1 Additional Practice Questions For Introduction to Economic Analysis

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1.1 Rational Choice

For the following choice environments and preference relations, decide if the relation is complete and transitive:

1. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( \max(x_1, x_2) \geq \max(y_1, y_2) \)
2. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( \max(x_1, x_2) > \max(y_1, y_2) \)
3. For \( X \subseteq \mathbb{R} \), \( x \succeq y \) if \( a/ \) \( x \) is a rational number and \( y \) is not or \( b/ \) if neither or both \( x \) and \( y \) are rational but \( x \geq y \)
4. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( x_1 > y_2 \)
5. For \( X \subseteq \mathbb{R} \), \( x \succeq y \) if and only if \( x - y \geq 2 \)
6. For \( X \subseteq \mathbb{R}^3 \), \( x \succeq y \) if and only if \( x_i > y_i \) for 2 of the 3 \( i \)
7. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( x_1 + x_2 = y_1y_2 \)
8. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( \sqrt{(x_1 - 3)^2 + (x_2 - 7)^2} \geq \sqrt{(y_1 - 3)^2 + (y_2 - 7)^2} \)
9. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( x_1 + x_2 \geq y_1 + y_2 \) and \( x_1x_2 \geq y_1y_2 \)
10. For \( X \subseteq \mathbb{R}^2 \), \( x \succeq y \) if and only if \( x_1 + x_2 \geq y_1 + y_2 \) or \( x_1x_2 \geq y_1y_2 \)
11. For \( X \subseteq \mathbb{R}^2_{++} \), \( x \succeq y \) if and only if \( x_1 + x_2 \geq y_1 + y_2 \) and \( x_1x_2 \geq y_1y_2 \)

For the following preferences, determine whether or not there is a utility representation, and if there is, write down two utility functions that represent the preferences:

1. When choosing between bundles of containing both apples and oranges, I always prefer the bundle with the most apples
2. When choosing between bundles containing both apples and oranges, I prefer the bundle with the most items of fruit
3. When choosing between bundles containing both apples and oranges, I prefer the bundle that is closest to having the same number of apples and oranges
4. When choosing between bundles containing both apples and oranges, I flip a coin. If it comes up heads, I prefer the bundle with the most apples. If it comes up tails I prefer the bundle with the most oranges
Solve consumer problems of people with the following utility functions. In each case you will have to construct the budget constraint, assuming that any good \( x_i \) is available at price \( p_i \), and there is \( M \) amount of money available

1. \( u(x_1, x_2, x_3) = x_1^\alpha x_2^\beta x_3^{1-\alpha-\beta} \)
2. \( u(x_1, x_2) = 3 \log x_1 + 2 \log x_2 \)
3. \( u(x_1, x_2) = \min(x_1, x_2) \)
4. \( u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}} \)
5. \( u(x_1, x_2) = x_1 + x_2^4 \)
6. \( u(x_1, x_2) = x_1 + x_2 \)

### 1.2 Equilibrium and Efficiency

Below are utility functions and allocations for different economies. In each case, give an equation for the contract curve and solve for the competitive equilibrium. In each case show explicitly the three steps for solving for a competitive equilibrium (find demand functions, substitute into feasibility and solve for prices, then substitute back into demand function to solve for allocations)

1. \( U_A(x_1^A, x_2^A) = (x_1^A)^\alpha (x_2^A)^{(1-\alpha)}, U_B(x_1^B, x_2^B) = (x_1^B)^\beta (x_2^B)^{(1-\beta)}, (w_1^A, w_2^A) = (3, 7), (w_1^B, w_2^B) = (2, 4) \)
2. \( U_A(x_1^A, x_2^A) = 4 \log x_1^A + 5 \log x_2^A, U_B(x_1^B, x_2^B) = 2 \log x_1^B + \log x_2^B, (w_1^A, w_2^A) = (3, 7), (w_1^B, w_2^B) = (2, 4) \)
3. \( U_A(x_1^A, x_2^A, x_3^A) = 4 \log x_1^A + 5 \log x_2^A + \log x_3^A, U_B(x_1^B, x_2^B, x_3^B) = 3 \log x_1^B + \log x_2^B + 2 \log x_3^B, (w_1^A, w_2^A, w_3^A) = (2, 1, 0), (w_1^B, w_2^B, w_3^B) = (1, 1, 1) \)
4. \( U_A(x_1^A, x_2^A) = 4 \log x_1^A + 5 \log x_2^A, U_B(x_1^B, x_2^B) = \min(x_1^B, 2x_2^B), (w_1^A, w_2^A) = (1, 2), (w_1^B, w_2^B) = (2, 3) \)
5. \( U_A(x_1^A, x_2^A) = 3x_1^A + x_2^A, U_B(x_1^B, x_2^B) = \min(x_1^B, 2x_2^B), (w_1^A, w_2^A) = (1, 2), (w_1^B, w_2^B) = (2, 3) \)

For the following economies, calculate the competitive equilibrium prices and allocations

1. \( U_A(x_1^A, x_2^A) = (x_1^A)^\gamma (x_2^A)^{(1-\gamma)}, U_B(x_1^B, x_2^B) = (x_1^B)^\beta (x_2^B)^{(1-\beta)}, U_C(x_1^C, x_2^C) = (x_1^C)^\gamma (x_2^C)^{(1-\gamma)}, (w_1^A, w_2^A) = (1, 2), (w_1^B, w_2^B) = (2, 1), (w_1^C, w_2^C) = (2, 2) \)
2. \( U_A(x_1^A, x_2^A) = \log x_1^A + 4 \log x_2^A, U_B(x_1^B, x_2^B) = 2 \log x_1^B + 3 \log x_2^B, U_C(x_1^C, x_2^C) = 3 \log x_1^C + 2 \log x_2^C, (w_1^A, w_2^A) = (4, 4), (w_1^B, w_2^B) = (2, 1), (w_1^C, w_2^C) = (2, 2) \)
For the following economy, show that the first welfare theorem doesn’t hold
(in other words, calculate the competitive equilibrium, calculate the set of Pareto
optimal allocations and show that the competitive equilibrium is not pareto
optimal)

1. $U_A(x^1_A, x^2_A) = (x^1_A)\alpha(x^2_A)^{(1-\alpha)} + x^1_A, U_B(x^1_B, x^2_B) = (x^1_B)^\beta(x^2_B)^{(1-\beta)} + x^1_A,
   (w^1_A, w^2_A) = (3, 7), (w^1_B, w^2_B) = (2, 4)$

1.3 Time and Uncertainty

For the following intertemporal utility functions, solve the consumer’s problem
(in other words, solve for the demand of each good in each period as a function of
prices. To do this, you will have to construct a budget constraint for the
agent. Assume that the agent has an income of $w_i$ in each period and that
there is an interest rate $1 + r_i$ between period $i$ and $i + 1$). In each case,
calculate savings as a function of interests rate(s), and decide how savings will
change as the interest rate changes.

1. $U = u(c_1) + \beta u(c_2)$ where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$
2. $U = u(c_1) + \beta u(c_2) + \beta^2 u(c_3)$ where $u(c) = \log c$. (i.e. there are three
   periods)
3. $U = x_1 x_2 + \beta x_1 x_2$. Here there are 2 goods available in each period.
   Assume that, in each time period, the price of $x_1$ is $p_1$ and the price of
   good $x_2$ is $p_2$
4. $U = c_1 + \beta c_2$ (here you should consider three cases, where $\beta$ is greater
   than, the same as, or less than $1 + r$

For question 1 above, assume that there is inflation is 10% between the two
periods (i.e. there is a price $p_1$, $p_2$ for the good in the two periods, and that
$p_2 = 1.10p_1$. First assume that the agents income is denominated in real terms.
What is the equilibrium nominal interest rate. Now assume that the agent’s
income in denominated in nominal terms. What is the agent’s income?

For the following economies, solve for the competitive equilibrium interest
rate(s) and allocations

1. $U_A = u(c^1_A) + \beta u(c^2_A), U_B = u(c^1_B) + \beta u(c^2_B)$, where the instantaneous utility
   functions are logarithmic, and with endowments $(w^1_A, w^2_A), (w^1_B, w^2_B)$
2. As above, but let $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $(w^1_A, w^2_A) = (1, 3), (w^1_B, w^2_B) = (2, 1)$
3. $U_A = u(c^1_A) + \beta u(c^2_A) + \beta^2 u(c^3_A), U_B = u(c^1_B) + \beta u(c^2_B) + \beta^2 u(c^3_B)$, where
   the instantaneous utility functions are logarithmic, and with endowments $(w^1_A, w^2_A, w^3_A), (w^1_B, w^2_B, w^3_B)$
4. \( U_A = \log c_A^1 + \beta \log c_A^2, U_B = c_B^1 + \beta c_B^2 \) and endowments \((w_A^1, w_A^2) = (1, 3), (w_B^1, w_B^2) = (2, 1)\)

Imagine that there are 3 states of the world: A, B and C which occur with probability \(p, q\) and \((1 - p - q)\) respectively. What is the expected value of the following financial instruments? What is the variance of the expected payoff of these financial instruments? What is the covariance between the first financial instrument and each of the others? If the agent was risk neutral, how much would they pay for each of these financial instruments?

1. A share that pays off 1 unit in state A
2. A bond which pays off 1 unit in states A, B and C
3. A share that pays off 1 unit in state A and C
4. Insurance that pays off 1 unit in state B and -1 units in any other state
5. A share that pays off \(\frac{1}{3p}\) in state A, \(\frac{1}{3q}\) in state B and \(\frac{1}{3(1-p-q)}\) in state C

For each of the above assets, write down the expected utility of holding such an asset. If the agent has a logarithmic utility function, what is the expected utility of each asset?

Prove (analytically, I.E. without graphs), that an agent with a concave utility function must be risk averse

\((\text{Difficult})\) Consider an economy with two states of the world and two agents, A and B. In state 1, agent A gets 10 units of good and agent B gets 0 units. In state 2, A gets 0 units and B gets 15 units. Say that there exists a financial instruments which each agent can buy or sell which promises to pay the bearer 1 unit in state 1 at the cost of \(p\) units in state 2. Each agent has a utility function \(u(c) = \frac{c^{1-\sigma}}{1-\sigma}\)

1. Work how many units of the financial instrument each agent will demand as a function of \(p\). (remember this can be positive or negative as the agents are allowed to buy and sell the asset)
2. Solve for the equilibrium value of \(p\). (note that feasibility states that the total demand for the financial asset must be zero, as if one agent buys a unit of the asset, the other must sell it to him)
3. Work out the allocations of each agent in equilibrium
4. Repeat steps 1-3, but this time assume that the utility function of agent B is \(u(c) = \frac{c^{1-\sigma}}{1-\sigma}\)
1.4 Growth

Show that, for a Cobb-Douglas production functions, \( AL_t^\alpha K_t^{1-\alpha} \) the marginal product of capital is increasing in labour and decreasing in capital, and the marginal product of labour is decreasing in labour and increasing in capital.