Centralized versus Over-The-Counter Markets*

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Abstract

The opacity of over-the-counter (OTC) markets – in which a large number of financial products including credit derivatives trade – appears to have played a central role in the ongoing financial crisis. We model such OTC markets in general equilibrium with default as markets for risk sharing where agents’ financial positions are not mutually observable. In this setting, there is excess “leverage” in that parties in OTC contracts take on short positions that lead to levels of default risk that are higher than Pareto-efficient ones. Thus, OTC markets feature a counterparty risk externality that we show can lead to ex ante productive inefficiency. This externality is absent when trading is organized via a centralized counterparty, e.g., an exchange observing all trades.
1 Introduction and motivation

Most financial contracts are arrangements between two parties to deliver goods or cash in amounts and at times that depend upon uncertain future events. Designed appropriately, financial contracts facilitate risk-sharing in the economy. There may be many risks in such contracts, but one additional risk to be evaluated at the time of contracting is the risk that the counterparty will not fulfill its future obligations. This counterparty risk is difficult to evaluate because the exposure of the counterparty to various risks is generally not public information. Contractual terms such as prices, interest rates and collateral that affect the terms of trade can be tailored to mitigate counterparty risk, but the extent to which this can be achieved, and how efficiently so, depends in general on how contracts are traded.

One possible trading infrastructure is an over-the-counter (OTC) market in which each party trades with another, subject to a bankruptcy code that determines how counterparty defaults will be resolved.\(^1\) A key feature of OTC markets is their opacity. In particular, even within a set of specific contracts, for example, credit default swaps, no trading party has full knowledge of positions of others. We show theoretically in this paper that such opacity of exposures in OTC markets leads to an important risk spillover – a “counterparty risk externality”\(^2\) – that leads to excessive “leverage” in the form of short positions that collect premium upfront but default ex post and result in inefficient levels of risk-sharing.

Counterparty risk externality is the effect that the default risk on one contract will be increased if the counterparty agrees to the same contract with another agent because the second contract increases the probability that the counterparty will be unable to perform on the first one. Put simply, the default risk on one deal depends on what else is done. The intuition for our result concerning the inefficiency of OTC markets is that in OTC markets it is not at all transparent what else is being done. Hence, counterparties cannot charge price schedules that effectively penalize the creation of counterparty risk. This makes it likely that excessively large short positions will be built by some institutions without the full knowledge of other market participants. In general, when such institutions were to default, their counterparties would

\(^1\)The bankruptcy code may be specified in the contract or adhere to a uniformly applicable corporate bankruptcy code.

\(^2\)The term “counterparty risk externality” is as employed by Acharya and Engle (2009). A part of the discussion below, especially related to A.I.G. is also based on that article.
also incur significant losses, creating systemic risk in the economy or more formally, inefficient ex-ante risk-sharing.

For example, in September 2008, it became known that A.I.G.’s liquidity position was inadequate given that it had written credit default swaps (bespoke CDS) for many investors guaranteeing protection against default on mortgage-backed products. Each investor realized that the value of A.I.G.’s protection was dramatically reduced on its individual guarantee. Investors demanded increased collateral – essentially posting of extra cash – which A.I.G. was unable to provide and the Treasury had to take over A.I.G. The counterparty risks were so widespread globally that a default would probably have spurred many other defaults generating a downward spiral.

The A.I.G. example illustrates well the cost that large OTC exposures can impose on the system when a large institution operating in OTC markets defaults on its obligations. But, more importantly, it also raises the question of whether A.I.G.’s true risk as a counterparty was reflected by investors in prices and risk controls for protections they purchased from A.I.G. We argue that the opacity of the OTC markets in which these credit derivatives trade was primarily responsible for allowing the build-up of such large exposures in the first place.

While a number of financial innovations in fixed income and credit markets have traded until now in OTC markets, many products linked to commodity and equities have traded successfully on centralized exchanges. A distinguishing feature of centralized exchange relative to OTC trading is that even though individual agents still do not see each others’ trades, there is a centralized counterparty – the exchange – that sees all trades (at least on all products traded on that particular exchange). Crucially, this enables the exchange to offer individual parties pricing schedules (e.g., collateral arrangements) for trades that are contingent not just on observable or public characteristics (e.g., credit ratings) but also on its own knowledge of other trades (e.g., net positions in CDS contracts).

We show formally that when trading is organized in the form of a centralized exchange, the conditioning of contract terms for each party on its overall positions is sufficient to get that party to internalize the counterparty risk externality of its trades. In other words, the moral hazard that a party wants to take on excessively short positions – collect cash today and default tomorrow – is counteracted by the fact that they face a steeper price schedule by so doing. Effectively, centralized trading creates an intermediary for whom agents’ trades are not opaque and this is sufficient to achieve the efficient
risk-sharing outcome.

1.1 Model and results

We derive these results in a competitive general equilibrium (GE) model with two periods but allowing for the possibility of default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005). There is a single financial asset, which can be interpreted as a contingent claim on future states of the world, and agents can take long or short positions in the asset. Trades are collateralized by agents’ endowments. When an agent has short positions that cannot be met by the pledgeable fraction of endowment, there is default. The possibility of default (the option to exercise limited liability, to be precise) implies that long and short positions do not yield the same payoff and indeed that there is counterparty risk in trading. We assume a natural bankruptcy rule that illustrates why counterparty risk potentially arises in such a setting. In particular, in any given state of the world, the payoff to long positions is determined pro-rata across positions based on delivery from short positions.\(^3\) This rationing of payments implies that each trade imposes a payoff externality on other trades. We call this spillover as “counterparty risk externality.”

In this setup, we consider two trading structures and ask whether counterparty risk externality leads to inefficient risk-sharing. For pedagogical purposes, we start with the centralized exchange in which a competitive central counterparty observes all trades and sets pricing schedules for agents that are conditional on this knowledge. We show that the competitive equilibrium in the economy with a centralized exchange is constrained Pareto efficient.

Next, we consider the OTC setting in which trades are not mutually observed and thus pricing schedules faced by agents are not conditional on their other trades (even though they might be conditioned on public information about their type, e.g., their endowment level or equivalently their credit rating). We study two different cases, one in which a centralized bankruptcy mechanism operates to distribute the cash flow delivered on the short positions of the asset pro-rata with respect to the long positions and one in which the bankruptcy mechanism is bilateral. We show that, in either case,

\(^3\)In particular, in case of a centralized bankruptcy mechanism, delivery from short positions is the sum total of full payments from non-defaulting parties and partial payments from defaulting parties; whereas in case of a bilateral bankruptcy mechanism, delivery on short positions is just the partial payment from the defaulting counterparty.
The competitive equilibrium in the OTC economy is generically inefficient compared to constrained Pareto efficient outcome.

The inefficiency in the OTC setting manifests as excessively large short positions as agents do not internalize the default risk these positions impose on other trades in the economy. Intuitively, as long as there is a “risk premium” on the insurance contract (e.g., because the risk being insured is aggregate in nature) and/or the costs of defaulting are not excessively large, the insurer perceives a benefit from building up short positions and defaulting ex post. We interpret this outcome as characterizing excessive “leverage" from an ex-ante standpoint. Interestingly, this implies a lower cost of insurance per unit of promised insurance payoff since the realized insurance payoff is smaller when insurer is more likely to default. In our model, we capture the resulting inefficiency in the form of deadweight costs of bankruptcy, but more generally, it could also manifest as excessive systemic risk from an ex-post standpoint.

Put together, these results imply that centralized markets such as an exchange are an efficient regulatory response to the moral hazard that in the absence of perfect observability of trades, agents have incentives to take on short positions that allow them to consume today and default tomorrow.

Our analysis also makes it precise that it is the opacity or lack of transparency of the OTC markets that leads to ex-ante inefficiency in terms of excessively large short positions or leverage. The inefficiency of OTC in fact obtains with a centralized bankruptcy mechanism as well as with a bilateral bankruptcy resolution mechanism.

As an extension, we allow agents to alter their production schedules. In this case, the moral hazard of excessive leverage in the OTC case translates into an additional inefficiency in terms of excessive production. This result clarifies that the inefficiency of OTC markets extends beyond just inefficient risk-sharing. An example of this inefficiency could be ex-ante systemic risk. Suppose that there is insurance being provided on economy-wide mortgage default rates. This would carry a significant risk premium, giving rise to perverse insurer incentives to default. Thus, in equilibrium, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of mortgages and the insured would feed the excessive creation of the housing stock backing such mortgages.\footnote{This may be a partial explanation of the role played by credit default swap insurances and A.I.G. in fueling the credit boom preceding the crisis of 2007-09.}
In future, we plan to consider bilateral OTC markets in the presence of a "large" individual agent that effectively observes the trades of all others but whose trades are not seen by others. Such an agent would enjoy monopoly rents in the OTC setting, which in turn would reduce private incentives in the economy to coordinate on a centralized trading platform and achieve Pareto improvement. Furthermore, informal discussions motivated by the model suggest extensions in which excessive leverage and excessive production can lead to a "bubble" in the market for a collateral good (e.g., the housing stock), a subsequent crash upon realization of adverse shocks, and a breakdown of credit markets in those states. Finally, our analysis suggests that the possibility of regulatory forbearance of "too big to fail" positions can result in ex-ante inefficiencies even with centralized exchange trading.

The remainder of the paper is structured as follows. Section 2 provides a simple example of the counterparty risk externality and the insurance provision with default risk. Section 3 presents our basic GE model, the centralized exchange case, the OTC case, the Pareto efficient case, and the welfare analysis. Section 4 discusses several possible extensions of the model. Section 5 considers the policy implications of our model for OTC versus centralized trading. Section 6 relates our work to existing literature. Section 7 concludes.

2 Example

Consider a two-period economy with three types of agents. There are two states of the world at $t = 1$, denoted by Good (G) and Bad (B). The probabilities of these states are $p$ and $(1 - p)$, respectively. Agents’ endowments in the two states are denoted as $w^i(s)$, $i = 1, 2, 3$, and $s = G, B$, and their initial endowments are denoted as $w^i_0$. We assume throughout that initial endowments are large enough that there are no default considerations at $t = 0$. For simplicity, we also assume that

$$w^1(G) > w^2(G) > w^3(G) = 0,$$

and

$$w^1(B) = w^2(B) = 0 < w^3(B).$$

In other words, agents of type 1 and 2 have endowment in good state of the economy, but none in the bad state, whereas agents of type 3 are endowed in the bad state but not in the good state.
The utility of agents of each type satisfies the mean-variance utility structure:

\[ E[u(x_0, x(s))] = x_0 + E(x(s)) - \frac{\gamma}{2} \text{var}(x(s)), \]

where \( x_0 \) is the residual endowment at \( t = 0 \), and \( x(s) \) is the realized endowment at \( t = 1 \), taking account of trades structured at \( t = 0 \) and materialized at \( t = 1 \).

We assume that the only traded contract is an “insurance” (or a credit default protection) which resembles a put option on the bad state of the economy. The contractual payoff of the contract is \( R(G) = 0 \) and \( R(B) > 0 \). For simplicity, we will refer to \( R(B) \) simply as \( R \). Importantly, the economy will allow for default so that the actual payoff on the contract need not coincide with \( R \). The insurance contract must be paid for at \( t = 0 \) and we denote its price as \( q \).

To highlight our main point, we consider agents 1 and 2 purchasing insurance contract from agents 3. We denote the long positions of agents 1 and 2 as \( z_i^0, i = 1, 2 \), and the short position of agents 3 as \( z_3^0 \). Note that the only agents that can default given our assumptions are agents 3. We assume that in case they default, they suffer a linear non-pecuniary penalty as a function of the positions defaulted upon, whose pecuniary equivalent in the bad state is given by \( \epsilon z_3^0 \). Broadly speaking, this penalty can be interpreted as loss of continuation of franchise value in a multi-period setting.

Suppose the realized positions on the long positions in state \( B \) is \( R^+ \leq R \). Then, the \( t = 0 \) payoffs to the three agents are

\[ (x_0^1, x_0^2, x_0^3) = (w_0^1 - z_1^1 q, w_0^2 - z_2^2 q, w_0^3 + z_3^3 q), \]

and \( t = 1 \) payoffs in good and bad states are given respectively as

\[ [x^1(G), x^2(G), x^3(G)] = [w^1(G), w^2(G), 0], \]

and

\[ [x^1(B), x^2(B), x^3(B)] = [R^+ z_1^1, R^+ z_2^2, w^3(B) - R^+ z_3^3 - \epsilon z_3^3 1_D], \]

where \( 1_D \) is an indicator variable which takes on value of one if there is default \( (R^+ < R) \) and zero otherwise.

We assume that trading is over-the-counter (OTC) so that agents do not observe the size of the trades put on by other agents and hence prices
cannot be conditioned on these. In other words, all agents take the price per unit of insurance as a given constant (and not a schedule depending on total insurance sold by agents 3 in the economy). Agents are fully rational, however, and anticipate correctly the likelihood of default, and its consequent effect on the realized payoff on the insurance contract ($R^+$) relative to the promised payoff ($\bar{R}$).

Then, equilibrium in the economy is characterized by the trading positions, the payoff on the insurance contract (involving the possibility of default) and the cost of insurance: $(z^1, z^2, z^3, R^+, q)$, such that

1. Each agent maximizes its expected utility by choosing its trade positions (as we describe below);
2. Market for insurance clears: $z^3 = z^1 + z^2$; and,
3. In case of default, (we assume that) there is pro-rata sharing of agents 3’s total endowment between the long positions of agents 1 and 2:

$$R^+ = \frac{w^3(B)}{z^1 + z^2}.$$

Now, consider agent 1’s maximization problem:

$$\max_{x^1} w_0^1 - z^1 q + pw_1^1(G) + (1 - p)R^+ z^1 - \frac{\gamma}{2} \text{var}(x^1(s)),$$

where

$$\text{var}(x^1(s)) = \rho(1 - \rho)[w_1^1(G) - R^+ z^1]^2.$$

Then, the first-order condition for agent 1 implies that

$$z^1(R^+, q) = \frac{1}{R^+} \left[ w_1^1(G) + \frac{(1 - p)R^+ - q}{\gamma p(1 - \rho)R^+} \right]. \quad (1)$$

Similarly, we obtain for agent 2’s long position that:

$$z^2(R^+, q) = \frac{1}{R^+} \left[ w_2^2(G) + \frac{(1 - p)R^+ - q}{\gamma p(1 - \rho)R^+} \right] \quad (2)$$

In other words, all else equal, agents 1 and 2 purchase more insurance if they have greater endowment in the good state and less so if the cost of insurance rises.
On the other hand, the more insurance agent 3 sells the higher are its incentives to default in state $B$. To clarify agent 3 choice with regards to default, consider first the case in which it cannot default. In this case its problem is

$$\max_{z^3} w_0^3 + z^3 q + (1 - p)[w^3(B) - Rz^3] - \frac{\gamma}{2} p(1 - p)[w^3(B) - Rz^3]^2,$$

which yields

$$z_{ND}^3 = \frac{1}{R} \left[ w^3(B) - \frac{(1 - p)R - q}{\gamma p(1 - p)R} \right]. \quad (3)$$

In the limit for no default costs, $\epsilon = 0$, agent 3, holding the position $z_{ND}^3$ will in fact not default in equilibrium only if

$$w^3(B) \geq Rz_{ND}^3,$$

which turns out equivalent to requiring that $q < (1 - p)R$. This condition has the intuitive interpretation that the insurer has incentives not to default ex post only if the price of insurance is smaller than the expected payoff on the insurance, or in other words, that there is no “risk premium” in the insurance price. This will, however, not hold in equilibrium in general, whenever the insurance is against a risk that cannot be fully diversified away.\footnote{This is potentially an important point as it explains why there is the default moral hazard on part of insurers for credit default swaps which invariably contain at least some portion of aggregate risk (or in the case of A.I.G., almost all portion) since default is inherently a macroeconomic phenomenon. In contrast, there is less risk of such a moral hazard for traditional insurance businesses: Traditional insurances are on risks such as death, accidents, etc., which are easily diversified away across agents in the economy, so that insurers simply earn the actuarially fair premium and do not earn a significant risk premium.}

More generally, let’s consider then the problem of agent 3, the insurer, when we explicitly allow for default at costs $\epsilon > 0$:

$$\max_{z^3} w_0^3 + z^3 q - (1 - p)\epsilon z^3 - \frac{\gamma}{2} p(1 - p)(\epsilon z^3)^2.$$  

Clearly, the insurer pledges the entire endowment in the bad state at $t = 1$ in order to collect as much insurance premium as possible at $t = 0$.\footnote{Note that the no-default condition now takes the form:}

$$w^3(B) \geq (R + \epsilon)z^3.$$
from the first-order condition,

\[ z^3 = \frac{q - (1 - p)\epsilon}{\gamma p(1 - p)\epsilon^2}. \]  

(4)

Substituting for \((z^1, z^2, z^3)\) in the market-clearing and bankruptcy conditions of the equilibrium yields two equations in the realized insurance payoff \(R^+\) and insurance price \(q\) which can be solved to characterize the equilibrium:

\[
R^+(q) = \frac{w^3(B)\gamma p(1 - p)\epsilon^2}{q - (1 - p)\epsilon},
\]

(5)

\[
w^3(B) = w^1(G) + w^2(G) + \frac{2}{\gamma p} - \frac{2q}{\gamma p(1 - p)R^+}.
\]

(6)

We obtain a quadratic equation in the cost of insurance \(q\), which we solve numerically.

### 2.1 Numerical example

We parametrize the above economy with \(w^1(G) = 10\), \(w^2(G) = 5\), and \(w^3(B) = 10\). We set \(p = 0.9\) and vary \(\epsilon\) in the range \([0.1, 1.0]\). Figures 1, 2 and 3 plot respectively the equilibrium quantity of insurance sold \((z^3)\), its realized payoff \((R^+)\), and its price \((q)\), all as a function of \(\epsilon\), the deadweight cost of default.

There is a critical value of \(\epsilon\) below which defaults take place and this value is around 0.548. Above this value, there is no default. Interestingly, for all \(\epsilon\) smaller than this threshold value, the equilibrium is effectively the same as far as risk-sharing is concerned. In particular, agents 3 transfer all their endowment in the bad state at \(t = 1\) to agents 1 and 2.

To be precise, the equilibrium utilities (relative to \(t = 0\) endowments) are \((U^1, U^2, U^3) = (-1.97, -0.84, 1.35)\) regardless of \(\epsilon\) in the default range. However, this is not true of equilibrium quantity of insurance and its price.

For example, when \(\epsilon = 0.5\), the quantities traded are \((z^1, z^2) = (8.22, 2.74)\) with \(z^3 = z^1 + z^2\), there is 9% default on the contract \((R^+ = 0.91)\), and insurance price is \(q = 0.30\).

In contrast, with \(\epsilon = 0.01\), the quantities traded become much larger: \((z^1, z^2) = (410.95, 136.98)\), there is 98% default on the contract \((R^+ = 0.02)\), and insurance price is \(q = 0.01\).
In other words, as the default incentives for agent 3 become stronger, there is greater quantity of insurance sold. Thus, there is greater default and greater deadweight costs suffered by agents 3. In turn, the equilibrium insurance price is smaller too. Default by the insurer lowers the price of insurance since the payoff on the contract is rationally anticipated by those purchasing insurance to be smaller. Put simply, the quality of insurance has gone down given default risk of the insurer.

The potential inefficiency of the equilibrium in example above stems from excessive deadweight costs of agent 3’s bankruptcy. More generally, the high quantity of insurance sold may also be welfare-reducing if insurance has a moral hazard effect on part of agents 1 and 2, in terms of their changing productive investments towards aggregate risky assets.

3 The model

We now formalize the above numerical example for an OTC market with default risk and explain why a centralized exchange improves upon it.

To start with, we develop a general equilibrium (GE) model without production or intermediation, effectively, a two-period GE model with default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005). We then extend it to allow for the different information structures of centralized versus OTC markets, is enough to illustrate the main issues.

Agents and endowments The economy is populated by \( i = 1, \ldots, I \) types of agents. Let \( x^i_0 \) be consumption of agent \( i \) at time 0. Let \( s = 1, \ldots, S \) denote the states of uncertainty in the economy, which are realized at time 1. State \( s \) occurs with probability \( p_s \), \( \sum_s p_s = 1 \). Let \( x^i_1 \) be agent \( i \)'s consumption at time 1, a random variable over the state space \( S \): \( x^i_1(s) \), for \( s \in S \). Let \( w^i_0 \) be the endowment of agent \( i \) at time 0; and \( w^i_1(s) \) her endowment at time 1 in state \( s \). The utility of agent \( i \) over consumption in state \( s \) is denoted as \( u^i(x^i_0, x^i_1(s)) \) and belongs to the von-Neumann Morgenstern class of expected utility functions.

Financial asset and trading We assume, for simplicity, that only one financial asset is traded in this economy, an asset whose payoff is \( R \), a non-
negative vector in $S$. The payoff $R$ is exogenous. We can imagine it representing a derivative contract, e.g., a credit default swap.

**Default risk** Let $z_i^+$ and $z_i^-$ be the long and short positions, respectively, of agent $i$ in financial markets. Agents selling the asset might default on their required payments. In particular, agent $i$’s short positions are collateralized by the pledgeable fraction $\alpha$ of her endowment at time 1. In other words, in the event of default, creditors (counterparties holding long positions on the asset with the defaulting party) have recourse only to a fraction $\alpha \in [0,1]$ of the debtor’s endowment $w_i^1(s)$. Only a fraction of the debtor’s endowment can be pledged as collateral, for instance because a part of the endowment is agent-specific. Other than the defaulting agent simply losing her collateral to counterparties, default is assumed to have a small direct deadweight cost $\varepsilon z_i^-$ that is proportional on the position defaulted upon. Deadweight costs of default will serve the convenient purpose of providing a bound to short positions on the asset. Note that the recovery for each long position depends on the bankruptcy resolution which we will specify below.

### 3.1 Centralized exchange economy

We first study an economy in which all asset trades are operated by a competitive centralized “exchange.” In essence, the exchange is a centralized counterparty that observes all trades and conditions contract terms for individual agents on these trades. First, we explain how the exchange resolves default, the default condition for each agent, and next, how the exchange conditions contract terms taking account of incentives of agents to default.

**Bankruptcy resolution** We assume that no creditor has direct privileged recourse to a debtor’s collateral, in case of default. The centralized exchange, on the other hand, has full recourse to the debtors’ pledgeable collateral. Furthermore, the exchange operates as a bankruptcy mechanism, by distributing the cash flow of the short positions of the asset, pro-rata with respect to the long positions. To be precise, there are short positions that deliver on the contracts in a given state of the world, and there are others that default. At equilibrium, the sum total of these cash flows is distributed pro-rata among the holders of long positions. In other words, the exchange can be interpreted as guaranteeing all trades but requiring that those members of the
exchange who benefit from the guarantee on any given default make capital contributions to cover the exchange’s cost of providing the guarantee.

**The default condition** An agent of type $i$ with (long, short) portfolio position $(z^i_+, z^i_-)$ will default in period 1 in state $s$ if her income after assets pay off is smaller than the non-pledgeable fraction of her endowment. Let $R_+(s)$ denote the payoff in state $s$ of her long asset portfolio. The payoff $R_+(s)$ is taken as given by each agent, though it is endogenously determined at equilibrium, depending on the default rate in the economy at equilibrium (as shown later).

Then, agent $i$ defaults on its short positions iff:

$$w^i_1(s) + R_+(s)z^i_+ - R(s)z^i_- < (1 - \alpha) w^i_1(s) - \varepsilon z^i_-.$$  \hspace{1cm} (7)

Let $I^d(z_+, z_-; i, s)$ be an indicator variable taking on value one if agent $i$ with the portfolio $(z_+, z_-)$ will default at equilibrium in state $s$, and zero otherwise.\footnote{If $R_+(s) \geq 0$ and hence any agent $i$ defaulting for some $s$ must have $z^i_- > 0$, or in other words, have some short position in the asset. In other words, $I^d(z_+, 0; i, s) = 0$.}

$$I^d(z_+, z_-; i, s) = \begin{cases} 
1 & \text{if } w^i_1(s) + R_+(s)z^i_+ - R(s)z^i_- < (1 - \alpha) w^i_1(s) - \varepsilon z^i_- \\
0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (8)

Finally, let $I^{nd}(z_+, z_-; i, s) = 1 - I^d(z_+, z_-; i, s)$.

**Equilibrium payoffs on long and short positions:** Since all long positions share pro-rata the payments from defaulting and non-defaulting short positions, the equilibrium payoff of long positions in state $s$, depends upon $(z_+, z_-)$, the vectors of long and short positions of agents, respectively. We denote the payoff as $R_+(z_+, z_-; s)$, and it is given by

$$R_+(z_+, z_-; s) = \sum_i \left[ \alpha w^i_1(s) \cdot I^d(z^i_+, z^i_-; i, s) + R(s)z^i_- \cdot I^{nd}(z^i_+, z^i_-; i, s) \right] .$$  \hspace{1cm} (9)

Note that in case of autarky, that is, with no trading ($\sum_i z^i_+ = \sum_i z^i_- = 0$), we assume that $R_+(z_+, z_-; s) = R(s)$.\footnote{If $R_+(s) \geq 0$ and hence any agent $i$ defaulting for some $s$ must have $z^i_- > 0$, or in other words, have some short position in the asset. In other words, $I^d(z_+, 0; i, s) = 0$.}
Importantly, while the payout on long position is identical for all agents, this is not the case for short position of agents. In particular, the equilibrium payout of any short position by agent \( i \) can be computed as follows: for agent \( i \) with trading positions \((z^i_+, z^i_-)\), the payoff on short position is:

\[
R^i_-(z^i_+, z^i_-; s) = \begin{cases} 
\frac{\alpha w^i(s)}{z^i_-} & \text{if } \text{I}^d(z^i_+, z^i_-; i, s) = 1 \\
R(s) & \text{otherwise}
\end{cases}
\]  

(10)

**Prices**  We assume that prices are set in a competitive manner. Specifically, agents are price-takers and so is the exchange (for instance because many could operate the same exchange or because there are many market-makers performing the role of the exchange).

Since the payoff on the long positions is symmetric across agents, the pricing for long positions is straightforward to describe. Taking as given the return on her long portfolio, \( R_+(s) \), an agent \( i \) faces a given bid price \( q_+ \).

However, the payoff on the short positions is not symmetric across agents. This requires us to modify the price-taking assumption on short positions in an important manner (that is similar in spirit to modifications in Acharya and Bisin, 2008, and Bisin, Gottardi and Ruta, 2009). Note that agents holding short positions on the asset might default on their required payments, and their default condition depends on their type (endowment) and on their total trading positions. Since the exchange observes agents’ types and total trading positions, it offers a different ask price to different agents, reflecting the probability of default implied by their characteristics, that is, their endowment as well as their trading positions. Consequently, the exchange offers and in turn the agents face the ask price map \( q^i_-(z^i_+, z^i_-) \).

**Budget constraints**  The budget constraints of agent \( i \) are then given by the following equations for consumption at date 0 and in states \( s \in S \) at date
\[
x^+_i + q^+_i z^+_i - q^-_i (z^+_i, z^-_i) z^-_i = w^+_0
\]
\[
x^+_i (s) = \max \left\{ w^+_1 (s) + R_+ (s) z^+_i - R (s) z^-_i, (1 - \alpha) w^+_1 (s) - \varepsilon z^-_i \right\}
\]
where \( z^+_i, z^-_i \geq 0 \).

The agents’ demand functions for going long and short on the asset, \((z^+_i, z^-_i)\), are obtained from their optimization, taking the price \(q^+_i\) and the price map \(q^-_i (z^+_i, z^-_i)\) as given. Note that, typically, agents will have both long and short positions in their portfolios, as the payoffs of long and short positions are not perfectly collinear, because of the default option embedded in short positions.

**The exchange’s problem** We turn next to the decision problem of the competitive centralized exchange, which controls the supply of the asset to agents. Let the supply offered by the exchange to agent \(i\) for long and short positions be denoted \((\theta^+_i, \theta^-_i)\), and let \((\theta^+_i, \theta^-_i) = \left( \begin{array}{c} \ldots \\ \theta^+_i \\ \ldots \\ \theta^-_i \\ \ldots \\ \end{array} \right)\) denote the vector of the exchange’s offered supplies.

Then, given the supplies, the exchange can compute the cash flow of the long positions of agents: \(^9\)
\[
R_+ (\theta^+_i, \theta^-_i; s) = \sum_i \left[ \alpha w^+_1 (s) \cdot I^d (\theta^+_i, \theta^-_i; i, s) + R (s) \theta^+_i \cdot I^{nd} (\theta^+_i, \theta^-_i; i, s) \right] \sum_i \theta^+_i
\]
with \(R_+ (\theta^+_i, \theta^-_i; s) = R (s)\) in case of autarky when there is no trading \((\sum \theta^+_i = \sum \theta^-_i = 0)\).

Let
\[
m^i (s) = MRS^i (s) \equiv p_i \frac{u^i_1 (x^+_0, x^+_1 (s))}{u^i_0 (x^+_0, x^+_1 (s))}
\]
\(^8\)To avoid encumbering the notation, we assume here and in the rest of the paper that each agent of type \(i\) will, in equilibrium choose the same portfolio positions \((z^+_i, z^-_i)\). In fact, existence of an equilibrium might require allowing identical agents to choose different portfolios to guarantee enough continuity to aggregate demand.

\(^9\)To avoid an even more cumbersome notation, we avoid an explicit distinction between the individual portfolio of an agent \(i\) and the aggregate portfolio of the agents \(i\) trading with the exchange, which are the same at equilibrium (which implicitly imposes symmetry).
denote the marginal rate of substitution between date 0 and state \(s\) at date 1 for agents of type \(i\) at equilibrium, where \(u_i'\) is the first derivative of the utility function \(u_i(x_0^i, x_1^i)\) with respect to consumption at date \(t\).

For any long and short positions offered, \((\theta_+, \theta_-)\), the exchange prices a unitary long position as \(\max_i E \left( m^i R_+ (\theta_+, \theta_-) \right)\), taking as given the stochastic discount factor of the agent \(i\) who values it the most at the margin, that is the agent who would acquire it if offered. Note that \(m^i\) is an \(S\)-dimensional vector (formally defined below while setting up the competitive equilibrium).

Similarly, given \((\theta^i_+, \theta^i_-)\), the exchange can compute the cash flow of the short position of agent \(i\) as:

\[
R^i_- (\theta^i_+, \theta^i_-; s) = \begin{cases} \\
\frac{\alpha w^i(s)}{\theta_-^-} \text{if } I^d(\theta^i_+, \theta^i_-; i, s) = 1 \\
R(s) \text{ otherwise} 
\end{cases}
\]  

(14)

Then, the exchange prices a unitary short position of agent \(i\) as:

\[
q^i_- (\theta^i_+, \theta^i_-) = E \left( m^i R^i_- (\theta^i_+, \theta^i_-) \right)
\]  

(15)

taking as given the stochastic discount factor \(m^i\) of the agent \(i\) whom the position is offered to. Again, note that \(m^i\) is an \(S\)-dimensional vector (formally defined below while setting up the competitive equilibrium).

To summarize, a competitive exchange takes as given the stochastic discount factors \(\{m^i_-\}\) which price respectively at equilibrium, the long positions of the asset and the short positions of the asset of agent \(i\). Crucially, the exchange anticipates the compositional effects on default risk of portfolios of different agent types, that is, it recognizes how each agent \(i\)'s incentives to default are affected by its positions \((\theta_+, \theta_-)\) and how that affects the payoffs on the long position \((R_+ (\theta_+, \theta_-))\) and the short positions of each agent \((R_-^i (\theta^i_+, \theta^i_-))\).

Thus, the exchange solves the following problem:

\[
\max_{\{\theta^i_+, \theta^i_-\}} \sum_i \left[ E \left( m^i R_+ (\theta^i_+, \theta^i_-) \right) \theta^i_+ \right. - \left. E \left( m^i R^i_- (\theta^i_+, \theta^i_-) \right) \theta^i_- \right]
\]  

(16)

s.t.

\[
\sum_i \left( \theta^i_+ - \theta^i_- \right) = 0.
\]  

(17)
Competitive equilibrium  At competitive equilibrium, the portfolios demanded by the agents are offered by the competitive exchange and markets clear:

$$\theta_+^i = z_+^i, \quad \theta_-^i = z_-^i, \quad \forall i,$$  \hspace{1cm} (18)

and the price maps and returns anticipated by agents are consistent with those perceived by the exchange:

$$q_+ = \max_i E \left( m_i^i R_+(z_+^i, z_-^i) \right),$$ \hspace{1cm} (19)

$$q_-^i(z_+^i, z_-^i) = E \left( m_i^i R_+^i(z_+^i, z_-^i) \right), \quad \text{and}$$ \hspace{1cm} (20)

$$R_+(s) = R_+(z_+^i, z_-^i; s).$$ \hspace{1cm} (21)

3.2 OTC markets

Let us now assume that trading does not occur on a centralized exchange (or any exchange for that matter), but is instead intermediated in Over The Counter (OTC) markets. We model OTC markets as standard competitive markets with no centralized exchange. We consider in turn two different bankruptcy resolution mechanisms, a centralized one and a bilateral one. However, we first derive welfare properties of the OTC markets with centralized bankruptcy and compare them to centralized exchange, and then introduce the bilateral bankruptcy. This sequence is for pedagogical reasons as well as to highlight that it is not the bilateral nature of OTC markets that is the driving force behind the welfare analysis.

3.2.1 OTC markets with centralized bankruptcy

We assume that no creditor has privileged recourse to a debtor’s collateral in case of default. Nonetheless, a bankruptcy mechanism operates to distribute the cash flow delivered on the short positions of the asset (full cash flow or endowment recovered in case of default) pro-rata with respect to the long positions. Thus, as far as bankruptcy resolution is concerned, OTC markets with centralized bankruptcy are comparable to a centralized exchange.

Prices  As before, let $z_+^i$ and $z_-^i$ be the long and short positions, respectively, of agent $i$ in the asset. The important point is that OTC markets have no centralized clearing mechanism (nor a centralized registry) and hence the
trades of each agent $i$, $(z^i_+, z^i_-)$, are *not* observed in OTC markets by other agents. Thus, from an ex-ante standpoint, no agent in OTC markets can “net” other agents’ positions or offer to an agent $i$ a price schedule that reflects the default probability implied by her overall positions. Note that the bankruptcy mechanism allows for such netting in case of default ex post, when it is assumed that all trades are revealed, but absent observation of trades ex ante, such netting cannot be conditioned on trades while setting prices.

However, long and short positions will in general still be associated with distinct prices, $q_+$ and $q_-$ respectively. The ask price depends on the agent’s type $i$, as the type determines agent’s endowment which is public knowledge and affects her probability of default. Importantly though, the ask price for agent $i$ is *not* conditioned on her trades. This is the primary distinction between OTC and centralized markets: contract terms (prices, interest rates, collateral requirements, etc.) are not conditioned on agents’ trades in the case of OTC markets whereas they are in case of a centralized exchange.

**Budget constraints** Any agent $i$ takes as given the price and return of her long positions in the asset. The budget constraints of agent $i$ are thus given by:

$$x^i_0 + q_+ z^i_+ - q_-' z^- = w^i_0$$

$$x^i_1(s) = \max \{ w^i_1(s) + R_+(s) z^i_+, R(s) z^-_+, (1 - \alpha) w^i_1(s) - \varepsilon z^-_+ \}$$

where $z^i_+, z^-_+ \geq 0$.

**Competitive equilibrium** At the competitive equilibrium, financial markets clear:

$$\sum_i z^i_+ - z^-_+ = 0.$$  (23)

Furthermore, at equilibrium, the payoff $R_+(s)$ must satisfy the condition:

$$R_+(s) = R_+(z_+, z^-; s) = \frac{\sum_i \left[ \alpha w^i_1(s) \cdot I^d(z^i_+, z^-_+; i, s) + R(s) z^-_+ \cdot I^{nd}(z^i_+, z^-_+; i, s) \right]}{\sum_i z^i_+}, \forall s$$  (24)
with $R_+(s) = R(s)$ in case of autarky ($\sum_i z^i_+ = \sum_i z^i_- = 0$).

Equilibrium prices are such that

$$q_+ = E(m_+ R_+), \text{ for } m_+ = m^{i_+} \text{ with } i_+ \in \arg \max E(m^i R_i)$$  \hspace{1cm} (25)

$$q_- = E(m^i R^i_-(s))$$ \hspace{1cm} (26)

with

$$R^i_-(s) = R^i_+(z^i_+, z^i_-; s) = \begin{cases} \frac{\alpha w^i(s)}{z^i_-} i f \ I^d(z^i_+, z^i_-; i, s) = 1 \\ R(s) \text{ otherwise} \end{cases}$$ \hspace{1cm} (27)

### 3.3 Welfare

How does the competitive equilibrium under OTC markets compare in terms of efficiency properties to the competitive equilibrium under centralized exchange? To answer this question, we write down the constrained Pareto efficient outcome as the solution to the following problem:

$$\max_{(x^i_0, x^i_1, z^i_+, z^i_-)} \sum_i \lambda^i E \left( u^i(x^i_0, x^i_1) \right)$$ \hspace{1cm} (28)

s.t.

$$\sum_i x^i_0 - w^i_0 = 0$$ \hspace{1cm} (29)

$$\sum_i x^i_1 - w^i_1 = 0,$$ \hspace{1cm} (30)

$$x^i_1(s) = \max \{w^i_1(s) + R_+(s) z^i_+ - R(s) z^i_-, (1 - \alpha) w^i_1(s) - \varepsilon z^i_- \}, \forall i, s$$ \hspace{1cm} (31)

$$R_+(s) = \frac{\sum_i \alpha w^i_1(s) \cdot I^d(i, s) + R(s) z^i_- \cdot I^{nd}(i, s)}{\sum_i z^i_+}$$ \hspace{1cm} (32)

where $\lambda^i$ is the Pareto weight associated to agents of type $i$, $I^d(i, s)$ is the indicator variable corresponding to default of agent $i$ in state $s$, as before $I^{nd}(i, s) = 1 - I^d(i, s)$, and $R_+(s) = R(s)$ if $\sum_i z^i_+ = \sum_i z^i_- = 0$.

This is the standard constrained efficiency problem for a GE economy once it is assumed that default is not controlled by the planner. The constraint

$$x^i_1(s) = \max \{w^i_1(s) + R_+(s) z^i_+ - R(s) z^i_-, (1 - \alpha) w^i_1(s) - \varepsilon z^i_- \}, \forall i, s$$ \hspace{1cm} (34)
serves two purposes:

(i) it restricts the planner’s allocations to those that can be achieved with the limited financial instruments available in the economy; and

(ii) it accounts for the fact that each agent can choose to default, in each state $s$:


default state $s$ is $(1 - \alpha) w_i^0(s) - \varepsilon z_i^\perp$, the non-pledgeable fraction of endowment net of the deadweight costs.

3.4 Results

We can derive the following results on the constrained efficiency of the centralized exchange economy and the (generic) constrained inefficiency of the economy with OTC markets:

**Proposition 1.** Any competitive equilibrium of the centralized exchange economy is constrained Pareto optimal.

The intuition for efficiency of the centralized exchange economy is that each agent $i$ that is short on the asset faces a price $q_i^+(z_i^\perp, z_i^\perp)$ that is conditioned on her positions. Consequently, she internalizes the effect of her default on the payoff of long positions on the asset $R_i^+(s)$. The observability of all trades by the exchange and its conditioning of prices based on this information enables the economy with default risk to get agents to internalize the costs they impose (in terms of inefficient risk-sharing) on other agents due to positions that lead to “excessive" defaults ex post.

In striking contrast,

**Proposition 2.** Competitive equilibria of the centralized exchange economy cannot be robustly supported with OTC markets.$^{11}$ More specifically, any competitive equilibrium of the centralized exchange economy in which default occurs with positive probability cannot be supported with OTC markets.

The intuition is that in OTC markets, each agent $i$ that is short on the asset faces a price $q_i^+$ that is not conditioned on her positions. Consequently, she

\begin{equation}
 u^i(x_i^0, x_i^1(s)) \geq u^i(x_i^0, (1 - \alpha) w_i^1(s)).
\end{equation}

\footnote{Formally, by robustly we mean: for a open set of economies parametrized by agents endowments and preferences.}

$^{10}$Formally, the constraint includes the incentive compatibility constraint for each agent’s choice of default:

\begin{equation}
 u^i(x_i^0, x_i^1(s)) \geq u^i(x_i^0, (1 - \alpha) w_i^1(s)).
\end{equation}

$^{11}$Formally, the constraint includes the incentive compatibility constraint for each agent’s choice of default:
does not internalize the effect of her default on the payoff of long positions on the asset $R_+$. Furthermore, let the leverage of agent $i$, $L^i$, be defined as the value of her (promised) debt divided by the value of her endowment

$$L^i = \frac{E(m^i R z_i^i)}{E(m^i w^i_1)}.$$ 

Then,

**Proposition 3.** *For deadweight costs $\varepsilon$ small enough, any competitive equilibrium of the OTC markets economy is characterized, by weakly greater (and robustly by strictly greater) leverage and default with respect to centralized exchange economies.*

Since ask prices in OTC markets do not penalize the short positions for their own incentives to default, agents have incentives to exceed the Pareto efficient short positions. Indeed, the proof of these main propositions in the Appendix shows that as long as price on the short position is positive, which is robustly the case in equilibrium, there is incentive to go excessively short, collect the premia up front, and default ex post. This increases the equilibrium default rate and leads to inefficient risk-sharing. In particular, for efficient risk-sharing, it is in general necessary to be able to commit to future payoffs on financial assets, but in OTC markets, such commitment is not enforced through prices and incentives to go excessively short and default dilute the claims of shorting agent’s counterparties.

**Remark.** If $\varepsilon = 0$, $z_+^i$ is unbounded and, strictly speaking, the economy has no equilibrium. This is just a extreme case, which is of interest to identify the “force” towards borrowing and default built into our model of OTC markets. Positive deadweight costs, $\varepsilon > 0$, guarantee existence.

Finally, it is also the case that a competitive equilibrium of the OTC markets economy with centralized bankruptcy is robustly not constrained Pareto efficient.

The proof of this statement, however, requires some complex differential computations and is omitted. It is an adaptation of that in Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis (2001).
Opacity and counterparty risk externality When combined together, Propositions 1, 2, and 3 imply that centralized markets such as an exchange are an efficient response to the moral hazard that in the absence of perfect observability of trades, agents have incentives to take on short positions that allow them to consume today and default tomorrow. Our analysis, especially in Propositions 2 and 3, makes it precise that it is the opacity or lack of transparency of the OTC markets that leads to ex ante inefficiency in terms of excessively large short positions or leverage. We call this inefficiency as “counterparty risk externality” since it stems from counterparty risk, the risk of default of the short party on long positions, and excessive level of such counterparty risk lowers the payoff to all long positions in the economy, constituting an important negative externality on economy-wide risk sharing.

3.5 OTC markets with bilateral bankruptcy

In this section, we formalize that our results on constrained inefficiency of OTC markets are robust to considering a bilateral bankruptcy resolution.

We continue modeling bilateral OTC markets as standard competitive markets with no centralized exchange where no creditor has privileged recourse to a debtor’s collateral in case of default, but a bilateral bankruptcy mechanism operates to distribute the collateral-based recovery on each agent in default, pro-rata with respect to the agent’s creditors. This latter feature is different from a centralized bankruptcy in which recoveries on all defaulting short positions were assumed to be pooled and distributed pro-rata to the long positions.

Trading and prices Agents enter into bilateral contracts. Let $z^+_{ij}$ be long positions of agents of type $i$ sold by agents of type $j$. Let $z^-_i$ be short positions of agents of type $i$ (all short positions are symmetric for the agents shorting the asset, independently of the counterparty). As before, the trades of each agent $i$, $(\{z^+_{ij}\}, z^-_i)$, are not observed in OTC markets by other agents.

Long and short bilateral positions will in general be traded at a price $q^i$, where the apex $j$ denotes the type of the agent in the short position. Importantly though, while the price depends on the type of the shorting agent, it is not conditioned on agents’ other trades, which are not observed.

Budget constraints The budget constraints of agent $i$ are thus given by:
\[ x_0^i + q^j \sum_{j \in I \setminus \{i\}} z_{ij}^j - q^i z_0^i = w_0^i \]
\[ x_1^i(s) = \max \left\{ w_1^i(s) + \sum_{j \in I \setminus \{i\}} R_+^j(s)z_{ij}^j - R(s)z_0^j, (1 - \alpha) w_1^i(s) - \varepsilon z_0^j \right\} \]

where \( z_{ij}^j, z_0^j \geq 0, \forall j \in I \setminus \{i\} \),

and \( R_+^j(s) \) is the payoff to the long positions where the counterparty is agent \( j \).

**Competitive equilibrium** Each agent \( i \) determines the demands for long and short positions by maximizing their objectives, taking as given the payoff of long positions on the asset, \( R_+^j \), for any possible counterparty \( j \in I \setminus \{i\} \).

At equilibrium, however, the payoff \( R_+^j \), for any \( j \in I \), must satisfy the consistency condition:

\[ R_+^j(s) = \begin{cases} \frac{\omega_i^j(s)}{\sum_{j' \in I \setminus \{i\}} z_{j'}^j}, & \text{if } I^d(j, s) = 1; \\ R(s) & \text{otherwise} \end{cases} \]

where \( \forall i \),

\[ I^d(i, s) = 1 \text{ iff } w_1^i(s) + \sum_{j \in I \setminus \{i\}} R_+^j(s)z_{ij}^j - R(s)z_0^j < (1 - \alpha) w_1^i(s) \]

and where \( (z_{ij}^j, z_0^j)_{j \in I \setminus \{i\}} \) are evaluated at equilibrium.

At the competitive equilibrium, furthermore, financial markets clear:

\[ \sum_{j \in I \setminus \{i\}} z_{ij}^j - z_0^j = 0. \]

**Results** We do not formally show the following results, but the bilateral nature of OTC markets considered above does not qualitatively affect any of the welfare properties of OTC markets relative to centralized exchanges. In particular, competitive equilibrium of centralized trading cannot be supported as a competitive equilibrium of bilateral OTC markets; and, for deadweight costs of default that are sufficiently small, bilateral OTC markets feature excessive leverage and default relative to centralized trading.
To summarize, it is the opacity of OTC markets which gives rise to the moral hazard of agents wanting to take excessive leverage through short positions, rather than the bilateral or centralized nature of resolution of bankruptcy.

4 Production risk

In our whole analysis the aggregate endowment of the economy, \( \sum_{i \in I} w^i_0 \) at time 0 and \( \sum_{i \in I} w^i_s \) in each state \( s \in S \), has been kept constant. Consequently, the inefficiency of the institution of over-the-counter markets, has only distributional effects: borrowers have incentives to take on excessive leverage, asset prices rationally reflect equilibrium leverage, and hence borrowing-lending and insurance markets endogenously fail to serve their purpose. Therefore, resources are mis-allocated in equilibrium. Nonetheless, no resources are lost in the aggregate.

This ceases to be the case if we allow for production in the economy, as we proceed to show in this section.\(^\text{12}\) Suppose each agent is endowed with a production function \( f \) which transforms consumption goods at time 0 into consumption goods at time 1. More precisely, consider the following technology. Let \( K = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_A \end{pmatrix} \) denote a capital allocation vector over \( A \) activities (e.g., projects), so that \( k_1 + k_2 + \ldots + k_A = k \). The production function can then be defined as the output in state \( s \) given capital allocation \( K \): \( f(s, K), \forall s.\(^\text{13}\)

Note that, by allowing for multiple technological activities \( (A \geq 2) \), this formulation allows for some control of the agents over the distribution of capital across activities and hence over the probability distribution of outcomes, that is, over production risk.

With this technology in place the equilibrium analysis of centralized exchange and OTC economies is extended to production. For instance, the

\(^{12}\)While we restrict to an economy with “backyard production” on the part of agents, for simplicity, the analysis directly extends to a firm production economy.

\(^{13}\)We assume \( f \) is continuously differentiable, strongly increasing, and strictly quasi-concave.
budget constraints in the economy with a centralized exchange become:

\[ x_0^i + q_+ z_+^i - q_-(z_+^i, z_-^i) z_-^i = w_0^i - k_i \]

\[ x_1^i(s) = \max \left\{ w_1^i(s) + R_+(s) z_+^i - R(s) z_-^i + f(s, K_i), (1 - \alpha) w_1^i(s) \right\} \]

and agent \( i \) chooses a non-negative portfolio \((z_+^i, z_-^i)\) as well as a non-negative capital allocation \( K_i \). Budget constraints in the OTC economies are similarly formulated.

It is easy to show that, in the production economy,

1. A centralized exchange continues to decentralize constrained Pareto efficient allocations; and
2. The generic inefficiency of OTC markets manifests itself with overproduction and excessive production risk taking.

An example of this inefficient risk-taking might have been the effect of credit default swap insurances sold by A.I.G. to a large number of financial firms in the United States and the Europe on tail risk of corporate bond and loan portfolios and mortgage-backed securities. This is tantamount to selling insurance on economy-wide default risk. Our model implies that in such a setting, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of default risk and the insured would feed the excessive creation of the housing stock and corporate assets backing such pools of aggregate default risk.

5 OTC markets and the crisis

We now consider the implications of our model for the debates raised by the financial crisis of 2007-09 on the desirability of over-the-counter versus centralized trading. In particular, analysis of the role played by OTC markets in the ongoing financial crisis has led to several reform proposals.\(^ {14} \) Our theoretical analysis can help provide a normative comparison of these proposals.

For example, Acharya, Engle, Figlewski, Lynch and Subrahmanyam (2009) divide the proposals into requiring a (i) centralized registry with no disclosure to market participants; (ii) centralized counterparty with no disclosure

\(^ {14} \text{See Stulz (2009) for a summary of the dimensions along which OTC markets for credit derivatives likely contributed, and did not contribute, to the crisis.} \)
(except aggregates) to market participants; and (iii) exchange with public disclosure of prices and volumes. Our theoretical analysis makes it clear that a centralized registry by itself is not sufficient as it only gives regulators ex-post access to trade-level information but does not counteract the ex-ante moral hazard of institutions wanting to take on excessive leverage. Both centralized counterparty and exchange suffice on this ground but it is the centralized counterparty that is crucial rather than an exchange; in other words, it is sufficient that one party sees all trades and sets price schedules and risk controls conditional on that information, and it is not necessary to disclose information on all trades to individual agents.

Regulatory reforms announced in March 2009, and since then approved by the House in the United States, involve significant changes to the trading infrastructure of OTC markets, with the objective of reducing systemic risk in the financial sector. Under the proposed reforms, mature and standardized credit derivatives such as the credit default swaps (CDS) and indices linked to the CDS will be traded through a centralized counterparty; there is no proposal yet to mandate that these be traded on an exchange. Regulators will gain unfettered access to information on prices, volumes and exposures from the centralized counterparties, but the proposals do not require that such information be made public. While some aggregate information will be disseminated to all market participants, such as the recent data published by the Depository Trust and Clearing Corporation (DTCC) on all live positions in credit derivatives, full transparency is being required only for regulatory usage.

Our results suggest that these proposed changes are likely to be adequate except that many financial products such as the customized or “bespoke” collateralized debt and loan obligations (CDOs and CLOs) are not amenable to centralized clearing. The OTC markets will surely exist if only to trade these remaining contracts. It might seem that these bespoke products can be ignored from a risk point of view, however nearly all the problem legacy assets are of this type and these we now know are extremely risky systemically. On these assets, our results suggest that further trade-level transparency among market participants will likely be required in at least some form to improve the assessment and pricing of counterparty risk.
6 Related Literature

The bilateral nature of contracts in the OTC markets has been stressed in the recent literature on the subject. Duffie, Garleanu and Pedersen (2005, 2007) focus on search frictions, dynamic bargaining and valuation in OTC markets; Caballero and Simsek (2009) analyze the role of complexity introduced by bilateral connections and their role in causing financial panics and crises; and, Golosov, Lorenzoni, and Tsyvinski (2009) examine what kind of bilateral contracts will get formed when agents have private information about their endowment shocks.

Specifically in the context of insurance provision through financial contracts, the literature (e.g., Duffee and Zhou, 2001, Acharya and Johnson, 2007, Parlour and Winton, 2008) has largely focused on moral hazard on part of the insured due to presence of information frictions, rather than moral hazard on part of the insurer, the latter being the focus of our paper. On this focus, our paper is more closely related to Allen and Carletti (2006), Thompson (2009) and Zawadowski (2009).

Allen and Carletti (2006) consider contagion from insurance sector to the financial sector when there is credit risk transfer, but they do not consider agency-theoretic issues. In contrast, we allow for default incentives of the insurer and model credit risk transfer more generally as risk-sharing through financial contracts in a GE setting. Zawadowski (2009) analyzes counterparty risk in entangled financial systems. The system is “entangled” because banks hedge risks using bilateral OTC contracts but do not internalize the cost of their own failure on other banks (through counterparty risk exposures). As a result of this network externality, banks purchase less insurance against low probability events. Thus, there is less insurance in his model whereas due to moral hazard on part of the insurer, there is in fact excessive insurance in our setup but it is of low quality and entails default by the insurer. Thompson (2009) considers the moral hazard of default on part of the insurer when there is credit risk transfer in the financial sector. His focus is on analyzing how this moral hazard provides incentives to the insured parties to reveal information about their type, so that the two agency problems interact and reduce each others’ adversity.

More specifically on the benefits of OTC versus centralized markets, our analysis did not consider practical issues relating to the extent of netting of positions that is possible when markets are OTC versus when there is a centralized exchange but only for a part of the space of financial products.
Duffie and Zhu (2009) explain that for a centralized exchange for credit default swaps to reduce counterparty risk more than in the OTC setting, it would require netting not just across CDS but also across other products such as interest-rate swaps. In our model, the primary role of the exchange is not to reduce or eliminate counterparty risk but to improve its price by aggregating information on trades.

We conjecture that if there were a centralized registry of positions (centralized or OTC) that is observed by different exchanges and market participants, then the pricing (or collateral) arrangements would be efficient ex ante, and so would be the levels of ex-post default risk. This is related to Leitner (2009)'s result that a clearinghouse-style mechanism, allowing each party to declare its trades and revealing publicly those that hit pre-specified position limits, can prevent agents from promising the same asset to multiple counterparties and then defaulting.

7 Conclusion

We focused on symmetric information about states of the world in our analysis of centralized versus over the counter markets. However, the model can be extended to relax this assumption. In particular, it is possible to add adverse selection to the model, e.g., in the form of unobservable probability distributions over $S$, the uncertainty states at time $1$. The model would combine features of Rothschild and Stiglitz (1976) and Akerlof (1970), with separating equilibria in the economy with centralized exchanges, and excessive lemons trading (in the form of risky short positions) in the case of OTC markets.\footnote{Santos and Scheinkman (2001) have adverse selection as well in their model of competition of exchanges.} It remains an important exercise for future to confirm that the relative inefficiency of OTC markets relative to centralized markets persists in this setting. Indeed, we conjecture that the inefficiency of OTC markets will be exacerbated in a setting with adverse selection.

References


Appendix

Proof of Proposition 1. The proof proceeds by contradiction. Let \((x_0^i, x_1^i, z_+^i, z_-^i)\) denote an equilibrium of the economy with centralized exchange and let \((\tilde{x}_0^i, \tilde{x}_1^i, \tilde{z}_+^i, \tilde{z}_-^i)\) denote the constrained Pareto optimal allocation which dominates the equilibrium allocation. The allocation \((\tilde{x}_0^i, \tilde{x}_1^i, \tilde{z}_+^i, \tilde{z}_-^i)\) must not have been budget feasible. That is,

\[
\begin{align*}
\tilde{x}_0^i + q_+ \tilde{z}_+^i - q_-(\tilde{z}_+^i, \tilde{z}_-^i)\tilde{z}_-^i - w_0^i & \geq \\
x_0^i + q_+ z_+^i - q_-(z_+^i, z_-^i)z_-^i - w_0^i
\end{align*}
\]

with \(>\) for at least one agent \(i\). Summing over \(i\):

\[
\sum_i (\tilde{x}_0^i - x_0^i) + \sum_i q_+ \tilde{z}_+^i - q_-(\tilde{z}_+^i, \tilde{z}_-^i)\tilde{z}_-^i - q_+ z_+^i + q_-(z_+^i, z_-^i)z_-^i > 0
\]

But, since market clearing must hold for both \((x_0^i, x_1^i, z_+^i, z_-^i)\) and \((\tilde{x}_0^i, \tilde{x}_1^i, \tilde{z}_+^i, \tilde{z}_-^i)\), \(\sum_i (\tilde{x}_0^i - x_0^i) = 0\), and we obtain:

\[
\sum_i q_+ \tilde{z}_+^i - q_-(\tilde{z}_+^i, \tilde{z}_-^i)\tilde{z}_-^i - q_+ z_+^i + q_-(z_+^i, z_-^i)z_-^i > 0.
\]

At equilibrium the exchange supplies \(\theta_+^i = z_+^i, \theta_-^i = z_-^i\), for any \(i\), which must generate weakly higher profits than the supply \(\tilde{\theta}_+^i = \tilde{z}_+^i, \tilde{\theta}_-^i = \tilde{z}_-^i\), for any \(i\). At equilibrium, moreover, the exchange internalizes the price maps,

\[
\begin{align*}
q_+ &= \max E \left[ m^i R_+ (\theta_+^i, \theta_-^i) \right], \\
q_- (\theta_+^i, \theta_-^i) &= E \left[ m^i R_- (\theta_+^i, \theta_-^i) \theta_-^i) \right]
\end{align*}
\]

Therefore,

\[
\sum_i q_+ \tilde{z}_+^i - q_-(\tilde{z}_+^i, \tilde{z}_-^i)\tilde{z}_-^i - q_+ z_+^i + q_-(z_+^i, z_-^i)z_-^i > 0
\]

implies

\[
\sum_i E \left[ m^i \left( R_+ (\tilde{\theta}_+^i, \tilde{\theta}_-) \tilde{\theta}_+^i - R_+ (\theta_+^i, \theta_-^i) \theta_+^i \right) - m^i \left( R_- (\tilde{\theta}_+^i, \tilde{\theta}_-) \tilde{\theta}_-^i - R_- (\theta_+^i, \theta_-^i) \theta_-^i \right) \right] > 0
\]

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which contradicts the fact that $\theta^*_+ = z^*_+, \theta^*_- = z^*_-$, for any $i$, maximizes the objective function of the firm. \hfill \square

**Proof of Proposition 2.** Assume $\varepsilon = 0$ and the proof below extends by continuity to $\varepsilon$ sufficiently small.

Let $(z^*_+, z^*_-) \in \mathbb{R}^2$ be the equilibrium portfolio for agent $i$ in a centralized exchange economy. Let $S(i) \subseteq S$ denote the subset of the states of uncertainty in which, at equilibrium, an agent $i$ will default. Then, $S(i)$ is robustly non-empty. Furthermore, if $S(i)$ is non-empty, then $z^*_- > 0$. For any economy such that $S(i)$ is non-empty (and $z^*_- > 0$) for some $i$, at equilibrium of the centralized exchange economy, we must have

$$q^i_-(z^*_+, z^*_-) = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z^*_-} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s).$$

Suppose, by contradiction, that such a competitive equilibrium of the centralized exchange economy can be supported with OTC markets. Then it is necessarily supported by price $q^i_- = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z^*_-} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, such that $q^i_-(z^*_+, z^*_-) \in \mathbb{R}^2$ at the equilibrium portfolio $(z^*_+, z^*_-)$. It is straightforward to see that in this case, at price $q^i_-$ agent $i$ prefers a portfolio $(z^*_+, z^*_- + dz)$, for some $dz > 0$. This is because the marginal valuation of the discounted repayment of a unitary extra short portfolio $dz$, $\sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z^*_-}$, depends negatively on $z^*_-$; while the price obtained at time 0 from the same unitary extra short portfolio, $dz$, $q^i_-$, does not. Since the portfolio $(z^*_+, z^*_- + dz)$ is budget feasible, a contradiction is reached. This is the case for any equilibrium of the centralized exchange economy such that $S(i)$ is non-empty, for some $i$, and hence the contradiction holds robustly. \hfill \square

**Proof of Proposition 3.** Assume $\varepsilon = 0$ and the proof below extends by continuity to $\varepsilon$ sufficiently small.

The “weakly greater" part of the statement is straightforward. We turn to prove the robustly “strictly greater" part. Consider the robust subset of economies for which, at a centralized exchange economy equilibrium, $S(i)$ is non-empty. An argument analogous to the one in the proof of Proposition 2 guarantees that, for these economies, when $\varepsilon$ is small enough, at an equilibrium with OTC markets, $S(i) = S$. Agents $i$, in other words, default in all
states $s \in S$. This proves that default is robustly strictly greater with OTC than with centralized exchange.

Consider such an equilibrium with OTC, to study leverage. At equilibrium it must be that $q_+^i > 0$. Suppose on the contrary that $q_+^i \leq 0$. In this case, we claim agents $i$ would rather choose $z_i^- = 0$ and hence would trivially not default. In fact, if $S(i) = S$, and $q_+^i \leq 0$, agents $i$ would consume

$$x_0^i = w_0^i - q_+^i z_+^i + q_-^i z_-^i$$
$$x_1^i(s) = (1 - \alpha) w_1^i(s) - \varepsilon z_-^i,$$

But it is easy to see that $q_+ \geq 0$, and hence

$$x_0^i \leq w_0^i + q_-^i z_-^i \leq w_0^i$$
$$x_1^i(s) = (1 - \alpha) w_1^i(s) - \varepsilon z_-^i$$

By resorting to autarchy, $z_-^i = z_+^i = 0$, instead agents $i$ would guarantee themselves

$$x_0^i = w_0^i$$
$$x_1^i(s) = w_1^i(s)$$

which they prefer. Prices such that $q_-^i \leq 0$ therefore imply no default. This is the case for all agents of all types $i$. But then $R_+(s) = R(s)$, for all $s \in S$ and $z_+^i$ is robustly $> 0$, for some $i$, for prices satisfying $q_+ = E[mR]$ with $m(s) = \min_i MRS^i(s)$, if $\sum z_+^i = \sum z_-^i = 0$ for any $i$. At an equilibrium with OTC markets, therefore, it must be that $q_-^i > 0$. In this case $z_-^i$ grows unbounded as $\varepsilon \rightarrow 0$. This proves that leverage is robustly strictly greater with OTC markets than with centralized exchange for $\varepsilon$ small enough. ■
Figure 1: The quantity of insurance sold ($z^3$) as a function of the deadweight cost of default ($\epsilon$).
Figure 2: The realized payoff on the insurance ($R'$) as a function of the deadweight cost of default ($\varepsilon$)
Figure 3: The equilibrium price of insurance (q) as a function of the deadweight cost of default (ε)