On the Joint Evolution of Culture and Institutions: Online Appendix

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In this appendix we study the examples in Section 5. In particular, we explicit assumptions and work out results in their algebraic complexity.

**Example 1: Elites, workers, and extractive institutions**

Let workers be group $i = 1$ and the elite be $i = 2$. Both groups can transform labor one-for-one into private consumption goods. Let $a^i$ denote labor exerted by any member of group $i$. Let $s$ denote the initial resources each elite member is endowed with. Let $p$ denote the (income) tax rate and $G$ the public good provided by fiscal institutions. Preferences for group $i$ are represented by the following utility function:

$$ u^i(a^i, G, p) = u^i(a^i(1 - p) + s) + \theta^i v(1 - a^i) + \Omega \cdot G. $$

Our characterization of the distinction between workers and the elite in terms of cultural values requires that:

i) the parameter $\theta^i$ representing the preference for leisure satisfies $\theta^1 < \theta^2$.

Furthermore we assume extreme preferences for leisure of the elite, $\theta^2 > \frac{u'(s)}{v'(1)} > 1 = \theta^1$. In this case, members of the elite never work, $a^2 = 0$, and consume their resources, $s$.

The optimal behavior of the mass workers is determined by:

$$ a^1(a^1) = \arg \max_a u((1 - p)a + s) + v(1 - a) $$

and thus

$$ a(p) \begin{cases} 
(0, 1) & \text{and determined by } (1 - p)u'(1 - p)(1 + a) = v'(1 - a) \text{ when } p \leq p_0 \\
0 & \text{when } p \geq p_0 
\end{cases} $$

with $p_0$ the tax rate over which the optimal effort is equal to 0:

$$ p_0 = 1 - \frac{v'(1)}{u'(s)} $$

**The societal equilibrium policy:**

The societal equilibrium policy $p(\beta, q)$ is characterized by the following conditions

$$ p \in \arg \max_{p \in [0, 1]} W(p, a^1, a^2, G) = \beta u^1(a^1, G, p) + (1 - \beta)u^2(a^2; G) \quad (1) $$

s.c. $G = p [qa^1 + (1 - q)a^2]$, for given $a^1, a^2$. 

1
\[
\begin{align*}
    a^1 &= a(p) = \arg \max_{a} u((1 - p)a + s) + v(1 - a) \\
    a^2 &= 0 = \arg \max_{a} u(s + a(1 - p) + s) + \theta^2 v(1 - a)
\end{align*}
\]

We then get the following characterization:

**Characterization of societal equilibrium policy:**

Assume \( u'(s + a(0)) < \Omega < u'(s) \), there exists two thresholds \( \underline{\beta}(q) \) and \( \overline{\beta}(q) \) such that \( \underline{\beta}(q) < \overline{\beta}(q) \), for all \( q \in [0, 1] \) and such that

i) For \( 0 \leq \beta < \underline{\beta}(q) \), \( p(\beta, q) \in [p_0, 1] \), \( a(p) = 0 \) and \( G = 0 \).

ii) For \( \beta \in (\underline{\beta}(q), \overline{\beta}(q)) \) then \( p(\beta, q) \in (0, p_0) \) and \( a(p) > 0 \) and \( G > 0 \). \( p(\beta, q) \) is decreasing in \( \beta \) and decreasing in \( q \).

iii) For \( \beta \geq \overline{\beta}(q) \), then \( p(\beta, q) = 0 \), \( a = a(0) > 0 \) and \( G = 0 \).

**Proof.** i) Substituting (2) into the first order condition of problem (1) provides the following conditions characterizing a stable interior societal equilibrium policy:

\[
\Xi(p, q, \beta) = -\beta \cdot u'((1 - p)a(p) + s) \cdot a(p) + \Omega q \cdot a(p) = 0
\]

Thus when \( a(p) \neq 0 \) (that is for \( p \leq p_0 \)), this is equivalent to

\[
\Theta(p, q, \beta) = -\beta \cdot u'((1 - p)a(p) + s) + \Omega q = 0
\]

Now the function \( \Theta(p, q, \beta) \) is decreasing in \( p \) as long as \( a(p) \) is decreasing in \( p \) (something that is ensured when the utility function \( u(.) \) satisfies \( \frac{-u''(x)x}{u'(x)} < 1 \)). As well we have:

\[
\Theta(0, q, \beta) = -\beta \cdot u'(a(0) + s) + \Omega q < 0
\]

if and only if

\[
\beta > \overline{\beta}(q) = \frac{\Omega q}{u'(a(0) + s)}
\]

Note that \( \overline{\beta}(q) \) is increasing in \( q \). It follows immediately that for \( \beta \geq \overline{\beta}(q) \), \( \Theta(p, q, \beta) < 0 \) for all \( p \in [0, 1] \) and the only possible societal equilibrium is the corner solution \( p(\beta, q) = 0 \).

ii) Similarly note that

\[
\Theta(p_0, q, \beta) = -\beta \cdot u'(s) + \Omega q < 0
\]

when \( \beta > \underline{\beta}(q) = \frac{\Omega q}{u'(s)} \). It is easy to see that \( \underline{\beta}(q) < \overline{\beta}(q) \). Hence for \( \beta \in (\underline{\beta}(q), \overline{\beta}(q)) \), we then get that \( \Theta(0, q, \beta) > 0 \), \( \Theta(p_0, q, \beta) < 0 \) and \( \Theta(p, q, \beta) \) decreasing in \( p \). Therefore there exists a unique \( p(\beta, q) \in (0, p_0) \) such that \( \Theta(p, q, \beta) = 0 \). At this point we obviously have \( a(p) > 0 \) and \( G = q \cdot a(p) > 0 \). Moreover, one can immediately see that \( \Theta \beta(p, q, \beta) < 0 \) and that \( \Theta q(p, q, \beta, \beta) = \Omega > 0 \) and thus \( p(\beta, q) \) is decreasing in \( \beta \) and increasing in \( q \).

iii) Finally note that when \( \beta < \underline{\beta}(q) \), \( \Theta(p_0, q, \beta) > 0 \) and \( \Xi(p, q, \beta) > 0 \) for all \( p < p_0 \). From this it follows the best response of the policy maker to any effort of workers \( a(p) > 0 \) is to choose
a policy \( p \geq p_0 \). Given that with \( a(p) = 0 \), the policy maker is indifferent with any policy \( p \geq p_0 \). Moreover for all policies \( p \geq p_0 \), one has \( a(p) = 0 \). It follows therefore that for \( \beta < \beta(q) \) \( p(\beta, q) \) can be any policy in \([p_0, 1]\) associated to \( a = 0 \), no production and no provision of the public good (ie. \( G = 0 \)). A natural selection of this equilibrium correspondence is \( p(\beta, q) = p_0 \). ■

**Societal commitment equilibrium policy:**

Denote now

\[
\tilde{W}(p, \beta, q) = \beta u^1 \left( a^1(p), G, p \right) + (1 - \beta) u^2 \left( a^2(p), G, p \right)
\]

\[
= \beta \cdot u(a(p)(1 - p) + s) + v(1 - a(p)) + (1 - \beta) \cdot [u(s) + \theta^2 v(1)]
\]

+ \( \Omega q \cdot [p \cdot a(p)] \)

with \( G = q \cdot [p \cdot a(p)] \). Then the societal commitment equilibrium policy \( p^{\text{com}}(\beta, q) \) for any value of \( (\beta, q) \in [0, 1]^2 \) is the solution of the following program:

\[
p \in \arg \max_p \tilde{W}(p, \beta, q).
\]

We get the first order condition characterizing an interior solution as

\[
\tilde{W}_p(p, \beta, q) = -\beta \cdot u' ((1 - p)a(p) + s) \cdot a(p)
\]

+ \( \Omega q \cdot [a(p) + pa_p(p)] \)

We assume that \( \tilde{W}(p, \beta, q) \) is strictly concave in \( p \). This holds when \(|v''|\) is large enough and \( p \cdot a(p) \) is concave in \( p^1 \). Then we have the following characterization:

\[1\]Indeed

\[
\tilde{W}_{pp} = -\beta \left[ u'' (-a + (1 - p)a_p) a + a_p a_p \right]
\]

+ \( \Omega [\lambda + (1 - \lambda)q] \cdot [pa(p)]'' \)

\[= -\beta \left[ -u''(a)^2 + [u''(1 - p)a + u'] a_p \right]
\]

+ \( \Omega [\lambda + (1 - \lambda)q] \cdot [pa(p)]'' \)

differentiation of the workers’ first order condition provides:

\[
\frac{da^1}{dp} = \frac{u' + (1 - p)au''}{u''(1 - p)^2 + v''}
\]

now after substitution, one gets

\[
-u''(a)^2 + [u''(1 - p)a + u'] a_p = \frac{(u')^2 + 2(1 - p)u' u'' a - v'' u''(a)^2}{u''(1 - p)^2 + v''} < 0
\]

when \(|v''|\) large enough. Hence \( \tilde{W}_{pp} < 0 \) when \(|v''|\) is large enough and \( pa(p) \) is concave in \( p \).
Characterization of societal commitment equilibrium policy:
i) For $\beta < \beta(q)$, $p^{\text{com}}(\beta, q) \in (0, p^{\text{max}}]$ and is characterized by the following equation:

$$-\beta \cdot u'((1 - p)a(p) + s) \cdot a(p) + \Omega q \cdot [a(p) + pa_p(p)] = 0$$

and $p^{\text{max}} = \arg \max_p pa(p)$.

ii) For $\beta \geq \beta(q)$, $p^{\text{com}}(\beta, q) = 0$.

iii) $p^{\text{com}}(\beta, q)$ is decreasing in $\beta$ and increasing in $q$.

iv) $p^{\text{com}}(\beta, q) < p(\beta, q), \forall \beta < \beta(q)$, and $q \in (0, 1]$.

Proof. i) It is easy to see that for $\beta < \beta(q)$,

$$\tilde{W}_p(0, \beta, q) = -\beta \cdot u'(a(0) + s) \cdot a(0) + \Omega q \cdot [a(0)] > 0$$

and

$$\tilde{W}_p(p^{\text{max}}, \beta, q) = -\beta \cdot u'((1 - p^{\text{max}})a(p^{\text{max}})) \cdot a(p^{\text{max}}) \leq 0$$

Thus $p^{\text{com}}(\beta, q) \in (0, p^{\text{max}}]$ is obtained and is characterized by $\tilde{W}_p(p, \beta, q) = 0$.

ii) Conversely for $\beta \geq \beta(q)$, $\tilde{W}_p(0, \beta, q) \leq 0$ and $p^{\text{com}}(\beta, q) = 0$. Finally given that $\tilde{W}_p(\beta, p, \beta, q) < 0$ one has $p^{\text{com}}(\beta, q)$ is decreasing in $\beta$. Similarly $\tilde{W}_p(p^{\text{com}}, \beta, q) = \Omega \cdot [a(p) + pa_p(p)] > 0$ for $p < p^{\text{max}}$. Hence $p^{\text{com}}(\beta, q)$ is increasing in $q$.

iii) Finally at $p = p(\beta, q) \in (0, 1)$, it is easy to see that $\tilde{W}_p(p(\beta, q), \beta, q) = \Omega q \cdot [pa_p(p)] < 0$ for $\beta < \beta(q))$ and $q \in (0, 1]$. Given that $\tilde{W}(p, \beta, q)$ is strictly concave, this implies that $p(\beta, q) > p^{\text{com}}(\beta, q)$ for that range of parameters $\beta$ and $q$.

Cultural Dynamics:

- For $\beta \in (\beta(q), \beta(q))$, the parameters of cultural intolerance $\Delta V^1(p)$ and $\Delta V^2(p)$ are given by

$$\Delta V^1(p) = u((1 - p)a(p) + s) + v(1 - a(p))$$

$$- [u(s) + v(1)]$$

$$\Delta V^2(p) = u(s) + \theta^2 v(1)$$

$$- [u(s + (1 - p)a(p)) + \theta^2 v(1 - a(p))]$$
$\Delta V^1(p)$ is obviously decreasing in $p$. For $\Delta V^2(p)$ we get

$$\frac{d\Delta V^2}{dp} = -\left[-u'a + (u'(1-p) - \theta^2 v') a_p\right]$$

$$= u'a + (\theta^2 - 1)v'a_p$$

$$= u'a + (\theta^2 - 1)(1-p) \cdot u' \cdot \frac{u' + (1-p)au''}{u''(1-p)^2 + v''} > 0$$

when $|v''|$ large enough.

We conclude that $\Delta V^1(p)/\Delta V^2(p)$ is decreasing in $p$ when $v(.)$ is concave enough. In that region as $p = p(\beta, q)$ is in fact a decreasing function of $\beta$ and increasing function of $q$, it follows that $\Delta V^1/\Delta V^2$ is an increasing function of $\beta$ and an increasing function of $q$.

- For $\beta \geq \bar{\beta}(q)$, $\Delta V^1(p) = \Delta V^1(0)$ and $\Delta V^2(p) = \Delta V^2(0)$ are constant.
- For $\beta < \bar{\beta}(q)$, It is easy to see that $\Delta V^1(p_0) = \Delta V^2(p_0) = 0$. and there is no cultural dynamics in that region.

Cultural steady states are determined by:

$$\frac{\Delta V^1(p(\beta, q))}{\Delta V^2(p(\beta, q))} = \frac{q}{1-q} \text{ for } \beta \in (\underline{\beta}(q), \bar{\beta}(q))$$

$$\frac{\Delta V^1(0)}{\Delta V^2(0)} = \frac{q}{1-q} \text{ for } \beta \geq \bar{\beta}(q)$$

In the region $\beta \in (\underline{\beta}(q), \bar{\beta}(q))$ it is easy to see that when $|v''|$ large enough, this determines an upward sloping curve $q(\beta)$ in $\beta$. In the region $\beta > \bar{\beta}(q)$, whenever it exists, this determines a vertical sloping curve $q = q_0$.\(^2\)

Finally note that as $\beta \to \bar{\beta}(q)_+$

$$\Delta V^1(p) \simeq \Delta V^1(p_0) + (p - p_0) \left(\frac{d\Delta V^1}{dp}\right)_{p_0} + \frac{(p - p_0)^2}{2} \left(\frac{d^2\Delta V^1}{dp^2}\right)_{p_0}$$

$$\simeq -\frac{(p - p_0)^2}{2} \left[u'(s)\right] a'_p(p_0)$$

and

$$\Delta V^2(p) \simeq \Delta V^2(p_0) + (p - p_0) \left(\frac{d\Delta V^2}{dp}\right)_{p_0} + \frac{(p - p_0)^2}{2} \left(\frac{d^2\Delta V^2}{dp^2}\right)_{p_0}$$

$$\Delta V^2(p) \simeq (p - p_0) (\theta^2 - 1) a'_p(p_0) + \frac{(p - p_0)^2}{2} \left(\frac{d^2\Delta V^2}{dp^2}\right)_{p_0}$$

\(^2\)The condition for such vertical part to exist is simply

$$q_0 = \frac{\Delta V^1(0)}{\Delta V^1(0) + \Delta V^2(0)} < \frac{u'(a^1(0))}{\Omega}$$
Thus
\[
\lim_{\beta \to \beta(q)} \frac{\Delta V^1(p(\beta, q))}{\Delta V^2(p(\beta, q))} \simeq \lim_{\beta \to \beta(q)} \frac{-\frac{(p - p_0)}{2} [u'(s)] a'_p(p_0)}{(\theta^2 - 1) a'_p(p_0) + \frac{(p - p_0)}{2} \left( u^2 \frac{\Delta V^2}{dp^2} \right)_{p_0}} = 0
\]
and thus therefore the manifold \( q(\beta) \) characterizing the cultural steady states for \( \beta \in (\beta(q), \beta(q)) \) touches the region \( \beta \leq \beta(q) \) at the point \( q = 0 \) and \( \beta = 0 \).

The description on the two configurations of steady states (case a) and (case b) depends on whether the cultural manifold \( q(\beta) \) intersect the curve \( \beta(q) \) before the point \( \tilde{q} \) at which \( \beta(\tilde{q}) = 1 \). Notice that this point \( \tilde{q} \) is given by
\[
\tilde{q} = \frac{u'(a(0) + s)}{\Omega}
\]
Therefore we are in case a) when
\[
\frac{\Delta V^1(0)}{\Delta V^2(0)} < \frac{\tilde{q}}{1 - \tilde{q}}
\]
given that
\[
\Delta V^1(0) = u(a(0) + s) + v(1 - a(0)) - [u(s) + v(1)]
\]
\[
\Delta V^2(0) = u(s) + \theta^2 v(1)
\]
\[
- [u(s + a(0)) + \theta^2 v(1 - a(0))]
\]
and substituting the value of \( \tilde{q} \) and \( \Delta V^1(0) \) and \( \Delta V^2(0) \) into (3) and rearranging provides the following condition for case a): \( \Omega < \tilde{\Omega} \) with
\[
\tilde{\Omega} = u'(a(0) + s) \left[ 1 + \frac{\Delta V^2(0)}{\Delta V^1(0)} \right]
\]

**Example 1 - Extension: The transition away from extractive institutions.**

Preferences are represented by the following utility functions, respectively for workers and elites:
\[
u^1(a^1, T^1, p) = u(a^1(1 - p) + s^1 + T^1) + \theta^1 v(1 - a^1)
\]
\[
u^{2j}(a^{2j}, T^2, p) = u(a^{2j} + s^2 + T^2) + \theta^{2j} v(1 - a^{2j})
\]

Our characterization of the distinction between the political groups (workers and elites) and the cultural groups (bourgeois and aristocrats) in terms of cultural values and technologies requires that:
i) the parameter $\theta^2$ representing the preference for leisure of the elites satisfy $\theta^{2a} > \theta^{2b} = \theta^1$
ii) Initial resources $s_i$ satisfy: $s^1 = 0$, $s^2 = s > \bar{c}$
iii) Tax Transfers $T^i$ satisfy $T^1 = 0$, $T^2 = T$

We make the following regularity conditions: $u(.)$ and $v(.)$ are smooth increasing concave functions with the inada conditions $u'(0) = v'(0) = +\infty$, $u'(\infty) = 0$.

Moreover we assume that:

Assumption (P1): $\theta^{2a} > \frac{u'(s)}{v'(1)} > 1 = \theta^1$

This ensures that with no transfer from the workers to the elite, "bourgeois" elite members do work while "aristocrat" elite members do not.

**Optimal Behaviors of workers and elite members:**

1) Given a linear tax rate $p$, the optimal behavior $a^1(p)$ of the workers depends on whether the survival constraint $c^1 \geq \bar{c}$ is binding or not.

1i) Consider first the "non extractive" regime NE where the survival constraint is not binding. The optimal behavior $a^1 = a^1_{op}(p)$ of the workers is obtained by the following condition for $p < 1$:

$$u'((1-p)a^1) (1-p) = v'(1-a^1)$$

and $a^1_{op}(1) = 0$. Simple differentiation provides that $a^1_{op}(p)$ is decreasing in $p$ when the utility function $u(.)$ satisfies the following property:

Assumption (P2): $-xu''(x) < 1$ for all $x \geq 0$

Indeed

$$\frac{da^1_{op}}{dp} = \frac{u''(1-p)a^1_{op} + u'_1}{u_1''(1-p)^2 + v''_1} < 0$$

with the convenient notations $u'_1 = u'((1-p)a^1_{op})$, $u''_1 = u''((1-p)a^1_{op})$, $v''_1 = v''(1-a^1_{op})$.

1ii) Consider now the "extractive regime" E, where the survival constraint is binding. In that case we the workers' consumption is given by $c^1 = (1-p)a^1$ and the optimal behavior $a^1(p)$ of the workers is given by the relationship:

$$a^1 = \bar{a}^1(p) = \frac{\bar{c}}{1-p} \text{ for } p \leq 1 - \bar{c}$$

This regime prevails when the tax rate $p$ is larger than a threshold $\hat{p}$ given by the condition $a^1_{op}(\hat{p}) = \frac{\bar{c}}{1-\hat{p}}$ and characterized by the following condition $u'(\bar{c})(1-\hat{p}) = v'(1 - \frac{\bar{c}}{1-\hat{p}})$.
1iiii) The full characterization of the optimal behavior of the workers is then obtained as follows: assume that \( u'(\tau) > v'(1-\tau) \) (which ensures that \( a^1_{op}(0) > \bar{\tau} \)) and assumption (P2) holds, then the optimal effort of a worker writes as follows:

\[
\begin{align*}
\text{non extractive regime:} & \quad a^1(p) = a^1_{op}(p) \quad \text{for } p \in [0, \bar{p}] \\
\text{extractive regime:} & \quad a^1(p) = \bar{a}^1(p) = \frac{\bar{c}}{1-p} \quad \text{for } p \in [\bar{p}, 1-\bar{\tau}] 
\end{align*}
\]

Notice this effort \( a^1(p) \) is decreasing in \( p \) in the non extractive regime and is increasing in \( p \) in the extractive regime.

2) Given a lump sum transfer \( T \), the optimal behavior \( a^{2b}(T) \) of a "bourgeois" elite member (for an internal solution) is obtained by the following condition:

\[
u'(a^{2b} + T + s) = v'(1 - a^{2b})
\]

Assumption (P1) ensures that \( a^{2b}(0) > 0 \) while there exists a transfer level \( T = u^{-1}(v'(1)) - s > 0 \) such that \( a^{2b}(T) = 0 \) for all \( T \geq \bar{T} \). Moreover it is immediate to see that

\[
a^{2b}_T = \frac{da^{2b}}{dT} = -\frac{u''_{2b}}{u''_{2b} + v''_{2b}} < 0 \quad \text{and} \quad 1 + a^{2b}_T = \frac{v''_{2b}}{u''_{2b} + v''_{2b}} > 0
\]

with the usual notations \( u''_{2b} = u''(a^{2b} + T + s) \), \( v''_{2b} = v''(1 - a^{2b}) \)

3) For an "aristocrat" elite member, given that \( \theta^2 v'(1) > u'(s) \), we immediately get \( a^{2a}(T) = 0 \) for all \( T \geq 0 \).

**Societal equilibrium policy:**

- Define the policy maker objective function

\[
W(p, a^1, a^{2b}, T) = \beta \left[ u((1-p)a^1) + v(1-a^1) \right] + (1-\beta) \cdot \left[ q \cdot (u(T + a^{2b} + s) + v(1 - a^{2b})) + (1-q) \cdot (u(T + s) + \theta v(1)) \right]
\]

Then the societal equilibrium policy \( p(\beta, q) \) is characterized by the following conditions:

\[
p \in \arg \max_{p \in [0,1-\bar{\tau}]} W(p, a^1, a^{2b}, pa^1\frac{1-\lambda}{\lambda})) \quad (6)
\]

for given \( a^1, a^{2b} \)

\[
a^1 = a^1(p); \quad a^{2b} = a^2(T), \quad a^{2a} = 0 \quad \text{and} \quad T = pa^1\frac{1-\lambda}{\lambda}
\]
Note that under our regularity conditions, for given $a^1, a^{2b}, W(p, a^1, a^{2b}, pa^{1-\lambda})$ is a well defined, smooth and concave function of $p$. Hence there is a unique well defined value $p \in [0, 1 - \tau]$ that solves problem (6).

To characterize a societal equilibrium policy $p(\beta, q)$, we can distinguish between the two possible regimes (extractive and non extractive).

1) Consider first a "non extractive" societal equilibrium. Such a "non extractive" societal equilibrium policy $p(\beta, q) \leq \hat{p}$ should then satisfy the following conditions:

$$p = 0 \text{ when } W_p'(0, a^1(0), a^{2b}(0), 0) \leq 0$$

$$p \in (0, \hat{p}) \text{ when } W_p'(p, a^1(p), a^{2b}(T(p)), T(p)) = 0$$

with $T(p) = pa^1(p)^{\frac{1-\lambda}{\lambda}}$.

To analyze the structure of a "non extractive" societal equilibria, observe first that

$$W'_p(p, a^1(p), a^{2b}(T(p)), T(p)) = a^1(p) \cdot \Psi(p, \beta, q)$$

with the "auxiliary" function $\Psi(p, \beta, q)$:

$$\Psi(p, \beta, q) = -\beta u'((1-p)a^1(p)) + (1-\beta)(1-\lambda)\frac{1-\lambda}{\lambda} \left[ qu'(T(p) + a^{2b}(T(p)) + s) \right] + (1-q)u'(T(p) + s)$$

and $T(p) = pa^1(p)^{\frac{1-\lambda}{\lambda}}$. We are now in a position to provide sufficient regularity conditions ensuring the existence and uniqueness of a "non extractive" societal equilibrium in this economy.

**Characterization of a "non extractive" societal equilibrium policy:** Assume that the function $\Psi(p, \beta, q)$ is convex in $p$. Then for all $q \in [0, 1]$, there exists $\beta^m(q) \in (0, 1)$ and $\bar{\beta}(q) \in (0, 1)$ with $(\beta^m(q) < \bar{\beta}(q))$ such that there exists a unique "non extractive" societal equilibrium $p(\beta, q) \in [0, \hat{p}]$. It is characterized in the following way:

- For $\beta \in (\beta^m(q), \bar{\beta}(q))$, $p(\beta, q) \in (0, \hat{p})$ and is a decreasing function in $\beta$ and $q$.
- For $\beta \geq \bar{\beta}(q)$, $p(\beta, q) = 0$.

**Proof.** Note first that $p = 0$ is a societal equilibrium when $\Psi(0, \beta, q) \leq 0$. This condition writes as:

$$\Psi(0, \beta, q) = -\beta u'(a^1(0)) + (1-\beta)\left(1-\lambda\right)\left[ qu'(a^{2b}(0) + s) + (1-q)u'(s) \right] \leq 0$$

which is satisfied if and only if $\beta \geq \bar{\beta}(q)$ where

$$\bar{\beta}(q) = \frac{(1-\lambda)\left[ qu'(a^{2b}(0) + s) + (1-q)u'(s) \right]}{(1-\lambda)\left[ qu'(a^{2b}(0) + s) + (1-q)u'(s) \right] + \lambda u'(a^1(0))}$$
which is clearly a decreasing function of $q$, as $u'(a^{2b}(0) + s) < u'(s)$.

- Now consider $\beta < \beta(q)$. We have $\Psi(0, \beta, q) > 0$ and $\Psi(\hat{p}, \beta, q) \leq 0$ with

$$
\Psi(\hat{p}, \beta, q) = -\beta u'(\tau) + (1 - \beta) \left( \frac{1 - \lambda}{\lambda} \right) \left[ q u'\left( \frac{\hat{p} - \pi}{\lambda} \right) + a^{2b}(\frac{\hat{p} - \pi}{\lambda}) + s \right] < 0
$$

when

$$
\frac{(1 - \lambda)}{\lambda} \left[ q u'\left( T(\hat{p}) + a^{2b}(T(\hat{p})) + s \right) + (1 - q) u'(T(\hat{p}) + s) \right] \leq \beta \left[ u'(\tau) + \frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^{2b}(T(\hat{p})) + s) + (1 - q) u'(T(\hat{p}) + s) \right] \right]
$$

with $T(\hat{p}) = \frac{\hat{p} - \pi}{\lambda}$. This inequality writes as

$$
\beta \geq \beta^m(q) = \frac{\frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^{2b}(T(\hat{p})) + s) + (1 - q) u'(T(\hat{p}) + s) \right]}{u'(\tau) + \frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^{2b}(T(\hat{p})) + s) + (1 - q) u'(T(\hat{p}) + s) \right]}
$$

It is then easy to see that

$$
\frac{\partial \beta^m(q)}{\partial q} \propto \frac{(1 - \lambda)}{\lambda} \left[ u'(T(\hat{p}) + a^{2b}(T(\hat{p}) + s) - u'(T(\hat{p}) + s) \right] \left[ u'(\tau) + \frac{(1 - \lambda)}{\lambda} u'(T(\hat{p}) + s) \right] - \frac{(1 - \lambda)}{\lambda} \left[ u'(T(\hat{p}) + a^{2b}(T(\hat{p}) + s) - u'(T(\hat{p}) + s) \right] \frac{(1 - \lambda)}{\lambda} u'(T(\hat{p}) + s) \right] \propto \frac{(1 - \lambda)}{\lambda} \left[ u'(T(\hat{p}) + a^{2b}(T(\hat{p}) + s) - u'(T(\hat{p}) + s) \right] u'(\tau) < 0
$$

and $\beta^m(q)$ is a decreasing function of $q$.

As the function $\Psi(p, \beta, q)$ is continuously differentiable on $p \in [0, \pi]$, there exists an interior value $p \in (0, \pi)$ such that $\Psi(p, \beta, q) = 0$ and at which $\Psi_p(p, \beta, q) < 0$. Unicity of such a point is ensured when the function $\Psi(p, \beta, q)$ is convex in $p$. Indeed simple differentiation provides that

$$
\Psi_p = -\beta u''\left( (1 - p) a^{1}(p) \right) \left[ (1 - p) a^{1}_p - a^{1} \right] + (1 - \beta) \left( \frac{1 - \lambda}{\lambda} \right) \left[ q^b u''(T + a^{2b}(T) + s) \left( 1 + a^{2b}_T \right) + (1 - q^b) u''(T + s) \right] \frac{dT}{dp}
$$

and $dT/dp = [a^1(p) + pa^1_p(p)] \frac{1 - \lambda}{\lambda}$. Then using (4) and (5), it is immediate to see that $\Psi_p(0, \beta, q) < 0$ and

$$
\Psi_p(\tilde{p}, \beta, q) = -\beta u''(\tau) \left[ (1 - \tilde{p}) a^1_p - \frac{\tau}{1 - \tilde{p}} \right] + (1 - \beta) \left( \frac{1 - \lambda}{\lambda} \right) \left[ q^b u''(\tilde{T} + a^{2b}(\tilde{T}) + s) \left( 1 + a^{2b}_T \right) + (1 - q^b) u''(\tilde{T} + s) \right] \frac{dT}{dp} < 0
$$

when $\tilde{p} < p^\max = \arg \max_p T(p)$ and therefore $\frac{dT}{dp} > 0$.³

³We assume that $\tau$ is high enough to ensure $\tilde{p} < p^\max = \arg \max_p T(p)$.
- When $\Psi(p, \beta, q)$ is convex in $p$, $\Psi_p$ is an increasing function of $p$ and given that $\Psi_p(0, \beta, q) < 0$ and $\Psi_p(\hat{p}, \beta, q) < 0$, therefore for all $p \in (0, \hat{p})$, $\Psi_p(p, \beta, q) < 0$ and $\Psi(p, \beta, q)$ is a decreasing function of $p$. As for $\beta \in (\beta^m(q), \overline{\beta}(q))$, we know that $\Psi(0, \beta, q) > 0$ and $\Psi(\hat{p}, \beta, q) < 0$. Hence there exists a unique $p \in (0, \hat{p})$ such that $\Psi(p, \beta, q) = 0$.

- Finally regularity conditions for the function $\Psi(p, \beta, q)$ to be convex in $p$ are $u''' \geq 0$, $v''' \geq 0$ and the tax revenue $T(p)$ sufficiently concave in the tax rate $p$.

- The fact that for $\beta \in (\beta^m(q), \overline{\beta}(q))$, $p(\beta, q)$ is a decreasing function of $\beta$ and $q$ comes immediately from the fact that $\Psi(p, \beta, q)$ is a decreasing function of $\beta$ and $q$.

2) Consider now the "extractive" societal equilibrium policy. It is characterized by:

$$p \in \arg \max_{p \in [0, 1]} W(p, a^1, a^{2b}, pa^1 1 - \frac{\lambda}{\lambda})$$

for given $a^1, a^{2b}$

$$a^1 = \frac{\overline{c}}{1 - p}; \ a^{2b} = a^2(T), \ a^{2a} = 0 \text{ and } T = T(p) = \frac{p\overline{c}}{1 - p} 1 - \frac{\lambda}{\lambda}$$

observe now that for $p \geq \hat{p}$

$$W'_p(p, a^1(p), a^{2b}(T(p)), T(p)) = \frac{\overline{c}}{1 - p} \widetilde{\Psi}(p, \beta, q)$$

with the "auxiliary" function $\widetilde{\Psi}(p, \beta, q)$:

$$\widetilde{\Psi}(p, \beta, q) = -\beta u'(\overline{c})$$

$$(1 - \beta) \left( \frac{1 - \lambda}{\lambda} \left[ qu'(T(p) + a^{2b}(T(p)) + s) + (1 - q)u'(T(p) + s) \right] \right)$$

note that $\Psi(\hat{p}, \beta, q) = \widetilde{\Psi}(\hat{p}, \beta, q) > 0$ for all $\beta < \beta^m(q)$. Moreover

$$\widetilde{\Psi}_p(p, \beta, q) = (1 - \beta) \left( \frac{1 - \lambda}{\lambda} \left[ qu''(T(p) + a^{2b}(T(p)) + s)(1 + a^{2b}_T) + (1 - q)u''(T(p) + s) \right] \frac{dT}{dp} \right)$$

---

4Indicated differentiation provides:

$$\Psi_{pp} = -\beta u''' \left( (1 - p)a_p^1 - a^1 \right)^2 - \beta u'' (2a_p^1 + (1 - p)a_{pp}^1)$$

$$(1 - \beta) \left( \frac{1 - \lambda}{\lambda} \left[ qu''' \left( (1 + a^{2b}_T)^2 + q^a u_{2b} a^2_{2b} a^{2b}_T + (1 - q^b)u_{2a} \right) \right] \frac{dT}{dp} \right)^2$$

$$(1 - \beta) \left( \frac{1 - \lambda}{\lambda} \left[ q^b u_{2b} a^{2b}_T + (1 - q^b)u_{2a} \right] \frac{dT}{dp} \right)^2$$

Now it can be seen that when $u'' < 0$ and $u''' = 0$ and $0 < v'''$ and $2v'' + (1 - p)v''' < 0$, one has $-2a_p^1 + (1 - p)a_{pp}^1 > 0$ and $a_{2b}^2 > 0$, implying the convexity of $\Psi$. 

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and therefore
\[ \hat{\Psi}(p, \beta, q) < 0 \text{ for } p > \hat{p} \]

Moreover
\[
\hat{\Psi}(1 - \bar{c}, \beta, q) = -\beta u'(\bar{c}) + (1 - \beta) \frac{(1 - \lambda)}{\lambda} \left[ qu'((1 - \bar{c}) \frac{1 - \lambda}{A} + a^b((1 - \bar{c}) \frac{1 - \lambda}{A}) + s) \right] + (1 - q) u'((1 - \bar{c}) \frac{1 - \lambda}{A} + s)
\]

which is positive if \((1 - \beta)\frac{1 - \lambda}{\lambda} > \beta u'(\bar{c})\) or

\[
\beta \leq \beta(q) = \frac{\frac{(1 - \lambda)}{\lambda} \left[ qu'((1 - \bar{c}) \frac{1 - \lambda}{A} + a^b((1 - \bar{c}) \frac{1 - \lambda}{A}) + s) + (1 - q) u'((1 - \bar{c}) \frac{1 - \lambda}{A} + s) \right]}{u'(\bar{c}) + \frac{(1 - \lambda)}{\lambda} \left[ qu'((1 - \bar{c}) \frac{1 - \lambda}{A} + a^b((1 - \bar{c}) \frac{1 - \lambda}{A}) + s) + (1 - q) u'((1 - \bar{c}) \frac{1 - \lambda}{A} + s) \right]}
\]

which is also decreasing in \(q\) and \(\beta(q) < \beta^m(q)\). Thus

when \(\beta \leq \beta(q)\), \(\hat{\Psi}(p, \beta, q) > 0\) for all \(p \in (\hat{p}, 1 - \bar{c})\) and the societal equilibrium policy is \(p(\beta, q) = 1 - \bar{c}\).

For \(\beta \in (\beta(q), \beta^m(q))\), there exists a unique \(p(\beta, q) \in (\hat{p}, 1 - \bar{c})\) such that \(\hat{\Psi}(p(\beta, q), \beta, q) = 0\). \(p(\beta, q)\) is the extractive societal equilibrium policy.

**Characterization of "extractive" societal equilibrium policy:**

- For all \(q \in [0, 1]\), there exists \(\beta(q) \in (0, 1)\) with \(\beta(q) < \beta^m(q)\) such that for \(\beta < \beta^m(q)\) there exists a unique "extractive" societal equilibrium \(p(\beta, q) \in \left[\hat{p}, 1 - \bar{c}\right]\). It is characterized in the following way:

- For \(\beta \in (\beta(q), \beta^m(q))\) \(p(\beta, q) \in (\hat{p}, 1 - \bar{c})\) and is a decreasing function in \(\beta\) and \(q\). Workers are at their survival consumption constraint and provide an equilibrium effort \(\frac{\beta}{1 - p(\beta, q)} < 1\)

- For \(\beta \leq \beta(q)\), \(p(\beta, q) = 1 - \bar{c}\). Workers are at their survival consumption constraint and provide an equilibrium effort equal to 1.

**Societal commitment equilibrium policy:**

Denote in the same way:

\[
\tilde{W}(p, \beta, q) = \beta \left[ u((1 - p)a^1(p)) + v(1 - a^1(p)) \right] + (1 - \beta) \left[ q \cdot \left[ u(T(p) + a^2b(T(p)) + s) + v(1 - a^2b(T(p))) \right] \right]
\]

with

\[
T(p) = pa^1(p) \frac{1 - \lambda}{\lambda}
\]
The societal commitment equilibrium $p^{\text{com}}(\beta, q)$ for any value of $\beta \in [0, 1]$ is the solution of the following program:

$$p \in \arg \max_{\tilde{W}}(p, \beta, q)$$

Note that the function $\tilde{W}(p, \beta, q)$ takes different shapes depending on whether we are in a "non extractive" or an "extractive" regime. More precisely, we have:

- for $p < \tilde{p}$, the "non extractive" objective function $\tilde{W}(p, \beta, q) = \tilde{W}^{NE}(p, \beta, q)$ given by

$$\tilde{W}^{NE}(p, \beta, q) = \beta \left[ u((1-p)a^{1}_{op}(p)) + v(1-a^{1}_{op}(p)) \right]$$

$$+ (1-\beta) \left[ q \cdot u(T(p) + a^{2b}(T(p) + s) + v(1-a^{2b}(T(p)))) + (1-q) \cdot (u(T(p) + s) + \theta v(1)) \right]$$

and $T(p) = pa^{1}_{op}(p)\frac{1-\lambda}{\lambda}$.

- for $p > \tilde{p}$, we have the "extractive" objective function $\tilde{W}(p, \beta, q) = \tilde{W}^{E}(p, \beta, q)$ given by

$$\tilde{W}^{E}(p, \beta, q) = \beta \left[ u(\bar{r}) + v(1 - \frac{\bar{c}}{1-p}) \right]$$

$$+ (1-\beta) \left[ q \cdot u(T^{\beta}(p) + a^{2b}(T(p) + s) + v(1-a^{2b}(T(p)))) + (1-q) \cdot (u(T(p) + s) + \theta v(1)) \right]$$

with now $T(p) = \frac{\bar{r}p - \lambda}{1-p}$.

We then get the following characterization of the societal commitment equilibrium policy:

Characterization of societal commitment equilibrium policy:

When $T(p) = a^{1}_{op}(p)p$ is concave and reaching its maximum at some value $p^{\text{max}} > \tilde{p}$. Then $p = p^{\text{com}}(\beta, q)$ is a well defined function characterized in the following way. For all $q \in [0, 1]$, there exists a threshold $\hat{\beta}(q)$ with $\hat{\beta}(q) < \beta(q)$ such that:

- For $\beta \in [0, \hat{\beta}(q))$, $p^{\text{com}}(\beta, q) = p_{E}(\beta, q) \in [\tilde{p}, 1-\bar{c}]$ and is a decreasing function of $\beta$ and $q$. We are in the extractive regime.

- For $\beta \in (\hat{\beta}(q), \beta(q))$, $p^{\text{com}}(\beta, q) = p_{NE}(\beta, q) \in (0, \tilde{p})$ and is a decreasing function of $\beta$ and $q$. We are in the "non extractive" regime.

Proof. i) Consider first the "non extractive" objective function $\tilde{W}^{NE}(p, \beta, q)$. It is given by

$$\tilde{W}^{NE}(p, \beta, q) = \beta \left[ u((1-p)a^{1}_{op}(p)) + v(1-a^{1}_{op}(p)) \right]$$

$$+ (1-\beta) \left[ q \cdot u(T(p) + a^{2b}(T(p) + s) + v(1-a^{2b}(T(p)))) + (1-q) \cdot (u(T(p) + s) + \theta v(1)) \right]$$
with \( T(p) = pa_{op}^1(p) \frac{1-\lambda}{\lambda} \). Note that
\[
\tilde{W}^{NE}_p = -\beta u' \left( (1 - p) \, a_{op}^1(p) \right) \cdot a_{op}^1(p) \\
+ (1 - \beta) \frac{(1 - \lambda)}{\lambda} \left[ qu'(T(p) + a^2b(T(p)) + s) + (1 - q)u'(T(p) + s) \right] \frac{d[p_{op}^1(p)]}{dp}
\]

and when \( T(p) \) is concave, it is straightforward to see that \( \tilde{W}^{NE}(p, \beta, q) \) is a strictly concave function of \( p \).

ii) It is also a simple matter to observe that \( \tilde{W}^{NE}_p(0, \beta, q) \leq 0 \) if and only if \( \beta \geq \overline{\beta}(q) \). When \( \beta \geq \overline{\beta}(q) \) as \( \tilde{W}^{NE}_p(p, \beta, q) \) is a concave function of \( p \), we have that \( \tilde{W}^{NE}_p(p, \beta, q) < \tilde{W}^{NE}_p(0, \beta, q) \leq 0 \), and therefore \( \tilde{W}^{NE}_p(p, \beta, q) \) reaches its maximum at \( p_{NE}(\beta, q) = 0 \). For \( \beta < \overline{\beta}(q) \), one has \( \tilde{W}^{NE}_p(0, \beta, q) > 0 \). Moreover \( \tilde{W}^{NE}_p(p_{\text{max}}, \beta, q) < 0 \). Hence there is a unique value \( p_{NE}(\beta, q) \in (0, p_{\text{max}}] \) at which \( \tilde{W}^{NE}_p(p, \beta, q) \) reaches a maximum and it is determined by the first order condition \( \tilde{W}^{NE}_p(p, \beta, q) = 0 \). Obviously as \( \tilde{W}^{NE}_p(p, \beta, q) < 0 \) and \( \tilde{W}^{NE}_p(p, \beta, q) < 0 \), it follows immediately that \( p_{NE}(\beta, q) \) is decreasing both in \( \beta \) and \( q \).

iii) Consider next the "Extractive" objective function \( \tilde{W}(p, \beta, q) = \tilde{W}^E(p, \beta, q) \) given by
\[
\tilde{W}^E(p, \beta, q) = \beta \left[ u(\overline{e}) + v(1 - \frac{\overline{e}}{1 - p}) \right] \\
+ (1 - \beta) \left[ q \cdot \left( u(T^m(p) + a^2b(T(p)) + s) + v(1 - a^2b(T(p))) \right) \right] \\
+ (1 - q) \cdot (u(T(p) + s) + \theta v(1))
\]

with now \( T(p) = \frac{\overline{e} - \frac{1-\lambda}{\lambda}}{1-p} \). Differentiation of \( \tilde{W}^E(p, \beta, q) \) gives as well :
\[
\tilde{W}^E_p = \Lambda(p, \beta, q) \cdot \frac{\overline{e}}{(1-p)^2}
\]

with
\[
\Lambda(p, \beta, q) = -\beta u'(1 - \frac{\overline{e}}{1 - p}) \\
+ (1 - \beta) \frac{1 - \lambda}{\lambda} \left[ q \cdot \left( u'(T(p) + a^2b(T(p)) + s) \right) + (1 - q) \cdot (u'(T(p) + s)) \right]
\]

and \( \tilde{W}^E_p \geq 0 \) if and only if \( \Lambda(p, \beta, q) \geq 0 \).

It is easy to see that \( \Lambda(p, \beta, q) \) is decreasing in \( p \). Moreover for \( \beta > \overline{\beta}_{\text{max}}(q) \) such that
\[
\beta > \overline{\beta}_{\text{max}}(q) = \frac{1 - \lambda}{\lambda} \left[ \frac{q \cdot (u'(a^2b(0) + s)) + (1 - q) \cdot (u'(s))}{u'(1 - \overline{e}) + \frac{1 - \lambda}{\lambda} \left[ q \cdot (u'(a^2b(0) + s)) + (1 - q) \cdot (u'(s)) \right]}} \right]
\]

we have \( \Lambda(0, \beta, q) < 0 \) and therefore \( \tilde{W}^E_p < 0 \) for all \( p \in [0, 1 - \overline{e}] \). Note as well that \( \lim_{p \to 1 - \overline{e}} \Lambda(p, \beta, q) = -\infty \) for \( \beta > 0 \). Hence when \( \beta \leq \overline{\beta}_{\text{max}}(q) \), \( \Lambda(0, \beta, q) > 0 \) and there is a
unique $p = p_E(\beta, q) < 1 - \tau$ such that $\Lambda(p_E, \beta, q) = 0$ and the "extractive" objective function is maximized at this point $p_E(\beta, q)$. Moreover $p_E(\beta, q)$ is decreasing in $\beta$ and $q$. From this it follows that for all $\beta, q$ such that $p_E(\beta, q) \leq \hat{p}$, necessarily $\hat{W}_p^E \leq 0$ in the extractive region $p > \hat{p}$.

This last condition can be stated as $\beta \geq \overline{\beta}_M(q) \in [0, 1]$ with necessarily $\overline{\beta}_M(q) < \overline{\beta}_{\text{max}}(q)$. Conversely for $\beta \leq \overline{\beta}_M(q) < \overline{\beta}_{\text{max}}(q)$, the extractive objective function $\hat{W}_p^E$ is maximized at some $p_E(\beta, q) \in [\hat{p}, 1 - \tau]$ in the extractive policy region. Note as well that $\lim_{\beta \to 0} p_E(\beta, q) = 1 - \tau$.

iv) One may compute also the right side and left side derivative of the social objective function at the borderline of the "extractive" and the "non extractive" regimes $p = \hat{p}$. For $p = \hat{p}^-$, we have:

\[
\hat{W}_p^- = \hat{W}_p^{NE}(\hat{p}) = -\beta u'(\tau) \frac{\tau}{1 - \hat{p}} + (1 - \beta) \frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^2(T(\hat{p}) + s) + (1 - q) u'(T(\hat{p}) + s) \right] \frac{\tau}{1 - \hat{p}} + \frac{\beta d a^1(\hat{p})}{d p} \]

while for $p = \hat{p}^+$,

\[
\hat{W}_p^+ = \hat{W}_p^E(\hat{p}) = \beta \left[ -u'(1 - \frac{\tau}{1 - \hat{p}}) \frac{\tau}{(1 - \hat{p})^2} \right] + (1 - \beta) \frac{1 - \lambda}{\lambda} \left[ q u'(T(\hat{p}) + a^2(T(\hat{p}) + s) + (1 - q) u'(T(\hat{p}) + s) \right] \frac{\tau}{(1 - \hat{p})^2}
\]

Using the fact that $u'(\tau)(1 - \hat{p}) = u'(1 - \frac{\tau}{1 - \hat{p}})$, it follows that

\[
\hat{W}_p^- = \hat{W}_p^+ + (1 - \beta) \frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^2(T(\hat{p}) + s) + (1 - q) u'(T(\hat{p}) + s) \right] \frac{\tau}{1 - \hat{p}} + \frac{\beta d a^1(\hat{p})}{d p} \]

Note also that $\hat{W}_p^- > 0$ when

\[
\beta < \overline{\beta}_c(q) = \frac{(1 - \lambda)}{\lambda} \left[ q u'(T(\hat{p}) + a^2(T(\hat{p}) + s) + (1 - q) u'(T(\hat{p}) + s) \right] \frac{\tau}{1 - \hat{p}} + \frac{\beta d a^1(\hat{p})}{d p} \]

with $\overline{\beta}_c(q) < \overline{\beta}_M(q)$ (as $0 = \hat{W}_p^+(\overline{\beta}_M(q)) = \hat{W}_p^-(\overline{\beta}_M(q)) < \hat{W}_p^+(\overline{\beta}_c(q))$, and $\hat{W}_p^+(\beta)$ is a decreasing function of $\beta$).

This discussion tells us that the social policy objective function $\hat{W}(p, \beta, q)$ is a continuously differentiable function on the interval $p \in [0, \hat{p})$ and $(\hat{p}, 1 - \tau]$ but not at $\hat{p}$. Hence it is not necessarily a concave function of $p$ (see figure A.4).
v) Collecting the previous information, we have the following characterization for the global optimum:

- For $\beta \leq \beta_c(q)$, given that $\tilde{W}_p = \tilde{W}_p^{NE}(\tilde{p}, \beta, q) > 0$ and that the function $\tilde{W}_p^{NE}(p, \beta, q)$ is a concave function of $p$ in the interval $[0, \tilde{p}]$, it follows that for all $p \in [0, \tilde{p}]$ $\tilde{W}_p^{NE}(p, \beta, q) > 0$ and therefore the global optimum can only be in the "extractive" region $[\tilde{p}, 1 - \tau]$. Therefore $p^{com}(\beta, q) = p_E(\beta, q) \in [\tilde{p}, 1 - \tau]$.

- Similarly for $\beta \geq \beta_M(q)$, the optimum policy $p_E(\beta, q)$ of the "extractive" objective function $\tilde{W}_p^{E}(p, \beta, q)$ is smaller than $\tilde{p}$, Hence in the "extractive" region $p \in [\tilde{p}, 1 - \tau]$, necessarily $\tilde{W}_p^{E} \leq 0$, therefore the global optimum can only be in the "non extractive region $[0, \tilde{p}]$. As a consequence, $p^{com}(\beta, q) = p_N E(\beta, q) \in [0, \tilde{p})$ with $p_N E(\beta, q) = \arg \max_{[0,\tilde{p}]} \tilde{W}_p^{NE}(p, \beta, q)$.

- Finally consider the last case where $\beta \in (\beta_c(q), \beta_M(q))$. In such a situation, we need to compare $\max_{p \leq \tilde{p}} \tilde{W}_p^{NE}(p, \beta, q)$ to $\max_{\tilde{p} \leq p \leq 1 - \tau} \tilde{W}_p^{E}(p, \beta, q)$ to characterize the global optimum. For this consider the function

$$\Delta(\beta, q) = \tilde{W}_p^{NE}(p_{NE}(\beta, q), \beta, q) - \tilde{W}_p^{E}(p_E(\beta, q), \beta, q)$$

in the interval $\beta \in [\beta_c(q), \beta_M(q)]$. We see that $\Delta(\beta_c(q), q) < 0$ while $\Delta(\beta_M(q), q) > 0$. 

Figure A.4: Shape of the function $\tilde{W}(p, \beta, q)$
Moreover differentiation of $\Delta (\beta, d)$ gives:

\[
\Delta_\beta (\beta, q) = \widetilde{W}_\beta^{NE}(p_{NE}(\beta, q), \beta, q) - \tilde{W}_\beta^{E}(p_E(\beta, q), \beta, q) \quad (8)
\]

\[
= u((1 - p_{NE})a_{op}^1(p_{NE})) + v(1 - a_{op}^1(p_{NE}))
\]

\[
\quad - \left[ q \cdot (u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE})))) + (1 - q) \cdot (u(T_{op}(p_{NE}) + s) + \theta v(1))
\right] \\
\quad - \left[ u(\bar{c}) + v(1 - \frac{\bar{c}}{1 - p_E}) \right] \\
\quad + \left[ q \cdot (u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E)))) + (1 - q) \cdot (u(T_E(p_E) + s) + \theta v(1)) \right]
\]

with $T_{op}(p) = \frac{1 - \lambda}{\lambda} p a_{op}^1(p)$, $T_E(p) = \frac{1 - \lambda}{\lambda} \frac{p_E}{1 - p}$. Note that $p_{NE}(\beta, q) < \hat{p} < p_E(\beta, q)$ and as well that $\hat{p} < p_{max}$ where $p_{max} = \arg \max_p p a_{op}^1(p)$. Also we have $u((1 - p_{NE})a_{op}^1(p_{NE})) + v(1 - a_{op}^1(p_{NE})) > u(\bar{c}) + v(1 - \frac{\bar{c}}{1 - p_E})$ and that

\[
T_{op}(p_{NE}) = p_{NE} a_{op}^1(p_{NE}) \frac{1 - \lambda}{\lambda} < \frac{\hat{p} a_{op}^1(\hat{p})}{1 - \lambda} = \frac{\hat{p} c}{1 - \hat{p}} \frac{1 - \lambda}{\lambda} < \frac{p_E c}{1 - p_E} \frac{1 - \lambda}{\lambda} = T_E(p_E)
\]

From this it follows that in (8) the term

\[
q \cdot (u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE})))) + (1 - q) \cdot (u(T_{op}(p_{NE}) + s) + \theta v(1))
\]

is smaller than the term

\[
q \cdot (u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E)))) + (1 - q) \cdot (u(T_E(p_E) + s) + \theta v(1))
\]

Therefore $\Delta_\beta (\beta, q) > 0$ and there exists a unique threshold $\hat{\beta} (q) \in (\beta_c(q), \beta_M(q))$ such that $\Delta \left( \hat{\beta} (q), q \right) = 0$ and $\Delta_\beta \left( \hat{\beta} (q), q \right) > 0$. Moreover differentiation by $q$ provides

\[
\Delta_q (\beta, q) > 0 = \widetilde{W}_q^{NE}(p_{NE}(\beta, q), \beta, q) - \tilde{W}_q^{E}(p_E(\beta, q), \beta, q)
\]

\[
= (1 - \beta) \left[ \left( u(T_{op}(p_{NE}) + a^{2b}(T_{op}(p_{NE}) + s) + v(1 - a^{2b}(T_{op}(p_{NE})))) - (u(T_E(p_E) + a^{2b}(T_E(p_E) + s) + v(1 - a^{2b}(T_E(p_E))))
\right) \\
\right] \\
\left. - [(u(T_{op}(p_{NE}) + s) + \theta v(1)) - (u(T_E(p_E) + s) + \theta v(1))] \right]
\]

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Hence mined by the equation \( \Sigma (T) = u(T + s) - [u(T + a^2b(T) + s) + v(1 - a^2b(T))] \)

Then we obtain \( \Sigma' (T) = u'(T + s) - u'(T + a^2b(T) + s) > 0 \) and \( \Sigma (T) \) is increasing in \( T \).

As \( \Delta_q (\beta, q) = (1 - \beta) [\Sigma (T_E (p_E)) - \Sigma (T_{op} (p_{NE}))] \) and \( T_{op} (p_{NE}) < T_E (p_E) \), it follows that \( \Delta_q (\beta, q) > 0 \) and therefore \( \hat{\beta} (q) \) is decreasing in \( q \).

The previous discussion implies that for \( \beta \in \left( \bar{\beta} (q), \hat{\beta} (q) \right) \) the global optimum is in the "extractive" region \([\bar{\beta}, 1 - \bar{c}]\). with \( p^{\text{com}} (\beta, q) = p_E (\beta, q) \in (\bar{\beta}, 1 - \bar{c}] \), while for \( \beta \in \left( \hat{\beta} (q), \bar{\beta}_M (q) \right) \) the global optimum is in the "non extractive" region \([0, \bar{\beta}]\). with \( p^{\text{com}} (\beta, q) = p_{NE} (\beta, q) \in (0, \bar{\beta}] \).

Finally at \( \beta = \hat{\beta} (q) \), the objective function is maximized both at \( p_E (\hat{\beta} (q), q) > \bar{p} \) and at \( p_{NE} (\hat{\beta} (q), q) < \bar{p} \). The optimal policy can be any randomization between these two policies.

Collecting all the results in i), ii) and v) gives the characterization of the societal commitment equilibrium policy.

\[ \Box \]

**Comparison between** \( p(\beta, q) \) **and** \( p^{\text{com}} (\beta, q) \):

- At \( \beta \geq \hat{\beta} (q) \) we have the societal equilibrium policy \( p(\beta, q) < \bar{p} \) and at such point \( \hat{W}_p^{NE} (p(\beta, q)) \) is equal to
  \[
  (1 - \beta) \cdot \left[ qu' (T (p)) + a^2b (T (p) + s) + (1 - q)u' (T (p) + s) \right] p \cdot a^1_p < 0
  \]

Hence \( p(\beta, q) > p_{NE} (\beta, q) = p^{\text{com}} (\beta, q) \).

- On the other hand for \( \beta \in \left( 0, \hat{\beta} (q) \right) \) the societal equilibrium policy \( p(\beta, q) > \bar{p} \) and determined by the equation
  \[
  \beta u' (\bar{v}) = (1 - \beta) \frac{1 - \lambda}{\lambda} \left[ qu' (T (p)) + a^2b (T (p) + s) \right] + (1 - q)u' (T (p) + s)
  \]
  at such point \( \hat{W}_p^{E} \) is equal to
  \[
  \hat{W}_p^{E} = \Lambda \left( p(\beta, q), \beta, q \right) \frac{\bar{v}}{(1 - p(\beta, q))^2}
  \]
  with \( \Lambda \left( p(\beta, q) \right) \) given by
  \[
  \Lambda \left( p(\beta, q) \right) = -\beta u' (1 - \frac{\bar{v}}{1 - p})
  \]
  \[
  +(1 - \beta) \frac{1 - \lambda}{\lambda} \left[ q \cdot \left( u' (T (p)) + a^2b (T (p) + s) \right) + (1 - q) \cdot (u' (T (p) + s)) \right]
  \]

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Thus at such point \( p(\beta, q) \) one has:

\[
\Lambda (p(\beta, q), \beta(q)) = \beta \left[ u'(\bar{\tau}) - v'(1 - \frac{\bar{\tau}}{1 - p}) \right]
\]

Given the convexity of \( \tilde{W}^E \) one has \( p(\beta, q) > p_E(\beta, q) \) if and only if \( \Lambda (p(\beta, q), \beta(q)) < 0 \). Denote \( \Phi(p) = u'(\bar{\tau}) - v'(1 - \frac{\bar{\tau}}{1 - p}) \). It is easy to check that \( \Phi(p) \) is decreasing in \( p \) and such \( \lim_{p \to 1 - \tau} \Phi(p) = -\infty \) and \( \Phi(\bar{\beta}) > 0^5 \). Thus there exists a unique value \( p^c \in (\bar{\beta}, 1 - \bar{\tau}) \) such that

\[
\Phi(p^c) = u'(\bar{\tau}) - v'(1 - \frac{\bar{\tau}}{1 - p^c}) = 0
\]

and

\[
p^c = 1 - \frac{\bar{\tau}}{1 - v'(1 - \frac{\bar{\tau}}{1 - p^c})}
\]  

(9)

Therefore for all \( q \in [0, 1] \), there exists a unique value \( \beta^e(q) \in (\hat{\beta}(q), \beta^m(q)) \) such that \( p(\beta^e, q) = p^c \). From this it follows that for \( \beta \geq \beta^e(q) \), one has \( p(\beta, q) \leq p(\beta^e, q) = p^c \), which implies \( \Lambda (p(\beta, q), \beta(q)) = \beta \Phi(p(\beta, q)) \geq 0 \), and thus \( p(\beta, q) \leq p_E(\beta, q) \). The curve \( p(\beta, q) \) crosses the curve \( p_E(\beta, q) \) from above at the point \( \beta^e(q) \) in the interval \((0, \beta^m(q))\).

Now for all \( q \in [0, 1] \) we know that \( p^\text{com}(\beta, q) = p_E(\beta, q) > \bar{\beta} \) for \( \beta < \hat{\beta}(q) \). Therefore two cases can occur. When \( \beta^e(q) < \hat{\beta}(q) \), then for \( \beta \in (\hat{\beta}(q), \beta^m(q)) \) we have \( p(\beta, q) \geq p_E(\beta, q) = p^\text{com}(\beta, q) \); while for \( \beta \in [\beta^e(q), \hat{\beta}(q)] \) \( p(\beta, q) \leq p_E(\beta, q) = p^\text{com}(\beta, q) \). When \( \beta^e(q) > \hat{\beta}(q) \), then for \( \beta \in (\hat{\beta}(q), \beta^m(q)) \), we have \( p(\beta, q) \geq p_E(\beta, q) = p^\text{com}(\beta, q) \).

Moreover for the set of points \( \beta \in (0, \beta^m(q)) \), \( p(\beta, q) = 1 - \bar{\tau} > p_E(\beta, q) = p^\text{com}(\beta, q) \). From this it follows the comparison between \( p(\beta, q) \) and \( p^\text{com}(\beta, q) \):

a) For \( \beta \in (0, \min \{ \beta^e(q), \hat{\beta}(q) \}) \) one has \( p(\beta, q) > p^\text{com}(\beta, q) \)

b) For \( \beta \in (\min \{ \beta^e(q), \hat{\beta}(q) \}, \hat{\beta}(q)) \) one has \( p(\beta, q) < p^\text{com}(\beta, q) \)

c) For \( \beta \in (\hat{\beta}(q), 1) \) one has \( p(\beta, q) > p^\text{com}(\beta, q) \)

Finally note that \( \beta^e(q) \) is given by

\[
p^c = 1 - \frac{\bar{\tau}}{1 - v'(1 - \frac{\bar{\tau}}{1 - p})} = p(\beta^e(q), q)
\]

and \( \beta^e(q) \) is decreasing in \( q \). QED.

Note that region b) can only occur when \( \beta^e(q) < \hat{\beta}(q) \). Conditions on the economic fundamentals can be given to guarantee such possibility for all values of \( q \in [0, 1] \) and as depicted in figure 5 in the main text.

---

5Indeed \( \lim_{p \to 1 - \tau} \Phi(p) = u'(\bar{\tau}) - v'(0) = -\infty \) (as \( v'(0) = +\infty \)) and

\[
\Phi(\bar{\beta}) = u'(\bar{\tau}) - v'(1 - \frac{\bar{\tau}}{1 - \bar{\beta}}) > u'(\bar{\tau}) (1 - \bar{\beta}) - v'(1 - \frac{\bar{\tau}}{1 - \bar{\beta}}) = 0
\]
Specifically $\beta^c(q)$ is characterized by the equation $\hat{\Psi}(p^c, \beta, 0) = 0$ or

$$
\beta^c(q) = \frac{(1-\lambda)}{\lambda} \frac{\left[q \cdot u'(T^c + a^{2b}(T^c) + s) + (1 - q) \cdot u'(T^c + s)\right]}{\left[u'(\bar{c}) + \frac{(1-\lambda)}{\lambda} \left[q \cdot u'(T^c + a^{2b}(T^c) + s) + (1 - q) \cdot u'(T^c + s)\right]\right] [1 - \epsilon(\hat{p})]
$$

with $T^c = \frac{p^c(1-\lambda)}{1-p^c}$. Moreover, pose $a^1(\hat{p}) = \frac{p}{1-p^c}$, $\epsilon(\hat{p}) = -\frac{\hat{p}}{a^1(\hat{p})} \frac{da^1(\hat{p})}{dp} > 0$ the effort’s elasticity of workers in the “non constrained regime” evaluated at $p = \hat{p}$ (when the survival constraint binds), and $\hat{T} = \frac{\hat{p}(1-\lambda)}{1-p}$. Then using (7), one gets that

$$
\bar{\beta}_c(q) = \frac{(1-\lambda)}{\lambda} \frac{\left[q \cdot u'(\hat{T} + a^{2b}(\hat{T}) + s) + (1 - q) \cdot u'(\hat{T} + s)\right]}{\left[u'(\bar{c}) + \frac{(1-\lambda)}{\lambda} \left[q \cdot u'(\hat{T} + a^{2b}(\hat{T}) + s) + (1 - q) \cdot u'(\hat{T} + s)\right]\right] [1 - \epsilon(\hat{p})]
$$

with $\epsilon(\hat{p}) < 1$ ($\hat{p}$ a decreasing function of $\bar{c}$, so we can assume that the survival constraint $\bar{c}$ to be large enough to ensure $\hat{p}$ to be on the ”good” side of the Laffer curve (e.g; $\hat{p} < p^\text{max} = \arg \max p \cdot a^1(p)$). Note as well that the function

$$
X(T, q) = \frac{(1-\lambda)}{\lambda} \left[q \cdot u'(T + a^{2b}(T) + s) + (1 - q) \cdot u'(T + s)\right]
$$

is a decreasing function of $T$ and $q$. Moreover

$$
X''Tq = \frac{(1-\lambda)}{\lambda} \left[u''(T + a^{2b}(T) + s)(1 + \frac{da^{2b}}{dT}) - u''(T + s)\right] < 0
$$

when $u'' \leq 0$. Inspection of (9), shows as well that $p^c$ is a decreasing function of $\bar{c}$. Therefore for a large enough survival constraint $\bar{c}$, $p^c$ is also on the good side of the laffer curve (e.g $p^c < p^\text{max} = \arg \max p \cdot a^1(p)$). Given this and the fact that $p^c > \hat{p}$ ,we then obtain that $\hat{T} < T^c$. Therefore $X^c(q) = \chi(T^c, q) < \chi(\hat{T}, q) = \hat{\chi}(q)$. Now

$$
\beta^c(q) = \frac{\chi^c(q) [1 - \epsilon(\hat{p})]}{u'(\bar{c}) + \chi^c(q) [1 - \epsilon(\hat{p})]}
$$

$$
\bar{\beta}_c(q) = \frac{\hat{\chi}(q) [1 - \epsilon(\hat{p})]}{u'(\bar{c}) + \hat{\chi}(q) [1 - \epsilon(\hat{p})]}
$$

Thus $\beta^c(q) < \bar{\beta}_c(q)$ will hold when

$$
\epsilon(\hat{p}) < \frac{\hat{\chi}(q) - \chi^c(q)}{\hat{\chi}(q)}
$$

(10)

now $\hat{\chi}(q) - \chi^c(q)$ is an increasing function of $q^6$ and $\hat{\chi}(q)$ is decreasing in $q$. Thus it follows that the function $[1 - \chi^c(q) / \hat{\chi}(q)]$ is increasing in $q$. Condition (10) is satisfied for all $q \in [0, 1]$ when $\epsilon(\hat{p}) < 1 - \chi^c(0) / \hat{\chi}(0)$ This rewrites as

$$
\epsilon(\hat{p}) < 1 - \frac{u'(T^c + s)}{u'(\hat{T} + s)}
$$

$^6$ $\hat{\chi}(q) - \chi^c(q) = \chi_q(T^c, q) - \chi_q(T^c, q) > 0$ as $\chi'' \chi_q < 0$. 20
Again the RHS of this equation is an increasing function of $s$ when $u'' \leq 0$. Therefore in this case, this last condition will be satisfied when $s$ is large enough.

To summarize, when $u'' \leq 0$, the survival constraint $\varepsilon$ strict enough, the elite endowment $s$ large enough, or the workers’ effort elasticity to taxation weak enough, then for all values of $q \in [0,1]$ $\beta^*(q) < \hat{\beta}(q)$, and therefore there always exists a region of type b) such that for $\beta \in (\beta^*(q),\hat{\beta}(q))$ $p(\beta,q) < p^{com}(\beta,q)$.

**Cultural Dynamics:**

The parameters of cultural intolerance of the two types of elite members (bourgeois and aristocrats) can be written simply as:

$$\Delta V^b(p) = u\left(T(p) + a^{2b}(T(p)) + s\right) + v(1 - a^{2b}(T(p)))$$

$$- \left[u(T(p) + s) + v(1)\right]$$

$$\Delta V^a(p) = u(T(p) + s) + \theta v(1)$$

$$- \left[u\left(T(p) + a^{2b}(T(p)) + s\right) + \theta v(1 - a^{2b}(T(p))\right]$$

We then have

$$\Delta V^b(p) = \left[u'\left(T(p) + a^{2b}(T(p)) + s\right) - u'\left(T(p) + s\right)\right] \frac{dT}{dp}$$

The bracket term is negative and therefore $\Delta V^b(p)$ has the sign opposite to $dT/dp$.

Similarly

$$\Delta V^a(p) = \left[u\left(T(p) + a^{2b}(T(p)) + s\right) - u\left(T(p) + s\right)\right] \frac{dT}{dp}$$

The first term inside the bracket is positive while the second term is negative as $a^{2b}_T < 0$. However the larger the degree of concavity of $v(.)$, the smaller is $\|a^{2b}_T\|$. It follows that when $|v''|$ large enough, the bracket term has also positive sign and therefore $\Delta V^a(p)$ has the sign of $dT/dp$.

From this discussion, it follows that when $|v''|$ is large enough, $\Delta V^b/\Delta V^a$ has the same sign of variation as $-dT/dp$. As for both in the "non extractive" region (ie. $p \leq \bar{p}$) and in the "extractive" region (ie. $p \geq \bar{p}$) we have $dT/dp > 0$, then $\Delta V^b(p)/\Delta V^a(p)$ is decreasing in $p$. Now given that $p(\beta,q)$ is a decreasing function of $\beta$ and $q$, that $p(0,q) = 1 - \varepsilon$, that $p(\bar{\beta}(q),q) = 0$, then for $\beta \in [0,\bar{\beta}(q)]$, we get that $\Delta V^b/\Delta V^a$ is increasing in $\beta$ and in $q$. Moreover for $\beta \geq \bar{\beta}(q)$, $p(\beta,q) = 0$ and $\Delta V^b(p)/\Delta V^a(p)$ is a constant given by

$$\frac{\Delta V^b(0)}{\Delta V^a(0)} = \frac{u\left(a^{2b}(0) + s\right) - u\left(s\right) - \left[v(1) - v(1 - a^{2b}(0))\right]}{\theta v(1 - v(1 - a^{2b}(0))) - \left[u(a^{2b}(0) + s) - u\left(s\right)\right]}$$
Cultural steady states are determined by:
\[
\frac{\Delta V^b(p(\beta, q))}{\Delta V^a(p(\beta, q))} = \frac{q}{1 - q}
\] (11)

In the region \( \beta > \beta(q) \), there is a unique solution \( q^* \) of equation (11) independent from \( \beta \) (as \( \Delta V^b/\Delta V^a \) is just a constant in that region). In the region \( \beta \in [0, \beta(q)] \), given that \( p(\beta, q) \) is decreasing in \( q \), there could exist more than one value of \( q \) satisfying equation (11) in the relevant range of parameter \( \beta \). Note however that the LHS of (11) is always a strictly positive and bounded continuous function of \( q \in [0, 1] \) while the RHS is a continuous function vanishing to 0 at \( q = 0 \) and unbounded at \( q = 1 \). Hence by continuity, there is always one value of \( q(\beta) \) that satisfies (11) and is necessarily increasing in \( \beta \). Finally the point \( q^* \) such that the curve \( q(\beta) \) crosses \( \beta(q) \), by continuity is determined by
\[
\frac{\Delta V^b(0)}{\Delta V^a(0)} = \frac{q^*}{1 - q^*}
\]

QED.

**Example 2: Civic culture and Institutions**

We use parametric forms for the utility cost functions of undertaking civic control \( a \) and civic participation \( e \)

\[
C(a) = \phi_A \frac{a^{1+\epsilon_A}}{1 + \epsilon_A} \text{ with } \epsilon_A > 0
\]

\[
\Phi(e) = \phi_E \frac{e^{1+\epsilon_E}}{1 + \epsilon_E} \text{ with } \epsilon_E > 0
\]

We can therefore pose the preference profiles of the different agents as:

- **civic minded workers:** \( U^{1c}(e^{1c}, a^{1c}, p, E) = \omega - p + \nu(p(1 - \mu)) + \kappa E \)
  
  \[
  + [p(1 - \mu)] e^{1c} - \phi_E \frac{(e^{1c})^{1+\epsilon_E}}{1+\epsilon_E} - \phi_A \frac{(a^{1c})^{1+\epsilon_A}}{1+\epsilon_A} - \alpha (\mu p) \cdot (1 - a^{1c}) - \phi_A \frac{(a^{1c})^{1+\epsilon_A}}{1+\epsilon_A}
  \]

- **passive worker:** \( U^{1p}(e^{1p}, a^{1p}, E) = \omega - p + \nu(p(1 - \mu)) + \kappa E \)
  
  \[
  - \phi_E \frac{(e^{1p})^{1+\epsilon_E}}{1+\epsilon_E} - \phi_A \frac{(a^{1p})^{1+\epsilon_A}}{1+\epsilon_A}
  \]

- **Elite member:** \( U^2(p, E, A) = \omega - p + (1 - \theta \cdot (\lambda qa^{1c})) \cdot \mu p + \nu(p(1 - \mu)) + \kappa E \)

Therefore the optimal action of a "civic minded" worker is obtained from

\[
\max_{e^{1c}, a^{1c}} U^{1c}(e^{1c}, a^{1c}, p, E)
\]

\footnote{If there are more than one solution, there exists an odd number \( 2k + 1 \) of solutions \( (q_{2j+1}(\beta))_{j \in [0, k]} \) with \( q_{2j-1}(\beta) \) increasing in \( \beta \) and \( q_{2j}(\beta) \) decreasing in \( \beta \) for \( j \in [0, k] \).}
which provides the optimal behaviors

$$e^{1c}(p) = \left[ \frac{p(1-\mu)}{\phi_E} \right]^{\frac{1}{\epsilon_E}} \quad \text{and} \quad a^{1c}(p) = \left[ \frac{\alpha \cdot (\mu p)}{\phi_A} \right]^{\frac{1}{\epsilon_A}}$$

(12)

**Characterization of the societal equilibrium \( p(\beta, q) \):**

Denote next for given institutions \( \beta \), the policy maker objective function as:

$$W(p, A, E, e^{1c}, e^{1p}, a^{1c}, a^{1p}) = \beta \left\{ qU^{1c} + (1-q)U^{1p} \right\} + (1-\beta)U^2$$

with \( A = \lambda \cdot [q \cdot a^{1c} + (1-q) \cdot a^{1p}] \)

\( E = \lambda \cdot [q \cdot e^{1c} + (1-q) \cdot e^{1p}] \)

The first order condition of the policymaker for an interior solution writes as

$$W_p(p, A, E, e^{1c}, e^{1p}, a^{1c}, a^{1p}) = 0$$

or

$$-1 + (1-\mu)v'(p(1-\mu)) + \beta q \left[ (1-\mu)e^{1c} - \mu \alpha (1-a^{1c}) \right] + (1-\beta)\mu \left[ 1 - \theta \cdot \lambda q a^{1c} \right] = 0$$

Substitution of the optimal individual behaviors of the private agents provides the condition for an interior societal equilibrium policy \( p(\beta, q) \) \( \Psi(p, \beta, q) = 0 \) with

$$\Psi(p, \beta, q) = -1 + (1-\mu)v'(p(1-\mu)) + \beta q \left[ (1-\mu)e^{1c}(p) - \mu \alpha (1-a^{1c}(p)) \right] + (1-\beta)\mu \left[ 1 - \theta \cdot \lambda q a^{1c}(p) \right]$$

\( \Psi(0, \beta, q) = \infty \) as \( v'(0) = \infty \) and \( \Psi(\omega, \beta, q) < 0 \) for large enough \( \omega \) when \( \phi_E \) large enough that the following condition is satisfied. \( v'(\omega(1-\mu)) < \left[ 1 - \left( \frac{\omega(1-\mu)}{\phi_E} \right)^{\frac{1}{\epsilon_E}} \right] \cdot (1-\mu) \).

As well substitution of (12) gives immediately that

$$\Psi_p(p, \beta, q) = (1-\mu) \left( \beta q \frac{1}{\epsilon_E} \left[ (1-\mu) \right]^{\frac{1}{\epsilon_E}} p^{\frac{1}{\epsilon_E}-1} + (1-\mu)v''(p(1-\mu)) \right)$$

$$+ \beta \mu q \alpha \frac{1}{\epsilon_A} \left[ \frac{\mu \alpha}{\phi_A} \right]^{\frac{1}{\epsilon_A}} p^{\frac{1}{\epsilon_A}-1} - (1-\beta)\theta \cdot \lambda q \mu \frac{1}{\epsilon_A} \left[ \frac{\mu \alpha}{\phi_A} \right]^{\frac{1}{\epsilon_A}} p^{\frac{1}{\epsilon_A}-1}$$

Hence \( \Psi_p \) is negative when \( v(\cdot) \) concave enough and \( \phi_E \) and \( \phi_A \) are large enough. From this it follows that there exists a unique societal equilibrium policy \( p(\beta, q) \in (0, \omega) \) when \( v(\cdot) \) is concave enough and \( \omega, \phi_E \) and \( \phi_A \) are large enough.
Differentiation provides then immediately that

\[ \Psi_\beta = q \left[ (1 - \mu)e^{1c}(p) - \mu \alpha (1 - a^{1c}(p)) \right] - \mu \left[ 1 - \theta \cdot \lambda a^{1c}(p) \right] \]
\[ \Psi_q = \beta \left[ (1 - \mu)e^{1c}(p) - \mu \alpha (1 - a^{1c}(p)) \right] - (1 - \beta)\theta \cdot \lambda a^{1c}(p) \]

(13)

Now it can be seen that

\[ (1 - \mu)e^{1c}(p) - \mu \alpha \cdot (1 - a^{1c}(p)) = (1 - \mu) \left[ p \left( \frac{1 - \mu}{\phi_E} \right) \frac{1}{\tau_E} - \mu \alpha \cdot \left( 1 - \left[ \frac{\alpha \cdot (\mu p)}{\phi_A} \right] \frac{1}{\tau_A} \right) \right) \]

consequently \((1 - \mu)e^{1c}(p) - \mu \alpha \cdot (1 - a^{1c}(p)) < 0\) when \(\phi_E\) and \(\phi_A\) are large enough that the following condition \((1 - \mu) \left[ \omega \left( \frac{1 - \mu}{\phi_E} \right) \frac{1}{\tau_E} + \mu \alpha \left[ \frac{\alpha \cdot (\mu p)}{\phi_A} \right] \frac{1}{\tau_A} \right] < \mu \alpha\) is satisfied.

From this and (13) we obtain that \(\Psi_\beta\) and \(\Psi_q\) are negative when \(\phi_E\) and \(\phi_A\) are large enough. Differentiation of the societal equilibrium policy condition \(\Psi(p, \beta, q) = 0\) provides then immediately that:

\[ \frac{\partial p}{\partial \beta} = \frac{\Psi_\beta}{-\Psi_p} < 0 \quad \text{and} \quad \frac{\partial p}{\partial q} = \frac{\Psi_q}{-\Psi_p} < 0 \]

Characterization of the commitment societal equilibrium \(p^{com}(\beta, q)\):

Consider the policy objective function for given institutions \(\beta\) as:

\[ \tilde{W}(p, \beta, q) = \beta \left\{ qU^{1c} + (1 - q)U^{1p} \right\} + (1 - \beta)U^2(p, A) \]

with \(A = \lambda \cdot q \cdot a^{1c}(p)\)
\(E = \lambda \cdot q \cdot e^{1c}(p)\)
\(e^{1c}(p) = \left[ p \left( \frac{1 - \mu}{\phi_E} \right) \frac{1}{\tau_E} \right] \quad \text{and} \quad a^{1c}(p) = \left[ \frac{\alpha \cdot (\mu p)}{\phi_A} \right] \frac{1}{\tau_A} \)

one obtains the societal equilibrium with commitment \(p^{com}(\beta, q)\) for any value of \((\beta, q) \in [0, 1]^2\) as the solution of the following program:

\[ p \in \arg \max_{p \in [0, \omega]} \tilde{W}(p, \beta, q) \]

The first order condition characterizing an interior solution for \(p^{com}(\beta, q)\) is given by:

\[ \tilde{W}_p = \kappa \lambda q e^{1c} + \beta q (1 - \mu) e^{1c} - 1 + (1 - \mu) e'(p(1 - \mu)) \]
\[- \beta q \mu \alpha (1 - a^{1c}) + (1 - \beta) \mu \left[ 1 - \theta \lambda a^{1c} - p \lambda q \theta a^{1c}_p \right] \]

Differentiating another time provides
\[
\tilde{W}_{pp} = \kappa \lambda q e_p^{1c} + \beta q (1 - \mu) e_p^{1c} + (1 - \mu)^2 i''(p(1 - \mu))
+ \beta q \mu \alpha a_p^{1c} - (1 - \beta) \mu q \theta [2a_p^{1c} + pa_p^{1c}]
\]

or after substitution
\[
\tilde{W}_{pp} = \kappa \lambda q \frac{1 - \epsilon_E}{(\epsilon_E)^2} \left[ \frac{1}{\phi_E} \right] \frac{1}{p^{1 - \epsilon_E}} + \beta q (1 - \mu) \frac{1}{\epsilon_E} \left[ \frac{(1 - \mu)}{\phi_E} \right] \frac{1}{p^{1 - 1}}
+ (1 - \mu)^2 i''(p(1 - \mu)) + \beta q \mu \alpha \left[ \frac{1}{\epsilon_A} \right] \left[ \frac{1}{\phi_A} \right] \left[ \frac{(1 - \mu)}{\phi_E} \right] \frac{1}{p^{1 - 1}} - (1 - \beta) \mu q \theta (1 + \epsilon_A) \frac{1}{\phi_A} \frac{1}{p^{1 - 1}}
\]

Again when \( \nu(.) \) is concave enough and \( \phi_E \) and \( \phi_A \) are large enough, we obtain that \( \tilde{W}_{pp} < 0 \) and the function \( \tilde{W}(p, \beta, q) \) is a strictly concave in \( p \), ensuring the existence of the societal equilibrium with commitment \( p^{com}(\beta, q) \). Moreover the cross derivative \( \tilde{W}_{p\beta} \) writes as
\[
\tilde{W}_{p\beta} = q \left[ (1 - \mu) e_p^{1c} - \mu (1 - a_p^{1c}) \right] - \mu \theta \lambda q \left[ a_p^{1c} + pa_p^{1c} \right] < 0
\]

Hence \( \tilde{W}_{p\beta} < 0 \) when \( (1 - \mu) e_p^{1c} - \mu (1 - a_p^{1c}) < 0 \), something that is ensured when \( \phi_E \) and \( \phi_A \) are large enough. Consequently \( p^{com}(\beta, q) \) is decreasing in \( \beta \).

**Comparison between \( p^{com}(\beta, q) \) and \( p(\beta, q) \)**

Note that at the point \( p(\beta, q) \),
\[
\tilde{W}_p(p(\beta, q)) = \kappa \lambda q \cdot e_p^{1c} - (1 - \beta) \lambda q \theta \cdot p a_p^{1c}
\]
\[
= \lambda q \left[ \kappa \cdot \frac{1}{\epsilon_E} \left[ \frac{(1 - \mu)}{\phi_E} \right] \frac{1}{p^{1 - \epsilon_E}} - (1 - \beta) \theta \cdot \frac{1}{\epsilon_A} \left[ \frac{\mu \alpha}{\phi_A} \right] \frac{1}{\phi_E} \frac{1}{p^{1 - 1}} \right]
\]
\[
= \lambda q p^{1 - 1} \left[ \kappa \cdot \frac{1}{\epsilon_E} \left[ \frac{(1 - \mu)}{\phi_E} \right] \frac{1}{p^{1 - 1}} - (1 - \beta) \theta \cdot \frac{1}{\epsilon_A} \left[ \frac{\mu \alpha}{\phi_A} \right] \frac{1}{\phi_E} p^{(1 - 1)} \right]
\]

and assume \( \frac{1}{\epsilon_A} - \frac{1}{\epsilon_E} > 0 \) or \( \epsilon_E > \epsilon_A \) (ie. civic monitoring is more sensitive to public leakages than civic participation is sensitive to public good provision). Then the function
\[
\Theta(\beta, q) = (1 - \beta) \theta \cdot \frac{1}{\epsilon_A} \left[ \frac{\mu \alpha}{\phi_A} \right] \left[ \frac{1}{\phi_E} \right] p \left( \beta, q \right) \left( \frac{1}{\phi_A} - \frac{1}{\epsilon_E} \right)^{+1}
\]

is decreasing in \( \beta \) and \( q \). Thus the value \( \hat{\beta}(q) \) such that
\[
\Theta(\beta, q) = \sigma = \kappa \cdot \frac{1}{\epsilon_E} \left[ \frac{(1 - \mu)}{\phi_E} \right] \frac{1}{\phi_E}
\]

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is decreasing in q. Moreover $\tilde{W}_p (p (\beta, q), \beta, q) \geq 0$ if and only if $\Theta (\beta, q) \leq \sigma$ or $\beta \geq \tilde{\beta} (q)$. Thus as $\tilde{W}(p, \beta, q)$ is a concave function and reaches a maximum at $p^{com} (\beta, q)$, one finally obtains $p^{com} (\beta, q) \gtrless p (\beta, q)$ if and only if $\beta \gtrless \tilde{\beta} (q)$.

Collection of the previous discussion on the properties of $p^{com} (\beta, q)$ and $p (\beta, q)$ provides figure 8 in the main text.

Characterization of the cultural manifold $q (\beta)$

\[
\begin{align*}
\Delta V^{1c} &= [p(1 - \mu)] e^{1c} - \Phi_E (e^{1c}) + \alpha (\mu p) a^{1c} - \Phi_A (a^{1c}) \\
\Delta V^{1p} &= \Phi_E (e^{1c}) + \Phi_A (a^{1c}) \\
\Delta V^{1c} / \Delta V^{1p} &= \frac{[p(1 - \mu)] e^{1c} - \Phi_E (e^{1c}) + \alpha (\mu p) a^{1c} - \Phi_A (a^{1c})}{\Phi_E (e^{1c}) + \Phi_A (a^{1c})}
\end{align*}
\]

or

\[
\begin{align*}
\Delta V^{1c} / \Delta V^{1p} &= \frac{\Phi_E (e^{1c})}{\Phi_A (a^{1c})} \left[ \frac{\Phi_E (e^{1c})}{\Phi_A (a^{1c})} e^{1c} - 1 + \frac{\Phi_A (a^{1c})}{\Phi_A (a^{1c})} - 1 \right] \\
&= \frac{\Phi_E (e^{1c})}{\Phi_A (a^{1c})} + 1 \\
\end{align*}
\]

which is an increasing function of $\frac{\Phi_E (e^{1c})}{\Phi_A (a^{1c})}$ as $\epsilon_E > \epsilon_A$. Now

\[
\frac{\Phi_E (e^{1c})}{\Phi_A (a^{1c})} = \frac{\phi_E (e^{1c})}{\phi_A (a^{1c})} \frac{1 + \epsilon_A (e^{1c})^{1+\epsilon_E}}{1 + \epsilon_A (a^{1c})^{1+\epsilon_A}} \\
= \frac{\frac{1}{\phi_E} \frac{1}{\phi_A} (1 - \mu)^{\epsilon_E} \epsilon_A (1 + \epsilon_A)^{1+\epsilon_A}}{\phi_A (a^{1c})^{1+\epsilon_A}} p^\left( \frac{\epsilon_E}{\epsilon_A} \right)
\]

which is a decreasing function of $p$. Thus $\frac{\Delta V^{1c}}{\Delta V^{1p}} = \Gamma (p)$ is a decreasing function of $p$.

Thus the cultural manifold $q (\beta)$ is characterized by the relationship:

\[
\Gamma (p (\beta, q)) = \frac{q}{1 - q}
\]

There always exists at least solution $q = q (\beta)$ to this equation as $\Pi(q) = \Gamma (p (\beta, q)) - \frac{q}{1 - \frac{q}{1 - q}}$ is a continuous function of $q$ such that $\Pi(0) > 0$ and $\Pi(1) < 1$. When the solution $q = q (\beta)$
is unique, one necessarily has $\Pi_q(q = q(\beta)) = \Gamma'p_q - \frac{1}{(1-q)^2} < 0$ and consequently the cultural manifold is depicted by a well defined function $q = q(\beta)$ such that

$$\frac{dq}{d\beta} = \frac{\Gamma'p_3}{(1-q)^2} - \Gamma'p_q > 0$$

as $\Gamma' < 0$ and $p_3 < 0$.

The solution to (14) may have more than one solution. In that case the sign of $\Pi_q$ at these solutions alternate between negative and positive, starting with a negative sign for the smallest solution. The branches (in even number) of the cultural manifold associated to solutions with $\Pi_q < 0$ are upward sloping in $\beta$ while those (in uneven number) for which $\Pi_q > 0$ are downward sloping in $\beta$.

**Example 3: Property Rights and Conflict**

Given there is random matching before each contest game, one may compute the expected payoff of "conflict-prone" and "conflict-averse" individuals as

$$\Omega_1(p, q) = p\omega + 2(1-p)\omega \left[ \frac{q}{4} + (1-q)\left(\frac{c^2}{c^1 + c^2}\right)^2 \right] - F$$

$$\Omega_2(p, q) = p\omega + 2(1-p)\omega \left[ q\left(\frac{c^1}{c^1 + c^2}\right)^2 + \frac{(1-q)}{4} \right]$$

Writing $c^1 = c$ and $c^2 = c(1+\alpha)$ with $a > 0$. Then (15) rewrites as

$$\Omega_1(p, q) = p\omega + 2(1-p)\omega \left[ \frac{q}{4} + (1-q)\left(\frac{1+\alpha}{2+\alpha}\right)^2 \right] - F$$

$$\Omega_2(p, q) = p\omega + 2(1-p)\omega \left[ q\left(\frac{1}{2+\alpha}\right)^2 + \frac{(1-q)}{4} \right]$$

It is immediate to see that $\Omega_1(p, q)$ is decreasing in $q$ and that

$$\frac{\partial \Omega_1(p, q)}{\partial p} \geq 0 \iff q \geq \bar{q}(\alpha) = \frac{(1+\alpha)^2 - \frac{1}{2}}{(1+\alpha(2+\alpha))^2 - \frac{1}{4}}$$

and $\Omega_2(p, q)$ is decreasing in $q$ and that

$$\frac{\partial \Omega_2(p, q)}{\partial p} \leq 0$$
Property rights costs $C(p)$ satisfy: $C(0) = 0, C(p)$ is increasing convex (ie. $C'(p) \geq 0, C''(p) > 0$ and $C'(0) = 0, C'(1) = +\infty$).

The societal equilibrium

To compute the societal equilibrium outcome, note first that the expected payoff of an agent of "conflict-prone" type in a societal equilibrium is given by:

$$G_1(p, q, a^{11}, a^{12}, a^{21}) = p\omega + q \left( 2(1 - p)\omega \frac{a^{11}}{a^{11} + a^{11}} - c^1 a^{11} \right) + (1 - q) \left( 2(1 - p)\omega \frac{a^{12}}{a^{12} + a^{21}} - c^1 a^{12} \right) - F$$

and that of an agent of type "conflict averse",

$$G_2(p, q, a^{21}, a^{12}, a^{22}) = p\omega + q \left( 2(1 - p)\omega \frac{a^{21}}{a^{21} + a^{12}} - c^2 a^{21} \right) + (1 - q) \left( 2(1 - p)\omega \frac{a^{22}}{a^{22} + a^{22}} - c^2 a^{22} \right)$$

where $a^{11}, a^{12}, a^{21}, a^{22}$ are respectively the Nash contest efforts of a "conflict-prone" type 1 agent playing against another type 1 agent, a type-1 agent playing against a "conflict-averse" type-2 agent, a type-2 agent playing against a type-1 agent, and a type-2 agent playing against another type 2 agent.

The social planner in the policy game will then choose $p$ to solve the following problem:

$$\max_{\gamma} \beta G_1(p, q, a^{11}, a^{12}, a^{21}) + (1 - \beta) G_2(p, q, a^{21}, a^{12}, a^{22}) - C(p)$$

taking as given the values of $a^{11}, a^{12}, a^{21}, a^{22}$. One gets the following FOC:

$$\beta \omega \left[ 1 - 2 \left( q \frac{a^{11}}{a^{11} + a^{11}} + (1 - q) \frac{a^{12}}{a^{12} + a^{21}} \right) \right] + (1 - \beta) \omega \left[ 1 - 2 \left( q \frac{a^{21}}{a^{21} + a^{12}} + (1 - q) \frac{a^{22}}{a^{22} + a^{22}} \right) \right] = C'(p)$$

with the Nash equilibrium levels of contest efforts obtained from:

$$a^{11} = \frac{2(1 - p)\omega}{4c}, \quad a^{22} = \frac{2(1 - p)\omega}{4c(1 + \alpha)}, \quad a^{12} = 2(1 - p)\omega \frac{1 + \alpha}{c(2 + \alpha)^2}, \quad a^{21} = 2(1 - p)\omega \frac{1}{c(2 + \alpha)^2}$$

Hence the societal equilibrium level of property right protection is characterized by the following condition:

$$\beta \omega \left[ 1 - 2 \left( \frac{q}{2} + (1 - q) \frac{1 + \alpha}{2 + \alpha} \right) \right] + (1 - \beta) \omega \left[ 1 - 2 \left( \frac{q}{2} + \frac{(1 - q)}{2} \right) \right] = C'(p) \quad (17)$$
It is easy to see that:

**Characterization of societal equilibrium policy:** When $\beta < q$, the societal equilibrium policy outcome involves strictly positive protection of property right with $p(\beta, q) > 0$. Moreover $p(\beta, q)$ is decreasing in $\beta$ and increasing in $q$. When $\beta \geq q$, there is no property right protection in the societal equilibrium (ie. $p = 0$)

**The societal commitment equilibrium:**

The societal commitment equilibrium with property rights protection $p$ satisfies the following program:

$$
\max_p \beta \Omega_1(p, q) + (1 - \beta) \Omega_2(p, q)
$$

the FOC of this problem writes as:

$$
\beta \omega \left[ 1 - 2 \left( \frac{q}{4} + (1 - q) \left( \frac{1 + \alpha}{2 + \alpha} \right)^2 \right) \right] + (1 - \beta) \omega \left[ 1 - 2 \left( q \left( \frac{1}{2 + \alpha} \right)^2 + \frac{1 - q}{4} \right) \right] = C'(p_{com})
$$

(18)

One easily gets

**Characterization of societal commitment equilibrium policy:** Denote $\phi(\alpha) = \frac{1 + \alpha}{2 + \alpha}$ and assume that $1/\sqrt{2} < \phi(\alpha)$, i) then there exist a threshold $\bar{q}(\alpha) \in ]0, 1[$ and an increasing function $\beta = \bar{\beta}(q)$ with $\bar{\beta}(0) < 1$ such that the societal commitment equilibrium involves “no-property rights” (ie. $p_{com} = 0$) if and only if $(\beta, q) \in [0, 1]^2$ are such that $q < \bar{q}(\alpha)$ and $\beta \geq \bar{\beta}(q)$.

ii) When the societal commitment equilibrium policy $p_{com}(\beta, q) > 0$, then $p_{com}(\beta, q)$ is decreasing in $\beta$ and increasing in $q$.

iii) One has $p(\beta, q) \leq p_{com}(\beta, q)$.

**Proof.** C ) Inspection of the FOC reveals that

$$
p_{com} = 0 \quad \text{when} \quad \beta \geq \bar{\beta}(q) = \frac{\frac{1}{4} + q \left[ \frac{1}{4} - (1 - \phi(\alpha))^2 \right]}{q \left[ \frac{1}{4} - (1 - \phi(\alpha))^2 \right] + (1 - q) \left[ \phi(\alpha)^2 - \frac{1}{4} \right]}
$$

with $\bar{\beta}(q) > q$ for all $q \in [0, 1]$

$$
\phi(\alpha) = \frac{1 + \alpha}{2 + \alpha}
$$
is an increasing function of $\alpha$. Notice as well that for all $\alpha > 0$, one has
\[
\frac{1}{4} - (1 - \phi(\alpha))^2 > 0 \text{ and } \phi(\alpha)^2 + (1 - \phi(\alpha))^2 > \frac{1}{2}
\]
Moreover $\tilde{\beta}(q) = 1$ at a value $\tilde{q}(\alpha) \in (0, 1)$ given by
\[
\tilde{q}(\alpha) = \frac{\phi(\alpha)^2 - \frac{1}{2}}{\phi(\alpha)^2 - \frac{1}{4}}
\]
Hence it follows that a societal equilibrium with commitment region of "no-property rights" exists (ie. $p^{com} = 0$) if and only if
\[
\frac{1}{\sqrt{2}} < \phi(\alpha) \text{ and } q < \tilde{q}(\alpha)
\]
It is also immediate to see that $\tilde{\beta}(q)$ is increasing in $q$ with
\[
\tilde{\beta}(0) = \frac{1}{\phi(\alpha)^2 - \frac{1}{4}} \text{ and } \tilde{\beta}(1) > 1
\]
ii) Differentiation immediately provides that for an interior solution of property rights $p^{com}(\beta, q)$ one has:
\[
\frac{\partial p^{com}}{\partial \beta} < 0 \text{ and } \frac{\partial p^{com}}{\partial q} > 0
\]
iii) To show that $p(\beta, q) \leq p^{com}(\beta, q)$, consider the difference of the LHS of the two equations (18) and (17):
\[
\beta \omega \left[ 1 - 2 \left( \frac{q}{4} + (1 - q) \left( \frac{1 + \alpha}{2 + \alpha} \right)^2 \right) \right] + (1 - \beta) \omega \left[ 1 - 2 \left( q \left( \frac{1}{2 + \alpha} \right)^2 + \frac{1 - q}{4} \right) \right] - \beta \omega \left[ 1 - 2 \left( \frac{q}{2 + \alpha} + (1 - q) \frac{1 + \alpha}{2 + \alpha} \right) \right] - (1 - \beta) \omega \left[ 1 - 2 \left( \frac{q}{2 + \alpha} + \frac{(1 - q)}{2} \right) \right]
\]
which gives:
\[
2\beta \omega \left[ \frac{q}{2} + (1 - q) \frac{1 + \alpha}{2 + \alpha} - \left( \frac{q}{4} + (1 - q) \left( \frac{1 + \alpha}{2 + \alpha} \right)^2 \right) \right]
\]
\[
+ 2(1 - \beta) \omega \left[ \left( \frac{q}{2 + \alpha} + (1 - q) \frac{1}{2} \right) - \left( q \left( \frac{1}{2 + \alpha} \right)^2 + \frac{1 - q}{4} \right) \right]
\]
or finally
\[
2\beta \omega \left[ \frac{q}{4} + (1 - q) \frac{1 + \alpha}{(2 + \alpha)^2} \right] + 2(1 - \beta) \omega \left[ \frac{q}{2 + \alpha} + \frac{(1 - q)}{4} \right] > 0
\]
Hence $C'(p^{com}) > C'(p)$ and the result $p(\beta, q) < p^{com}(\beta, q)$ for the case of interior solutions. Obviously for $\beta > \tilde{\beta}(q)$ one has $p(\beta, q) = p^{com}(\beta, q) = 0$. QED

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Given that “conflict-prone” individuals are not always in favor of property right protection, an increase of their weight in the social welfare function implies a lower societal equilibrium with commitment value of \( p \). Also the larger the fraction of “conflict-prone” individuals, the larger the occurrence of rent-dissipation for the ”conflict prone” agents and the larger the risk of rent expropriation for the ”conflict-averse” individuals. Hence the more efficient it is to commit to protection of property rights. The previous discussion suggests that there may actually be a region in which the societal equilibrium with commitment is characterized by no property right protection.

Cultural dynamics:

Turning to cultural dynamics, we have the socialization incentives \( \Delta V_1(p, q) \) of "violence-prone" individuals as:

\[
\Delta V_1(p, q) = q \left( 2(1 - p) \frac{\omega}{2} - c^1 a^{11} \right) + (1 - q) \left( 2(1 - p) \frac{\omega}{2 + \alpha} - c^1 a^{12} \right) - F
\]

\[
- \left[ q \left( 2(1 - p) \frac{\omega}{2 + \alpha} - c^1 a^{21} \right) + (1 - q) \left( 2(1 - p) \frac{\omega}{2} - c^1 a^{22} \right) \right]
\]

which after substitution of \( a^{11} = \frac{2(1-p)\omega}{4c}, a^{22} = \frac{2(1-p)\omega}{4c(1+\alpha)}, a^{12} = 2(1-p)\omega \frac{1+\alpha}{c(2+\alpha)^2}, \)

\( a^{21} = 2(1-p)\omega \frac{1}{c(2+\alpha)^2} \) provides

\[
\Delta V^1 = 2(1 - p)\omega \left[ q \left( \frac{1}{4} \right) + (1 - q) \left( \frac{1 + \alpha}{2 + \alpha} \right)^2 \right] - F
\]

\[
-2(1-p)\omega \left[ q \frac{1 + \alpha}{(2 + \alpha)^2} + (1 - q) \frac{1 + 2\alpha}{4(1 + \alpha)} \right]
\]

\[
= 2(1 - p)\omega \left[ q \left( \frac{1}{4} - \frac{1 + \alpha}{(2 + \alpha)^2} \right) + (1 - q) \left( \frac{1 + \alpha}{2 + \alpha} \right)^2 - \frac{1 + 2\alpha}{4(1 + \alpha)} \right] - F
\]

\[
= \frac{2(1-p)\omega \alpha^2}{4(1+\alpha)(2+\alpha)^2} [(3 + \alpha) - q(2 + \alpha)] - F
\]

Now for the ”conflict-averse” individual, one similarly has:

\[
\Delta V^2 = q \left( 2(1 - p) \frac{\omega}{2 + \alpha} - c^2 a^{21} \right) + (1 - q) \left( 2(1 - p) \frac{\omega}{2} - c^2 a^{22} \right)
\]

\[
- \left[ q \left( 2(1 - p) \frac{\omega}{2} - c^2 a^{11} \right) + (1 - q) \left( 2(1 - p) \frac{\omega}{2 + \alpha} - c^2 a^{12} \right) - F \right]
\]
or

\[ \Delta V^2 = 2(1-p)\omega \left[ q \frac{1}{(2+\alpha)^2} + (1-q) \frac{1}{4} \right] - 2(1-p)\omega \left[ q \left( \frac{1}{2} - (1+\alpha) \frac{1}{4} \right) + (1-q) \frac{1+\alpha}{(2+\alpha)^2} \right] + F \]

\[ = 2(1-p)\omega \left[ q \frac{\alpha^2}{4(2+\alpha)} + \frac{\alpha^2}{4(2+\alpha)^2} \right] + F > 0 \]

Finally for the locus \( \dot{q}_t = 0 \), one can compute:

\[ \Delta V^1 \Delta V^2 = \frac{\alpha^2}{4(1+\alpha)(2+\alpha)^2} \left[ (3+\alpha) - q(2+\alpha) \right] - \frac{F}{2(1-p)\omega} \]

\[ = \frac{\alpha^2}{4(1+\alpha)(2+\alpha)^2} \left[ (3+\alpha) - q(2+\alpha) \right] - \frac{F}{2(1-p)\omega} \]

\[ = \frac{\alpha^2}{4(1+\alpha)(2+\alpha)^2} \left[ (3+\alpha) - q(2+\alpha) \right] - \frac{F}{2(1-p)\omega} \]

or finally

\[ \frac{\Delta V^1}{\Delta V^2} = \frac{1}{(1+\alpha)} \left[ (2+\alpha)(1-q) + 1 \right] - \frac{2F(2+\alpha)^2}{(1-p)\omega\alpha^2} = \Phi(q,p,\alpha) \]

It is a simple matter to see that \( \Phi(q,p,\alpha) \) is a decreasing function of \( p \) and \( q \). We assume that \( F/\omega \) is small enough

\[ \frac{F}{\omega} < \frac{(1-p(0,0))}{2(1+\alpha)} \left( \frac{\alpha}{2+\alpha} \right)^2 [3+\alpha] \] (19)

to ensure that for all \( \beta, q \in [0,1]^2 \), one has \( \Delta V^1/\Delta V^2 \) to be strictly positive\(^8\). The characterization of the interior cultural steady state is obtained from :

\[ \Phi(q,p(\beta,q),\alpha) = \frac{q}{1-q} \] (20)

It is immediate to see that the LHS of equation (20) is a decreasing function of \( q \) (as \( p(\beta,q) \) is increasing in \( q \) and \( \Phi(q,p,\alpha) \) is decreasing in \( p \)). The RHS of (20) is increasing in \( q \) and goes from 0 to \( \infty \) when \( q \) goes from 0 to 1. Hence given by (19) that \( \Phi(0,p(\beta,0),\alpha) > 0 \), then it is immediate to see that equation (20) has a unique solution \( q(\beta) \) and that \( q(\beta) < 1/2 \). Moreover \( q(\beta) \) is an increasing function of \( \beta \). Moreover there is a unique value \( \hat{\beta}(\alpha) \) such that \( q(\beta) = \beta \).

Indeed such \( \beta \) is determined by

\[ \Phi(\beta,p(\beta,\beta),\alpha) = \Phi(\beta,0,\alpha) = \frac{\beta}{1-\beta} \]

or after substitution

\[ \frac{1-\beta}{(1+\alpha)} \left[ (2+\alpha)(1-\beta) + 1 \right] - \beta (2+\alpha) + 1 = \frac{2F(2+\alpha)^2}{\omega\alpha^2} \] (21)

\(^8\)Otherwise we could get the possibility of a cultural steady state without "conflict-prone" individuals.
Denote the LHS of (21) as a function $\Sigma(\beta, \alpha)$. Simple differentiation shows that $\Sigma(\beta, \alpha)$ is decreasing in $\beta$ and takes value $\Sigma(0, \alpha) = \frac{4 + \alpha}{1 + \alpha} > 0$ and $\Sigma(1, \alpha) = -((2 + \alpha) + 1) < 0$. Now (19) implies
\[
\frac{2F(2 + \alpha)^2}{\omega \alpha^2} < \frac{(1 - p(0,0))}{(1 + \alpha)}[3 + \alpha] < \frac{4 + \alpha}{1 + \alpha} = \Sigma(0)
\]
Therefore there exists a unique value $\hat{\beta}(\alpha) \in (0, 1)$ satisfying (21). To such value $\hat{\beta}(\alpha)$, there exists a corresponding unique value $\hat{q}(\alpha) = q(\hat{\beta}(\alpha))$. QED

Institutional and cultural joint dynamics

We have the following results that correspond to the phase diagrams 11a) and 11b):

**Result**: i) The dynamics of case 1 as shown in the phase diagram (11a) (ie. $\hat{q}(\alpha) < \tilde{q}(\alpha)$) holds for $\alpha = \frac{c_2}{c_1} - 1$ large enough.

ii) The dynamics of case 2 as shown in the phase diagram (11b) (ie. $\hat{q}(\alpha) > \tilde{q}(\alpha)$) holds for $\alpha = \frac{c_2}{c_1} - 1$ close enough to $\alpha_{\min}$.

**Proof.** Consider
\[
\Sigma(\tilde{q}(\alpha), \alpha) = \frac{1 - \tilde{q}(\alpha)}{(1 + \alpha)}[(2 + \alpha)(1 - \tilde{q}(\alpha)) + 1] - \tilde{q}(\alpha)(\tilde{q}(\alpha)(2 + \alpha) + 1)
\]
recalling that $\tilde{q}(\alpha_{\min}) = 0$ and $\lim_{\alpha \to \infty} \tilde{q}(\alpha) = \frac{2}{3}$ with
\[
\alpha_{\min} = \frac{2 - \sqrt{2}}{\sqrt{2} - 1}
\]
then it follows that
\[
\Sigma(\tilde{q}(\alpha_{\min}, \alpha_{\min})) = \frac{(3 + \alpha_{\min})}{(1 + \alpha_{\min})} > \frac{(1 - p(0,0))}{(1 + \alpha_{\min})}[3 + \alpha_{\min}] > \frac{2F(2 + \alpha_{\min})^2}{\omega \alpha_{\min}^2}
\]
and therefore that $\tilde{q}(\alpha_{\min}) < \hat{\beta}(\alpha_{\min}) = \hat{q}(\alpha_{\min})$ and this holds as well for $\alpha$ close enough to $\alpha_{\min}$ by continuity.

Similarly
\[
\lim_{\alpha \to \infty} \left[ \Sigma(\tilde{q}(\alpha), \alpha) - \frac{2F(2 + \alpha)^2}{\omega \alpha^2} \right] = \lim_{\alpha \to \infty} \Sigma(\tilde{q}(\alpha), \alpha) - \frac{2F}{\omega} = -\infty
\]
Thus for $\alpha$ large enough, one has $\Sigma(\tilde{q}(\alpha), \alpha) < 0 < \frac{2F(2 + \alpha)^2}{\omega \alpha^2} = \Sigma(\tilde{q}(\alpha), \alpha)$ and therefore $\tilde{q}(\alpha) < \hat{q}(\alpha)$. QED