done within the minority group. Alternatively we could assume that oblique socialization may also involve role models from the mainstream group. We discuss briefly the implications of such extension in the conclusion.
This cultural transmission mechanism introduced by Cavalli Sforza and Feldman (1981) is then summarily represented by the following system of equations for $P^{ij}$, the probability that a child from a family with trait $i$ is socialized to trait $j$ for $i, j \in \{N, I\}, i \neq j$ and $q^N = q$ and $q^I = 1 - q$.

\[
\begin{align*}
P^{ii} &= d^i + (1 - d^i)q^i \\
P^{ij} &= (1 - d^i)(1 - q^i)
\end{align*}
\]

(3)

From this, it is clear that the minority group population remains stationary, and the (continuous) dynamics of evolution of the fraction of "non integrated" minority individuals (endowed with the peci\-ficial trait $B$) is easily obtained as (Bisin and Verdier 2001, 2010):

\[
\dot{q} = q(1 - q) \left[ d^N - d^I \right]
\]

(4)

The parental rate of socialization $d^N$ and $d^I$ are obtained from parents’ choices of cultural transmission. These are motivated by imperfect empathy, namely a form of altruism biased towards the parents’ own cultural traits: parents care about their children’s choices, but they evaluate them using their own (the parents’ - not the children’s) preferences. More specifically, Let $U^{ii}, U^{ij}$ denote the utility to a cultural trait $i$ parent of a type $j$ child\textsuperscript{10}. When $U^{ii} \geq U^{ij}$ parents have an incentive to socialize their children to their own cultural trait. Socialization requires parental resources, e.g., time spent with children, private school tuition, church contribution, and so on. Denote by $C(d^i)$ the socialization costs, where $d^i$ is the probability of direct socialization of parents with trait $i$ to the $i$ trait. The value of parental socialization choice is then obtained from the following maximization program:\textsuperscript{11}

\[
W^i(q^i) = \max_{d^i \in [0,1]} -C(d^i) + P^{ii}U^{ii}(q^i) + P^{ij}U^{ij}(q^i), \quad \text{s. t. } 1), \text{ and } 2).
\]

Assuming for simplicity some quadratic socialization costs, $C(d^i) = \frac{1}{2} (d^i)^2$, we obtain the optimal effort of socialization:

\[
d^i = d(q^i, \Delta U^i) = (1 - q^i)\Delta U^i,
\]

(5)

where $\Delta U^i = U^{ii} - U^{ij}$ measures the relative value for a parent with trait $i$ to share with his/her child the same cultural trait. To analyze the cultural incentives to transmit the two

\textsuperscript{10}Because of the fact that individuals interact socially (they vote jointly on public good provision) $U^{ii}, U^{ij}$ will be functions of $q^i$.

\textsuperscript{11}The socialization choice of parents is independent of their individual consumption of private and public goods because of preference separability.
traits $N$ and $I$, we assume that the consumption of the two public goods $G^a$ and $G^b$, involves a deliberate choice at a small “cognitive” cost $f$ to the individual.\(^{12}\)

Now for a parent of type $I$ at time $t$ with income $\omega^P$, the relative value of having a child of his own type rather than of the other type $N$ is simply obtained by $\Delta U^I = U^{II} - U^{IN}$ with

$$
U^{II} = (1 - \tau^*_e)\omega^P - x_e^b - f + V^a(\tau^*_e \tau^*_e) \\
U^{IN} = (1 - \tau^*_e)\omega^P - x_e^b - f
$$

Typically $U^{II}$ is the utility to have a child of type $I$ as perceived by a parent of type $I$ and $U^{IN}$ is the utility to have a child of type $N$ as perceived by a parent of type when the anticipated political equilibrium of the child generation is characterized by a tax rate $\tau^*_e$, a provision of the mainstream public good $\tau^*_e \tau^*_e$ and a minority contribution $x_e^b$. We assume that parents are “politically” myopic in their cultural transmission process and that they consider that the political equilibrium of their generation will apply as well to the society of their children.\(^{14}\) In such a case: $\tau^*_e = \tau^*_c$, $\tau^*_e \tau^*_e = g^*_A(q_t, \mu)$, and $x_e^b = x_t^b = \hat{x}(q_t, \mu, \theta)$. Letting $f \to 0$ this finally gives:

$$
\Delta U^I = U^{II} - U^{IN} = V^a(g^*_A(q_t, \mu))
$$

where $g^*_A(q_t, \mu)$ is the equilibrium provision of mainstream public good voted in a society with a fraction $q_t$ of non integrated individuals in the minority group $B$.

By a similar reasoning, one obtains

$$
\Delta U^N = U^{NN} - U^{NI} = V^b(\hat{x}(q_t, \mu, \theta)) = V^b(V_G^b - 1(\frac{\Phi(q_t, \theta)}{\mu})
$$

\(^{12}\)Although the two public goods have some non rivalry access property, what matters for parents’ incentives to transmit their own trait depends on the differential way children decide to consume one or the other public good compared to them. We model this decision choice by making individuals pay a (small) private cost $f$ to choose to consume $G^a$ or $G^b$. Then we let $f$ tend to zero. (See Bisin and Verdier 2000a) for a similar feature).

\(^{13}\)We assume that there is no social mobility in this society. Hence the income of a child is the same as the income of the parent. Introducing the possibility for endogenous social mobility is an interesting avenue for future research.

\(^{14}\)This assumption simplifies the dynamic analysis of cultural transmission ensuring that there are no forward looking considerations. Such dimensions are well known to generate complex dynamic features with multiple rational expectations equilibrium paths (see Bisin and Verdier 2000a) for some discussion along these lines).
For convenience, we suppress the dependence on \( \theta \) in the notations and simply denote \( S^b(q) = V^b(I_G^{-1}(\Phi(q))) \) with \( \Phi(q) = 1 + \frac{\theta(1-q)}{1-\theta q} \). Substitution of (5), (6), and (7) into (4) provides finally the cultural dynamics inside the community

\[
\dot{q} = q(1-q) \left[ S^b(q)(1-q) - V^a(g_A^* (q, \mu))q \right]
\]

with \( g_A^* (q, \mu) = \hat{g}^P (\mu) \) when \( q \leq \overline{q}(\mu) \), and \( g_A^* (q, \mu) = \hat{g}^R (\mu) \) when \( q > \overline{q}(\mu) \).

3 Politico-cultural steady states

We are now in a position to analyze the politico-cultural dynamics of the society. For this, we characterize the structure of the politico-cultural steady states associated to the cultural dynamics as defined by (8). Clearly, the system exhibits the two corner steady states \( q = 0 \) and \( q = 1 \) which can be shown to be locally unstable. An interior steady state is then characterized by the condition:

\[
S^b(q) = V^a(g_A^* (q, \mu)) \frac{q}{1-q}
\]

Under the condition that the utility function \( V^b(\cdot) \) is enough concave\(^{15} \), it can be easily seen that for a given value \( g > 0 \), there is a unique solution \( q^* (g) \) such that

\[
S(q) = V^a(q) \frac{q}{1-q}
\]

Such solution \( q^* (g) \) is decreasing in \( g \). Indeed, a given level of general public good \( g \) makes it more attractive for the minority group to integrate culturally into the mainstream values. This provides a smaller long run value of the fraction of individuals in community \( B \) who have a preference for the community specific public good.

[Figure (1) about it].

Figure 1 depict the solution \( q^* (g) \). The LHS of equation (9) is drawn as an increasing concave locus \( SS \), the RHS of equation (9) is depicted by the increasing convex locus \( GG \).

\(^{15}\)Specifically when the function \( V^b(\cdot) \) satisfies the following property:

\[
[V^b_G(y)]^2 > V^b_G(y)V^b_{GGG}(y)
\]

one can easily show that the function \( S^b(q) \) is increasing concave with \( S^b(0) = 0 \), and \( S^b(0) = +\infty \). \( S(0) = 0 \). Then it follows that for a given value \( g > 0 \), there is a unique solution \( q^* (g) \) such that \( S(q) = V^a(g) \frac{q}{1-q} \).
As shown the unique steady state $q^*(g)$ is obtained at the intersection of $SS$ and $GG$. An increase in $g$ leads to a upward shift of the locus $GG$, leading therefore to a smaller value of $q^*(g)$.

Noting that $g^*_\lambda(q, \mu)$ can take only two values: $\hat{y}^P(\mu)$ and $\hat{y}^R(\mu)$, the previous discussion indicates two potential steady state values $q^{P*}(\mu) = q^*(\hat{y}^P(\mu))$ and $q^{R*}(\mu) = q^*(\hat{y}^R(\mu))$ with $q^{P*}(\mu) < q^{R*}(\mu)$ as $\hat{y}^P(\mu) > \hat{y}^R(\mu)$ for all $\mu \in [0, 1]$.

Now to be consistent with the political equilibrium structure, one should also have that $g^*_\lambda(q, \mu) = \hat{y}^P(\mu)$ prevails when $q < \bar{\tau}(\mu)$ while $g^*_\lambda(q, \mu) = \hat{y}^R(\mu)$ prevails when $q > \bar{\tau}(\mu)$. From this the phase diagram of the cultural dynamics can be easily drawn as depicted in figure 2a, 2b, and 2c depending on how the curve $V^\alpha(g^*_\lambda(q, \mu)) \frac{d}{d\tau}$ crosses the locus $SS$.

Figures 2a), 2b) and 2c) about it

Figure 2a) shows the case where the intersection occurs only once at a point $q^{P*}(\mu) < \bar{\tau}(\mu)$. $q^{P*}(\mu)$ is the unique stable cultural steady state characterized by high integration of the members of community $B$ to "mainstream" preferences. This is accompanied by high provision of the general public good $G^a$ and low provision of the community public good $G^b$. The politico-cultural dynamics of this case are easy to describe depending on the initial "cultural integration" level $q_0$ of the community. When the minority group is already quite integrated (i.e., $q_0 < \bar{\tau}(\mu)$) then the median voter remains permanently the poor individual. Conversely, when the community is not well integrated at the start (i.e., $q_0 > \bar{\tau}(\mu)$), the median voter is initially the rich individual that votes for less general public good $A$ than the poor agent. The provision of such public good however is high enough to induce the minority group to integrate (to have the fraction $q_t$ decreasing overtime). As the fraction $q_t$ gets below the threshold $\bar{\tau}(\mu)$, the median voter shifts from the rich to the poor, increasing thereby the provision of the general public good. This further reinforces the integration process of the minority with a fraction $q_t$ converging towards the lower steady state value $q^{P*}(\mu)$. Note that when the community is already very integrated (i.e., $q_0 < q^{P*}(\mu)$), then the so-called cultural substitutability of the transmission process (Bisin and Verdier 2001) implies that there is some degree of cultural resilience of the minority group. This makes the fraction of individuals with a preference for the community specific good to converge again towards $q^{P*}(\mu)$.

Figure 2c) shows conversely the situation where there is unique intersection point $q^{R*}(\mu)$ with $q^{R*}(\mu) > \bar{\tau}(\mu)$. $q^{R*}(\mu)$ is then the unique stable cultural steady state characterized by low provision of the general public good $G^a$ and high provision of the community public
good $G^h$. For a community sufficiently non integrated, (ie. $q_0 > \bar{\tau}(\mu)$), the pivotal agent in the political process remains indefinitely the rich agent, and there is permanently little provision of the general public good $G^a$. When $q_0 < \bar{\tau}(\mu)$, the median voter is initially the poor agent who is in favor of a relatively large level of general public good in society. This is not however large enough to counteract the strong motive of the minority members to transmit their own specific cultural trait. As a result, the fraction $q_t$ of ”non integrated” individuals keeps increasing, crossing at some point the threshold $\bar{\tau}(\mu)$. This induces a shift of the pivotal political agent towards the rich individual. A smaller provision of general public good follows. As a consequence, this reinforces the cultural transmission process of the minority community towards a low integration level $q^{R*}(\mu)$.

Finally figure 2b) shows the situation where the locus $SS$ intersect two times the curve $V^a(g^{*}) = \frac{1}{1-q}$, respectively on the lower branch (for $q < \bar{\tau}(\mu)$) and on the higher branch (for $q > \bar{\tau}(\mu)$). This situation depicts the possibility of multiple politico-cultural steady state situations. Depending on whether the minority community $B$ is initially well integrated (ie. $q_0 < \bar{\tau}(\mu)$) or not (ie. $q_0 > \bar{\tau}(\mu)$), the equilibrium trajectory is different. In the first case, there is a process of integration with convergence towards a high level of integration $q^{P*}(\mu)$. In the second case, there is a process of cultural differentiation with $q_t$ converging to the higher value $q^{R*}(\mu)$.

The structure of the politico-cultural long run equilibria depends crucially on the size of the minority group. Notice first that the dependence of the equilibrium values $q^{P*}(\mu)$ and $q^{R*}(\mu)$ on the community size $\mu$ is a priori ambiguous. On the one hand, an increase in $\mu$ leads to a larger tax base for the ”mainstream” public good $G^a$, making the mainstream trait more attractive. On the other hand, a larger community group $\mu$ leads to a larger provision of the community public good $G^b$ which makes trait $B$ more attractive. To avoid a long taxonomy discussion, we assume that in the relevant range for $\mu \in [0,1]$, the community public good effect dominates the tax base effect. Therefore a larger minority group $\mu$ is naturally more likely to sustain a larger fraction of ”non integrated” individuals. We formulate this as the following condition:

$16$Condition C: $q^{P*}(\mu)$ and $q^{R*}(\mu)$ are increasing functions of $\mu \in [0,1]$

Then we have the following proposition:

---

$16$This condition holds when $V^a(.)$ is relatively concave compared to $V^b(.)$ and that the economy already provides a relatively large amount of ”mainstream” public good without the existence of the minority group $B$. 

16
Proposition 1: Under assumption C, we have the following characterization:

1) When $\bar{q}(1) > q^*(\hat{g}^R(1))$, for all $\mu \in [0,1]$ then there is a unique stable steady state politico-cultural equilibrium characterized by ”high integration” (ie. with $q^*(\mu) = q^P^*(\mu) < \bar{q}(\mu)$)

2) When $q^*(\hat{g}^P(1)) \leq \bar{q}(1) < q^*(\hat{g}^R(1))$, there exists a size threshold $\mu_L < 1$ such that:
   2i) for $\mu < \mu_L$ we are in a situation as described in 1).
   2ii) for $\mu \in [\mu_L, 1]$, there are two stable steady state politico-cultural equilibria. In addition to a ”high integration” equilibrium as described in 1), there is also a ”low integration” equilibrium (ie. with $q^*(\mu) = q^R^*(\mu) > \bar{q}(\mu)$).

3) When $\bar{q}(1) \leq q^*(\hat{g}^P(1))$, there exists two size thresholds $\mu_L$ and $\mu_H$ with $\mu_L < \mu_H < 1$ such that:
   3i) For $\mu < \mu_L$ we are in a situation as described in 1).
   3ii) For $\mu \in [\mu_L, \mu_H]$ we are in a situation as described in 2ii).
   3iii) For $\mu \in [\mu_H, 1]$ there is a unique steady state politico-cultural equilibrium characterized by ”low integration” (ie. $q^*(\mu) = q^R^*(\mu) > \bar{q}(\mu)$).

Proposition 1 is illustrated in figures 3a) 3b) and 3c) depicting the three possible cases. In all vignettes the threshold curve $\bar{q}(\mu)$ is drawn describing the threshold above which ”non integrated” individuals in community $B$ induce a shift of the pivotal agent from being poor to being rich with respect to the provision of the general public good $g_A$. The larger the size of the minority community, the smaller this threshold.

Figures 3a) 3b) and 3c) about here

The two cultural steady state loci $q^P^*(\mu)$ and $q^R^*(\mu)$ are also drawn. As already noticed, these curves are increasing in $\mu$. A larger size of the poorer community $B$ induces less willingness by the median voter to finance the provision of the general public good. This in turn makes the transmission of the ”integrated” cultural trait $I$ less attractive to members of the minority community. A larger fraction $q$ of individuals with the ”non integrated” trait $N$ prevails in the long run. Note that the locus $q^R^*(\mu)$ is always above the other locus $q^P^*(\mu)$. When the pivotal agent in society is richer, there is less provision of the general public good. This leads again to less integration to the mainstream culture over generations of members in the minority community $B$. Finally, note that to get the consistency between the cultural dynamics and the political equilibrium at each period, the locus $q^P^*(\mu)$ is the relevant part when $q^P^*(\mu) < \bar{q}(\mu)$ while $q^R^*(\mu)$ is the relevant one for $q^R^*(\mu) > \bar{q}(\mu)$.

Figure 3a) shows the case where for all values of $\mu$, the long run equilibrium is only a ”high integration” equilibrium. Figure 3b) shows the case where after a threshold $\mu_L$, the society is
characterized by two steady state politico-cultural equilibria: one with high integration and one with low integration. Finally figure 3c) shows the case where that situation occurs for $\mu \in [\mu_L, \mu_H]$, while for $\mu > \mu_H$, the only long run steady state is a low integration equilibrium one. As highlighted in these figures, the situation of multiple politico-cultural steady states is more likely to occur, everything else been equal, when the size of the minority group is intermediate.

4 Comparative dynamics

In this section we investigate how changes in social conditions (inequality, poverty) interact with cultural transmission to determine the nature of the politico-cultural trajectories and the extent of redistribution induced by the political system.\textsuperscript{17}

- **Change in income inequality (or in the wage gap) $\Delta\omega$**

  Clearly aggregate income $\varpi$ can be written as a function of the income gap $\Delta\omega = \omega^R - \omega^P$

  $$\varpi = \varpi(\mu) = (\lambda_A^R + \mu \lambda_B^R)\Delta\omega + \omega^P (1 + \mu)$$

  which is increasing in $\Delta\omega$ (the degree of inequality or the wage skill gap). Fixing $\omega^P$, we obtain the relative positions $\omega^R/\varpi$ and $\omega^P/\varpi$ which are respectively increasing and decreasing in $\Delta\omega$. It follows from this that the general public good provision $q^R(\mu)$ is decreasing in $\Delta\omega$ and that $q^{R*}(\mu)$ is increasing in $\Delta\omega$. Similarly, $q^P(\mu)$ is increasing in $\Delta\omega$ and $q^{P*}(\mu)$ is decreasing in $\Delta\omega$. Note as well that the threshold $\varpi(\mu)$ determining the politically pivotal agent remains unchanged.

  Figure 4) about here

  The shift is illustrated in figure 4). An increase in inequality leads to a greater impact of a shift of the pivotal agent in terms of the provision of the general public good: The rich wants to reduce its provision while the poor wants to increase it. Because of the social complementarities between the political equilibrium and the cultural dynamics, all else been equal, an increase in $\Delta\omega$ makes the equilibrium trajectory more sensitive to the initial conditions of the minority group. Indeed a change in initial conditions may induce the

\textsuperscript{17}One could also consider the comparative dynamics on the degree of representativeness $\theta$ of integrated individuals inside the minority group. It is easy to show that an increase in $\theta$ makes it more likely to end up in a “high integration” politico-cultural equilibrium.
median agent to shift from the poor to the rich. In a society with a large value of \( \Delta \omega \) this induces a larger shift of general public good provision than in a society with a lower value of \( \Delta \omega \). This larger shift from high provision to low provision (or vice versa) triggers stronger cultural dynamics in opposite direction for the minority group. As a result, the configuration of parameters with multiple long run politico-cultural steady states is enlarged and there is a bigger impact of initial conditions in terms of cultural differentiation.

- **Change in the poverty rate of the minority group** \( \lambda_B^P \)

  Consider now a change in \( \lambda_B^P \) (i.e. increased poverty inside the minority group) \( \tilde{g}^R (\mu) \) and \( \tilde{g}^P (\mu) \) are decreasing when \( \lambda_B^P \) increases as both \( \tilde{\omega}^R \) and \( \tilde{\omega}^P \) get larger.\(^{18}\) On figure 2, this implies that the curve \( V^a(g) \frac{d}{dg} \) is shifted down and \( q^P^* (\mu) \) and \( q^R^* (\mu) \) are increased (whenever they exist). At the same time, the threshold \( \overline{\tau} (\mu) \) is pivoting counter-clockwise at the point \( \mu = 2\lambda_B^P - 1 \) (at which \( \overline{\tau} (\mu) = 1 \)). Therefore the part of \( \overline{\tau} (\mu) < 1 \) is shifted out with a positive shock on \( \lambda_B^P \).\(^{19}\) An increase of poverty inside the minority group has therefore subtle and ambiguous impacts on the steady state pattern of cultural differentiation of the minority group. Indeed the mainstream society is less willing to supply the general public good (both the poor and the rich agent) as the tax base is reduced, but at the same time the poor agent is more likely to be the pivotal agent leading therefore to more provision of the general public good.

This is illustrated in figure 5) where the locus of cultural steady states before and after the shock on \( \lambda_B^P \) are respectively depicted in red and black colours. In case (1), at a minority size \( \mu_1 \), the system exhibits two cultural steady states with "high integration" at \( H_1 \) and "low integration" at \( L_1 \) before the change in \( \lambda_B^P \), and only a unique "low integration" steady state at \( H_1' \) after the change. If the social system was converging towards the "low integration" \( L_1 \), the increase in poverty of the minority group favors cultural integration towards point \( H_1' \). Indeed the shock in \( \lambda_B^P \) shifts the integrated minority members to be poorer in average. This is enough to affect the political equilibrium and to induce the median voter to switch from

\(^{18}\) Indeed it is easy to see that an increase in \( \lambda_B^P = 1 - \lambda_B^P \) leads to a reduction of \( \overline{\omega} \) and therefore an increase in \( \omega^R / \overline{\omega} \) and \( \omega^P / \overline{\omega} \).

\(^{19}\) Indeed

\[ \overline{\tau} (\mu) = 1 + \frac{2\lambda_B^P - 1 - \mu}{2\lambda_B^P \mu} \]

Hence \( \overline{\tau} (\mu) < 1 \) if and only if \( \mu > 2\lambda_B^P - 1 \).
a rich "mainstream" or "integrated" agent to a poor individual demanding more general public good $G^\mu$. The resulting cultural dynamics lead to a "high integration" equilibrium steady state at $H'$. Conversely in case (2) at a minority size $\mu_2 > \mu_1$, a positive shock on $\lambda_B^P$ moves the system from a unique "low integration" steady state at $L_2$ to the possibility of two cultural steady states with "high integration" at $H'_2$ and "low integration" at $L'_2$ after the change in $\lambda_B^P$. If the social system was converging before the poverty shock towards the "low integration" equilibrium $L_2$ from an initial point like $O_2$ (above the new threshold line $\bar{T}(\mu))$, the increase in poverty leads to further cultural differentiation towards point $L'_2$ characterized by a larger fraction of "non integrated" individuals in the long run. If however the system was converging towards $L_2$ from a point like $O'_2$ (below the new threshold line $\bar{T}(\mu)$), then the poverty shock on the minority induces cultural convergence towards a "high integration" equilibrium at $H'_2$.

The preceding discussion suggests that an increase in poverty of the minority group leads to a larger (resp. smaller) impact of initial conditions on final steady states outcomes for larger (resp. smaller) minority groups. In other words, larger and initially more culturally differentiated minority groups tend to become even less integrated after an increase in poverty, while smaller and initially less culturally differentiated groups tend to become even more integrated after such a shock.

- **Change in the poverty rate of the mainstream group $\lambda_A^P$**

A change in $\lambda_A^P$ (ie. increased poverty inside the mainstream group) has very similar effects as a change in $\lambda_B^P$. Again both $\bar{g}^R(\mu)$ and $\bar{g}^P(\mu)$ get smaller when $\lambda_A^P$ increases (as $\tilde{w}^R$ and $\tilde{w}^P$ are increasing in $\lambda_A^P$). The steady state values $q^P(\mu)$ and $q^R(\mu)$ are consequently larger (whenever they exist). At the same time, the threshold $\bar{T}(\mu)$ above which there is a shift of the political pivotal agent from a poor agent to a rich one is also increased. As in the previous comparative dynamics, an increase of poverty inside the mainstream group has therefore ambiguous impacts on the steady state pattern of cultural differentiation of the minority group.\textsuperscript{20}

This discussion is summarized by the following proposition:

\textsuperscript{20}The ambiguity comes from the same reasons; The tax base is reduced and therefore the mainstream society is less willing to supply the general public good. At the same time though, a "mainstream" or minority "integrated" poor agent is also more likely to be politically pivotal and to demand a larger provision of the general public good.
Proposition 2  

i) An increase in income inequality $\Delta \omega$ between rich and poor makes it more likely to have multiple politico-cultural steady state equilibria and increases the sensitivity of the final cultural integration pattern to the initial conditions in terms of cultural differentiation.  

ii) An increase in the degree of poverty $\lambda_B^P$ of the minority community or $\lambda_A^P$ of the mainstream group has ambiguous effects on cultural integration. Higher poverty is more likely to reduce (resp. increase) long run cultural integration when the minority group is large and initially less integrated (resp. small and initially more integrated).

5  Multiple minority groups

So far, we assumed that there was only one minority cultural group interacting with the mainstream society. In this section, we consider the possibility for more than one minority group. Although such groups may value different specific public goods, they exert on each other some positive externalities in terms of cultural evolution because of their interactions on political decisions related to the general public good. Cultural differentiation may then be easier to sustain in the long run. At the same time though, controlling for the total size of the minority groups, an increase in the number of minority groups (cultural fragmentation) also induces a dilution effect as each group is less able to sustain a reasonable amount of its own group specific public good. This creates an opposite force reducing overtime the cultural resilience of each group and therefore the pattern of long run cultural differentiation. We analyze these issues in the following sections.

5.1 Two minority groups

Let us start with two symmetric poor minority groups $z \in \{B, C\}$ of size $\mu/2$ with the same distribution of income $\omega^R$ and $\omega^P$ and same relative fractions $\lambda^R_z = \lambda^R_m$ and $\lambda^P_z = \lambda^P_m$, (with $\lambda^R_m + \lambda^P_m = 1$). Assume again that these groups are poorer than the mainstream group with $1/2 < \lambda_A^R < \lambda_m^P$. Individuals in each group $z \in \{B, C\}$ are differentiated along their public good preferences: a fraction $1 - q^z$ of individuals are ”culturally integrated” to the mainstream society and share the same preferences as individuals in the mainstream group $D$. The residual fraction $q^z$ is ”non integrated”, does not value the ”mainstream” public good $G^m$ and has preferences over another group specific public good $G^z$ only produced inside community $z$. Preferences of such individuals write as:

$$u^z(c, G^z) = c + V^z(G^z)$$
Assume as well the same shape for the community good preference function of both minority groups \( z \in \{ B, C \} \): \( V^z(G) = V^m(G) \). The government budget constraint again writes as \( \tau \omega = G \) and total income is \( \omega = (\lambda^A + \mu \lambda^B_m) \omega^R + (\lambda^B + \mu \lambda^R_m) \omega^P \). The utility of each type of agents in each minority group writes as follows:

for group \( z \): the "integrated" individuals (ie. cultural type \( I \)) have the same policy preferences

\[
\bar{w}^A(\tau, \omega, \omega^z, x^z) = \left[ (1 - \tau) \omega - x^z \right] + V^a(\tau \omega)
\]

where \( x^z \) is the individual contribution inside community \( z \) to produce good \( G^z \).

Given that \( G^z = \sum_{i \in B} x^z = \frac{1}{2} x^z \), the "non integrated" individuals (ie. cultural type \( N^z \)) in group \( z \) have the following policy preferences

\[
w^z(\tau, \omega, \omega^z, x^z) = \left[ (1 - \tau) \omega - x^z \right] + V^m(\frac{\mu}{2} x^z)
\]

Inside group \( z \), the individual contribution to produce good \( G^z \) is again decided by a cultural leader and is enforced inside the community. This cultural leader decides about the contribution of each individual weighting for simplicity equally those of type \( I \) and those of type \( N^z \). That is we assume that \( \theta = 1/2 \) in each group. Then we have the following problem characterizing the optimal provision of \( G^z \)

\[
\max_{x^z} \left[ (1 - q^z) \cdot \sum_{i \in R, P} \bar{w}^A(\tau, \omega, \omega^z, x^z) \right] + q^z \cdot \sum_{i \in R, P} w^z(\tau, \omega, \omega^z, x^z)
\]

or equivalently

\[
\max_{x^z} -x^z + q^z V^m(\frac{\mu}{2} x^z)
\]

This provides the following optimal individual contribution \( \hat{x}^z(q^z, \mu) = \frac{2V^{m-1}(2 \mu q^z)}{\mu} \) and community public good \( \hat{G}^z(q^z, \mu) = V^m_G(\frac{2}{\mu q^z}) \).

It is also immediate to get the preferred level of provision of the mainstream public good \( \hat{g}_{j,k}^i \) for an individual with income \( \omega^j \) \( i \in \{ R, P \} \) in group \( j \in \{ A, B, C \} \), and cultural type \( k \in \{ I, N^z \} \) when that individual belongs to group \( z \in \{ B, C \} \).

- For a mainstream individual or a culturally integrated minority individual of group \( z \) we get as before

\[
\hat{g}_{j,k}^i = \hat{g}_{l,k}^i = \hat{g}^i = V^m_G(\frac{\omega^i}{\omega}) \text{ for } z \in \{ B, C \}.
\]

- For a non culturally integrated individual of group \( z \), we have \( \hat{g}_{k,N^z}^i = 0 \) whatever his income \( \omega^j \) \( i \in \{ R, P \} \). Given that we have \( 0 = \hat{g}_{k,N^z}^i < \hat{g}^R < \hat{g}^P \), the median voter of this economy is then easily obtained. Noting again the threshold
\[
\bar{\pi}(\mu) = \frac{2\lambda_A^P - 1 + (2\lambda_{R}^P - 1)\mu}{\lambda_{R}^P \mu} > 0
\]

we have the following. When \( q^B + q^C < \bar{\pi}(\mu) \), the median voter is given by the mainstream/integrated poor and the equilibrium provision of good \( G^a \) is obtained as \( g_A^*(\mu) = \tilde{g}^P(\mu) \). When \( q^B + q^C > \bar{\pi}(\mu) \), the median voter is given by the mainstream rich and the provision of good \( G^a \) is obtained as \( g_A^*(\mu) = \tilde{g}^R(\mu) < \tilde{g}^P(\mu) \).

Consider now the process of cultural evolution within each minority community \( z, \in \{B, C\} \). By a similar reasoning as in section 2, the evolution of the fraction of ”non integrated” individuals in group \( z \) follows a dynamic equation:

\[
\dot{q}^z = q^z(1 - q^z) \left[ S^m(\frac{q^z}{2})(1 - q^z) - V^a(g_A^*(q^z + q^{z'}, \mu))q^z \right]
\]

and \( g_A^*(q^z + q^{z'}, \mu) = \begin{cases} \tilde{g}^P(\mu) \text{ when } q^z + q^{z'} \leq \bar{\pi}(\mu) \text{ and } z \neq z' \in \{B, C\} \\ \tilde{g}^R(\mu) \text{ when } q^z + q^{z'} > \bar{\pi}(\mu) \end{cases} \)

with \( S^m(q) = V^m(V_G^{m-1}(\frac{1}{\mu})) \) \( ^{21} \)

For a given level of general public good \( g \), denote the unique solution \( q_2^*(g) \) such that

\[
S^m(\frac{q}{2}) = V^a(g) \frac{q}{1 - q}
\] (10)

Given that the equilibrium level of general public good \( g_A^*(q^z + q^{z'}, \mu) \) can only take two values \( \tilde{g}^P(\mu) \) and \( \tilde{g}^R(\mu) \), equation (11) provides two solutions \( q_2^{R*}(\mu) = q_2^*(\tilde{g}^P(\mu)) \) and \( q_2^{R*}(\mu) = q_2^*(\tilde{g}^R(\mu)) \). We also know that \( q_2^{R*}(\mu) < q_2^{R*}(\mu) \) as \( \tilde{g}^P(\mu) > \tilde{g}^R(\mu) \) for all \( \mu \in [0, 1] \).

Figure 6) shows the stable steady state manifold \( \tilde{q}^z(q^z) \) for a given group \( z \) as a function of the fraction of ”non integrated” individuals \( q^{z'} \) of the other group \( z' \). The line \( q^z + q^{z'} = \bar{\pi}(\mu) \) separates the region of the square \([0, 1]^2\) in two regions. Above the line, there is low provision of general public good \( \tilde{g}^R(\mu) \) and low cultural integration with the fraction \( q^z \) of ”non integrated” individuals converging towards \( q_2^{R*}(\mu) = q_2^*(\tilde{g}^R(\mu)) \). Below the line, there is high provision of the general public good \( \tilde{g}^P(\mu) \) and high cultural integration and \( q^z \) converging

\( ^{21} \)This function can easily be shown to be increasing concave in \( q \) and to satisfy \( S^m(0) = 0 \) and \( S^m(0) = +\infty \).
towards \( q^B_2 (\mu) = q^C_2 (\mu) \). The steady state manifolds of the two groups are depicted as curve \( BB \) for group \( B \) and curve \( CC \) for group \( C \). 

Putting up together these two curves, figures 7 a), 7 b), 7 c) provide the phase diagrams of the joint cultural evolution of the two minority groups associated to (10)

Figures 7a) 7b) and 7c) about here

Three cases may occur. The first two cases describe situations where the manifolds intersect only once at a locally stable steady state of the system (10): case a) with a high integration cultural steady state and \( q^B_* = q^C_* = q^P_2 (\mu) \); and case b) with a low integration cultural steady state and \( q^B_* = q^C_* = q^R_2 (\mu) \). The third possibility depicted in case c) reflects the case of multiple politico-cultural steady states where the manifolds have two stable intersection points \( L \) and \( H \) respectively at a low and a high integration equilibrium.

Interestingly, the phase diagram reveals a dynamic cultural complementarity between the two minority groups. The existence of a relatively non integrated minority group facilitates the persistence of non integration of the other group. The reason for this comes from the fact that a non integrated minority group induces a political externality on the other group. Indeed it contributes to the shift of the political pivotal agent to vote for a low provision of the general public good. This in turn reduces the cultural integration of both groups overtime.

To see that in the most intuitive way, consider for instance figure 6c) with the possibility of two long run stable politico-cultural equilibria at points \( L \) and \( H \). Suppose that the initial cultural conditions of minority group \( B \) is \( q^B(0) \) and such that \( q^B(0) < \bar{q}(\mu) \). When the other group \( C \) starts with very few ”non integrated” individuals (ie. \( q^C(0) \) close to 0), then point \( O_i \) associated to the initial cultural conditions, lies below the line \( q^B + q^C = \bar{q}(\mu) \). The median voter is a poor integrated individual demanding a high level of the general public good \( g_A = q^P_2 (\mu) \). Because of this, point \( O_i \) belongs to the basin of attraction of the high integration steady state point \( H \). Group \( B \) therefore converges to the high integration equilibrium characterized by \( q^B_* = q^C_* = q^P_2 (\mu) \). Conversely, when group \( C \) starts with a significant number of non integrated individuals \( q^C(0) \) such that \( q^C(0) > \bar{q}(\mu) - q^B(0) \), then the political pivotal agent is a rich integrated individual who demands less general public

\[ \bar{q} (q^z) = \begin{cases} q^P_2 (\mu) & \text{for } q^z < \bar{q}(\mu) - q^P_2 (\mu) \\ q^R_2 (\mu) & \text{for } q^z > \bar{q}(\mu) - q^R_2 (\mu) \end{cases} \]

\[ ^22 \text{Given (10), the steady state manifold for group } z \text{ is formally obtained as} \]
good provision \( g_A = q'^R_s (\mu) \). As a consequence, the initial condition point \( O_2 \) lies above the line \( q'^R + q'^C = \overline{q}(\mu) \) and the system converges towards a low integration equilibrium \( L \) characterized by a high fraction of non integrated individuals in each minority group.

This discussion suggests that the capacity for cultural resilience of one group may crucially depend on the degree of cultural resilience of other minority groups in the same society. Even when these groups do not directly influence each other in the cultural transmission process, they impose however on each other a positive externality through the political economy of general public goods. These in turn indirectly affect the cultural integration to the mainstream culture.

### 5.2 Multiple minority groups and cultural fragmentation

Interestingly, the previous analysis can be extended to \( N \) symmetric minority groups \( z \in \{1, \ldots, N\} \) each of size \( \mu_i/N \) with the same distribution of income \( (\omega^R \text{ and } \omega^P) \) and relative fractions \( \lambda^R_m \) and \( \lambda^P_m \) (with \( \lambda^R_m + \lambda^P_m = 1 \)) and again assuming that \( 1/2 < \lambda^P_A < \lambda^P_m \). Preferences of integrated individuals in each group \( z \) are the same as before while preferences of non integrated individuals in group \( z \) over their group specific public good \( G^z \) are given by

\[
u^z(c, G^z) = c + V^x(G^z) \quad \text{for all } z \in \{1, \ldots, N\} \]

Following the same line of reasoning as in the previous section, one immediately gets the following. When \( \sum_{z=1}^{\infty} q^z < \overline{q}(\mu) \), the mainstream/integrated poor is the median voter and the equilibrium provision of good \( G^z \) is given by \( g^*_A (\mu) = \hat{g}^P(\mu) \). Conversely when \( \sum_{z=1}^{\infty} q^z > \overline{q}(\mu) \), the mainstream rich is the median voter and the provision of good \( G^z \) is now given by \( g^*_A (\mu) = \hat{g}^R(\mu) < \hat{g}^P(\mu) \). Notice that \( \overline{q}(\mu) \) does not depend on \( N \) and again is decreasing in \( \mu \) as \( 2\lambda^P_A - 1 > 0 \).

The process of cultural evolution within each minority community \( z \in \{1, \ldots, N\} \) is obtained in the same way as before and consequently writes as

\[
\frac{\dot{q}^z}{q^z} = q^z(1 - q^z) \left[ S^m(\frac{q^z}{N})(1 - q^z) - V^x(g^*_A)q^z \right] \quad \text{for all } z \in \{1, \ldots, N\}
\]

and

\[
g^*_A = \begin{cases} 
\hat{g}^P(\mu) & \text{when } \sum_{z'=1}^{\infty} q^{z'} < \overline{q}(\mu) \\
\hat{g}^R(\mu) & \text{when } \sum_{z'=1}^{\infty} q^{z'} > \overline{q}(\mu)
\end{cases}
\]

As before, for a given value of general public good \( g \), it is easy to see that each group \( z \)
exhibits a stable cultural steady state $q_N^*(g)$ such that

$$S\left(\frac{q}{N}\right) = V^a(g) \frac{q}{1-q} \quad (11)$$

Simple differentiation shows that $q_N^*(g)$ is decreasing in the number of groups $N$. Maintaining constant the total size of the population, a larger number of minority groups means that each of them is smaller. Because of this dilution effect, each minority group $z$ produces less of his own specific group public good $G^z$. This leads to weaker incentives to transmit the "non integrated" cultural trait in this community and consequently to a smaller steady state fraction of non integrated individuals. Consequently cultural fragmentation favors the adoption of the mainstream cultural trait.

Notice then that $g = g_A^*$ can take two values $\hat{g}^P(\mu)$ and $\hat{g}^R(\mu)$ corresponding respectively to a "high integration" and a "low integration" politico-cultural equilibrium. In each minority group $z$, this provides two equilibrium fractions $q_N^*(\hat{g}^P(\mu))$ and $q_N^*(\hat{g}^R(\mu))$ with $q_N^*(\hat{g}^R(\mu)) < q_N^*(\hat{g}^R(\mu))$. To be consistent with equilibrium voting, the "high integration" equilibrium $q_N^*(\hat{g}^P(\mu))$ prevails when $\sum_{z=1}^N q_N^*(\hat{g}^P(\mu)) = Nq_N^*(\hat{g}^P(\mu)) < \frac{\mu}{N}$, while conversely the "low integration" equilibrium $q_N^*(\hat{g}^R(\mu))$ prevails when $Nq_N^*(\hat{g}^R(\mu)) > \frac{\mu}{N}$.

As is easily seen, cultural fragmentation (an increase in $N$) generates two forces going in opposite direction with respect to cultural integration. As already mentioned there is a "dilution" effect: each minority group being smaller in size, the provision of each group specific public good $G^z$ is smaller. This in turn reduces the cultural resilience of the group specific trait in the cultural transmission process and the steady state fraction of non integrated individuals $q_N^*(\hat{g}^R(\mu))$ is decreasing in $N$.

At the same time though, more fragmentation also means that it is easier to shift the median voter from a poor to a rich individual, as all "non integrated" minority individuals make a unified political coalition against the provision of the general public good. Given that such a shift reduces the political equilibrium provision of general public good, the cultural resilience of each minority trait is enhanced as well as the likelihood to get a low integration politico-cultural steady state equilibrium.

When is it that we can ensure that high cultural integration is the only possible long run outcome? To see that note that the condition for a low integration equilibrium to exist writes as $q_N^*(\hat{g}^R(\mu)) > \frac{\mu}{N}$. After substitution this rewrites as

$$S\left(\frac{\frac{\mu}{N^2}}{N^2}\right) > V^a(\hat{g}^R(\mu)) \frac{\frac{\mu}{N}}{N - \frac{\mu}{N}} \quad (12)$$

It can be seen (see the appendix) that this last inequality cannot be satisfied for $N$ large enough. The dilution effect is stronger than the political shifting effect when the degree of
cultural fragmentation is large enough. A direct implication of this is the fact that cultural integration is more likely to occur in the long run in multicultural societies composed of many small groups than a few significant ones.

6 Conclusion

The questions of cultural identity and insertion of minority groups within modern welfare state institutions are hot issues right now in many western countries. In Europe, in the background of islam radicalization and terrorism, migration flows of political refugees from Syria and Irak have generated important political tensions and the rise of extremist parties in several countries (Hungary, Greece, Austria, as well as France, and even Germany). In particular, the question of how religious communitarianism can be compatible with secular modern state institutions has been at the heart of political debates. In the context of globalization and technological change creating important income disparities within and across nations (Bourguignon 2015, Piketty 2014, Facchini and Couvreur 2015), political pressures for vertical redistribution interact with the demands for horizontal redistribution associated with these immigration flows. Such interactions are crucially affecting the sustainability of welfare state systems in culturally diverse societies.

In this paper, we provided a simple dynamic political economy framework to analyze these issues, and to discuss the implications for public goods provision, redistribution, and cultural integration. Our analysis suggests a dynamic complementarity between the political economy of redistribution, and the process of cultural integration of minority groups in a mainstream society. By shifting political coalitions, cultural diversity may erode the support for large scale general public good provision and vertical redistribution from the rich to the poor. Dynamically, the attractiveness of adopting the mainstream culture is reduced, and minority groups are more likely to remain culturally non integrated. Importantly, this in turn has negative feedbacks effects on the future demands for redistribution and general public good provision over the next generations.

Our model indicates the possibility for multiple politico-cultural steady state trajectories leading to very different outcomes in terms of integration and redistribution. The long run political sustainability of vertical redistribution and general public goods provision in a multicultural society then crucially depends on the initial degree of cultural differentiation and fragmentation of the society. Interestingly, our framework also suggests that an increase in income inequality, associated say to globalization or technical change, makes even more salient the sensitivity of long run outcomes to these initial conditions. This feature may then
help understand why the political stakes to monitor the extent of first generation immigration flows might be currently so high in western countries facing increasing vertical inequalities and economic stagnation.

Obviously, our analysis left a number of important issues that may be incorporated in future research. First, we adopted a political economy set-up of a median voter model. While this is convenient to capture important dimensions of the debate on redistribution in multicultural societies, it would be worth introducing richer political dimensions such as probabilistic voting, endogenous political participation and/or lobbying influences by specific groups. Introducing these dimensions may alter the conditions for high integration and low integration trajectories. As long as some non integrated minority individuals do participate in the political decision making process of provision of general redistribution or public goods, we may expect our main argument that cultural diversity affects the pattern of public policies in a way that reinforces the preservation of such diversity, to be present in these extended contexts.

We also assumed that socialization only occurred inside the minority group (through parents and role models from the community). Our framework can be easily extended to the context where socialization by role models from the mainstream group may also occur (see oneline appendix). In that case, it can be shown that full cultural integration obtains for a small enough minority group. However, when the minority group is large enough, there is still scope for cultural differentiation at any equilibrium level of the general public good provided by the political system, and the results of this paper are qualitatively preserved.

One may think to introduce richer economic and institutional contexts, taking into account the impact of minority groups on labor market outcomes, and conversely the role of market discrimination on the incentives to culturally assimilate. As well, one may consider the role of education as a mechanism producing jointly economic and cultural integration outcomes, and eventually affecting social mobility inside the society. These extensions would naturally lead to endogenous dynamics of income inequality, providing new political economy interactions between cultural integration and vertical redistribution.

Finally our framework only considered a ”universalist” public policy (the provision of a general public good). One could alternatively consider the determination of a ”communautarian” policy whereby the community good is partly publicly funded through a given share of tax revenue. Such extension would permit an investigation of the political economy of multiculturalist policies and their implications for the equilibrium trajectories of level cultural integration. While clearly beyond the scope of the present paper, we hope that the framework outlined here can be a stepping stone to analyse these issues in future research.
References


Figure 1: cultural steady state
Figure 2a): high integration equilibrium
Figure 2b): multiple equilibria
Figure 2c): Low integration equilibrium
Figure 3a): Only High Integration Equilibrium
Figure 3b): High and Low Integration Equilibria
Figure 3c): High and Low Integration Equilibria
Multiple Equilibria

Figure 4a): increasing income inequality $\Delta \omega$
Figure 4b): increasing poverty rate $\lambda_B^P$ in the minority group
Figure 5: Two minority groups

\[ q^z + q^{z'} = \overline{q}(\mu) \]
Figure 6a): High cultural integration steady state
Figure 6b): Low cultural integration steady state
Figure 6c): multiple cultural integration steady states
Appendix: Proofs

- **Stability of corner steady states:** It is immediate to see that \( q = 1 \) is locally instable. Under the conditions \( V^b(0) = 0 \); \( V_G^b(0) = +\infty \), it is easy to conclude that \( S^b(0) = 0 \), and \( S^\prime(0) = +\infty \). From this a second order development show that the steady state \( q = 0 \) is also unstable. QED.

- **Concavity of the function** \( S(q) \): Recall that \( S^b(q) = V^b(V_G^{b-1}(\frac{\Phi(q)}{\mu})) \) with \( \Phi(q) = 1 + \frac{\vartheta(1-q)}{1-\vartheta q} \). Differentiation gives immediately:

\[
S'(q) = \frac{\Phi(q) \Phi'(q)}{\mu^2} \frac{1}{V_G^b(V_G^{b-1}(\frac{\Phi(q)}{\mu}))} > 0
\]

given that \( \Phi'(q) = -\frac{\vartheta}{(1-\vartheta q)^2} < 0 \) and that \( \Phi''(q) = \frac{2\vartheta}{(1-\vartheta q)^3} > 0 \). Moreover \( S'(0) = +\infty \). Differentiating another time provides:

\[
S''(q) = \frac{(\Phi')^2 + \Phi\Phi''}{\mu^2 V_G^b} - \frac{\Phi\Phi'}{\mu^2 (V_G^b)^3} \frac{\Phi'}{\mu} = \frac{1}{\mu^2 V_G^b} \left[ (\Phi')^2 + \Phi\Phi'' - \frac{\Phi(\Phi')^2}{\mu} \frac{V_G^b}{(V_G^b)^2} \right]
\]

which using the fact that \( \Phi(q) = \mu V_G^b(\mu x^b) \), can be rewritten as

\[
S''(q) = \frac{1}{\mu^2 V_G^b} \left[ \Phi\Phi'' + (\Phi')^2 \left(1 - \frac{V_G^b V_G^{GGG}}{(V_G^b)^2}\right) \right]
\]

From \( \Phi'' > 0 \), it follows immediate that with the condition \([V''(y)]^2 > V'(y)V'''(y)\), the bracket term is positive and \( S(q) \) is increasing concave. QED.

- **Proof of proposition 1:** The proof is immediate. It comes from the simple fact that \( \overline{\pi}(\mu) \) is a decreasing function of \( \mu \) with \( \lim_0 \overline{\pi}(\mu) = +\infty \) and that \( q^{P*}(\mu) \) and \( q^{R*}(\mu) \) are increasing functions of the size of the poor minority group \( \mu \). Indeed as \( \mu \) increases, the average income \( \overline{\pi}(\mu) \) in the economy decreases. This leads to less provision of the general public good \( g_A \) (whether the poor or the rich agents are pivotal in the vote decision) and therefore a larger long run fraction \( q^* (\mu) \) of individuals in the minority community who keep their preferences for their community specific good \( g_B \). The conditions for the specific long run politico-cultural equilibria result simply from the fact that the functions \( \Theta^P(\mu) = q^{P*}(\mu) - \overline{\pi}(\mu) \) and \( \Theta^R(\mu) = q^{R*}(\mu) - \overline{\pi}(\mu) \) are increasing functions of \( \mu \), that \( \Theta^P(\mu) < \Theta^R(\mu) \) and the consistency conditions that a high integration equilibrium prevails if and only if \( \Theta^P(\mu) < 0 \) and a low integration equilibrium prevails when \( \Theta^R(\mu) > 0 \). QED.
• **Cultural Fragmentation with N minority groups:** The condition for a low integration equilibrium to exist writes as

\[ q_N^*(\tilde{g}^R(\mu)) > \frac{\overline{\tau}(\mu)}{N} \]

After substitution this rewrites as

\[ S(\frac{\overline{\tau}(\mu)}{N^2}) > V^a(\tilde{g}^R(\mu)) \frac{\overline{\tau}(\mu)}{N - \overline{\tau}(\mu)} \] (13)

Denote then the function

\[ \Psi(N) = (N - \overline{\tau}(\mu)) S(\frac{\overline{\tau}(\mu)}{N^2}) \text{ with } S(x) = V^b(V^{b-1}(\frac{1}{\mu x})) \]

(13) can be restated as

\[ \Psi(N) > V^a(\tilde{g}^R(\mu))\overline{\tau}(\mu) \] (14)

differentiation of \(\Psi(N)\) provides

\[ \Psi'(N) = V^b(V^{b-1}(\frac{N}{\mu^2})) + \frac{2(N - \overline{\tau}(\mu)) N^2}{(\mu^2)^2} V^b(\tilde{g}^{b-1}(\frac{N}{\mu^2})) \]

which is negative when \(N \to \infty\) as \(V^b(V^{b-1}(\frac{N}{\mu^2})) \to 0\) and the second term becomes infinitely negative. Hence for some \(N \geq N^\ast\), the \(\Psi(N)\) is decreasing in \(N\) and keeping on increasing \(N\) makes it less likely to have (14) satisfied. Therefore after some threshold \(N^\ast\), cultural fragmentation reduces the likelihood that a politico-cultural equilibrium steady state exhibits low cultural integration. \textbf{QED.}
nationalist), and a link between preferences for redistribution and national identification (nationalists are for less redistribution, at a given income level). We depart from Shayo’s paper on two dimensions. First, instead of a static social identity choice model a la Akerlof-Kranton (2000), our framework involves preferences dynamics, based on a micro-founded intergenerational cultural transmission process a la Bisin and Verdier (2001). Second, the focus of our analysis is different. We are concerned on how political outcomes on general redistribution or public good provision interacts with the dynamics of cultural integration and we highlight the comparative dynamics implications of income inequality changes on this interaction.

The paper is organized in the following way. Section 2 reviews the literature connected the paper. Section 3 presents the basic model, describing the political economy block and the cultural transmission framework. Section 4 discusses the nature of the equilibrium politico-cultural steady states. In section 5 we provide some comparative dynamics discussion with respect to income inequality, poverty rates and other parameters. Section 6 briefly discusses the extensions to direct cultural influence by the mainstream group, while section 7 presents the extension with multiple minority groups. Finally section 8 concludes. Proofs are relegated to an appendix.

2 A model of public good provision and cultural integration

We consider a simple economy composed of two social groups $A$ and $B$. Group $A$ of size $L_A$ normalized to 1 is the majority ”mainstream” group. It is composed of two types of individuals differentiated by their income levels: rich $R$, and $P$ with respective incomes $\omega^R > \omega^P$ and fractions $\lambda^R_A$ of rich and $\lambda^P_A$ of poor (with $\lambda^R_A + \lambda^P_A = 1$). Group $B$ is a minority group of size $\mu < 1$, composed also of rich and poor individuals with income $\omega^R_B$ and $\omega^P_B$. As it is often the case empirically we assume the minority group to be poorer on average than the mainstream group, with fractions of poor and rich $\lambda^P_B$, and $\lambda^R_B$ (with $\lambda^R_B + \lambda^P_B = 1$) such that $1/2 < \lambda^P_A < \lambda^P_B$. This ensures that both groups have a majority of poor and that the minority group $B$ is on average poorer than group $A$.

- Preferences:

  Individuals in group $A$ are concerned with the provision of a so-called ”mainstream” public good $G^A$. We view $G^A$ as a public good in the large sense, such as public and secular
education, public secular health services provision, security and rule of law under a secular modern justice system, equal rights and access to goods and markets, protection against gender discrimination. Essentially we think of the type of general public good for which a broad consensus exists among ”mainstream” individuals who belong to a typical socio-liberal modern democracy. The preferences take the following simple quasi-linear form over private consumption and the mainstream public good:

\[ u(c, G) = c + V^a(G^a) \]

with \( V^a(.) \) an increasing concave function such that \( V^a(0) = 0 \), \( V^a_G(0) = +\infty \) and \( V^b_G(\infty) = 0 \).

Individuals in group \( B \) constitute a minority community and are differentiated along their public good preferences: a fraction \( 1 - q \) of these individuals are ”culturally integrated” to the mainstream society, and as such share the same preferences as the mainstream group \( A \). The residual fraction \( q \) of individuals in group \( B \) are ”culturally non integrated” and do not value the ”mainstream” public good \( G \). They have preferences over some other community specific public good \( G^b \) that can only be produced inside community \( B \). This public good \( G^b \) can be associated to the provision or support for group specific activities and services, such as ethnic or religious education, marriage and social matching making, informal dispute settlement or arbitration services.

The preferences of the ”non integrated” individuals in group \( B \) write as:

\[ u^b(c, G^b) = c + V^b(G^b) \]

with \( V^b(.) \) an increasing strictly concave function such that \( V^b(0) = 0 \), and the derivatives satisfy \( V^b_G(0) = +\infty, V^b_G(\infty) = 0 \).

The mainstream good \( G^a \) is provided by majority voting of the whole society through proportional taxation on income (at a flat tax rate \( \tau \)). The government budget constraint writes simply as \( \tau \bar{\omega} = \bar{G}^a \) with total income \( \bar{\omega} = (\lambda_A^R + \mu \lambda_B^R)\omega^R + (\lambda_A^P + \mu \lambda_B^P)\omega^P \).

The amount of specific community \( B \) good \( G^b \) is decided by some community organization or leader, out of contributions obtained inside group \( B \). Consistent with Prummer and Siedlarek (2016), and Verdier and Zenou (2015, 2016), we recognize here the role of community leaders as important socializing agents and public good providers in immigrant or minority

\[ ^2\text{Prummer and Siedlarek (2016) highlights the importance of social connections inside the minority group as an important factor of cultural resilience. Verdier and Zenou (2015, 2016) consider the strategic and forward looking implications of cultural leaders for the process of cultural integration of minority groups. These papers do not consider the political economy issues analyzed in this paper.} \]
communities. These leaders are typically incarnated by immigrant religious organizations and their agents (Imams, Rabbis or Priests)\(^3\) or foreign-language media \(^4\).

Moreover we assume that all individuals in group \(B\) follow the prescription of the community leader and contribute to the group specific public good \(G^b\), even if they have no taste for that good (ie. are "culturally integrated"). The idea here is the fact that all individuals of group \(B\) are somewhat subject to some informal community mechanism that enforces the payment of the contributions for good \(G^b\). In particular, "culturally integrated" individuals may feel compelled to follow the community leader’s suggestion, although they do not value privately good \(G^b\). This captures the fact that individuals are socially embedded into community networks through which provision is publicly observed by others. Default of doing this could trigger some penalty cost associated to a loss of reputation, social pressure or ostracization. Another possibility is the fact that "culturally integrated" individuals have internalized a psychological cost to deviate from the obligation to contribute to the community public good even when they do not have individual preferences over such goods.\(^5\)

The important feature from a political economy perspective is the fact that a "culturally integrated" individual may still express his "mainstream" preferences in the secret of the ballot when voting over the provision of the general public good \(G^a\).\(^6\)

Substituting the public budget constraint, one obtains the utility of each type of agents:

- for group \(A\):

\[
w^A(\tau, \omega^i, \omega) = (1 - \tau)\omega^i + V^a(\tau\omega)
\]

\(^3\)For instance in Turkish communities in Germany the leadership role falls to Imams and in particular the “DITIB”, an institution of the Turkish government (Yasar (2012)). Imams part of such institution influence the norms and values of their communities in Germany. Through their sermons and teachings they provide group specific services (religious education, community services and counseling), that affect the assimilation of Turkish immigrants. To finance such services, monetary contributions are asked in the community (Ceylan (2010)).

\(^4\)For instance in Hispanic communities in the US, Spanish-language media provide spanish communication services in the community and have an influence on the attitude towards integration (Portes and Sensenbrenner (1993)), Subervi-Velez (1986)).

\(^5\)Hence implicitly we assume that there is a latent characteristic that still attaches the "culturally integrated " individuals to their minority community, even when they have shifted to mainstream public good preferences.

\(^6\)The model could be extended to allow "culturally integrated" individuals to choose whether or not to pay the contribution \(x^b\), at a psychological or social cost \(C\) if they do not pay. In such a case, when \(C\) is large enough, the community leader will always choose an equilibrium contribution \(x^b\) that satisfies the incentive compatibility constraint \(x^b < C\) inducing the "culturally integrated" individual to contribute to the group specific public good.
- for group \( B \): the "integrated" individuals have the same policy preference but contribute to the group specific public good \( G^b \).

\[
\tilde{w}^A \left( \tau, \omega^i, \overline{\omega}, x^b \right) = \left( 1 - \tau \right) \omega^i - x^b \right] + V^a(\tau \overline{\omega})
\]

where \( x^b \) is the individual contribution inside community \( B \) to produce good \( G^b \).

Given \( G^b = \sum_{i \in B} x^{bi} = \mu x^b \), the "non integrated" individuals in group \( B \) have the following policy preferences

\[
w^B \left( \tau, \omega^i, \overline{\omega}, x^b \right) = \left( 1 - \tau \right) \omega^i - x^b \right] + V^b(\mu x^b)
\]

- **Provision of the community specific public good**

We assume that the cultural leader inside group \( B \) decides about the contributions of all individuals, weighting the "integrated" and "non integrated" types with respective weights \( \theta \) and \( 1 - \theta \). Typically \( \theta \) reflects the degree of openness of the community leader towards the "mainstream" culture. Alternatively, \( \theta \) could also reflect the internal political economy representation of progressive (pro integration) versus conservative (pro non integration) factions inside the community institution. We then have the following maximization problem

\[
\max_{x^b} \theta \left[ (1 - q) \cdot \tilde{w}^A \left( \tau, \omega^i, \overline{\omega}, x^b \right) \right] + (1 - \theta) \left[ q \cdot w^B \left( \tau, \omega^i, \overline{\omega}, x^b \right) \right]
\]

or equivalently

\[
\max_{x^b} \left[ \theta(1 - q) + (1 - \theta) q \right] x^b + (1 - \theta) q V^b(\mu x^b)
\]

Denote \( \Phi(q, \theta) = 1 + \frac{\theta(1-q)}{1-\theta} \cdot \). Using the first order condition of (1), we then obtain the optimal level of contribution \( \hat{x}(q, \mu, \theta) = \frac{V_G^{-1}(\Phi(q, \theta))}{\mu} \) and level of community specific public good \( \hat{G}^b(q, \mu, \theta) = V_G^{-1}(\Phi(q, \theta)) \mu \).

- **Provision of the "mainstream" public good \( G^a \)**

In each period \( t \), the whole society decides about the provision of the "mainstream" or general public good \( G^a \) by majority voting. To determine the political economy equilibrium it is therefore useful to consider the preferred provision of the mainstream public good for the different groups (members of the majority group \( A \), integrated and non integrated individuals of the minority group \( B \)). For this denote \( \hat{g}_{i,k}^j \) the level of general public good \( G^a \) preferred by an individual with income \( \omega^i \) \( i \in \{ R, P \} \) (ie. \( R \) for rich, \( P \) for poor) , in group \( j \in \{ A, B \} \)
(ie. A for mainstream, B for minority) and cultural type $k \in \{I, N\}$ (ie. I for ”integrated”, N for ”non integrated”) if he belongs to group B.

Consider then an individual with income $\omega^i$ $i \in \{R, P\}$ from the mainstream group A, or from the minority group B and culturally integrated (ie. $k = I$). Substituting the public budget constraint into the indirect utility of such agent, it is straightforward to see that his preferred level of $G^a$ is given by:

$$\arg \max_g \left\{ \omega^i - \frac{\omega^i g}{\overline{w}} + V^a(g) \right\}$$

the solution of which is given by $\hat{g}^R_A = \hat{g}^R_{B,I} = \hat{g}^i$ for $i \in \{R, P\}$. With our notations, we therefore have $\hat{g}^R_A = \hat{g}^R_{B,I} = \hat{g}^i$ for $i \in \{R, P\}$.

Conversely for a ”non integrated” individual (type $N$) that belongs to group B, we obviously get $\hat{g}^R_{B,N} = 0$ whatever his income level $i \in \{R, P\}$. From this, we can easily rank the preferred levels of general public goods $\hat{g}^i_{j,k}$ of the different types of agents as:

$$\hat{g}^R_{B,N} = \hat{g}^R_{B,N} = 0 < \hat{g}^R = \hat{g}^R_{B,I} \leq \hat{g}^P = \hat{g}^P_{B,I}$$

Note that aggregate income $\overline{\omega} = \overline{\omega}(\mu) = \left(\lambda_A^R + \mu \lambda_B^R\right)\omega^R + \left(\lambda_A^P + \mu \lambda_B^P\right)\omega^P$ is increasing in $\mu$. Consequently the optimal policy levels $\hat{g}^R = \hat{g}^R(\mu)$ and $\hat{g}^P = \hat{g}^P(\mu)$ are also increasing in $\mu$, reflecting a standard size effect of public good provision.

Given that the preferences of the agents satisfy the single crossing property, the equilibrium is therefore obtained by the preferred policy of the median voter. To characterize this median voter, denote $\overline{\tau}(\mu)$ the threshold such that

$$\lambda_A^P + \lambda_B^P \mu(1 - q) = 1 - \lambda_A^P + (1 - \lambda_B^P)\mu(1 - q) + \mu q$$

The left hand side of this equation reflects the total number of individuals in favor of the policy platform $\hat{g}^P$ rather than $\hat{g}^R$, while the right hand side reflects the total number of individuals that support $\hat{g}^R$ rather than $\hat{g}^P$. Then $\overline{\tau}(\mu)$ is given by

$$\overline{\tau}(\mu) = 1 + \frac{2\lambda_A^P - 1 - \mu}{2\lambda_B^P \mu} > 0$$

It is just the fraction of integrated individuals in group $B$ that ensures that the two political platforms $\hat{g}^R$ and $\hat{g}^P$ get the same amount of voting support. Note that $\overline{\tau}(\mu)$ is decreasing in $\mu$ as we assumed that $2\lambda_A^P - 1 > 0$.

When $q < \overline{\tau}(\mu)$, the median voter is easily characterized as a mainstream/ or minority ”integrated” poor and the equilibrium provision of good $G^a$ is given by $g^+_A(q, \mu) = \hat{g}^P(\mu)$.  

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Conversely, when \( q > \tau(\mu) \), the median voter is a mainstream rich individual and the provision of good \( G^a \) is now given by \( g^*(q, \mu) = \tilde{g}^R(\mu) < \tilde{g}^P(\mu) \). Typically, when the number of "non integrated" minority individuals is sufficiently large, there is less equilibrium provision of the mainstream public good \( G^a \), and there is as well less redistribution (through the financing of that public good across income class groups).

- **Cultural evolution inside the minority community** \( B \).

Consider now the pattern of cultural integration of the minority group \( B \).\(^7\) For this we embed our previous model of political economy of public goods provision into a framework of cultural transmission a la Bisin and Verdier (2001)\(^8\) in which individuals in the minority group can acquire either the mainstream trait \( A \) (and be "culturally integrated") or the minority trait \( B \) (and be culturally "non integrated") by an intergenerational process of cultural transmission. Parents play an active socialization role in such a process.

Specifically, at each point of time \( t \), the timing is the following: 1) well socialized individuals vote, contribute and decide which public good to consume (mainstream or group specific) depending on their preferences; 2) then each minority individual before dying has one child and there is cultural transmission of the two traits (integrated \( I \) or non integrated \( N \)) inside the minority group \( B \). Cultural transmission is the result of direct vertical (parental) socialization and horizontal/oblique socialization inside the minority group.\(^9\) More precisely,

i) direct vertical socialization to the parent’s trait, say \( i \in \{I, N\} \), occurs with probability \( d^i \);

ii) if a child from a family with trait \( i \) is not directly socialized, which occurs with probability \( 1 - d^i \), he/she is horizontally/obliquely socialized by picking the trait of a role model chosen randomly in the minority population (i.e., he/she picks the "non integrated" trait \( N \) with probability \( q \) and the "integrated" trait \( I \) with probability \( 1 - q \)).

\(^7\)See also Lazear (1999) and Kónya (2005) for models of cultural assimilation of migrants respectively in a static and a dynamic context, but without endogenous political economy considerations.

\(^8\)For an economic approach see a serie of papers by Bisin and Verdier (1998, 2000a,b), 2001) which build upon the work of Cavalli-Sforza and Feldman (1981) in evolutionary biology and Boyd and Richerson (1985) in anthropology; see also Bisin and Verdier (2010) for a survey of the literature.

\(^9\)We assume here that socialization to the integrated trait (i.e. value the general public good) can only be done within the minority group. Alternatively we could assume that oblique socialization may also involve role models from the mainstream group. We discuss briefly the implications of such extension in the conclusion.
This cultural transmission mechanism introduced by Cavalli Sforza and Feldman (1981) is then summarily represented by the following system of equations for $P_{ij}$, the probability that a child from a family with trait $i$ is socialized to trait $j$ for $i, j \in \{N, I\}, i \neq j$ and $q^N = q$ and $q^I = 1 - q$.

$$
P_{ii} = d^i + (1 - d^i)q^i
$$
$$
P_{ij} = (1 - d^i)(1 - q^i)
$$

From this, it is clear that the minority group population remains stationary, and the (continuous) dynamics of evolution of the fraction of "non integrated" minority individuals (endowed with the peci

The parental rate of socialization $d^N$ and $d^I$ are obtained from parents’ choices of cultural transmission. These are motivated by imperfect empathy, namely a form of altruism biased towards the parents’ own cultural traits: parents care about their children’s choices, but they evaluate them using their own (the parents’ - not the children’s) preferences. More specifically, Let $U^{ii}, U^{ij}$ denote the utility to a cultural trait $i$ parent of a type $j$ child. When $U^{ii} \geq U^{ij}$ parents have an incentive to socialize their children to their own cultural trait. Socialization requires parental resources, e.g., time spent with children, private school tuition, church contribution, and so on. Denote by $C(d^i)$ the socialization costs, where $d^i$ is the probability of direct socialization of parents with trait $i$ to the $i$ trait. The value of parental socialization choice is then obtained from the following maximization program:

$$
W^i(q^i) = \max_{d^i \in [0,1]} -C(d^i) + P^{ii}U^{ii}(q^i) + P^{ij}U^{ij}(q^i), \quad \text{s. t. 1), and 2)}.
$$

Assuming for simplicity some quadratic socialization costs, $C(d^i) = \frac{1}{2}(d^i)^2$, we obtain the optimal effort of socialization:

$$
d^i = d(q^i, \Delta U^i) = (1 - q^i)\Delta U^i,
$$

where $\Delta U^i = U^{ii} - U^{ij}$ measures the relative value for a parent with trait $i$ to share with his/her child the same cultural trait. To analyze the cultural incentives to transmit the two

\[10\]Because of the fact that individuals interact socially (they vote jointly on public good provision) $U^{ii}, U^{ij}$ will be functions of $q^i$.

\[11\]The socialization choice of parents is independent of their individual consumption of private and public goods because of preference separability.
traits $N$ and $I$, we assume that the consumption of the two public goods $G^a$ and $G^b$, involves a deliberate choice at a small “cognitive” cost $f$ to the individual.\textsuperscript{12}

Now for a parent of type $I$ at time $w$ with income $\omega^P$, the relative value of having a child of his own type rather than of the other type $N$ is simply obtained by $\Delta U^I = U^{II} - U^{IN}$ with

\[
U^{II} = (1 - \tau^*_e)\omega^P - x^b_e - f + V^a(\tau^*e)\]
\[
U^{IN} = (1 - \tau^*_e)\omega^P - x^b_e - f
\]

Typically $U^{II}$ is the utility to have a child of type $I$ as perceived by a parent of type $I$ and $U^{IN}$ is the utility to have a child of type $N$ as perceived by a parent of type when the anticipated political equilibrium of the child generation is characterized by a tax rate $\tau^*_e$, a provision of the mainstream public good $\tau^*e$ and a minority contribution $x^b_e$. We assume that parents are "politically" myopic in their cultural transmission process and that they consider that the political equilibrium of their generation will apply as well to the society of their children.\textsuperscript{14} In such a case : $\tau^*_e = \tau^*_t$, $\tau^*e = g^*_A(q_t, \mu)$, and $x^b_e = x^b_l = \hat{x}(q_t, \mu, \theta)$. Letting $f \to 0$ this finally gives:

\[
\Delta U^I = U^{II} - U^{IN} = V^a(g^*_A(q_t, \mu)) \tag{6}
\]

where $g^*_A(q_t, \mu)$ is the equilibrium provision of mainstream public good voted in a society with a fraction $q_t$ of non integrated individuals in the minority group $B$.

By a similar reasoning, one obtains

\[
\Delta U^N = U^{NN} - U^{NI} = V^b(\mu\hat{x}(q_t, \mu, \theta)) = V^b(V_G^{b-1}(\Phi(q_t, \theta) / \mu) \tag{7}
\]

\textsuperscript{12}Although the two public goods have some non rivalry access property, what matters for parents' incentives to transmit their own trait depends on the differential way children decide to consume one or the other public good.compared to them. We model this decision choice by making individuals pay a (small) private cost $f$ to choose to consume $G^a$ or $G^b$. Then we let $f$ tend to zero. (See Bisin and Verdier 2000a) for a similar feature).

\textsuperscript{13}We assume that there is no social mobility in this society. Hence the income of a child is the same as the income of the parent. Introducing the possibility for endogenous social mobility is an interesting avenue for future research.

\textsuperscript{14}This assumption simplifies the dynamic analysis of cultural transmission ensuring that there are no forward looking considerations. Such dimensions are well known to generate complex dynamic features with multiple rational expectations equilibrium paths (see Bisin and Verdier 2000a) for some discussion along these lines).
For convenience, we suppress the dependence on $\theta$ in the notations and simply denote $S^b(q) = V^b(1 - \frac{\Phi(q)}{\mu})$ with $\Phi(q) = 1 + \frac{\theta(1-q)}{1-\theta q}$. Substitution of (5), (6), and (7) into (4) provides finally the cultural dynamics inside the community

$$\dot{q} = q(1-q) \left[ S^b(q)(1-q) - V^u(g^*_A(q, \mu))q \right]$$

with $g^*_A(q, \mu) = g^P(\mu)$ when $q \leq \Phi(\mu)$, and $g^*_A(q, \mu) = g^R(\mu)$ when $q > \Phi(\mu)$.

### 3 Politico-cultural steady states

We are now in a position to analyze the politico-cultural dynamics of the society. For this, we characterize the structure of the politico-cultural steady states associated to the cultural dynamics as defined by (8). Clearly, the system exhibits the two corner steady states $q = 0$ and $q = 1$ which can be shown to be locally unstable. An interior steady state is then characterized by the condition:

$$S^b(q) = V^u(g^*_A(q, \mu)) \frac{q}{1-q}$$

Under the condition that the utility function $V^b(.)$ is enough concave\(^{15}\), it can be easily seen that for a given value $g > 0$, there is a unique solution $q^*(g)$ such that

$$S(q) = V^u(g) \frac{q}{1-q}$$

Such solution $q^*(g)$ is decreasing in $g$. Indeed, a given level of general public good $g$ makes it more attractive for the minority group to integrate culturally into the mainstream values. This provides a smaller long run value of the fraction of individuals in community $B$ who have a preference for the community specific public good.

[Figure (1) about it].

Figure 1 depict the solution $q^*(g)$. The LHS of equation (9) is drawn as an increasing concave locus $SS$, the RHS of equation (9) is depicted by the increasing convex locus $GG$.

\(^{15}\)Specifically when the function $V^b(.)$ satisfies the following property:

$$[V^b_G(y)]^2 > V^b_C(y)V^b_{GG}$$

one can easily show that the function $S^b(q)$ is increasing concave with $S^b(0) = 0$, and $S^b(1) = +\infty$. $S(0) = 0$. Then it follows that for a given value $g > 0$, there is a unique solution $q^*(g)$ such that $S(q) = V^u(g) \frac{q}{1-q}$. 

14
As shown the unique steady state $q^*(g)$ is obtained at the intersection of $SS$ and $GG$. An increase in $g$ leads to a upward shift of the locus $GG$, leading therefore to a smaller value of $q^*(g)$.

Noting that $g^*_N(q, \mu)$ can take only two values: $\tilde{g}^P(\mu)$ and $\tilde{g}^R(\mu)$, the previous discussion indicates two potential steady state values $q^{P*}(\mu) = q^*(\tilde{g}^P(\mu))$ and $q^{R*}(\mu) = q^*(\tilde{g}^R(\mu))$ with $q^{P*}(\mu) < q^{R*}(\mu)$ as $\tilde{g}^P(\mu) > \tilde{g}^R(\mu)$ for all $\mu \in [0, 1]$.

Now to be consistent with the political equilibrium structure, one should also have that $g^*_N(q, \mu) = \tilde{g}^P(\mu)$ prevails when $q < \overline{q}(\mu)$ while $g^*_N(q, \mu) = \tilde{g}^R(\mu)$ prevails when $q > \overline{q}(\mu)$. From this the phase diagram of the cultural dynamics can be easily drawn as depicted in figure 2a) 2b), and 2c) depending on how the curve $V^a(g^*_N(q, \mu)) \frac{q}{1-q}$ crosses the locus $SS$.

Figures 2a), 2b) and 2c) about it

Figure 2a) shows the case where the intersection occurs only once at a point $q^{P*}(\mu) < \overline{q}(\mu)$. $q^{P*}(\mu)$ is the unique stable cultural steady state characterized by high integration of the members of community $B$ to "mainstream" preferences. This is accompanied by high provision of the general public good $G^a$ and low provision of the community public good $G^b$. The politico-cultural dynamics of this case are easy to describe depending on the initial "cultural integration" level $q_0$ of the community. When the minority group is already quite integrated (ie. $q_0 < \overline{q}(\mu)$) then the median voter remains permanently the poor individual. Conversely, when the community is not well integrated at the start (ie. $q_0 > \overline{q}(\mu)$), the median voter is initially the rich individual that votes for less general public good $A$ than the poor agent. The provision of such public good however is high enough to induce the minority group to integrate (to have the fraction $q_t$ decreasing overtime). As the fraction $q_t$ gets below the threshold $\overline{q}(\mu)$, the median voter shifts from the rich to the poor, increasing thereby the provision of the general public good. This further reinforces the integration process of the minority with a fraction $q_t$ converging towards the lower steady state value $q^{P*}(\mu)$. Note that when the community is already very integrated (ie. $q_0 < q^{P*}(\mu)$), then the so-called cultural substitutability of the transmission process (Bisin and Verdier 2001) implies that there is some degree of cultural resilience of the minority group. This makes the fraction of individuals with a preference for the community specific good to converge again towards $q^{P*}(\mu)$.

Figure 2c) shows conversely the situation where there is unique intersection point $q^{R*}(\mu)$ with $q^{R*}(\mu) > \overline{q}(\mu)$. $q^{R*}(\mu)$ is then the unique stable cultural steady state characterized by low provision of the general public good $G^a$ and high provision of the community public
good $G^h$. For a community sufficiently non integrated, (ie. $q_0 > \bar{q}(\mu)$), the pivotal agent in the political process remains indefinitely the rich agent, and there is permanently little provision of the general public good $G^a$. When $q_0 < \bar{q}(\mu)$, the median voter is initially the poor agent who is in favor of a relatively large level of general public good in society. This is not however large enough to counteract the strong motive of the minority members to transmit their own specific cultural trait. As a result, the fraction $q_t$ of ”non integrated” individuals keeps increasing, crossing at some point the threshold $\bar{q}(\mu)$. This induces a shift of the pivotal political agent towards the rich individual. A smaller provision of general public good follows. As a consequence, this reinforces the cultural transmission process of the minority community towards a low integration level $q^{R*}(\mu)$.

Finally figure 2b) shows the situation where the locus $SS$ intersect two times the curve $V^a(g^*) \frac{a}{1-q}$, respectively on the lower branch (for $q < \bar{q}(\mu)$) and on the higher branch (for $q > \bar{q}(\mu)$). This situation depicts the possibility of multiple politico-cultural steady state situations. Depending on whether the minority community $B$ is initially well integrated (ie. $q_0 < \bar{q}(\mu)$) or not (ie. $q_0 > \bar{q}(\mu)$), the equilibrium trajectory is different. In the first case, there is a process of integration with convergence towards a high level of integration $q^{P*}(\mu)$. In the second case, there is a process of cultural differentiation with $q_t$ converging to the higher value $q^{R*}(\mu)$.

The structure of the politico-cultural long run equilibria depends crucially on the size of the minority group. Notice first that the dependence of the equilibrium values $q^{P*}(\mu)$ and $q^{R*}(\mu)$ on the community size $\mu$ is a priori ambiguous. On the one hand, an increase in $\mu$ leads to a larger tax base for the ”mainstream” public good $G^a$, making the mainstream trait more attractive. On the other hand, a larger community group $\mu$ leads to a larger provision of the community public good $G^h$ which makes trait $B$ more attractive. To avoid a long taxonomy discussion, we assume that in the relevant range for $\mu \in [0, 1]$, the community public good effect dominates the tax base effect. Therefore a larger minority group $\mu$ is naturally more likely to sustain a larger fraction of ”non integrated” individuals. We formulate this as the following condition:16

**Condition C:** $q^{P*}(\mu)$ and $q^{R*}(\mu)$ are increasing functions of $\mu \in [0, 1]$

Then we have the following proposition:

---

16This condition holds when $V^a(.)$ is relatively concave compared to $V^b(.)$ and that the economy already provides a relatively large amount of ”mainstream” public good without the existence of the minority group $B$. 

16
Proposition 1: Under assumption C, we have the following characterization:

1) When \( \overline{\sigma}(1) > q^*(\hat{g}^R(1)) \), for all \( \mu \in [0,1] \) then there is a unique stable steady state politico-cultural equilibrium characterized by "high integration" (ie. \( q^*(\mu) = q^{P*}(\mu) < \overline{\sigma}(\mu) \))

2) When \( q^*(\hat{g}^P(1)) \leq \overline{\sigma}(1) < q^*(\hat{g}^R(1)) \), there exists a size threshold \( \mu_L < 1 \) such that:
   2i) for \( \mu < \mu_L \) we are in a situation as described in 1).
   2ii) for \( \mu \in [\mu_L,1] \) there are two stable steady state politico-cultural equilibria. In addition to a "high integration" equilibrium as described in 1), there is also a "low integration" equilibrium (ie. \( q^*(\mu) = q^{R*}(\mu) > \overline{\sigma}(\mu) \)).

3) When \( \overline{\sigma}(1) \leq q^*(\hat{g}^P(1)) \), there exists two size thresholds \( \mu_L \) and \( \mu_H \) with \( \mu_L < \mu_H < 1 \) such that:
   3i) For \( \mu < \mu_L \) we are in a situation as described in 1).
   3ii) For \( \mu \in [\mu_L,\mu_H] \) we are in a situation as described in 2ii).
   3iii) For \( \mu \in [\mu_H,1] \) there is a unique steady state politico-cultural equilibrium characterized by "low integration" (ie. \( q^*(\mu) = q^{R*}(\mu) > \overline{\sigma}(\mu) \)).

Proposition 1 is illustrated in figures 3a) 3b) and 3c) depicting the three possible cases. In all vignettes the threshold curve \( \overline{\sigma}(\mu) \) is drawn describing the threshold above which "non integrated" individuals in community \( B \) induce a shift of the pivotal agent from being poor to being rich with respect to the provision of the general public good \( g_A \). The larger the size of the minority community, the smaller this threshold.

Figures 3a) 3b) and 3c) about here

The two cultural steady state loci \( q^{P*}(\mu) \) and \( q^{R*}(\mu) \) are also drawn. As already noticed, these curves are increasing in \( \mu \). A larger size of the poorer community \( B \) induces less willingness by the median voter to finance the provision of the general public good. This in turn makes the transmission of the "integrated" cultural trait \( I \) less attractive to members of the minority community. A larger fraction \( q \) of individuals with the "non integrated" trait \( N \) prevails in the long run. Note that the locus \( q^{R*}(\mu) \) is always above the other locus \( q^{P*}(\mu) \). When the pivotal agent in society is richer, there is less provision of the general public good. This leads again to less integration to the mainstream culture over generations of members in the minority community \( B \). Finally, note that to get the consistency between the cultural dynamics and the political equilibrium at each period, the locus \( q^{P*}(\mu) \) is the relevant part when \( q^{P*}(\mu) < \overline{\sigma}(\mu) \) while \( q^{R*}(\mu) \) is the relevant one for \( q^{R*}(\mu) > \overline{\sigma}(\mu) \).

Figure 3a) shows the case where for all values of \( \mu \), the long run equilibrium is only a "high integration" equilibrium. Figure 3b) shows the case where after a threshold \( \mu_L \), the society is
characterized by two steady state politico-cultural equilibria: one with high integration and one with low integration. Finally figure 3c) shows the case where that situation occurs for $\mu \in [\mu_L, \mu_H]$, while for $\mu > \mu_H$, the only long run steady state is a low integration equilibrium one. As highlighted in these figures, the situation of multiple politico-cultural steady states is more likely to occur, everything else been equal, when the size of the minority group is intermediate.

4 Comparative dynamics

In this section we investigate how changes in social conditions (inequality, poverty) interact with cultural transmission to determine the nature of the politico-cultural trajectories and the extent of redistribution induced by the political system.\footnote{One could also consider the comparative dynamics on the degree of representativeness $\theta$ of integrated individuals inside the minority group. It is easy to show that an increase in $\theta$ makes it more likely to end up in a "high integration" politico-cultural equilibrium.}

- Change in income inequality (or in the wage gap) $\Delta\omega$

Clearly aggregate income $\overline{\sigma}$ can be written as a function of the income gap $\Delta\omega = \omega^R - \omega^P$

$$\overline{\sigma} = \overline{\sigma}(\mu) = (\lambda^R_A + \mu \lambda^R_B)\Delta\omega + \omega^P(1 + \mu)$$

which is increasing in $\Delta\omega$ (the degree of inequality or the wage skill gap). Fixing $\omega^P$, we obtain the relative positions $\omega^R/\overline{\sigma}$ and $\omega^P/\overline{\sigma}$ which are respectively increasing and decreasing in $\Delta\omega$. It follows from this that the general public good provision $\tilde{g}^R(\mu)$ is decreasing in $\Delta\omega$ and that $q^{R*}(\mu)$ is increasing in $\Delta\omega$. Similarly, $\tilde{g}^P(\mu)$ is increasing in $\Delta\omega$ and $q^{P*}(\mu)$ is decreasing in $\Delta\omega$. Note as well that the threshold $\overline{q}(\mu)$ determining the politically pivotal agent remains unchanged.

Figure 4) about here

The shift is illustrated in figure 4). An increase in inequality leads to a greater impact of a shift of the pivotal agent in terms of the provision of the general public good: The rich wants to reduce its provision while the poor wants to increase it. Because of the social complementarities between the political equilibrium and the cultural dynamics, all else been equal, an increase in $\Delta\omega$ makes the equilibrium trajectory more sensitive to the initial conditions of the minority group. Indeed a change in initial conditions may induce the
median agent to shift from the poor to the rich. In a society with a large value of $\Delta \omega$ this induces a larger shift of general public good provision than in a society with a lower value of $\Delta \omega$. This larger shift from high provision to low provision (or vice versa) triggers stronger cultural dynamics in opposite direction for the minority group. As a result, the configuration of parameters with multiple long run politico-cultural steady states is enlarged and there is a bigger impact of initial conditions in terms of cultural differentiation.

- **Change in the poverty rate of the minority group $\lambda_B^P$**

Consider now a change in $\lambda_B^P$ (ie. increased poverty inside the minority group) $\hat{g}^R (\mu)$ and $\hat{g}^P (\mu)$ are decreasing when $\lambda_B^P$ increases as both $\frac{\omega^R}{\bar{\omega}}$ and $\frac{\omega^P}{\bar{\omega}}$ get larger.\(^{18}\) On figure 2, this implies that the curve $V^a(g) \frac{\omega^P}{\bar{\omega}}$ is shifted down and $q^P* (\mu)$ and $q^R* (\mu)$ are increased (whenever they exist). At the same time, the threshold $\overline{\tau} (\mu)$ is pivoting counter-clockwise at the point $\mu = 2\lambda_A^P - 1$ (at which $\overline{\tau} (\mu) = 1$). Therefore the part of $\overline{\tau} (\mu) < 1$ is shifted out with a positive shock on $\lambda_B^P$.\(^{19}\) An increase of poverty inside the minority group has therefore subtle and ambiguous impacts on the steady state pattern of cultural differentiation of the minority group. Indeed the mainstream society is less willing to supply the general public good (both the poor and the rich agent) as the tax base is reduced, but at the same time the poor agent is more likely to be the pivotal agent leading therefore to more provision of the general public good.

![Figure 5](image_url) about here

This is illustrated in figure 5) where the locus of cultural steady states before and after the shock on $\lambda_B^P$ are respectively depicted in red and black colours. In case (1), at a minority size $\mu_1$, the system exhibits two cultural steady states with ”high integration” at $H_1$ and ”low integration” at $L_1$ before the change in $\lambda_B^P$, and only a unique ”low integration” steady state at $H_1'$ after the change. If the social system was converging towards the ”low integration” $L_1$, the increase in poverty of the minority group favors cultural integration towards point $H_1'$. Indeed the shock in $\lambda_B^P$ shifts the integrated minority members to be poorer in average. This is enough to affect the political equilibrium and to induce the median voter to switch from

\[^{18}\text{Indeed it is easy to see that an increase in } \lambda_B^P = 1 - \lambda_B^P \text{ leads to a reduction of } \bar{\omega} \text{ and therefore an increase in } \omega^R/\bar{\omega} \text{ and } \omega^P/\bar{\omega}.\]

\[^{19}\text{Indeed } \overline{\tau} (\mu) = 1 + \frac{2\lambda_A^P - 1 - \mu}{2\lambda_A^P \mu} \]

Hence $\overline{\tau} (\mu) < 1$ if and only if $\mu > 2\lambda_A^P - 1$.\(^{19}\)
a rich "mainstream" or "integrated" agent to a poor individual demanding more general public good $G^p$. The resulting cultural dynamics lead to a "high integration" equilibrium steady state at $H'_1$.

Conversely in case (2) at a minority size $\mu_2 > \mu_1$, a positive shock on $\lambda_B^p$ moves the system from a unique "low integration" steady state at $L_2$ to the possibility of two cultural steady states with "high integration" at $H'_2$ and "low integration" at $L'_2$ after the change in $\lambda_B^p$. If the social system was converging before the poverty shock towards the "low integration" equilibrium $L_2$ from an initial point like $O_2$ (above the new threshold line $\bar{\pi}(\mu)$), the increase in poverty leads to further cultural differentiation towards point $L'_2$ characterized by a larger fraction of "non integrated" individuals in the long run. If however the system was converging towards $L_2$ from a point like $O'_2$ (below the new threshold line $\bar{\pi}(\mu)$), then the poverty shock on the minority induces cultural convergence towards a "high integration" equilibrium at $H'_2$.

The preceding discussion suggests that an increase in poverty of the minority group leads to a larger (resp. smaller) impact of initial conditions on final steady states outcomes for larger (resp. smaller) minority groups. In other words, larger and initially more culturally differentiated minority groups tend to become even less integrated after an increase in poverty, while smaller and initially less culturally differentiated groups tend to become even more integrated after such a shock.

- **Change in the poverty rate of the mainstream group $\lambda_A^p$**

A change in $\lambda_A^p$ (ie. increased poverty inside the mainstream group) has very similar effects as a change in $\lambda_B^p$. Again both $\bar{\pi}^R(\mu)$ and $\bar{\pi}^P(\mu)$ get smaller when $\lambda_A^p$ increases (as $\bar{\omega}^R$ and $\bar{\omega}^P$ are increasing in $\lambda_A^p$). The steady state values $q^{P*}(\mu)$ and $q^{R*}(\mu)$ are consequently larger (whenever they exist). At the same time, the threshold $\bar{\pi}(\mu)$ above which there is a shift of the political pivotal agent from a poor agent to a rich one is also increased. As in the previous comparative dynamics, an increase of poverty inside the mainstream group has therefore ambiguous impacts on the steady state pattern of cultural differentiation of the minority group.\(^{20}\)

This discussion is summarized by the following proposition:

\(^{20}\)The ambiguity comes from the same reasons; The tax base is reduced and therefore the mainstream society is less willing to supply the general public good. At the same time though, a "mainstream" or minority "integrated" poor agent is also more likely to be politically pivotal and to demand a larger provision of the general public good.
Proposition 2  i) An increase in income inequality $\Delta \omega$ between rich and poor makes it more likely to have multiple politico-cultural steady state equilibria and increases the sensitivity of the final cultural integration pattern to the initial conditions in terms of cultural differentiation. ii) An increase in the degree of poverty $\lambda_B^p$ of the minority community or $\lambda_A^p$ of the mainstream group has ambiguous effects on cultural integration. Higher poverty is more likely to reduce (resp. increase) long run cultural integration when the minority group is large and initially less integrated (resp. small and initially more integrated).

5 Multiple minority groups

So far, we assumed that there was only one minority cultural group interacting with the mainstream society. In this section, we consider the possibility for more than one minority group. Although such groups may value different specific public goods, they exert on each other some positive externalities in terms of cultural evolution because of their interactions on political decisions related to the general public good. Cultural differentiation may then be easier to sustain in the long run. At the same time though, controlling for the total size of the minority groups, an increase in the number of minority groups (cultural fragmentation) also induces a dilution effect as each group is less able to sustain a reasonable amount of its own group specific public good. This creates an opposite force reducing overtime the cultural resilience of each group and therefore the pattern of long run cultural differentiation. We analyze these issues in the following sections.

5.1 Two minority groups

Let us start with two symmetric poor minority groups $z \in \{B, C\}$ of size $\mu/2$ with the same distribution of income $\omega^R$ and $\omega^P$ and same relative fractions $\lambda_z^R = \lambda_m^R$, and $\lambda_z^P = \lambda_m^P$, (with $\lambda_z^R + \lambda_m^P = 1$). Assume again that these groups are poorer than the mainstream group with $1/2 < \lambda_A^R < \lambda_m^P$. Individuals in each group $z \in \{B, C\}$ are differentiated along their public good preferences: a fraction $1 - q^z$ of individuals are "culturally integrated" to the mainstream society and share the same preferences as individuals in the mainstream group $D$. The residual fraction $q^z$ is "non integrated", does not value the "mainstream" public good $G^m$ and has preferences over another group specific public good $G^z$ only produced inside community $z$. Preferences of such individuals write as:

$$u^z(c, G^z) = c + V^z(G^z)$$
Assume as well the same shape for the community good preference function of both minority groups \( z \in \{ B, C \} \): \( V^z(G) = V^m(G) \). The government budget constraint again writes as \( \tau \omega = G = G^m \) and total income is \( \omega = (\lambda_A^z + \mu \lambda^m_R) \omega^R + (\lambda_A^z + \mu \lambda^m_P) \omega^P \). The utility of each type of agents in each minority group writes as follows:

\[
Y^z(J) = Y^p(J).
\]

The government budget constraint again writes as

\[
\tau \omega = G = G^m
\]

and total income is

\[
\omega = (\lambda_A^z + \mu \lambda^m_R) \omega^R + (\lambda_A^z + \mu \lambda^m_P) \omega^P.
\]

The utility of each type of agents in each minority group writes as follows:

for group \( z \) : the "integrated" individuals (ie. cultural type \( I \)) have the same policy preferences

\[
\tilde{w}^z(\tau, \omega^i, \omega, x^z) = [(1 - \tau) \omega^i - x^z] + V^z(\tau \omega)
\]

where \( x^z \) is the individual contribution inside community \( z \) to produce good \( G^z \).

Given that \( G^z = \sum_{i \in B} x^z_i = \frac{M}{2} x^z \), the "non integrated" individuals (ie. cultural type \( N^z \)) in group \( z \) have the following policy preferences

\[
w^z(\tau, \omega^i, \omega, x^z) = [(1 - \tau) \omega^i - x^z] + V^z(\frac{M}{2} x^z)
\]

Inside group \( z \), the individual contribution to produce good \( G^z \) is again decided by a cultural leader and is enforced inside the community. This cultural leader decides about the contribution of each individual weighting for simplicity equally those of type \( I \) and those of type \( N^z \). That is we assume that \( \theta = 1/2 \) in each group. Then we have the following problem characterizing the optimal provision of \( G^z \)

\[
\max \frac{x^z}{(1 - q^z)} \cdot \sum_{i = R, P} \tilde{w}^z(\tau, \omega^i, \omega, x^z) + q^z \cdot \sum_{i = R, P} w^z(\tau, \omega^i, \omega, x^z)
\]

or equivalently

\[
\max \frac{x^z}{-x^z + q^z V^m(\frac{M}{2} x^z)}
\]

This provides the following optimal individual contribution \( \tilde{x}^z(q^z, \mu) = \frac{2V^m(\mu)(\frac{2}{\mu q^z})}{\mu} \) and community public good \( \tilde{G}^z(q^z, \mu) = V^m \frac{2V^m(\mu)(\frac{2}{\mu q^z})}{\mu} \).

It is also immediate to get the preferred level of provision of the mainstream public good \( \hat{g}_{j,k}^i \) for an individual with income \( \omega^i \) \( i \in \{ R, P \} \) in group \( j \in \{ A, B, C \} \), and cultural type \( k \in \{ I, N^z \} \) when that individual belongs to group \( z \in \{ B, C \} \).

- For a mainstream individual or a culturally integrated minority individual of group \( z \) we get as before

\[
\hat{g}_{A}^i = \hat{g}_{z,k}^i = \hat{g}^i = V^m \left( \frac{\omega^i}{\omega} \right) \quad \text{for} \quad z \in \{ B, C \}.
\]

- For a non culturally integrated individual of group \( z \), we have \( \hat{g}_{z,N^z}^i = 0 \) whatever his income \( \omega^i \) \( i \in \{ R, P \} \). Given that we have \( 0 = \hat{g}_{z,N^z}^i < \hat{g}^R < \hat{g}^P \), the median voter of this economy is then easily obtained. Noting again the threshold

22
$$\overline{\tau}(\mu) = \frac{2\lambda_A^p - 1 + (2\lambda_m^p - 1)\mu}{\lambda_m^p\mu} > 0$$

we have the following. When \(q^B + q^C < \overline{\tau}(\mu)\), the median voter is given by the mainstream/integrated poor and the equilibrium provision of good \(G^a\) is obtained as \(g_A^*(\mu) = \hat{g}^P(\mu)\). When \(q^B + q^C > \overline{\tau}(\mu)\), the median voter is given by the mainstream rich and the provision of good \(G^a\) is obtained as \(g_A^*(\mu) = \hat{g}^R(\mu) < \hat{g}^P(\mu)\).

Consider now the process of cultural evolution within each minority community \(z, \in \{B,C\}\). By a similar reasoning as in section 2, the evolution of the fraction of "non integrated" individuals in group \(z\) follows a dynamic equation:

$$q^z = q^z(1 - q^z)\left[S^m(q^2)\left(1 - q^z\right) - V^a(g_A^*(q^z + q^{z'}, \mu))q^z\right]$$

and \(g_A^*(q^z + q^{z'}, \mu) = \begin{cases} \hat{g}^P(\mu) & \text{when } q^z + q^{z'} \leq \overline{\tau}(\mu) \text{ and } z \neq z' \in \{B,C\} \\ \hat{g}^R(\mu) & \text{when } q^z + q^{z'} > \overline{\tau}(\mu) \end{cases}\)

with \(S^m(q) = V^m(V_G^{m-1}(\frac{1}{pq}))\) \(^{21}\)

For a given level of general public good \(g\), denote the unique solution \(q^*_2(g)\) such that

$$S^m(q^2) = V^a(\frac{q}{g}) - \frac{q}{1 - q} \quad (10)$$

Given that the equilibrium level of general public good \(g_A^*(q^z + q^{z'}, \mu)\) can only take two values \(\hat{g}^P(\mu)\) and \(\hat{g}^R(\mu)\), equation (11) provides two solutions \(q^*_2(\mu) = q^*_2(\hat{g}^P(\mu))\) and \(q^*_{2R}(\mu) = q^*_2(\hat{g}^R(\mu))\). We also know that \(q^*_2(\mu) < q^*_{2R}(\mu)\) as \(\hat{g}^P(\mu) > \hat{g}^R(\mu)\) for all \(\mu \in [0, 1]\).

Figure 6) shows the stable steady state manifold \(\overline{q}^z(q^z)\) for a given group \(z\) as a function of the fraction of "non integrated" individuals \(q^z\) of the other group \(z'\). The line \(q^z + q^{z'} = \overline{\tau}(\mu)\) separates the region of the square \([0,1]^2\) in two regions. Above the line, there is low provision of general public good \(\hat{g}^R(\mu)\) and low cultural integration with the fraction \(q^z\) of "non integrated" individuals converging towards \(q^*_2(\hat{g}^R(\mu))\). Below the line, there is high provision of the general public good \(\hat{g}^P(\mu)\) and high cultural integration and \(q^z\) converging

\(^{21}\)This function can easily be shown to be increasing concave in \(q\) and to satisfy \(S^m(0) = 0\) and \(S^{m_l}(0) = +\infty\).
towards \( q_2^B (\mu) = q_2^A (\bar{q}_2^B (\mu)) \). The steady state manifolds of the two groups are depicted as curve \( BB \) for group \( B \) and curve \( CC \) for group \( C \). \(^{22}\)

Putting up together these two curves, figures 7 a), 7 b), 7 c) provide the phase diagrams of the joint cultural evolution of the two minority groups associated to (10).

Figures 7a) 7b) and 7c) about here

Three cases may occur. The first two cases describe situations where the manifolds intersect only once at a locally stable steady state of the system (10): case a) with a high integration cultural steady state and \( q^{B*} = q^{C*} = q_2^{P*} (\mu) \); and case b) with a low integration cultural steady state and \( q^{B*} = q^{C*} = q_2^{R*} (\mu) \). The third possibility depicted in case c) reflects the case of multiple politico-cultural steady states where the manifolds have two stable intersection points \( L \) and \( H \) respectively at a low and a high integration equilibrium.

Interestingly, the phase diagram reveals a dynamic cultural complementarity between the two minority groups. The existence of a relatively non integrated minority group facilitates the persistence of non integration of the other group. The reason for this comes from the fact that a non integrated minority group induces a political externality on the other group. Indeed it contributes to the shift of the political pivotal agent to vote for a low provision of the general public good. This in turn reduces the cultural integration of both groups overtime.

To see that in the most intuitive way, consider for instance figure 6c) with the possibility of two long run stable politico-cultural equilibria at points \( L \) and \( H \). Suppose that the initial cultural conditions of minority group \( B \) is \( \overline{q}^B (0) \) and such that \( q^B (0) < \overline{q}^C (\mu) \). When the other group \( C \) starts with very few ”non integrated” individuals (ie. \( q^C (0) \) close to 0), then point \( O_1 \) associated to the initial cultural conditions, lies below the line \( q^B + q^C = \overline{q}^C (\mu) \). The median voter is a poor integrated individual demanding a high level of the general public good \( g_A = q_2^{P*} (\mu) \). Because of this, point \( O_1 \) belongs to the basin of attraction of the high integration steady state point \( H \). Group \( B \) therefore converges to the high integration equilibrium characterized by \( q^{B*} = q^{C*} = q_2^{P*} (\mu) \). Conversely, when group \( C \) starts with a significant number of non integrated individuals \( q^C (0) \) such that \( q^C (0) > \overline{q}^C (\mu) - q^B (0) \), then the political pivotal agent is a rich integrated individual who demands less general public

\(^{22}\)Given (10), the steady state manifold for group \( z \) is formally obtained as

\[
\overline{q}^z (q^z) = \begin{cases} 
q_2^{P*} (\mu) & \text{for } q^z < \overline{q} (\mu) - q_2^{P*} (\mu) \\
q_2^{R*} (\mu) & \text{for } q^z > \overline{q} (\mu) - q_2^{R*} (\mu)
\end{cases}
\]
good provision \( g_A = q^R_A(\mu) \). As a consequence, the initial condition point \( O_2 \) lies above the line \( q^R + q^C = \bar{\eta}(\mu) \) and the system converges towards a low integration equilibrium \( L \) characterized by a high fraction of non integrated individuals in each minority group.

This discussion suggests that the capacity for cultural resilience of one group may crucially depend on the degree of cultural resilience of other minority groups in the same society. Even when these groups do not directly influence each other in the cultural transmission process, they impose however on each other a positive externality through the political economy of general public goods. These in turn indirectly affect the cultural integration to the mainstream culture.

### 5.2 Multiple minority groups and cultural fragmentation

Interestingly, the previous analysis can be extended to \( N \) symmetric minority groups \( z \in \{1, \ldots, N\} \) each of size \( \mu_i/N \) with the same distribution of income \( (\omega^R, \omega^P) \) and relative fractions \( \lambda^R_m \), and \( \lambda^P_m \) (with \( \lambda^R_m + \lambda^P_m = 1 \)) and again assuming that \( 1/2 < \lambda^P_m < \lambda^P_m \). Preferences of integrated individuals in each group \( z \) are the same as before while preferences of non integrated individuals in group \( z \) over their group specific public good \( G^z \) are given by

\[
u^z(c, G^z) = c + V^{m_i}(G^z) \quad \text{for all } z \in \{1, \ldots, N\}
\]

Following the same line of reasoning as in the previous section, one immediately gets the following. When \( \sum_{z=1}^{N} q^z < \bar{\eta}(\mu) \), the mainstream/integrated poor is the median voter and the equilibrium provision of good \( G^o \) is given by \( g_A^*(\mu) = \hat{g}^P(\mu) \). Conversely when \( \sum_{z=1}^{N} q^z > \bar{\eta}(\mu) \), the mainstream rich is the median voter and the provision of good \( G^o \) is now given by \( g_A^*(\mu) = \hat{g}^R(\mu) < \hat{g}^P(\mu) \). Notice that \( \bar{\eta}(\mu) \) does not depend on \( N \) and again is decreasing in \( \mu \) as \( 2\lambda^P_m - 1 > 0 \).

The process of cultural evolution within each minority community \( z \in \{1, \ldots, N\} \) is obtained in the same way as before and consequently writes as

\[
\dot{q} = q^*(1 - q^*) \left[ S^m(\frac{q}{N})(1 - q^*) - V^a(g_A^*)q^z \right] \quad \text{for all } z \in \{1, \ldots, N\}
\]

and \( g_A^* = \begin{cases} 
\hat{g}^P(\mu) & \text{when } \sum_{z'=1}^{N} q^{z'} < \bar{\eta}(\mu) \\
\hat{g}^R(\mu) & \text{when } \sum_{z'=1}^{N} q^{z'} > \bar{\eta}(\mu)
\end{cases} \)

As before, for a given value of general public good \( g \), it is easy to see that each group \( z \)
exhibits a stable cultural steady state \(q_N^*(g)\) such that
\[
S\left(\frac{q}{N}\right) = V^u(g) \frac{q}{1 - q}
\]  
(11)

Simple differentiation shows that \(q_N^*(g)\) is decreasing in the number of groups \(N\). Maintaining constant the total size of the population, a larger number of minority groups means that each of them is smaller. Because of this dilution effect, each minority group \(z\) produces less of his own specific group public good \(G^z\). This leads to weaker incentives to transmit the ”non integrated” cultural trait in this community and consequently to a smaller steady state fraction of non integrated individuals. Consequently cultural fragmentation favors the adoption of the mainstream cultural trait.

Notice then that \(g = g_A^*\) can take two values \(\hat{g}^P(\mu)\) and \(\hat{g}^R(\mu)\) corresponding respectively to a ”high integration” and a ”low integration” politico-cultural equilibrium. In each minority group \(z\), this provides two equilibrium fractions \(q_N^*(\hat{g}^P(\mu))\) and \(q_N^*(\hat{g}^R(\mu))\) with \(q_N^*(\hat{g}^P(\mu)) < q_N^*(\hat{g}^R(\mu))\). To be consistent with equilibrium voting, the ”high integration” equilibrium \(q_N^*(\hat{g}^P(\mu))\) prevails when \(\sum_{z=1}^N q_N^*(\hat{g}^P(\mu)) = Nq_N^*(\hat{g}^P(\mu)) < \overline{q}(\mu)\), while conversely the ”low integration” equilibrium \(q_N^*(\hat{g}^R(\mu))\) prevails when \(Nq_N^*(\hat{g}^R(\mu)) > \overline{q}(\mu)\).

As is easily seen, cultural fragmentation (an increase in \(N\)) generates two forces going in opposite direction with respect to cultural integration. As already mentioned there is a ”dilution” effect: each minority group being smaller in size, the provision of each group specific public good \(G^z\) is smaller. This in turn reduces the cultural resilience of the group specific trait in the cultural transmission process and the steady state fraction of non integrated individuals \(q_N^*(\hat{g}^R(\mu))\) is decreasing in \(N\).

At the same time though, more fragmentation also means that it is easier to shift the median voter from a poor to a rich individual, as all ”non integrated” minority individuals make a unified political coalition against the provision of the general public good. Given that such a shift reduces the political equilibrium provision of general public good, the cultural resilience of each minority trait is enhanced as well as the likelihood to get a low integration politico-cultural steady state equilibrium.

When is it that we can ensure that high cultural integration is the only possible long run outcome? To see that note that the condition for a low integration equilibrium to exist writes as \(q_N^*(\hat{g}^R(\mu)) > \frac{\overline{q}(\mu)}{N^2}\). After substitution this rewrites as
\[
S\left(\frac{\overline{q}(\mu)}{N^2}\right) > V^u(\hat{g}^R(\mu)) \frac{\overline{q}(\mu)}{N - \overline{q}(\mu)}
\]  
(12)

It can be seen (see the appendix) that this last inequality cannot be satisfied for \(N\) large enough. The dilution effect is stronger than the political shifting effect when the degree of
cultural fragmentation is large enough. A direct implication of this is the fact that cultural integration is more likely to occur in the long run in multicultural societies composed of many small groups than a few significant ones.

6 Conclusion

The questions of cultural identity and insertion of minority groups within modern welfare state institutions are hot issues right now in many western countries. In Europe, in the background of islam radicalization and terrorism, migration flows of political refugees from Syria and Irak have generated important political tensions and the rise of extremist parties in several countries (Hungary, Greece, Austria, as well as France, and even Germany). In particular, the question of how religious communitarianism can be compatible with secular modern state institutions has been at the heart of political debates. In the context of globalization and technological change creating important income disparities within and across nations (Bourguignon 2015, Piketty 2014, Facchini and Couvreur 2015), political pressures for vertical redistribution interact with the demands for horizontal redistribution associated with these immigration flows. Such interactions are crucially affecting the sustainability of welfare state systems in culturally diverse societies.

In this paper, we provided a simple dynamic political economy framework to analyze these issues, and to discuss the implications for public goods provision, redistribution, and cultural integration. Our analysis suggests a dynamic complementarity between the political economy of redistribution, and the process of cultural integration of minority groups in a mainstream society. By shifting political coalitions, cultural diversity may erode the support for large scale general public good provision and vertical redistribution from the rich to the poor. Dynamically, the attractiveness of adopting the mainstream culture is reduced, and minority groups are more likely to remain culturally non integrated. Importantly, this in turn has negative feedbacks effects on the future demands for redistribution and general public good provision over the next generations.

Our model indicates the possibility for multiple politico-cultural steady state trajectories leading to very different outcomes in terms of integration and redistribution. The long run political sustainability of vertical redistribution and general public goods provision in a multicultural society then crucially depends on the initial degree of cultural differentiation and fragmentation of the society. Interestingly, our framework also suggests that an increase in income inequality, associated say to globalization or technical change, makes even more salient the sensitivity of long run outcomes to these initial conditions. This feature may then
help understand why the political stakes to monitor the extent of first generation immigration flows might be currently so high in western countries facing increasing vertical inequalities and economic stagnation.

Obviously, our analysis left a number of important issues that may be incorporated in future research. First, we adopted a political economy set-up of a median voter model. While this is convenient to capture important dimensions of the debate on redistribution in multicultural societies, it would be worth introducing richer political dimensions such as probabilistic voting, endogenous political participation and/or lobbying influences by specific groups. Introducing these dimensions may alter the conditions for high integration and low integration trajectories. As long as some non integrated minority individuals do participate in the political decision making process of provision of general redistribution or public goods, we may expect our main argument that cultural diversity affects the pattern of public policies in a way that reinforces the preservation of such diversity, to be present in these extended contexts.

We also assumed that socialization only occurred inside the minority group (through parents and role models from the community). Our framework can be easily extended to the context where socialization by role models from the mainstream group may also occur (see oneline appendix). In that case, it can be shown that full cultural integration obtains for a small enough minority group. However, when the minority group is large enough, there is still scope for cultural differentiation at any equilibrium level of the general public good provided by the political system, and the results of this paper are qualitatively preserved.

One may think to introduce richer economic and institutional contexts, taking into account the impact of minority groups on labor market outcomes, and conversely the role of market discrimination on the incentives to culturally assimilate. As well, one may consider the role of education as a mechanism producing jointly economic and cultural integration outcomes, and eventually affecting social mobility inside the society. These extensions would naturally lead to endogenous dynamics of income inequality, providing new political economy interactions between cultural integration and vertical redistribution.

Finally our framework only considered a "universalist" public policy (the provision of a general public good). One could alternatively consider the determination of a "communautarian" policy whereby the community good is partly publicly funded through a given share of tax revenue. Such extension would permit an investigation of the political economy of multiculturalist policies and their implications for the equilibrium trajectories of level cultural integration. While clearly beyond the scope of the present paper, we hope that the framework outlined here can be a stepping stone to analyse these issues in future research.
References


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- Ceylan, R. (2010), *Die Prediger des Islam Imam—Wer sie sind und was sie wirklich wollen*. Freiburg: Herder.
Figure 1: cultural steady state

\[ V^a(g) \frac{q}{1-q} \]

\[ S(q) \]

\[ q^*(g) \]
Figure 2a): high integration equilibrium
Figure 2b): multiple equilibria
Figure 2c): Low integration equilibrium
Figure 3a): Only High Integration Equilibrium
Figure 3b): High and Low Integration Equilibria
Figure 3c): High and Low Integration Equilibria
Figure 4a): increasing income inequality $\Delta \omega$
Figure 4b): increasing poverty rate $\lambda_B^P$ in the minority group
Figure 5: Two minority groups

\[ q^z + q^{z'} = \bar{q}(\mu) \]
Figure 6a): High cultural integration steady state
Figure 6b): Low cultural integration steady state
Figure 6c): multiple cultural integration steady states
Appendix: Proofs

- **Stability of corner steady states:** It is immediate to see that \( q = 1 \) is locally instable. Under the conditions \( V^b(0) = 0 \); \( V^b_0(0) = +\infty \), it is easy to conclude that \( S^b(0) = 0 \), and \( S^b(0) = +\infty \). From this a second order development show that the steady state \( q = 0 \) is also unstable. QED.

- **Concavity of the function** \( S(q) \): Recall that \( S^b(q) = V^b(\bar{V}_G^{-1}(\frac{\mu q}{\eta})) \) with \( \Phi(q) = 1 + \frac{\eta(1-q)}{(1-\eta)q} \). Differentiation gives immediately:

\[
S'(q) = \frac{\Phi(q)\Phi'(q)}{\mu^2} \frac{1}{V^b_G(\bar{V}_G^{-1}(\frac{\mu q}{\eta}))} > 0
\]

given that \( \Phi'(q) = -\frac{\eta}{(1-\eta)q} < 0 \) and that \( \Phi''(q) = \frac{2\eta}{(1-\eta)q^2} > 0 \). Moreover \( S'(0) = +\infty \). Differentiating another time provides:

\[
S''(q) = \frac{(\Phi')^2 + \Phi\Phi''}{\mu^2V^b_GG} - \frac{\Phi\Phi'}{\mu^2}\bar{V}_G^b \frac{\Phi'}{\mu} = \frac{1}{\mu^2V^b_GG} \left[ (\Phi')^2 + \Phi\Phi'' - \frac{\Phi(\Phi')^2}{\mu} \frac{V^b_GG}{V^b_GG_2} \right]
\]

which using the fact that \( \Phi(q) = \mu V^b_G(\mu q) \), can be rewritten as

\[
S''(q) = \frac{1}{\mu^2V^b_GG} \left[ \Phi\Phi'' + (\Phi')^2 \left( 1 - V^b_GV^b_{GG} \right) \right]
\]

From \( \Phi'' > 0 \), it follows immediatley that with the condition \([V''(y)]^2 > V'(y)V'''(y)\), the bracket term is positive and \( S(q) \) is increasing concave. QED.

- **Proof of proposition 1:** The proof is immediate. It comes from the simple fact that \( \bar{\pi}(\mu) \) is a decreasing function of \( \mu \) with \( \lim_0 \bar{\pi}(\mu) = +\infty \) and that \( q^{P*}(\mu) \) and \( q^{R*}(\mu) \) are increasing functions of the size of the poor minority group \( \mu \). Indeed as \( \mu \) increases, the average income \( \bar{\pi}(\mu) \) in the economy decreases. This leads to less provision of the general public good \( g_A \) (whether the poor or the rich agents are pivotal in the vote decision) and therefore a larger long run fraction \( q^* (\mu) \) of individuals in the minority community who keep their preferences for their community specific good \( g_B \). The conditions for the specific long run politico-cultural equilibria result simply from the fact that the functions \( \Theta^P(\mu) = q^{P*}(\mu) - \bar{\pi}(\mu) \) and \( \Theta^R(\mu) = q^{R*}(\mu) - \bar{\pi}(\mu) \) are increasing functions of \( \mu \), that \( \Theta^P(\mu) < \Theta^R(\mu) \) and the consistency conditions that a high integration equilibrium prevails if and only if \( \Theta^P(\mu) < 0 \) and a low integration equilibrium prevails when \( \Theta^R(\mu) > 0 \). QED.
• Cultural Fragmentation with $N$ minority groups: The condition for a low integration equilibrium to exist writes as

$$q_N^*(\bar{g}^R(\mu)) > \frac{\bar{\eta}(\mu)}{N}$$

After substitution this rewrites as

$$S\left(\frac{\bar{\eta}(\mu)}{N^2}\right) > V^a(\bar{g}^R(\mu)) \frac{\bar{\eta}(\mu)}{N - \bar{\eta}(\mu)} \tag{13}$$

Denote then the function

$$\Psi (N) = (N - \bar{\eta}(\mu)) S\left(\frac{\bar{\eta}(\mu)}{N^2}\right) \text{ with } S(x) = V^b(V^b - 1\left(\frac{1}{\mu x}\right))$$

(13) can be restated as

$$\Psi (N) > V^a(\bar{g}^R(\mu))\bar{\eta}(\mu) \tag{14}$$

differentiation of $\Psi (N)$ provides

$$\Psi'(N) = V^b(V^b - 1\left(\frac{N}{\mu\bar{\eta}}\right)) + \frac{2(N - \bar{\eta}(\mu)) N^2}{(\mu\bar{\eta})^2} V^b \left(V^b - 1\left(\frac{N}{\mu\bar{\eta}}\right)\right)$$

which is negative when $N \to \infty$ as $V^b(V^b - 1\left(\frac{N}{\mu\bar{\eta}}\right)) \to 0$ and the second term becomes infinitely negative. Hence for some $N \geq \bar{N}$, the $\Psi (N)$ is decreasing in $N$ and keeping on increasing $N$ makes it less likely to have (14) satisfied. Therefore after some threshold $\bar{N}$, cultural fragmentation reduces the likelihood that a politico-cultural equilibrium steady state exhibits low cultural integration. $\textbf{QED.}$