Introduction to Economic Analysis
Problem Set I

• Question 1
  - Consider two individuals A and B that have same height. Therefore neither A is taller than B, or B is taller than A. Hence this relation is not complete.
  - To verify transitivity, suppose A is taller than B and B is taller than C. Clearly A is taller than C. Hence, this relation is transitive.

Question 2
• \( x \succeq y \) if and only if \( x_1 < y_1 \) and \( x_2 \leq y_2 \).
  - Not complete. Consider the following counter example: \( x = (0, 1) \) and \( y = (1, 0) \). Although \( x_1 < y_1 \), \( x_2 > y_2 \). Therefore \( x = (0, 1) \) and \( y = (1, 0) \) can’t be compared.
  - Transitive. Pick \( x = (x_1, x_2) \), \( y = (y_1, y_2) \) and \( z = (z_1, z_2) \) and suppose that \( x \succeq y \) and \( y \succeq z \) towards showing that \( x \succeq z \). By assumption, \( x \succeq y \) there \( x_1 < y_1 \) and \( x_2 \leq y_2 \) and since \( y \succeq z \), \( y_1 < z_1 \) and \( y_2 \leq z_2 \). That is,
    \[
    x_1 < y_1 \text{ and } x_2 \leq y_2
    \]
    and
    \[
    y_1 < z_1 \text{ and } y_2 \leq z_2
    \]
    Hence,
    \[
    x_1 < z_1 \text{ and } x_2 \leq z_2
    \]
    Therefore, \( x \succeq y \) and \( y \succeq z \) imply that \( x \succeq z \).

• \( x \succeq y \) if and only if \( x_1 \leq y_1 \) and \( x_2 \leq y_2 \).
  - Not complete. Consider the following counter example: \( x = (0, 1) \) and \( y = (1, 0) \). Clearly, neither \( x_i \geq y_i \) for all \( i \) nor \( y_i \geq x_i \) for all \( i \). So, neither \( x \succeq y \) nor \( y \succeq x \). Hence, the bundles \( x = (0, 1) \) and \( y = (1, 0) \) can not be compared.
  - Transitive. Pick \( x = (x_1, x_2) \), \( y = (y_1, y_2) \) and \( z = (z_1, z_2) \) and suppose that \( x \succeq y \) and \( y \succeq z \) towards showing that \( x \succeq z \). By assumption, \( x \succeq y \) then \( x_i \leq y_i \) for all \( i \) and since \( y \succeq z \), \( y_i \leq z_i \) for all \( i \). That is,
    \[
    x_1 \leq y_1 \text{ and } x_2 \leq y_2
    \]
    and
    \[
    y_1 \leq z_1 \text{ and } y_2 \leq z_2
    \]
    Hence,
    \[
    x_1 \leq z_1 \text{ and } x_2 \leq z_2
    \]
    Therefore, \( x \succeq y \) and \( y \succeq z \) imply that \( x \succeq z \).
• \( x \succeq y \iff \text{Max}\{x_1, x_2\} \geq \text{Max}\{y_1, y_2\} \).

  – Complete: pick any \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \). Clearly, either
    \[
    \text{Max}\{x_1, x_2\} \geq \text{Max}\{y_1, y_2\}
    \]
    holds or,
    \[
    \text{Max}\{x_1, x_2\} \leq \text{Max}\{y_1, y_2\}
    \]
    holds or both. Hence, either \( x \succeq y \) or \( y \succeq x \) or both.

  – Transitive: pick any \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \) and suppose that \( x \succeq y \) and \( y \succeq z \). To show that transitivity we need that \( x \succeq z \). Since \( x \succeq y \)
    \[
    \text{Max}\{x_1, x_2\} \geq \text{Max}\{y_1, y_2\}
    \]
    and since \( y \succeq z \)
    \[
    \text{Max}\{y_1, y_2\} \geq \text{Max}\{z_1, z_2\}
    \]
    So, we conclude that
    \[
    \text{Max}\{x_1, x_2\} \geq \text{Max}\{z_1, z_2\}
    \]
    which implies that \( x \succeq z \). Therefore, \( x \succeq y \) and \( y \succeq z \) imply that \( x \succeq z \).

**Question 3**
(Note that in the following I changed the notation a little bit to simplify the solution.)

• In order to find the demanded bundle we need to choose \( x, y \) and \( z \) in order to maximize the utility function given the budget constraint. So we solve the following problem:

\[
\begin{align*}
\max_{x,y,z} & \quad x^a y^b z^{1-a-b} \\
\text{s.t.} & \quad p_1 x + p_2 y + p_3 z = m
\end{align*}
\]

Let's write the Lagrangian:

\[
L = x^a y^b z^{1-a-b} - \lambda (p_1 x + p_2 y + p_3 z - m)
\]

We take derivative wrt \( x, y, z \) and \( \lambda \).

\[
L_x = ax^{a-1} y^b z^{1-a-b} - \lambda p_1 = 0 \quad (1)
\]

\[
L_y = bx^a y^{b-1} z^{1-a-b} - \lambda p_2 = 0 \quad (2)
\]
\[ L_z = (1 - a - b)x^ay^bz^{a-b} - \lambda p_3 = 0 \]  \hspace{1cm} (3)

\[ L_\lambda = -(p_1x + p_2y + p_3z - m) = 0 \]  \hspace{1cm} (4)

From equations (1) and (2),

\[ \frac{ay}{bx} = \frac{p_1}{p_2} \]

or

\[ y = \frac{bp_1x}{ap_2} \]  \hspace{1cm} (5)

From equations (1) and (3),

\[ \frac{az}{(1 - a - b)x} = \frac{p_1}{p_3} \]

or

\[ z = \frac{(1 - a - b)p_1x}{ap_3} \]  \hspace{1cm} (6)

Since \( y \) and \( z \) are solved with respect to \( x \), we can plug equations (5) and (6) in the equation (4) and solve for \( x \):

\[ p_1x + p_2 \frac{bp_1x}{ap_2} + p_3 \frac{(1 - a - b)p_1x}{ap_3} = m \]

Rearranging,

\[ (p_1 + \frac{b}{a}p_1 + \frac{1 - a - b}{a}p_1)x = m \]

Hence,

\[ x = \frac{am}{p_1} \]  \hspace{1cm} (7)

and,

\[ y = \frac{bm}{p_2} \]  \hspace{1cm} (8)

and,

\[ z = \frac{(1 - a - b)m}{p_3} \]  \hspace{1cm} (9)

• A consumer with this utility function spends \( \frac{a}{m}x = a \) fraction of her income on good 1. This does not depend on the income, \( m \), or the price ratios, \( \frac{p_1}{p_2}, \frac{p_2}{p_3} \).