

Managerial Hedging and Equity Ownership¹

Viral V. Acharya
London Business School
and CEPR

Alberto Bisin
New York University

29 January 2003²

¹Acharya is at the Institute of Finance and Accounting, London Business School, 6 Sussex Place, Regent's Park, London - NW1 4SA, United Kingdom. Tel: +44 (0)20 7262 5050 x.3535. Email: vacharya@london.edu. Acharya is also a Research Affiliate of the Centre for Economic Policy Research (CEPR). Bisin is at the Department of Economics, New York University, 269 Mercer St., New York, NY 10003. Tel: (212) 998 8916. Email: alberto.bisin@nyu.edu.

²This paper was earlier titled "Entrepreneurial Incentives in Stock Market Economies." We thank Phillip Bond, Peter DeMarzo, James Dow, Doug Diamond, Douglas Gale, Pierro Gottardi, Denis Gromb, Eric Hilt, Ravi Jagannathan, Li Jin, Kose John, Martin Lettau, Antonio Mello, Gordon Phillips, Adriano Rampini, Raghu Sundaram, Paul Willen, Luigi Zingales, seminar participants at European Finance Association Meetings 2003, Annual Finance Association Meetings 2004, London Business School, Stern School of Business-NYU, Northwestern University, Universiteit van Amsterdam, University of Maryland at College Park, and Washington University-St. Louis for helpful discussions, and Nancy Kleinrock for editorial assistance. We are especially grateful to Yakov Amihud for detailed feedback, to an anonymous referee for comments which have greatly improved the paper, and to Hongjun Yan for excellent research assistance. Bisin thanks the C.V. Starr Center for Applied Economics for technical and financial support.

Abstract

Managerial Hedging and Equity Ownership

Risk-averse managers can hedge the aggregate component of their exposure to firm's cash flow risk by trading in financial markets, but cannot hedge their firm-specific exposure. This gives them incentives to pass up firm-specific projects in favor of standard projects that contain greater aggregate risk. Such risk substitution gives rise to excessive aggregate risk in stock markets and excessive correlation of returns across firms and sectors, thereby reducing the risk-sharing among stock market investors. Managerial ownership of the firm can be designed to mitigate this externality. We characterize the resulting endogenous relationship between the managerial ownership and the extent of aggregate risk in the firm's cash flows, and discuss its implications for existing empirical research.

Keywords: Managerial Compensation, Ownership, Diversification, Hedging, Aggregate Risk, Firm-specific Risk, Financial Innovation, Capital Asset Pricing Model (CAPM).

J.E.L. Classification Code: G31, G32, G10, D52, D62, J33.

1 Introduction

Corporate finance theory suggests that managers and entrepreneurs receive incentive compensation schemes to align their interests with those of the claimants of their firm. Such schemes determine the share of their own firm that managers and entrepreneurs must retain in their portfolios. Accordingly, these schemes restrict managers and entrepreneurs from freely trading their firm and often even correlated firms. However, risk-averse managers and entrepreneurs can, to an extent, enter financial markets and privately hedge their risk exposure to the firm (Bettis, Bizjak, and Lemmon, 1999, Ofek and Yermack, 2000). Indirect evidence shows that managers and entrepreneurs hedge aggregate risk exposure more effectively than firm-specific risk. For instance, Jin (2002) and Garvey and Milbourn (2002) find empirically that the pay-performance sensitivity of incentive contracts falls with the idiosyncratic risk of firm's cash flows but is invariant to the market risk. This finding is consistent with hedging by managers that trade market indexes, but not their firms.

If risk-averse managers can hedge aggregate risk exposure better than firm-specific exposure, they have an incentive to substitute the firm-specific risk of their firm's cash flows for aggregate risk. For example, they may pass up innovative projects with firm-specific risk in favor of standard projects that have greater aggregate risk.¹ This form of risk substitution enables managers to be better diversified, but has perverse implications for aggregate risk-sharing in a general equilibrium context: If all managers in the economy engage in such risk substitution, then the correlation of cash flows of different firms is enhanced, as is, in turn, the aggregate risk in stock markets. This effect, which we call the *diversification externality* of incentive compensation, has not been directly studied. We study the positive and normative effects of this externality by characterizing the optimal contracts that are designed to address it.

We introduce firms in an incomplete-markets, general-equilibrium Capital Asset Pricing Model (CAPM) economy. The fraction of their firm that managers and entrepreneurs retain in their portfolios, i.e., their equity ownership of the firm, is determined contractually. Contractual agreements cannot, however, restrict their trades in aggregate indexes. Once

¹Consider, for example, the following choice faced by the CEO of a pharmaceutical company: The CEO can invest company's funds in R&D activities directed towards the invention of a new drug. Alternatively, the CEO can invest these funds in boosting sales of drugs that already proliferate in the market. The risk from R&D activities is firm-specific. In contrast, the risk from sales of existing drugs is more aggregate in nature: It depends upon factors, such as global demand for drugs, which also affect the profits of other pharmaceutical companies.

The media has often expressed the view that managers tend to pass up risky projects when exposed to the cash flow risk of their firms. For example, the article *Do Large Stakes Inhibit CEOs? – Big holdings may curb risk-taking* (Business Week, May 6, 1996) reports that “[W]hile it's good for top executives to have equity stakes in their company, they may grow excessively cautious if their stakes become too large.”

the ownership structure of firms is designed, agents trade in financial markets and prices are determined. Subsequently, entrepreneurs and managers choose the technology of the firm. Firms can produce a given expected cash flow with a given total risk through the use of different technologies: Some technologies have greater betas with respect to the aggregate risk factor and thus have greater aggregate risk; others have lower betas with respect to the aggregate risk factor and thus have greater firm-specific risk. Technological innovation (modifying the ‘intrinsic’ initial aggregate risk beta of each firm’s project) is costly for entrepreneurs and managers. The resulting aggregate risk beta is not observed by the firm’s investors.

The choice of the firm’s technology potentially introduces a moral hazard. In equilibrium, managers retain a positive share of their own firm in their portfolios, for instance due to unmodelled incentive compensation. Because managers are risk-averse and can hedge only the aggregate risk exposure by trading in market indexes, they have an incentive to increase the aggregate risk beta of their firm’s cash flows: By doing so, they can reduce their exposure to unhedgeable firm-specific risks. Such managerial diversification occurs at the cost of reducing the firm’s market value, since the market price of the firm’s shares decreases in its aggregate risk beta.

We characterize the optimal ownership structure of firms in the face of such moral hazard (diversification externality) and the induced equilibrium risk composition of firms’ cash flows. We show that if the firm’s technology is intrinsically more loaded on the aggregate risk factors of the economy, for example in pro-cyclical industries, then the optimal ownership scheme provides managers with a lower equity holding of their firms. The diversification externality is particularly severe for firms with high intrinsic aggregate risk loadings. Thus, in equilibrium, a smaller managerial ownership share is optimal. Indeed, it may even be optimal for these firms to choose equity holding for managers that is smaller than the market share of the firm, the benchmark holding in the absence of any moral hazard.

The choice of risk composition of firms’ cash flows by managers endogenously affects the level of risk-sharing in the economy. We show that, in equilibrium, managers choose aggregate risk in their firm’s cash flows that exceeds the first-best level. However, the optimal ownership structure of the firm induces a level of aggregate risk in firms that is constrained (second-best) efficient.² That is, the ownership structure is efficient from the point of view of a planner who cannot internalize the diversification externality. Prices in financial markets are not only market clearing, but they also efficiently align the objectives of management and stockholders with those of the constrained social planner: Managers

²At the first-best, the social planner can choose both the technology of firms and their ownership structure. In contrast, at the second-best the social planner designs the firm’s ownership structure, but must let managers and entrepreneurs make technology decisions.

recognize that increasing the aggregate risk of the firm reduces the equilibrium price of the firm's shares; and, in equilibrium, the fraction of the firm's shares retained by managers induces them to choose the constrained efficient firm loadings.

We extend this analysis by first considering the effect of alternative moral-hazard problems, other than the diversification externality, on the equilibrium risk composition of firm's cash flows. We also discuss the empirical implications of this analysis. Next, we consider a setting with multiple sectors, whereby the aggregate risk factor can be interpreted as a stock market index. In this setting, we argue that the diversification externality gives rise to an excessive loading of the firm's stock returns on the index returns, and, in turn, that it generates an excessive correlation of returns across sectors. Finally, we show that the diversification externality is more severe the greater the extent of purely idiosyncratic risk in the firm's cash flows.

We identify a diversification externality that is similar in spirit to that first studied by Amihud and Lev (1981). They consider diversification by firms as a way for risk-averse managers to reduce their exposure to firm-specific risk; see also Lambert (1986). In contrast, we abstract from the incentives of managers to reduce *only* the firm-specific variance and concentrate instead on the incentives to substitute hedgeable components of risk of the firm's cash flows for those that cannot be hedged.

The design of entrepreneurial ownership and managerial compensation under asymmetric information and moral hazard has been examined extensively in the corporate finance literature. Diamond and Verrechia (1982) and Ramakrishnan and Thakor (1984) were the first to analyze moral hazard when the firm returns have systematic and idiosyncratic risks. These papers are cast in partial-equilibrium settings. Few general equilibrium analyses of the ownership structure of firms have been developed. Allen and Gale (1988, 1991) study the capital structure of firms in general equilibrium. However, they do not study economies with moral hazard. Magill and Quinzii (1998) and Ou-Yang (2002) consider the issue of moral hazard between entrepreneurs and investors in a general equilibrium setting. The set-up of these papers follows that of Amihud and Lev (1981): Entrepreneurs can only affect the variance of their firm's cash flows, rather than their correlation with aggregate risk, as in our case.

From a theoretical point of view, our main contribution is to embed the agency-theoretic approach of Fama and Miller (1972) and Jensen and Meckling (1976) into a general equilibrium model of the price of risk, such as the CAPM. In particular, we follow Willen (1997) in introducing incomplete financial markets and restricted participation in the CAPM economy. In addition, we introduce assets in positive net supply to capture a stock market economy.

Sections 2–4 contain the model. Section 5 presents the equilibrium choice of ownership

and risk composition. Section 6 addresses the efficiency of these equilibrium choices. Sections 7 and 8 discuss the empirical relevance of our results. Sections 9 and 10 consider extensions including an analysis of financial innovations that alter the hedging capability of entrepreneurs and managers. Section 11 concludes. Appendices A–C contain the closed-form expressions for the competitive equilibrium, the expression for welfare criterion, and the proofs of propositions, respectively.

2 The Model

We study a perfectly competitive two-period equilibrium economy in which the CAPM pricing rule can be derived. Our analysis is cast in a general-equilibrium setting in order to address the issue of efficiency in economies where the composition of risks is endogenous.

A subset of the agents in the economy, entrepreneurs and managers, make capital budgeting choices: At a private cost, they can choose their firm’s technology and affect the risk composition of cash flows and, hence, stock returns. The CAPM setting enables us to cast the capital budgeting choice faced by entrepreneurs and managers in terms of a choice of betas (i.e., the loadings of cash flows) onto traded risk factors: By choosing the betas of firm cash flows, entrepreneurs and managers determine the proportion of aggregate and firm-specific components in the total cash flow risk of firms.

Capital budgeting choices are affected by the equity ownership structure of the firms. For most of our analysis, entrepreneurs and managers are prohibited from trading the stock of their own firms and others in the same sector, but they can trade other financial assets. This endows entrepreneurs and managers with a preference for components of cash flow risk that can be hedged by trading in financial markets. The ownership structure is, in turn, the result of an optimal-contracting problem between entrepreneurs and investors, or between managers and stockholders. We consider different corporate governance structures and the contracting problems induced under these structures. A governance structure determines whether the firm is originally held by entrepreneurs, as in *owner-managed firms*, or by stockholders, as in *corporations*. In the case of a corporation, the firm is run by managers, that is, the firm is management-controlled. We concentrate on owner-managed firms for most of the paper. We show in Section 7 that our results extend to corporations.

An owner-managed firm is owned ex-ante by an entrepreneur. If the firm’s cash flow betas are observable and the entrepreneur can credibly commit to a choice of these betas when the firm is sold in the stock market, then no moral-hazard concerns arise. Consequently, the entrepreneur’s choice of ownership structure and the cash flow betas are both optimal. If instead the cash flow betas are not observed by the market (i.e., they are private information

of the entrepreneur) and the choice of these betas occurs *after* the firm is sold in the stock market, then the issue of moral hazard arises. In this case, the proportion of the firm that the entrepreneur retains determines the choice of the firm’s cash flow betas. Investors in the market rationally anticipate the mapping between the entrepreneur’s holding of the firm and the choice of betas. Thus, the market price of shares depends upon the publicly observed ownership structure of the firm. Entrepreneurs also realize that the firm’s value will depend on its ownership structure, understanding that discounted prices will be associated with ownership structures that impart incentives to increase the aggregate risk of cash flows.

We introduce formally the simplest version of the model with a representative firm, relegating technical details to Appendix A.

The CAPM Economy with a Firm: The economy is populated by H agents, who live for two periods, 0 and 1. Agent h ’s preferences are represented by a Constant Absolute Risk Aversion (CARA) utility function,

$$u^h(c_0^h, c_1^h) \equiv -\frac{1}{A}e^{-Ac_0^h} - \frac{1}{A}e^{-Ac_1^h}, \quad (1)$$

where c_0^h and c_1^h denote consumption at time 0 and 1, respectively; $A > 0$ is the absolute risk aversion coefficient, which is assumed to be the same for all agents.

Agent 1 in the economy is the representative entrepreneur. The remaining agents, $h = 2, \dots, H$, are the investors. The entrepreneur owns a firm, which has a technology that produces a random, normally distributed cash flow at time 1, y_1^1 , of the unique consumption good. To emphasize that this is the firm’s cash flow, we will often refer to it as y_1^f . The entrepreneur has a private endowment at time 0, y_0^1 , but no private endowment at time 1 save his ownership of the firm. Each investor $h = 2, \dots, H$ has an endowment y_0^h in period 0, and a random, normally distributed endowment y_1^h in period 1.

The economy’s risks are spanned by N orthogonal normally distributed factors, x_n , $n = 1, \dots, N$, $N \geq 2$. The firm’s cash flow is driven by an aggregate risk factor, x_1 , that is positively correlated with the aggregate endowment of investors, $\sum_{h=2}^H y_1^h$; and by a second risk factor, x_2 , that is orthogonal to x_1 and to the aggregate endowment of investors and is interpreted as the “corporate sector-specific” risk in the economy:³

$$y_1^f - E(y_1^f) \equiv \beta_1^f x_1 + \beta_2^f x_2. \quad (2)$$

³Risk factor x_1 is common to both the stock market (the “corporate sector”) and agents’ endowments (for instance, private business income and returns to human capital). For instance, x_1 could represent a general aggregate productivity index. Extending the analysis to account for multiple industrial sectors, as in Section 9.1, allows us to interpret x_1 more naturally as a general stock market index, while x_2 (and x_3 , x_4 , ...) represent the additional risk components of specific sectors.

Without loss of generality, we adopt the normalizations: $E(x_i) = 0$, $\text{var}(x_i) = 1$ for $i = 1, 2$. The firm's betas, β_1^f and β_2^f , measure the covariance of the firm's earnings, y_1^f , with risk factors x_1 and x_2 , respectively:

$$\beta_j^f = \text{cov}(y_1^f, x_j), \quad j = 1, 2. \quad (3)$$

For simplicity we suppose that $\beta_1^f, \beta_2^f > 0$. The betas of investor h , β_1^h and β_2^h , are defined similarly.

There are three financial markets: a riskless bond market, where asset 0 with deterministic payoff of 1 is traded, a market where the aggregate factor x_1 is directly traded, and the stock market where shares of the representative firm f are traded. The bond and the asset paying off the aggregate factor x_1 are in zero net supply. The fraction w of the firm sold in the stock market constitutes the positive supply of the stock. The remaining fraction $(1 - w)$ constitutes the equity ownership of the entrepreneur. If an N -dimensional factor structure drives risk where $N > 2$, then the economy is one of incomplete markets. Trading in financial and stock markets is restricted. In particular, we assume that the entrepreneur, after having placed w shares on the market, cannot trade the stock of his own firm.⁴ However, all agents can trade the riskless bond.

We treat the entrepreneur as a price-taker and the economy as competitive. In particular, we abstract from the ability of entrepreneurs to strategically affect the equilibrium prices. One can interpret the representative entrepreneur as one of a continuum of entrepreneurs. Furthermore, for ease of exposition, we assume a firm's cash flows are driven only by the aggregate and the corporate sector-specific risk factors, and not by any firm-specific risk factor. That is, we treat the representative firm as equivalent to the 'corporate sector' comprised of a continuum of identical firms. Therefore, the restriction that the entrepreneur cannot trade his own firm is a restriction that the entrepreneur cannot trade *any* firm in his sector. In Section 9.2, we distinguish between the firm and the sector by allowing the cash flows of each firm to contain both a sector-specific and a purely firm-specific risk factor. Crucial in this context is that either the entrepreneur cannot hedge his sector-specific risk in financial markets, or else he cannot hedge his firm-specific risk. In particular, to prevent the entrepreneur from diversifying his firm-specific risk, he must not be allowed to trade the idiosyncratic risk of all the other firms in the sector.

⁴We acknowledge that recent evidence in Bettis, Bizjak, and Lemmon (1999) and Ofek and Yermack (2000) suggests that managers might be able to partly circumvent such trading restrictions. We discuss the case in which managers and entrepreneurs can trade their own stock in Section 10.

3 Equilibrium

Our analysis of equilibrium proceeds recursively. First, given arbitrary equity ownership structures and cash flow betas on risk factors, we solve for the market equilibrium and induced CAPM pricing rule. Then, given the ownership structure, we analyze the capital budgeting problem, i.e., the entrepreneur's choice of betas for the given ownership structure. Finally, we study the optimal-contracting problem, which determines the ownership structure of the firm.

Competitive Equilibrium of the CAPM Economy: Given the price of the riskless bond, π_0 , the price of the aggregate factor, π_1 , and the price of the representative firm, p^f , each agent chooses (i) a consumption allocation at time 0, c_0^h , (ii) portfolio positions in the risk-free bond, θ_0^h , in the aggregate factor, θ_1^h , and in the firm, θ_f^h , and (iii) a consumption allocation at time 1, a random variable c_1^h , to maximize

$$E[u^h(c_0^h, c_1^h)] \equiv -\frac{1}{A}e^{-Ac_0^h} + E\left[-\frac{1}{A}e^{-Ac_1^h}\right]. \quad (4)$$

The budget constraints faced by the investor h , $h > 1$, are:

$$c_0^h = y_0^h - \pi_0\theta_0^h - \pi_1\theta_1^h - p^f\theta_f^h \quad (5)$$

$$c_1^h = y_1^h + \theta_0^h + \theta_1^h x_1 + \theta_f^h y_1^f. \quad (6)$$

The entrepreneur, agent $h = 1$, faces the additional constraint that he cannot trade his firm ($\theta_f^1 \equiv 0$), once he sells fraction w at date 0:

$$c_0^1 = y_0^1 + wp^f - \pi_0\theta_0^1 - \pi_1\theta_1^1 \quad (7)$$

$$c_1^1 = \theta_0^1 + \theta_1^1 x_1 + (1-w)y_1^f. \quad (8)$$

Note that the entrepreneur receives proceeds wp^f from selling fraction w of the firm at the market price of p^f .

A *competitive equilibrium* of the economy is a consumption allocation (c_0^h, c_1^h) , for all agents $h = 1, \dots, H$, that solves the problem of maximizing (4) subject to (5) and (6) for $h > 1$, and the problem of maximizing (4) subject to (7) and (8) for $h = 1$; and prices (π_0, π_1, p^f) such that consumption and financial markets clear:

$$\sum_{h=1}^H (c_0^h - y_0^h) \leq 0, \quad (9)$$

$$\sum_{h=1}^H (c_1^h - y_1^h) \leq 0 \text{ (with probability 1 over possible states at } t = 1), \text{ and} \quad (10)$$

$$\sum_{h=1}^H \theta_j^h = 0, \quad j = 0, 1; \quad \sum_{h=2}^H \theta_f^h = w. \quad (11)$$

Given the equity ownership structure of the firm, w , and its cash flow betas β_j^f , $j = 1, 2$, a competitive equilibrium is uniquely determined. We discuss below the salient features of the competitive equilibrium that we exploit in our analysis. Closed-form solutions for equilibrium allocations and prices are reported in Appendix A.

The factor structure of the firm's cash flow, equation (2), implies that the equilibrium price of the firm can be written as the composition of price of the deterministic component, the price of the aggregate risk component, and the implicit price of the corporate sector-specific risk of its cash flow:

$$p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (12)$$

where π_2 is equilibrium price of a portfolio paying off x_2 . Given our assumptions, a portfolio paying off x_2 can be replicated through the trading of available assets by all agents except the entrepreneur; the price π_2 can therefore be determined by no-arbitrage from π_0 , π_1 and p^f . It is convenient to express the properties of equilibrium pricing in terms of the factor prices, (π_0, π_1, π_2) .

At the competitive equilibrium, each agent holds three "funds": the bond, the portion of aggregate endowment that is exposed to traded risk factors (subject to the restricted participation constraints), and the unhedgeable component of the personal endowment. The positive supply of the firm's stock also translates into positive supplies s_j , $j = 0, 1, 2$, of the riskless bond and risk factors:

$$s_0 = w E(y_1^f), \quad (13)$$

$$s_j = w \beta_j^f, \quad j = 1, 2. \quad (14)$$

This follows also from the factor structure of firm's cash flow (equation 2).

Under this representation, a version of the cross-sectional beta pricing relationship holds: The price of factor j relative to the price of bond is proportional to the covariance of the factor with the aggregate endowment of the economy and to the positive supply of factor j . The aggregate endowment relevant for the pricing of factor j is the sum of the endowments

of the agents who can trade factor j . Formally,

$$\frac{\pi_1}{\pi_0} = E(x_1) - \frac{A}{H} \left[\text{cov} \left((1-w)y_1^1 + \sum_{h=2}^H y_1^h, x_1 \right) + w\beta_1^f \right] = -\frac{A}{H} \sum_{h=1}^H \beta_1^h, \quad (15)$$

$$\frac{\pi_2}{\pi_0} = E(x_2) - \frac{A}{H-1} \left[\text{cov} \left(\sum_{h=2}^H y_1^h, x_1 \right) + w\beta_2^f \right] = -\frac{A}{H-1} \left(\sum_{h=2}^H \beta_2^h + w\beta_2^f \right) \quad (16)$$

where we have employed the normalization that $E(x_j) = 0$, $j = 1, 2$. Because the entrepreneur cannot trade the stock of his firm, he (effectively) cannot trade sector-specific risk factor x_2 . The relevant aggregate endowment for price of factor x_2 thus excludes his holding of this risk $(1-w)\beta_2^f$. Recall also that asset x_1 is positively correlated with the aggregate endowment of investors, $\sum_{h=2}^H y_1^h$; the firm endowment y_1^f is positively loaded on asset x_1 ; and asset x_2 is orthogonal to the aggregate endowment of investors. These assumptions imply

$$\sum_{h=1}^H \beta_1^h > 0, \quad \sum_{h=2}^H \beta_2^h = 0. \quad (17)$$

Finally, in equilibrium, the expected utility of agent h is

$$E[u^h(c_0^h, c_1^h)] = -\frac{(1+\pi_0)}{A} e^{-Ac_0^h(w, \beta_1^f, \beta_2^f, p^f)}, \quad (18)$$

where we stress the fact that the equilibrium time-0 consumption depends on the ownership structure of the firm, its technology, and the price of the firm. This expected utility also depends on the induced equilibrium prices, (π_j) , $j = 0, 1, 2$, that we omit for parsimony.

4 Capital Budgeting and Equity Ownership Structure

The entrepreneur can, at a private non-pecuniary cost, choose the the risk composition of the firm's cash flows. Formally, the entrepreneur can choose the betas, β_1^f and β_2^f , the respective loadings of the firm's cash flows on the aggregate risk and the corporate sector-specific risk. For simplicity, we assume the entrepreneur's choice only affects the distribution of the variance of cash flows between the aggregate and the sector-specific risks, but does not alter their expected value or the total variance.⁵ That is, we assume that

$$(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}, \quad (19)$$

⁵Note that the equilibrium price of the firm is affected by the capital budgeting choice. In turn, the expected stock return on the firm is affected as well even though the expected cash flows are not.

Table 1: The Sequence of Events under Different Governance Structures

Governance Structure	Sequence of Events		
Benchmark	Entrepreneurs choose fraction w to sell	Entrepreneurs choose aggregate risk loading β_1^f observable	\Rightarrow All agents including entrepreneurs trade Markets clear, prices are determined
Owner-Managed Firms	Entrepreneurs choose fraction w to sell \Rightarrow	All agents including entrepreneurs trade Markets clear, prices are determined	\Rightarrow Entrepreneurs choose aggregate risk loading β_1^f unobservable
Corporations (Management-Controlled Firms)	Investors choose fraction w to retain \Rightarrow Managers awarded fraction $(1 - w)$	All agents including managers trade Markets clear, prices are determined	\Rightarrow Managers choose aggregate risk loading β_1^f unobservable

where \bar{V} , the total variance of the cash flow of the firm, is held constant.

The entrepreneur must exert a non-pecuniary costly *effort* to change the initial ‘intrinsic’ composition of the cash flow risk. We assume that the cost is non-pecuniary, and is measured in terms of the time-0 consumption good. More specifically, this cost enters the entrepreneur’s expected utility according to the multiplicative factor $e^{AC(\beta_1^f - \bar{\beta}_1^f)^2}$, $C > 0$; where $\bar{\beta}_1^f > 0$ denotes the ‘intrinsic’ level of β_1^f (only changes in β_1^f from its ‘intrinsic’ level need be considered in the costs, since the associated changes in β_2^f are determined via equation 19). These assumptions on the cost structure are made for analytical tractability. They imply that the quadratic cost, $C(\beta_1^f - \bar{\beta}_1^f)^2$, is subtracted from the certainty equivalent of entrepreneur’s time-1 consumption, as in typical CARA-Normal principal-agent set-ups, e.g., Holmstrom and Milgrom (1987) and Laffont and Martimort (2002), Chapter 9.5.2. Formally, net of capital budgeting costs, the entrepreneur’s expected utility at equilibrium (equation 18) is given by

$$-\frac{(1 + \pi_0)}{A} e^{-A[c_0^1(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2]} . \quad (20)$$

Finally, entrepreneurs choose their equity ownership share $(1 - w)$ optimally. Table 1 details the exact sequence of events (for the analysis of corporations, see Section 7).

4.1 Benchmark: No Moral Hazard

We first study the determination of the ownership structure and the firm's technology in the benchmark case in which (i) the entrepreneur owns the firm ex ante, and (ii) investors observe the choice of technology, β_j^f , $j = 1, 2$, so that the entrepreneur can commit to a technology choice when choosing the share w of the firm to sell. Since there is no moral hazard, the choices of β_j^f and w are effectively simultaneous. Given that the firm trades as a composite and not in a piecemeal manner for its different risk loadings, it is a strong assumption that investors observe the risk composition of firm cash flows. Nevertheless, this case serves as a useful benchmark.

When choosing w , entrepreneurs rationally anticipate the unit price p^f at which they can sell this share:

$$p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f.$$

Each entrepreneur can affect the price of his own single firm, p^f , by his choice of β_1^f through this mapping, but he cannot affect the bond prices or risk factor prices: these prices are determined at equilibrium by the aggregate choices of the continuum of entrepreneurs. That is, markets are competitive, and all agents including entrepreneurs are price takers: All agents rationally anticipate that the price of a single firm depends on its cash flow betas β_j^f , given the prices of traded assets in the economy.⁶

Formally, the representative entrepreneur chooses the share w of the firm to sell, as well as its technology β_1^f to maximize expected utility net of the exerted effort:

$$\max_{w, \beta_1^f, \beta_2^f} -\frac{(1+\pi_0)}{A} e^{-A[c_0^1(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2]} \quad (21)$$

subject to:

$$p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (22)$$

$$(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}, \quad (23)$$

given the equilibrium prices of the bond and the risk factors, π_0 , π_1 , and π_2 , respectively.

⁶As discussed in Section 2 and assumed in Section 3, the entrepreneur takes as given the price of the riskless bond, π_0 , the price of the aggregate risk factor, π_1 , and the price of the representative firm, p^f . The composition of p^f , equation (12), implies that, in addition to π_0 and π_1 , the entrepreneur effectively takes as given the price of the sector-specific risk factor, π_2 .

The price of entrepreneur's *own* firm is also denoted as p^f for parsimony of notation. The entrepreneur recognizes that this price depends on the risk composition of his firm's cash flows, for given prices of risk factors. In equilibrium, the price of each entrepreneur's firm equals the price of the representative firm.

4.2 Moral Hazard

In contrast to this benchmark case, consider now owner-managed firms where the technology choice is not observed by capital market investors. As a result, entrepreneurs cannot commit their technology choice, β_j^f , at the moment they choose the fraction w of their firm to sell in the market; they choose β_j^f after they choose w , and after agents have traded and markets have cleared. While the specific timing of the choice of β_j^f and trading in capital markets is somewhat arbitrary, crucial for our analysis is that the chosen β_j^f are not observed by investors in competitive markets.

Proceeding recursively, we first study the capital budgeting problem of entrepreneurs, which determines β_j^f for a given w . Since w is observed by investors, but β_j^f is not, entrepreneurs anticipate that the price of their own firm p^f will depend only on w and not on their specific choice of β_j^f . Therefore, for given w and p^f , the choice of cash flow betas maximizes the entrepreneur's expected utility net of the exerted effort:

$$\max_{\beta_1^f, \beta_2^f} -\frac{(1+\pi_0)}{A} e^{-A[c_0^l(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2]} \quad (24)$$

subject to

$$(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}. \quad (25)$$

Because the price of the firm p^f does not affect the solution of this capital budgeting problem, we denote the solution simply as $\beta_j^f(w)$.

We now consider the choice of equity ownership by entrepreneurs. An entrepreneur's proceeds from selling share w of his firm are wp^f . Hence, he perceives a direct effect of the choice of w on his proceeds while also being aware that only the ownership structure, w , is observed by investors. The entrepreneur expects investors to rationally anticipate the equilibrium map between ownership structure and the risk composition of the firm, which results from the solution of the capital budgeting problem, $\beta_j^f(w)$. The entrepreneur therefore also perceives an indirect effect of his choice of w on the price of the firm p^f (equation 12) through its effect on his future choice of β_j^f via the map $\beta_j^f(w)$.⁷

Formally, the entrepreneur chooses w to maximize the expected utility net of effort:

$$\max_w -\frac{(1+\pi_0)}{A} e^{-A[c_0^h(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2]} \quad (26)$$

⁷This equilibrium concept has been introduced in the context of general equilibrium theory with asymmetric information by Prescott and Townsend (1984). Magill and Quinzii (1998) adopt it in a related setting and refer to the anticipatory behavior of entrepreneurs as "rational conjectures." Bisin and Gottardi (1999) study a different equilibrium concept appropriate when the equity ownership structure is also not observable.

subject to:

$$p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (27)$$

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \quad (28)$$

given $\pi_j, j = 0, 1, 2$.

5 Equilibrium Equity Ownership and Risk

We characterize below (i) the entrepreneurial choice of the aggregate risk beta of the firm's cash flows, β_1^f ; and (ii) the optimal equity ownership of firms, measured by the fraction $(1 - w)$ retained by entrepreneurs. We first consider the benchmark case when investors can observe the firm's risk loadings.

Proposition 1 *For owner-managed firms with no moral hazard, in equilibrium, the loading on the aggregate risk factor, denoted β_1^* , is reduced from its initial value $\bar{\beta}_1^f$:*

$$\beta_1^* = \bar{\beta}_1^f - \frac{A\pi_0}{2CH(1 + \pi_0)} \sum_{h=2}^H \beta_1^h < \bar{\beta}_1^f. \quad (29)$$

Each entrepreneur sells fraction w^ of the firm, retaining fraction*

$$(1 - w^*) = \frac{1}{H}. \quad (30)$$

In the absence of moral hazard, each entrepreneur simply owns the market fraction of the firm. The entrepreneur rationally anticipates that increasing the aggregate risk of the firm, thereby reducing the firm-specific risk, reduces the equilibrium value of its shares (equation 12). Hence, in equilibrium, the entrepreneur optimally reduces the aggregate risk loading of the firm, choosing $\beta_1^* < \bar{\beta}_1^f$.

Now consider owner-managed firms when investors do not observe the firm's risk loadings. In this case, entrepreneurs do not fully internalize the cost borne by the rest of the economy due to an increase in their firm's aggregate risk beta. In particular, entrepreneurs privately prefer to increase their firm's aggregate risk beta in order to reduce the fraction of their own wealth that is composed of unhedgeable risk. Such risk-substitution is costly for investors: Investors' endowments are exposed to aggregate risk, but not to corporate sector-specific risk. The result is that investors can bear the corporate sector-specific risk supplied by the stock market at a lower welfare loss than they can bear the aggregate risk.

Entrepreneurs can, however, design the ownership structure to reduce the extent of this moral hazard, i.e., to create an incentive to decrease the cash flow beta on the aggregate risk factor. Moreover, if the moral-hazard component in the decision problem of entrepreneurs is not excessively large,⁸ we can characterize the equilibrium loading on aggregate risk, β_1^f . Furthermore, we can characterize the condition on the initial loading $\bar{\beta}_1^f$ that guarantees the equilibrium fraction of the firm retained by the entrepreneur is smaller than the market share. That is, the ownership structure is optimally designed to provide the entrepreneur “negative incentive compensation,” which forces the entrepreneur to sell more of the firm to investors than he would if he could commit to technology choices ex-ante.

Proposition 2 *For owner-managed firms with moral hazard, in equilibrium, the loading on the aggregate risk factor, β_1^{**} , is such that*

$$\beta_1^* < \beta_1^{**} < \bar{\beta}_1^f. \quad (31)$$

*Each entrepreneur sells a fraction w^{**} of the firm and retains a fraction $(1 - w^{**})$, where*

$$(1 - w^{**}) < (1 - w^*) \text{ if } \bar{\beta}_1^f > K \sum_{h=2}^H \beta_1^h \text{ where } K = 1 + \frac{A}{4CH^2}. \quad (32)$$

At equilibrium, the optimal choice of w induces entrepreneurs to decrease the aggregate cash flow beta of their firms, $\beta_1^{**} < \bar{\beta}_1^f$, but not fully to the level without moral hazard, $\beta_1^* < \beta_1^{**}$. When the initial ‘intrinsic’ aggregate risk beta of the firm $\bar{\beta}_1^f$ is sufficiently high and/or the aggregate risk beta of investors’ endowments $\sum_{h=2}^H \beta_1^h$ is sufficiently low, condition (32) is satisfied and entrepreneurs hold a smaller fraction of the firm compared to the benchmark case, $(1 - w^{**}) < (1 - w^*)$.

To better understand condition (32), suppose that the aggregate risk factor x_1 is perfectly correlated with $\sum_{h=2}^H y_1^h$, the non-corporate sector (investors’) endowment of the economy. That is, x_1 could be interpreted as the Gross Domestic Product (GDP) minus the corporate sector output, but normalized to have unit variance. Then, $\sum_{h=2}^H \beta_1^h$ equals $\sqrt{\bar{V}_{nc}}$, where \bar{V}_{nc} is the variance of the non-corporate sector endowment. Furthermore, $\bar{\beta}_1^f$ equals $\rho\sqrt{\bar{V}}$, where \bar{V} is the variance of the corporate sector endowment, and ρ is the correlation between corporate sector and non-corporate sector endowments. Finally, let H go to infinity keeping

⁸This is the case formally if, for instance, changing the risk composition of firms requires a large enough effort cost C at the margin. This sufficient condition ensures global concavity of the entrepreneur’s objective as a function of w , as discussed in footnote 19, Appendix C.

\bar{V}_{nc} and ρ constant.⁹ Then, K tends to unity, and condition (32) requires that

$$\rho^2 \bar{V} > \bar{V}_{nc}. \quad (33)$$

That is, condition (32) requires that the correlation of corporate sector cash flows and non-corporate sector endowments be high and that the variability of corporate sector cash flows be large relative to the variability of non-corporate sector endowments. Empirical evidence suggests that the corporate sector output of an economy is highly correlated with the non-corporate sector output, and is much more variable.¹⁰

Next, to better explain our characterization of $(1 - w^{**})$, we examine the diverse effects of ownership structure on the incentives of entrepreneurs to load cash flows on hedgeable aggregate risks. The argument is essentially based on the mixed partial derivative of entrepreneurial objective (equation 24) with respect to the aggregate cash flow beta, β_1^f , and the share retained, $(1 - w)$.

Entrepreneurs benefit at the margin from increasing β_1^f because it reduces their exposure to unhedgeable, firm-specific risk. Formally, the firm-specific risk that entrepreneurs bear is given by $(1 - w)^2[\bar{V} - (\beta_1^f)^2]$, which decreases as β_1^f rises at a rate that is increasing in $(1 - w)^2\beta_1^f$. Therefore, a *decrease* in the retained fraction $(1 - w)$ reduces the marginal benefit to entrepreneurs from increasing the aggregate cash flow beta β_1^f .

But entrepreneurs also face a cost at the margin from increasing β_1^f . Entrepreneurs re-balance the aggregate risk exposure of their personal portfolios by trading in the market for the aggregate risk: In equilibrium, they sell the aggregate risk component in their wealth, $(1 - w)\beta_1^f$, at the given price π_1 and retain only the aggregate component of such risk. Since aggregate risk is disliked by agents, it is sold at a negative price and its re-balancing is costly for entrepreneurs. Formally, entrepreneurs face a hedging cost of $\pi_1(1 - w)\beta_1^f$, which increases in magnitude with β_1^f at a rate that is increasing in $\pi_1(1 - w)$. Therefore, an *increase* in the retained fraction $(1 - w)$ makes it less attractive for them to increase the aggregate cash flow beta β_1^f .

The effectiveness of using ownership structure to pre-commit a reduction in the aggregate cash flow beta thus depends upon the relative strengths of these two conflicting effects.

⁹This can be achieved for example by distributing investors into a continuum of cohorts that are ranked by the correlation of investors' endowment with corporate sector endowment, the correlations ranging from a minimum negative value to a maximum positive value.

¹⁰For example, based on data from the National Income and Product Accounts Table, the de-trended corporate sector output (growth rate) in the United States during 1946–2003 is approximately 1.6 (1.3) times as variable as the de-trended non-corporate sector output (growth rate), where the non-corporate sector output is measured as the difference between the Gross Domestic Product and the corporate sector output. The corporate and the non-corporate sector outputs are almost perfectly correlated for the United States. These calculations suggest that condition (33) is satisfied for the United States.

The price of aggregate risk π_1 increases (in magnitude) with the aggregate risk beta of investors' endowments, $\sum_{h=2}^H \beta_1^h$. Therefore, when $\sum_{h=2}^H \beta_1^h$ is sufficiently low relative to $\bar{\beta}_1^f$, the cost of hedging aggregate risk is not too high and entrepreneurs can diversify easily by personal trading. In this case, the only feasible pre-commitment device is one that exposes entrepreneurs to less unhedgeable risk than in the benchmark case: Entrepreneurial ownership is lower than in the benchmark case, giving rise to a “negative incentive compensation,” and itself provides diversification for the entrepreneur.

However, when the aggregate risk exposure of the investors' endowment $\sum_{h=2}^H \beta_1^h$ is high, it is costly for entrepreneurs to sell aggregate risk in capital markets. The optimal pre-commitment device is now one where the entrepreneur retains a fraction of the firm that exceeds the market share. This induces the entrepreneur to diversify by trading in capital markets: Since the quantity of aggregate risk the entrepreneur has to sell increases in the aggregate risk beta of the firm, the entrepreneur is incentivized to choose a smaller aggregate beta. Formally, a sufficient condition for this case to arise is $\bar{\beta}_1^f < \sum_{h=2}^H \beta_1^h$.

In the case of moral hazard, there exists no closed-form characterization for the equilibrium dependence of equity ownership on the firm's initial aggregate risk loading. Nor is there a straightforward representation of the relationship between equity ownership and equilibrium risk loading. Instead, we have studied these relationships numerically to better illustrate the implications of Proposition 1 and 2. The results are collected in Figures 1 and 2. As depicted in Figure 1, entrepreneurs of firms with ‘intrinsic’ technologies that are relatively more loaded on aggregate risk (for instance, firms in pro-cyclical industries) hold a smaller fraction of the firm at equilibrium. Conversely, Figure 2 shows that firms whose owner-entrepreneurs hold a larger share of equity in equilibrium also contain less aggregate risk in equilibrium.

6 Welfare Properties

In this section, we address the following welfare questions: Do entrepreneurs hold too much or too little of their firms? Is there efficiency in the induced equilibrium loading of the firms' cash flows on the aggregate risk factor? Does the stock market contribute additional risk to the aggregate endowment risk of the economy? Is such additional risk inefficient? Not surprisingly, the presence of moral hazard implies that, in equilibrium, entrepreneurs diversify inefficiently by over-loading their firms on aggregate risk factors, relative to the first-best. However, the relevant welfare question is as follows: Could a social planner regulate the firms' equity ownership structure so as to improve aggregate welfare, given the constraint that entrepreneurs will then choose technology to maximize their expected utility?

In CAPM economies, it is convenient to measure the welfare associated with the equilibrium of an economy relative to a benchmark. We take the welfare of the autarkic economy as this benchmark, where agents only trade the bond (see Willen, 1997, and Acharya and Bisin, 2000) and no capital budgeting takes place. The welfare associated with the equilibrium of our economy, which we denote μ , is defined as the minimal aggregate transfer, in terms of time-0 consumption, needed to equate an agent's expected utility at equilibrium with his expected utility at autarky. Formally, let $[c_0, c_1] \equiv [c_0^h, c_1^h]_{h \in H}$ denote the competitive equilibrium allocation in the economy; and let $[c_0^a, c_1^a]$ be the equilibrium allocation at autarky. Let π_0 be the equilibrium price of the bond, and π_0^a the price of the bond at autarky. Let $U^h(c_0^h, c_1^h)$ denote the expected equilibrium utility of agent h , and let $U^{ah}(c_0^{ah}, c_1^{ah})$ be the corresponding expected utility at autarky. The aggregate compensating transfer, μ , is defined as

$$\mu = \sum_{h=1}^H \mu^h, \quad (34)$$

where the individual compensating transfer, μ^h , is given by the solution to

$$U^{a1}(c_0^{a1} + \mu^1, c_1^{a1}) = U^1(c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2, c_1^1), \quad \text{and} \quad (35)$$

$$U^{ah}(c_0^{ah} + \mu^h, c_1^{ah}) = U^h(c_0^h, c_1^h), \quad \text{for } h = 2, \dots, H. \quad (36)$$

We show in Appendix B that

$$\mu = -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C(\beta_1^f - \bar{\beta}_1^f)^2. \quad (37)$$

Therefore, an economy is more efficient with a low equilibrium price of the risk-free asset and a correspondingly high risk-free return. This is because the risk-free rate increases when precautionary savings fall. This occurs when financial markets serve to hedge away the majority of agents' risk exposures.

Efficiency of Equity Ownership and Risk Loadings: The fraction w of the firm held by capital market investors, and the loadings β_j^f of the firm's cash flows on the economy's risk factors, are *first-best efficient* if they maximize the aggregate welfare index μ , taking into account the effects of w and β_j^f on competitive equilibrium prices. Formally, the first-best efficient choices of w and β_j^f maximize μ :¹¹

$$\max_{w, \beta_1^f, \beta_2^f} -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C(\beta_1^f - \bar{\beta}_1^f)^2 \quad (38)$$

¹¹Note that the solution to the first-best problem as well as the constrained-efficiency problem is independent of π_0^a , so that the choice of benchmark in the definition of μ is arbitrary.

subject to

$$(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}, \quad (39)$$

where π_0 , the equilibrium price of risk-free asset, is given by equation (A.9), Appendix A.

Proposition 3 *For owner-managed firms with no moral hazard, the equilibrium fraction of the firm held by investors, w^* , and aggregate risk loading, β_1^* , are first-best efficient.*

In the absence of moral hazard, this result on the first-best efficiency is intuitive. Consider now the situation in which a moral hazard arises: owner-managed firms for which risk loadings are not observed by investors. In this case, first-best efficiency is too strong a welfare requirement.

For the equilibrium to satisfy constrained efficiency, (i) the maps $\beta_j^f(w)$, defined in equations (24)–(25) of Section 4.2, determine the risk factor loadings of the firm’s cash flows, while (ii) the fraction of the firm held by capital market investors w maximizes the aggregate welfare index μ , given $\beta_j^f(w)$ and taking into account the effects of w and β_j^f on competitive equilibrium prices. Formally, the constrained-efficient choice of w maximizes μ :

$$\max_w -\frac{H}{A} \ln \frac{1+\pi_0}{1+\pi_0^a} - C(\beta_1^f - \bar{\beta}_1^f)^2 \quad (40)$$

subject to

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \quad (41)$$

where π_0 , the equilibrium price of risk-free asset, is given by equation (A.9), Appendix A.

Proposition 4 *For owner-managed firms with moral hazard, the equilibrium fraction of the firm held by investors, w^{**} , and the aggregate risk loading, β_1^{**} , are constrained efficient.*

To summarize, the private choice of entrepreneurs leads to socially optimal (second-best) outcomes. That is, the price mechanism efficiently aligns the objectives of entrepreneurs with those of the (constrained) social planner when the former designs the equity ownership structure to pre-commit capital budgeting choices. Entrepreneurs, while price-takers for prices of the risk factors, nevertheless face a price schedule for the firms they own and manage. They recognize that increasing the aggregate risk of the firm reduces the equilibrium value of its shares. In equilibrium, motivated by the capital gains from reducing this aggregate risk component, entrepreneurs choose equity ownership structures that enable a pre-commitment of the (constrained) efficient choices of cash flow loadings on risk factors.

7 Corporations

We now extend the analysis to consider corporations. In this case, the ex-ante ownership of the firm is spread across investors. These stockholders hire a manager and choose the fraction $(1 - w)$ of equity with which to endow the manager. For a corporation, it is natural to interpret this stock grant as “incentive compensation.” For the sake of consistency however, we refer to it as the firm’s ownership structure. In addition, the manager must be given a time-0 compensation W (in terms of time-0 consumption good units), such that the manager’s utility from time-0 compensation and the stock grant equals his reservation utility value of \bar{W} . We assume that the payment of this time-0 compensation is borne equally by all stockholders. The manager chooses the firm’s cash flow betas after receiving the stock award and after trading has taken place.

The analysis of corporations mirrors the analysis of owner-managed firms. The capital budgeting problem again determines a map at equilibrium between managerial equity ownership, w , and the manager’s choice of risk composition, $\beta_j^f(w)$, $j = 1, 2$.¹² Stockholders then choose w and W to maximize the sum of their individual welfares, $\sum_{h=2}^H \mu^h$, where the individual compensating transfer, μ^h , is defined in equation (36) and characterized in equation (B.2), Appendix B:

$$\max_{w,W} \sum_{h=2}^H \mu^h \equiv \sum_{h=2}^H \left[c_0^h - c_0^{ah} - \frac{1}{A} \ln \frac{1+\pi_0}{1+\pi_0^a} \right] \quad (44)$$

subject to the manager’s ($h = 1$) reservation utility constraint

$$-\frac{(1+\pi_0)}{A} e^{-A[c_0^1(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2]} = \bar{W}, \quad (45)$$

and subject to:

$$p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (46)$$

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \quad (47)$$

given π_j , $j = 0, 1, 2$.

¹²The budget constraints of investors and managers, (5) and (7), respectively, are modified as:

$$c_0^h = y_0^h - \frac{W}{H-1} - \pi_0 \theta_0^h - \pi_1 \theta_1^h - p^f \theta_f^h, \quad h > 1 \quad (42)$$

$$c_0^1 = y_0^1 + W - \pi_0 \theta_0^1 - \pi_1 \theta_1^1 \quad (43)$$

However, for parsimony, we use the same notation $\beta_j^f(w)$ as that for owner-managed firms with moral hazard. We show in Appendix C that the entrepreneur’s choice β_j^f for a given w in an owner-managed firm is identical to the manager’s choice of β_j^f in a corporation for that same w .

Thus, as with owner-managed firms, corporations can use w to pre-commit to the ex-post choice of technology characterized by $\beta_j^f(w)$. Entrepreneurs in owner-managed firms and stockholders in corporations both rationally anticipate the effect of technology choice on the value of the firm. As a result, all else equal, the proportion of the firm awarded to managers and the cash flow betas in equilibrium are the same as those under the equilibrium for owner-managed firms. The two settings are thus isomorphic.

Proposition 5 *In the case of corporations, stockholders choose to retain for themselves the same fraction of the firm that an entrepreneur sells to the stock market in an owner-managed firm with moral hazard: w^{**} . As a consequence, at equilibrium managers hold a fraction $(1 - w^{**})$ of the firm and choose the same loading on the aggregate risk factor as entrepreneurs would in an owner-managed firm with moral hazard: β_1^{**} .*

It follows from Proposition 4 that, in the case of corporations too, the equilibrium fraction of the firm held by investors and the induced cash flow betas are constrained efficient.

8 Empirical Implications

Propositions 2 and 5 reveal that a sort of “negative incentive compensation” scheme might be required in the presence of the diversification externality: the fraction of the firm awarded to the manager might be smaller than the market share. This result should not be interpreted literally. In practice, ownership structure and compensation schemes must also address agency problems other than the diversification externality. For instance, one reason that managers are required to hold shares of their firms is to limit the incentive to seek private benefits when they choose projects with different expected cash flows. In such settings, our results only imply that a reduction in these shares must limit the effects of the diversification externality.

Importantly, we must impose a caveat on the results derived in Propositions 2 and 5 because of the presence of different agency problems. A firm’s ownership structure and risk composition are both *endogenous* in our analysis. Consider the negative relationship at equilibrium between the managerial equity ownership and the component of aggregate risk in the firm, illustrated in Figure 2. The depicted relationship assumes no moral-hazard problems aside from the diversification externality, or, more generally, that the extent of any such problems is flat across firms. Variations in the alternative moral hazards across firms induce a cross-sectional variation in the choice of ownership structure and incentive compensation. This induced variation is not related to the diversification externality, and hence is *exogenous* to our model. We next demonstrate that such an exogenous increase in

the managerial equity ownership can give rise to capital budgeting choices that increase the firm's aggregate risk loadings.

Proposition 6 *An exogenous increase in the manager's share of the firm, $(1-w)$, increases the aggregate risk loading $\beta_1^f(w)$, if the initial aggregate risk loading, $\bar{\beta}_1^f$, is high enough. In particular, this holds if¹³*

$$\bar{\beta}_1^f > K(w) \sum_{h=2}^H \beta_1^h \text{ where } K(w) = \max \left[0, \frac{1}{(2H(1-w) - 1)} \left(\frac{A(1-w)^2}{4C} + 1 \right) \right]. \quad (48)$$

Figure 3 illustrates Proposition 6 for the example economy studied in Section 5. The figure plots the equilibrium beta, $\beta_1^f(w)$, as a function of $(1-w)$, the ownership of the manager, for a firm whose initial aggregate risk loading $\bar{\beta}_1^f$ is high enough (satisfying the condition in Proposition 6). Note that $(1-w) = (1-w^{**}) + \epsilon$, where $(1-w^{**})$ is the endogenous component of ownership, and $\epsilon > 0$ is the exogenous component of ownership. The exogenous component of ownership is assumed to address an alternative moral-hazard problem such as the extraction of private benefits by the manager. All else equal, varying $(1-w)$ is thus equivalent to varying the extent of this moral-hazard problem. Figure 3 shows that increasing the ownership exogenously increases the aggregate risk beta for the firm whose 'intrinsic' aggregate risk loading is high.

We conclude that, in the presence of alternative moral-hazard problems, a positive equilibrium relationship might arise between the ownership $(1-w)$ and the aggregate risk loading $\beta_1^f(w)$. This is in stark contrast to the negative endogenous relationship between $(1-w^{**})$ and $\beta_1^f(w^{**})$ illustrated in Figure 2, when the only moral-hazard problem present is the diversification externality.

These results have relevance for interpretation of the extant empirical literature linking managerial ownership to incentives to diversify.

Interpretation of the Empirical Evidence: Our results suggest that establishing an empirical relationship between the extent of diversification activity and managerial ownership of the firm might be difficult. Empirically, the literature quantifies the diversifying activities of managers as R^2 in a regression of firm-level stock returns on market returns,

¹³Note that if $(1-w) < \frac{1}{2H}$, then $K(w) = 0$, and the condition is always satisfied. If $(1-w) > \frac{1}{2H}$, then $K(w) = \frac{1}{(2H(1-w)-1)} \left(\frac{A(1-w)^2}{4C} + 1 \right)$, and, in particular, $K(w^*) = K$, where K is defined in Proposition 2.

and then studies the relationship between a firm's R^2 and the extent of incentive compensation awarded to managers.¹⁴ Our theory implies a negative cross-sectional relationship between a firm's R^2 and the extent of managerial ownership or incentive compensation (see Figures 2 and 3) *only* once we condition for these other moral-hazard components. Such conditioning, which is necessary to isolate the endogenous and the exogenous components of management compensation, is clearly very difficult: The alternative moral-hazard problems are heterogeneously distributed across firms and are often unobservable.

Not surprisingly then, the extant empirical literature derives inconclusive findings about the relationship between a firm's R^2 value and incentive compensation. Amihud and Lev (1981) document a significant negative relationship between R^2 from equity accounting returns and the equity ownership of officers and directors. May (1995) reports that a firm's diversification is positively correlated with the ratio of the manager's value of share ownership to total wealth. Denis, Denis, and Sarin (1997) identify a negative relationship similar to Amihud and Lev (1981) between measures of diversification and equity ownership (OWN) of officers and directors. Denis, Denis, and Sarin also find that this negative relationship holds at low to moderate levels of OWN, whereas very high levels of OWN yield a positive relationship between diversification and OWN.

Aggarwal and Samwick (2003) test a model where diversification can arise either due to managerial risk-aversion or due to a managerial desire to "build empires." Their model determines incentive compensation endogenously, and their empirical results show a positive relationship between firm diversification and the extent of the manager's incentive compensation. They conclude that managers diversify in response to changes in empire-building motives rather than to reduce exposure to risk. However, Aggarwal and Samwick do not consider the presence of rent-extraction problems other than empire-building motives. For instance, managers can affect expected cash flows by seeking private benefits. This is a moral hazard that does not lead to diversification as building an empire does. The interpretation of their results is therefore problematic: If compensation has been designed to address such other rent-extraction problems, then a positive relationship between diversification and incentive compensation is perfectly consistent with diversification caused by managerial risk-aversion.

Our results on the endogenous and the exogenous aspects of the relationship between R^2 and managerial ownership help to uncover the challenge embedded in this empirical investigation.

¹⁴It should also be noted that an issue concerning the interpretation of the empirical evidence is that a higher R^2 can result from a substitution of firm-specific risk with aggregate risk, as we consider, and also from a reduction of firm-specific risk with no effect on aggregate risk.

9 Extensions

We extend the analysis to consider different possible factor structures. These extensions more fully exploit the generality of the CAPM economy, which is formally stated in Appendix D (available upon request); we have analyzed the special case with two risk factors thus far.¹⁵ We continue to interpret a firm as a representative firm, that is, essentially as a sector, and thus often refer to firms as sectors.

9.1 Multi-Sector Economy

Consider an economy and a stock market with two sectors, f and g . The economy's factor structure is composed of a common risk factor, x_1 , and two additional risk factors, x_2 and x_3 , which are orthogonal to the common factor. In this multi-sector economy, the common factor can be interpreted as a "stock market index," and the additional risk factors can be interpreted as the "sector-specific" risks. The cash flows of the two sectors in terms of this basic factor structure of the economy are as follows:

$$y_1^f - E(y_1^f) \equiv \beta_1^f x_1 + \beta_2^f x_2, \quad (49)$$

$$y_1^g - E(y_1^g) \equiv \beta_1^g x_1 + \beta_3^g x_3. \quad (50)$$

Entrepreneurs cannot trade the shares of their own firms, but can trade otherwise in the stock market: entrepreneurs in sector f (respectively in sector g) can trade factors x_1 and x_3 (respectively x_1 and x_2).

At equilibrium, entrepreneurs load their firms' cash flows on x_1 , the component of cash flows that is common with the stock market index and is correlated with the aggregate endowment risk. Consequently, the cash flows of firms traded in the stock market, and by implication the stock returns of these firms, are excessively correlated across sectors, in addition to being correlated with the index returns and the aggregate portfolio.

More formally, entrepreneurs in sector f would want to trade the stock of sector g only as a way to hedge a part of the endowment risk. Entrepreneurs in sector f do not have incentives to trade factor x_3 , which is uncorrelated with their wealth, in order to hedge the endowment risk. They trade in the stock market index x_1 only.¹⁶ The argument is symmetric for entrepreneurs in sector g . Again, in general, the diversification externality

¹⁵For sake of expositional simplicity, we do not state our results in this section as formal propositions. Formal statements and proofs are available from the authors upon request.

¹⁶In fact the equilibrium entrepreneurial ownership $(1 - w^{**})$ and the equilibrium loading β_1^{**} are chosen exactly as in Section 5 if the cost function for technology changes is commensurate.

and the excessive correlation of stock market returns across sectors is enhanced for firms and economies that employ high-powered incentive compensation schemes to address alternative agency problems.

9.2 Purely Idiosyncratic Risk in the Stock Market

Consider a firm in the single-sector economy studied thus far as in fact a continuum of identical firms of measure 1, indexed by $s \in (0, 1)$, and facing independent and identically distributed (i.i.d.) shocks.¹⁷ We perturb our basic decomposition of stock market returns as follows:

$$y_1^{f,s} - E(y_1^f) \equiv \beta_1^f x_1 + \beta_2^f (x_2 + x_2^s), \quad s \in (0, 1). \quad (51)$$

Factor x_2^s represents firm s 's purely idiosyncratic component: It is i.i.d. over s , uncorrelated with x_1 and x_2 , and it satisfies $E(x_2^s) = 0$ and $\text{var}(x_2^s) = \sigma$. An entrepreneur cannot trade the shares of his own firm and of other firms in his sector, but can trade in the stock market otherwise. Specifically, entrepreneurs can trade factors x_1 , but cannot trade $(x_2 + x_2^s)$: The entrepreneur in firm s must hold the sector-specific component of his firm, x_2 , as well as the purely idiosyncratic risk component, x_2^s .

Entrepreneurs have incentives to over-load their firms' risk onto the hedgeable component x_1 and away from the unhedgeable component $(x_2 + x_2^s)$. The resources entrepreneurs employ to reduce the loading of the firm on x_2^s are wasted from the point of view of the economy: Each unhedgeable unit of the firm carries a variance of $(1 + \sigma)$ when held by the entrepreneur; the same unit, when sold to investors, carries an effective variance of 1, as investors can diversify away x_2^s across the continuum of firms. Thus, in equilibrium the fraction of their firms that entrepreneurs hold decreases in σ , and the induced loading of each firm on the common stock market component x_1 increases in σ . In particular, for any $\sigma > 0$, this loading is greater than β_1^{**} , the equilibrium loading for $\sigma = 0$.

10 The Effect of Financial Innovations

We have treated financial markets as exogenous in our analysis. However, the incentives to increase aggregate risk loading of firms depend on the financial markets available in the

¹⁷Working with a continuum of firms that face i.i.d. shocks requires abusing the Law of Large Numbers. Working instead with a countable infinity of firms would avoid this abuse with no change to our analysis, but at a notational cost. See, for example, Al-Najjar (1995).

economy; entrepreneurs and managers will load their firms' returns on those aggregate risk factors that they are not restricted from trading in order to hedge their risk exposure. We now analyze the effect of introducing financial innovations that allow greater hedging of such risk exposure.

The impact depends on the specific form of the innovations. More precisely, innovations that allow the entrepreneurs and managers to hedge their sector-specific risk have a negative effect on incentives and thus possibly on welfare.¹⁸ In contrast, innovations that enable the entrepreneurs and managers to diversify only the firm-specific or purely idiosyncratic risks tend to have a positive effect on their incentives and overall welfare.

These points can be easily illustrated. Consider the entrepreneurs of firm f in Section 9.1. What is the effect of a financial innovation that allows these entrepreneurs to hedge x_2 , the risk of their sector? This financial innovation allows entrepreneurs to undo the incentives provided by the firm ownership structure to reduce the aggregate risk of the firm's cash flows! Their exposure to any sector-specific risk can be rebalanced after the firm is sold on the market. Consequently, in equilibrium the fraction of their firms that entrepreneurs hold coincides with the market share of the firm, $(1 - w^*)$; the loading on the common stock market component x_1 is not reduced at all and coincides with the initial loading $\bar{\beta}_1^f$. This result represents a complete lack of entrepreneurial activity.

In contrast, consider an innovation that allows entrepreneurs to hedge the idiosyncratic component of the firm's return, x_2^s , but not the sector-specific component, x_2 (based on the factor structure of Section 9.2). In this case, entrepreneurs hedge x_2^s , as do all other agents. As a consequence, x_2^s has no effect on the economy whatsoever, and, in particular, entrepreneurs do not shy away from undertaking this risk. Thus, in equilibrium, entrepreneurs hold a fraction $(1 - w^{**})$ of the firm; and the effect of the innovation is to reduce the loading on the common stock market component to β_1^{**} . It follows that this innovation in fact has a positive welfare effect on the economy: It enables greater risk-sharing amongst agents and reduces wasteful diversification activity.

This analysis of financial innovations that allow entrepreneurs to better hedge their firm's idiosyncratic risk could help explain the finding of Campbell, Lettau, Malkiel, and Xu (2001) that, over the period 1962–1987, the U.S. stock market as a whole did not become more

¹⁸Even though such financial innovations generally have negative welfare effects for the economy as a whole, financial markets tend to introduce such financial instruments: entrepreneurs and managers will in fact demand them. As previously discussed, to manage labor income risk for CEOs and senior executives, investment banks have created a sizable market to manufacture derivative products. Bettis, Bizjak, and Lemmon (1999) provide direct evidence for managerial demand for such financial innovations. Ofek and Yermack (2000) also document the managerial propensity to actively rebalance their portfolios within the restrictions on the sale of their shareholdings.

volatile, even though individual firm volatility increased substantially. The increase in individual firm volatility could result, at least in part, from such financial innovations having led managers and entrepreneurs to reduce wasteful activity aimed at reducing firm-specific risk. Campbell, Lettau, Malkiel, and Xu (2001) also document a decline in the correlation between individual stock returns. This is consistent with the reduced incentives of managers to substitute firm-specific risks with hedgeable economy-wide risks. Bettis, Bizjak, and Lemmon (1999) document that the purchases of zero-cost collars and equity swaps by corporate insiders are followed by an increase in the volatility of their stocks' returns. While Bettis, Bizjak, and Lemmon do not isolate the systematic and the idiosyncratic components of volatility, their evidence is potentially consistent with our argument.

11 Conclusions

In this paper, we examine implications on capital budgeting deriving from a manager's ability to hedge the aggregate risk components of his exposure to firm cash flows. In particular, we focus on the incentives of managers to reduce the firm-specific risk, for example, by passing up entrepreneurial activity in favor of more prosaic projects. We show that such risk-substitution increases aggregate risk in stock markets and reduces the ability of investors to share risks via stock markets. We characterize the optimal ownership structure of the firm designed to counteract this diversification externality and study its welfare properties.

Our objective has been to identify managerial incentives as an endogenous determinant of the extent of aggregate or economy-wide risk. Our integrated model could be of more general use in financial economics. Potential applications include analysis of cross-sectional and time-series variation in firm-level and market-level volatility, and studying changes in the risk composition of firms following corporate mergers and acquisitions.

Appendix A: Capital Asset Pricing Model (CAPM) Economy and Its Competitive Equilibrium

Consider the economy described in Section 2 and its competitive equilibrium defined in Section 3. It is convenient and formally equivalent to represent the competitive equilibrium in terms of the markets and the prices of the two risk factors, x_1 and x_2 : using the aggregate risk asset x_1 and the firm f , agents $h > 1$ can replicate the payoff of the sector-specific risk factor x_2 ; and a portfolio $(\theta_0, \theta_1, \theta_f)$ of any agent's positions in the bond, asset x_1 , and the firm f , maps one-to-one onto a portfolio $(\theta_0 + \theta_f E(y_1^f), \theta_1 + \beta_1^f \theta_f, \beta_2^f \theta_f)$ of the agent's positions in the bond, asset x_1 , and asset x_2 . For the entrepreneur ($h = 1$), a participation restriction against trading the firm translates into a participation restriction against trading the sector-specific risk factor x_2 . If H_j denotes the set of agents trading asset j and $|H_j|$ is the size of this set, then $H_1 = 1, \dots, H$, $|H_1| = H$, $H_2 = 2, \dots, H$, and $|H_2| = H - 1$. Let \mathcal{J}^h be the set of risky assets that the agent can trade: $\mathcal{J}^1 = \{1\}$, and $\mathcal{J}^h = \{1, 2\}$ for $h > 1$. Finally, the positive net supply of the firm translates into a positive net supply of both the factors and the risk-free asset: $s_0 = wE(y_1^f)$, and $s_j = w\beta_j^f$ for $j = 1, 2$.

The competitive equilibrium defined in equations (4)–(11) translates into a competitive equilibrium of the economy with orthogonal risk factors as follows: The problem of each agent h is to choose a consumption allocation at time 0, c_0^h ; portfolio positions in the risk-free bond and in all tradable assets, $[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}^h}$; and a consumption allocation at time 1, a random variable c_1^h ; so as to maximize the expected utility

$$E \left[u^h(c_0^h, c_1^h) \right] \equiv -\frac{1}{A} e^{-Ac_0^h} + E \left[-\frac{1}{A} e^{-Ac_1^h} \right] \quad (\text{A.1})$$

subject to the budget constraints and the restricted participation constraints:

$$c_0^h = y_0^h - \pi_0 \theta_0^h - \sum_{j=1}^2 \pi_j \theta_j^h, \quad h > 1, \quad (\text{A.2})$$

$$c_0^1 = y_0^1 + wp^f - \pi_0 \theta_0^1 - \pi_1 \theta_1^1, \quad (\text{A.3})$$

$$c_1^h = y_1^h + \theta_0^h + \sum_{j=1}^2 \theta_j^h x_j, \quad h > 1, \quad (\text{A.4})$$

$$c_1^1 = \theta_0^1 + \theta_1^1 x_1 + (1 - w)y_1^f. \quad (\text{A.5})$$

Definition 1 *A competitive equilibrium is a consumption allocation (c_0^h, c_1^h) , for all agents $h = 1, \dots, H$, that solves the problem of maximizing (A.1) subject to (A.2–A.5) at prices $[\pi_0, \pi_1, \pi_2]$, and such that consumption and financial markets clear*

$$\sum_h (c_0^h - y_0^h) \leq 0, \quad (\text{A.6})$$

$$\sum_h (c_1^h - y_1^h) \leq 0 \text{ (with probability 1 over all possible states at } t = 1), \text{ and} \quad (\text{A.7})$$

$$\sum_h \theta_j^h = s_j, \quad j = 0, 1, 2, \text{ where } s_j \text{ is the net supply of factor } j. \quad (\text{A.8})$$

Proposition 7 *The competitive equilibrium of the two-period CAPM economy defined by equations (A.1)–(A.8) is characterized by prices of assets (π_j), portfolio choices (θ_j^h), and consumption allocations (c_t^h), given below.*

$$\pi_0 = \exp \left\{ A(y_0 - Ey_1) + \frac{A^2}{2H} \sum_{h=1}^H \left[(1 - R_h^2) \text{var}(y_1^h) + \sum_{j \in J^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2 \right] \right\}, \quad (\text{A.9})$$

where

$$y_0 = \frac{1}{H} \sum_{h=1}^H y_0^h, \quad y_1 = \frac{1}{H} \sum_{h=1}^H y_1^h, \quad (\text{A.10})$$

$$\beta_1 = \text{cov} \left((1 - w)y_1^1 + \sum_{h=2}^H y_1^h, x_1 \right), \quad \beta_2 = \text{cov} \left(\sum_{h=2}^H y_1^h, x_2 \right), \quad (\text{A.11})$$

$$s_0 = wE(y_1^f), \quad s_j = w\beta_j^f, \quad j = 1, 2, \quad (\text{A.12})$$

$$\frac{\pi_j}{\pi_0} = E(x_j) - A \left(\beta_j + \frac{1}{|H_j|} s_j \right), \quad j = 1, 2, \quad (\text{A.13})$$

and for $h > 1$ (non-entrepreneurs),

$$R_h^2 \equiv \frac{\sum_{j=1}^2 (\beta_j^h)^2}{\text{var}(y_1^h)}, \quad (\text{A.14})$$

$$\theta_j^h = \left(\beta_j + \frac{1}{|H_j|} s_j \right) - \beta_j^h, \quad j = 1, 2, \quad (\text{A.15})$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h - E(y_1^h) - \sum_{j=1}^2 \pi_j \theta_j^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad (\text{A.16})$$

$$c_1^h = \theta_0^h + \sum_{j=1}^2 \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + \left(y_1^h - \sum_{j=1}^2 \beta_j^h x_j \right), \quad (\text{A.17})$$

$$\text{var}(c_1^h) = \text{var}(y_1^h) - \sum_{j=1}^2 (\beta_j^h)^2 + \sum_{j=1}^2 \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2, \quad (\text{A.18})$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + E(y_1^h) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h), \quad (\text{A.19})$$

and finally, for $h = f = 1$ (entrepreneur),

$$R_h^2 \equiv \frac{(1-w)^2 (\beta_1^f)^2}{\text{var}(y_1^h)}, \quad (\text{A.20})$$

$$\theta_1^h = \left(\beta_1 + \frac{1}{|H_1|} s_1 \right) - (1-w)\beta_1^f, \quad \text{and } \theta_2^h = 0, \quad (\text{A.21})$$

$$\theta_0^h = \frac{1}{1+\pi_0} \left(y_0^h + wp^f - (1-w)E(y_1^f) - \pi_1 \theta_1^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad (\text{A.22})$$

$$c_1^h = \theta_0^h + \left(\beta_1 + \frac{1}{|H_1|} s_1 \right) x_1 + (1-w) (y_1^f - \beta_1^f x_1), \quad (\text{A.23})$$

$$\text{var}(c_1^h) = (1-w)^2 \text{var}(y_1^f) - (1-w)^2 (\beta_1^f)^2 + \left(\beta_1 + \frac{1}{|H_1|} s_1 \right)^2, \quad (\text{A.24})$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + (1-w)E(y_1^f) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h). \quad (\text{A.25})$$

This equilibrium with positive supply of assets is similar to that without positive supply (see, Willen, 1997, and Acharya and Bisin, 2000), except that (i) the entrepreneur holds only a fraction $(1-w)$ of his firm, (ii) at time 0 the entrepreneur collects proceeds of wp^f from selling fraction w of his firm, and (iii) aggregate beta β_j in the zero-supply-of-assets case is replaced by $(\beta_j + \frac{1}{|H_j|} s_j)$ to reflect the positive supply of assets. A derivation of these competitive equilibrium properties is a special case of the derivation for the general CAPM economy outlined in Appendix D, available from the authors upon request.

Appendix B: Welfare Properties

We characterize the welfare measure used in the paper (individual h 's compensating transfer, μ^h , and the aggregate compensating transfer, μ) in terms of the equilibrium price of the bond, π_0 , and the exogenously given price of the bond at autarky, π_0^a .

Proposition 8 *The individual welfare of agent h is measured by*

$$\mu^1 = c_0^1 - c_0^{a1} - \frac{1}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C(\beta_1^f - \bar{\beta}_1^f)^2, \text{ and} \quad (\text{B.1})$$

$$\mu^h = c_0^h - c_0^{ah} - \frac{1}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a}, \text{ for } h = 2, \dots, H. \quad (\text{B.2})$$

The aggregate welfare of the economy is measured by

$$\mu = -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C(\beta_1^f - \bar{\beta}_1^f)^2. \quad (\text{B.3})$$

Proof: Consider the definitions of individual welfare in equations (35) and (36). Since

$$U^1(c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2, c_1^1) \equiv -\frac{(1 + \pi_0)}{A} e^{-A[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}, \quad (\text{B.4})$$

$$U^h(c_0^h, c_1^h) \equiv \frac{(1 + \pi_0)}{A} e^{-Ac_0^h}, \text{ for } h = 2, \dots, H, \text{ and,} \quad (\text{B.5})$$

$$U^{ah}(c_0^{ah} + \mu^h, c_1^{ah}) \equiv -\frac{(1 + \pi_0^a)}{A} e^{-Ac_0^{ah}}, \forall h, \quad (\text{B.6})$$

we obtain that μ^1 , the individual welfare of entrepreneur, satisfies

$$\frac{(1 + \pi_0)}{A} e^{-A[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]} = \frac{(1 + \pi_0^a)}{A} e^{-A(c_0^{a1} + \mu^1)}. \quad (\text{B.7})$$

Simplifying this, we obtain the expression for μ^1 in equation (B.1). The result for μ^h , $h = 2, \dots, H$, follows similarly. Finally, since $\mu = \sum_{h=1}^H \mu^h$, and $\sum_{h=1}^H c_0^h = \sum_{h=1}^H c_0^{ah} = \sum_{h=1}^H y_0^h$ by the market-clearing conditions under both economies, we obtain the expression for μ in equation (B.3). \diamond

Appendix C: Proofs of Propositions

Proof of Proposition 1. Under the benchmark case specified in equation (21), the entrepreneur simultaneously chooses the ownership structure, w , and the cash flow betas, β_1^f and β_2^f . We first consider the representative entrepreneur's choice of β_1^f for a given w , and thereafter the choice of optimal w , taking into account the choice of β_1^f ; β_2^f is determined by the constraint (23).

The entrepreneur is a price-taker and trades only in asset 0 and asset 1, but rationally anticipates the effect of cash flow betas on the price of the firm p^f (equation 22). Using the

competitive equilibrium outcomes (A.21)–(A.25) from Appendix A, we obtain

$$\frac{\partial}{\partial \beta_1^f} \left[c_0^1(w, \beta_1^f, \beta_2^f, p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f) - C(\beta_1^f - \bar{\beta}_1^f)^2 \right] \quad (\text{C.1})$$

$$= \frac{\partial}{\partial \beta_1^f} \left[\theta_0^1 - \frac{A}{2} \text{var}(c_1^1) \right] - 2C(\beta_1^f - \bar{\beta}_1^f) \quad (\text{C.2})$$

$$= \frac{\partial}{\partial \beta_1^f} \left[\frac{1}{1 + \pi_0} \left(w\pi_1 \beta_1^f + w\pi_2 \beta_2^f + (1 - w)\pi_1 \beta_1^f - \frac{A\pi_0}{2} \text{var}(c_1^1) \right) \right] - 2C(\beta_1^f - \bar{\beta}_1^f) \quad (\text{C.3})$$

$$= \frac{A\pi_0}{1 + \pi_0} \left[\left((1 - w)^2 + \frac{w^2}{H - 1} - \frac{1}{H} \right) \beta_1^f - \frac{1}{H} \sum_{h=2}^H \beta_1^h \right] - 2C(\beta_1^f - \bar{\beta}_1^f), \quad (\text{C.4})$$

where, to obtain equation (C.3), we substitute constraints (22) and (23) and the restricted participation constraints $\theta_2^1 \equiv 0$, $|H_1| = H$, and $|H_2| = H - 1$. Finally, to obtain equation (C.4) from equation (C.3), we substitute equilibrium prices and aggregate supplies using equations (A.11)–(A.13) and the maintained assumption (equation 17) that $\beta_2 = 0$. The optimal β_1^f for a given w sets the partial derivative in equation (C.4) to zero.

Consider next the choice of w given the choice of β_1^f .

$$\frac{d[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw} = \frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} \frac{d\beta_1^f}{dw} + \frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} \quad (\text{C.5})$$

$$= \frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} \quad \text{by the envelope theorem.} \quad (\text{C.6})$$

Since the entrepreneur is a price-taker, using equations (A.21)–(A.25) we obtain

$$\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} = \frac{\partial c_0^1}{\partial w} = \frac{\partial}{\partial w} \left[(1 - w)E(y_1^f) + \theta_0^1 - \frac{A}{2} \text{var}(c_1^1) \right], \quad \text{where} \quad (\text{C.7})$$

$$\begin{aligned} \theta_0^1 - \frac{A}{2} \text{var}(c_1^1) &= \frac{1}{1 + \pi_0} \left[y_0^1 - (1 - w)E(y_1^f) + \pi_0 w E(y_1^f) + \sum_{j=1}^2 \pi_j w \beta_j^f - \pi_1 \theta_1^1 \right] \\ &- \frac{A\pi_0}{2(1 + \pi_0)} \text{var}(c_1^1) - \frac{1}{A(1 + \pi_0)} \ln(\pi_0). \end{aligned} \quad (\text{C.8})$$

Simplifying using constraints (22) and (23), the maintained assumptions, and the expressions for equilibrium prices and aggregate supplies in equations (A.11)–(A.13), we obtain

$$\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} = -E(y_1^f) + \frac{1}{1 + \pi_0} \left[(1 + \pi_0)E(y_1^f) + \pi_2 \beta_2^f + A\pi_0(1 - w)(\beta_2^f)^2 \right]$$

$$= \frac{1}{1 + \pi_0} \left[\pi_2 \beta_2^f + A \pi_0 (1 - w) (\beta_2^f)^2 \right] \quad (\text{C.9})$$

$$= \frac{A \pi_0}{1 + \pi_0} (\beta_2^f)^2 \left(1 - w - \frac{w}{H - 1} \right). \quad (\text{C.10})$$

This first-order derivative is set to zero at $w = w^* \equiv (1 - \frac{1}{H})$. Furthermore,

$$\frac{d^2[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw^2} = \frac{d}{dw} \left(\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} \right) = \frac{\partial^2[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w^2} \quad (\text{C.11})$$

$$= -\frac{HA\pi_0}{(H-1)(1+\pi_0)} (\beta_2^f)^2 < 0, \text{ since} \quad (\text{C.12})$$

$$\frac{\partial^2[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f \partial w} = \frac{\partial}{\partial \beta_1^f} \left[\frac{A\pi_0}{1+\pi_0} (\beta_2^f)^2 \left(1 - w - \frac{w}{H-1} \right) \right] \quad (\text{C.13})$$

$$= -\frac{2A\pi_0}{1+\pi_0} \beta_1^f \left(1 - w - \frac{w}{H-1} \right) = 0 \text{ at } w = w^*. \quad (\text{C.14})$$

The equity ownership retained by the entrepreneur is thus given by $(1 - w^*) = \frac{1}{H}$. Substituting $w = w^*$ in equation (C.4) and setting it to zero, we find that the aggregate risk cash flow beta chosen by the entrepreneur is $\beta_1^* = \bar{\beta}_1^f - \frac{A\pi_0}{2CH(1+\pi_0)} \sum_{h=2}^H \beta_1^h < \bar{\beta}_1^f$. Note that at $w = w^*$, $\frac{\partial^2}{\partial \beta_1^f} [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2] = -\frac{2AC\pi_0}{1+\pi_0} < 0$, satisfying the optimality of β_1^* . \diamond

Since the proof of Proposition 3 (first-best) relies on the steps in the above proof, we present it next. The reader can jump directly to the proofs of Propositions 2 and 5.

Proof of Proposition 3. In the first-best specified in equation (38), the equity ownership structure w and cash flow beta β_1^f are chosen by the planner to maximize the compensating aggregate transfer, μ , given by equation (B.3). Consider first the choice of β_1^f by the planner for a given w . From equation (A.9) for π_0 at the competitive equilibrium, we obtain

$$\frac{\partial \mu}{\partial \beta_1^f} = -\frac{H}{A(1+\pi_0)} \frac{\partial \pi_0}{\partial \beta_1^f} - 2C(\beta_1^f - \bar{\beta}_1^f), \text{ where} \quad (\text{C.15})$$

$$\begin{aligned} \frac{\partial \pi_0}{\partial \beta_1^f} &= \frac{\pi_0 A^2}{2H} \frac{\partial}{\partial \beta_1^f} \left[(1-w)^2 \text{var}(y_1^f) - (1-w)^2 (\beta_1^f)^2 + \sum_{h=1}^H \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2 \right] \\ &= \frac{\pi_0 A^2}{2H} \frac{\partial}{\partial \beta_1^f} \left[-(1-w)^2 (\beta_1^f)^2 + \frac{1}{H} \left(\sum_{h=2}^H \beta_1^h + \beta_1^f \right)^2 + \frac{w^2}{H-1} (\beta_2^f)^2 \right] \end{aligned} \quad (\text{C.16})$$

$$= \frac{\pi_0 A^2}{H} \left[-(1-w)^2 \beta_1^f + \frac{1}{H} \left(\sum_{h=2}^H \beta_1^h + \beta_1^f \right) - \frac{w^2}{H-1} \beta_1^f \right], \quad (\text{C.17})$$

where we have employed constraint (39), conditions (A.11) and (A.12), and $\beta_2 = 0$. Thus,

$$\frac{\partial \mu}{\partial \beta_1^f} = \frac{A \pi_0}{1 + \pi_0} \left[\left((1-w)^2 + \frac{w^2}{H-1} - \frac{1}{H} \right) \beta_1^f - \frac{1}{H} \sum_{h=2}^H \beta_1^h \right] - 2C(\beta_1^f - \bar{\beta}_1^f), \quad (\text{C.18})$$

which is identical to the first-order condition for the choice of β_1^f by the entrepreneur in the benchmark case (equation C.4). Thus, for a given w , the first-best choice of β_1^f is the same as that of the entrepreneur in the absence of moral hazard.

Next, consider the planner's choice of w . By the envelope theorem, we obtain

$$\frac{d\mu}{dw} = \frac{\partial \mu}{\partial w} = - \frac{H}{A(1 + \pi_0)} \frac{\partial \pi_0}{\partial w}, \quad \text{where} \quad (\text{C.19})$$

$$\begin{aligned} \frac{\partial \pi_0}{\partial w} &= \pi_0 \frac{\partial}{\partial w} \left[\frac{A^2}{2H} \left((1-w)^2 \text{var}(y_1^f) - (1-w)^2 (\beta_1^f)^2 + \sum_{h=1}^H \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2 \right) \right] \\ &= \frac{\pi_0 A^2}{2H} \frac{\partial}{\partial w} \left[(1-w)^2 (\beta_2^f)^2 + \frac{1}{H} \left(\sum_{h=2}^H \beta_1^h + \beta_1^f \right)^2 + \frac{w^2}{H-1} (\beta_2^f)^2 \right] \end{aligned} \quad (\text{C.20})$$

$$= \frac{\pi_0 A^2}{2H} \left[-2(1-w)(\beta_2^f)^2 + \frac{2w}{H-1} (\beta_2^f)^2 \right] \quad (\text{C.21})$$

$$= \frac{\pi_0 A^2}{H} (\beta_2^f)^2 \left(1 - w - \frac{w}{H-1} \right). \quad (\text{C.22})$$

The above equations can be simplified to yield

$$\frac{d\mu}{dw} = \frac{A \pi_0}{1 + \pi_0} (\beta_2^f)^2 \left(1 - w - \frac{w}{H-1} \right), \quad (\text{C.23})$$

which is identical to the first-order condition for choice of w by the entrepreneur in the benchmark case (equation C.10). It follows that in the absence of moral hazard, the entrepreneur's choices of w^* and β_1^* are first-best efficient. \diamond

From an expositional standpoint, it is easier to present the proofs of Propositions 2, 4, 5, and 6, in an interleaved fashion.

Proofs of Propositions 2, 4, 5, and 6.

Sequence of Steps: First, we characterize the technology choice β_1^{**} and the ownership structure choice w^{**} for the case of owner-managed firm with moral hazard (Proposition 2) and for the case of corporations (Proposition 5). Here, we also prove Proposition 6. Next, we show the constrained efficiency of these choices (Proposition 4). Finally, we prove the remaining part of Propositions 2 and 5: the equilibrium aggregate risk loading in the moral-hazard cases exceeds the benchmark aggregate risk loading, i.e., $\beta_1^* < \beta_1^{**}$.

Step 1: First consider the entrepreneur's technology choice for owner-managed firms with moral hazard as specified in equation (24). The analysis differs from the proof of Proposition 1 (the case of no moral hazard) as follows: From the entrepreneur's standpoint, the firm's proceeds cannot be affected by a choice of betas, because the betas are not observed by investors. Formally, this implies that the constraint $p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f$ (equation 22) does not affect the capital budgeting problem in the case of moral hazard. Define $\hat{c}_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f)$, where p^f is treated as a lump sum constant in order to distinguish it from $c_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f)$.

Using the competitive equilibrium outcomes (A.21)–(A.25), we obtain

$$\frac{\partial}{\partial \beta_1^f} [c_0^1(w, \beta_1^f, \beta_2^f, p^f) - C(\beta_1^f - \bar{\beta}_1^f)^2] \quad (\text{C.24})$$

$$= \frac{\partial}{\partial \beta_1^f} \left[\theta_0^1 - \frac{A}{2} \text{var}(c_1^1) \right] - 2C(\beta_1^f - \bar{\beta}_1^f) \quad (\text{C.25})$$

$$= \frac{\partial}{\partial \beta_1^f} \left[\frac{1}{1 + \pi_0} \left((1 - w)\pi_1 \beta_1^f - \frac{A\pi_0}{2} \text{var}(c_1^1) \right) \right] - 2C(\beta_1^f - \bar{\beta}_1^f) \quad (\text{C.26})$$

$$= \frac{\pi_1(1 - w)}{1 + \pi_0} + \frac{A\pi_0}{1 + \pi_0} (1 - w)^2 \beta_1^f - 2C(\beta_1^f - \bar{\beta}_1^f) \quad (\text{C.27})$$

$$= \frac{A\pi_0}{1 + \pi_0} (1 - w) \left[\left(1 - w - \frac{1}{H} \right) \beta_1^f - \frac{1}{H} \sum_{h=2}^H \beta_1^h \right] - 2C(\beta_1^f - \bar{\beta}_1^f), \quad (\text{C.28})$$

where, to obtain equation (C.27), we substitute only the constraint (25) and the restricted participation constraints $\theta_2^1 \equiv 0$, $|H_1| = H$, and $|H_2| = H - 1$; finally, to obtain equation (C.28) from equation (C.27), we substitute equilibrium prices and aggregate supplies using equations (A.11)–(A.13) and the maintained assumption that $\beta_2 = 0$.

Let $\beta_1^f(w)$ be the choice of β_1^f that sets the partial derivative in equation (C.28) to zero. This is the solution to the capital budgeting problem in (24). In the case of corporations, the manager is first awarded a fraction $(1 - w)$ of the firm as a stock reward, and then the manager chooses β_1^f and β_2^f . Investors retain the remaining fraction w . The initial time-1 endowments for all agents are thus identical to their endowments in the case of owner-managed firms for the same fraction w of the firm sold on the stock market. The only change arises at time 0 due to the time-0 compensation W awarded to the manager to ensure that in equilibrium he earns the reservation utility of \bar{W} (see footnote 12). Since these transfers are treated as lump sum constants while undertaking the choice of technology, the managerial choice $\beta_1^f(w)$ is identical to that of the owner-manager with moral hazard.

Next, consider the choice of w by the entrepreneur of the owner-managed firm, as specified in equation (26). While choosing w , the rational expectation constraints apply both for the

firm value (equation 27) and for the effect of w on β_1^f (equation 28). Thus, we obtain

$$\frac{d[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw} = \frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} \frac{d\beta_1^f(w)}{dw} + \frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} \quad (\text{C.29})$$

where $c_0^1 \equiv c_0^1(w, \beta_1^f(w), \beta_2^f(w), p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f)$ as distinct from $\hat{c}_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f)$. We examine successively each of the three terms in the above equation.

(I) First, note that

$$\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w} = \frac{A\pi_0}{1 + \pi_0} (\beta_2^f)^2 \left(1 - w - \frac{w}{H-1}\right) = 0 \text{ at } w = w^*, \quad (\text{C.30})$$

as in the case of no moral hazard (see equation C.10 in the proof of Proposition 1, above).

(II) Second, we examine $\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f}$. Note that, unlike the case of no moral hazard, $\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} \neq 0$ (in general), since $\beta_1^f(w)$ is chosen treating p^f as a lump sum constant. That is, $\beta_1^f(w)$ is chosen such that $\frac{\partial[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} = 0$, which is not generally the same as $\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} = 0$. Thus, when $\beta_1^f = \beta_1^f(w)$,

$$\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} = \frac{\partial[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} - \frac{\partial(\hat{c}_0^1 - c_0^1)}{\partial\beta_1^f} = -\frac{\partial(\hat{c}_0^1 - c_0^1)}{\partial\beta_1^f}, \quad (\text{C.31})$$

$$\text{where } \frac{\partial(\hat{c}_0^1 - c_0^1)}{\partial\beta_1^f} = \frac{\partial}{\partial\beta_1^f} \left[\frac{1}{1 + \pi_0} (w p^f - \pi_0 w E(y_1^f) - \pi_1 w \beta_1^f - \pi_2 w \beta_2^f) \right] \quad (\text{C.32})$$

$$= \frac{1}{1 + \pi_0} \left[-\pi_1 w - \pi_2 w \left(-\frac{\beta_1^f}{\beta_2^f} \right) \right] \quad (\text{C.33})$$

$$= \frac{A\pi_0 w}{1 + \pi_0} \left[\frac{1}{H} \left(\sum_{h=2}^H \beta_1^h + \beta_1^f \right) - \frac{w}{H-1} \beta_1^f \right] \quad (\text{C.34})$$

$$= \frac{A\pi_0 w}{H(1 + \pi_0)} \left[\sum_{h=2}^H \beta_1^h + \left(1 - \frac{w}{w^*}\right) \beta_1^f \right], \quad (\text{C.35})$$

where we have employed the expressions for equilibrium prices and aggregate supplies in equations (A.11)–(A.13). It follows that at $w = w^*$, $\frac{\partial[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} < 0$, since $\sum_{h=2}^H \beta_1^h > 0$.

(III) Finally, we examine $\frac{d\beta_1^f(w)}{dw}$. Since $\frac{\partial[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^f} = 0$ and $\frac{\partial^2[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^{f2}} < 0$ at $\beta_1^f = \beta_1^f(w)$ by the optimality of $\beta_1^f(w)$ under moral hazard, it follows that

$$\frac{\partial^2[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial\beta_1^{f2}} \frac{d\beta_1^f(w)}{dw} + \frac{\partial^2[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w \partial\beta_1^f} = 0 \quad (\text{C.36})$$

$$\Rightarrow \text{sign} \left(\frac{d\beta_1^f(w)}{dw} \right) = \text{sign} \left(\frac{\partial^2 [\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w \partial \beta_1^f} \right). \quad (\text{C.37})$$

From equation (C.28), we obtain

$$\frac{\partial^2 [\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w \partial \beta_1^f} = \frac{A\pi_0}{1 + \pi_0} \left[-2(1-w)\beta_1^f + \frac{1}{H} \left(\sum_{h=2}^H \beta_1^h + \beta_1^f \right) \right], \quad (\text{C.38})$$

where $\beta_1^f = \beta_1^f(w)$. It follows that at $w = w^* \equiv (1 - 1/H)$,

$$\frac{\partial^2 [\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w \partial \beta_1^f} = \frac{A\pi_0}{H(1 + \pi_0)} \left[\sum_{h=2}^H \beta_1^h - \beta_1^f(w) \right]. \quad (\text{C.39})$$

Thus, $\frac{d\beta_1^f(w)}{dw} < 0$ at $w = w^*$ if and only if $\beta_1^f(w^*) > \sum_{h=2}^H \beta_1^h$. Substituting $w = (1 - 1/H)$ in equation (C.28) and equating it to zero yields

$$\beta_1^f(w^*) = \bar{\beta}_1^f - \frac{A\pi_0}{2CH^2(1 + \pi_0)} \sum_{h=2}^H \beta_1^h < \bar{\beta}_1^f. \quad (\text{C.40})$$

It follows from (I), (II), and (III) above that at $w = w^*$, $\frac{d[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw} > 0$ iff $\bar{\beta}_1^f > K(\pi_0) \sum_{h=2}^H \beta_1^h$, where $K(\pi_0) = A\pi_0 / (2CH^2(1 + \pi_0)) + 1$. If the cost parameter C is sufficiently high, then the function $[\hat{c}_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]$ is globally concave.¹⁹ Then, the optimal ownership structure under moral hazard is w^{**} , where $w^{**} > w^*$ iff $\bar{\beta}_1^f > K(\pi_0) \sum_{h=2}^H \beta_1^h$.

Furthermore, since $\beta_1^f(w^*) < \bar{\beta}_1^f$, and in both cases above w^{**} is chosen to reduce the aggregate risk loading from its value at w^* , it follows that $\beta_1^{f**} = \beta_1^f(w^{**}) < \beta_1^f(w^*) < \bar{\beta}_1^f$.

¹⁹Define function $g(w, \beta_1^f(w)) \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f) - C(\beta_1^f - \bar{\beta}_1^f)^2$, where $\beta_1^f = \beta_1^f(w)$ and $(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}$. Then,

$$\frac{d^2 g}{dw^2} = \frac{\partial^2 g}{\partial w^2} + 2 \frac{\partial^2 g}{\partial \beta_1^f \partial w} \frac{d\beta_1^f(w)}{dw} + \frac{\partial^2 g}{\partial \beta_1^f{}^2} \left(\frac{d\beta_1^f(w)}{dw} \right)^2 + \frac{d^2 \beta_1^f(w)}{dw^2} \frac{\partial g}{\partial \beta_1^f}. \quad (\text{C.41})$$

Note the following: (i) From equation (C.12), $\frac{\partial^2 g}{\partial w^2} < 0, \forall w$; this is the global concavity of the entrepreneur's objective as a function of w under the benchmark case. (ii) $\frac{\partial^2 g}{\partial \beta_1^f{}^2} = \frac{\partial^2 c_0^1}{\partial \beta_1^f{}^2} - C < 0$ for C sufficiently large. (iii) Using equation (C.42), it can be shown that $\frac{d\beta_1^f(w)}{dw} \rightarrow 0$ as $C \rightarrow \infty$. (iv) Similarly, it can be shown that $\frac{d^2 \beta_1^f(w)}{dw^2} \rightarrow 0$ as $C \rightarrow \infty$. (v) From equation (C.13), $\frac{\partial^2 g}{\partial \beta_1^f \partial w}$ is independent of C . Finally, (vi) from equation (C.4), $\frac{\partial g}{\partial \beta_1^f}$ is bounded. It follows that for C sufficiently large, $\frac{d^2 g}{dw^2} < 0, \forall w$, i.e., if the moral-hazard problem is not "too severe," then as a function of w the objective of the entrepreneur under the moral-hazard case is a "perturbation" around the globally concave objective under no moral hazard.

Since $\pi_0 \in [0, 1]$, $K(\pi_0) \in [1, K]$, where $K \equiv 1 + A/(4CH^2)$. This proves Proposition 2 (except for $\beta_1^* < \beta_1^{**}$, which is proven in Step 3).

For Proposition 6, equating the first-order derivative in (C.28) to zero, we obtain the general solution to the capital budgeting problem under moral hazard:

$$\beta_1^f(w) = K_1(w)\bar{\beta}_1^f - K_2(w) \sum_{h=2}^H \beta_1^h, \text{ where} \quad (\text{C.42})$$

$$K_1(w) = \left[1 - \frac{A\pi_0(1-w)}{2C(1+\pi_0)} \left(1 - w - \frac{1}{H} \right) \right]^{-1}, \text{ and} \quad (\text{C.43})$$

$$K_2(w) = \frac{A\pi_0(1-w)}{2CH(1+\pi_0)} K_1(w). \quad (\text{C.44})$$

From equations (C.37) and (C.38), it follows that $\beta_1^f(w)$ is decreasing in w iff $(2H(1-w) - 1)\beta_1^f(w) > \sum_{h=2}^H \beta_1^h$. Substituting for $\beta_1^f(w)$ from (C.42) and recognizing that $\pi_0 \in [0, 1]$, a sufficient condition for $\beta_1^f(w)$ to be decreasing in w is

$$\bar{\beta}_1^f > K(w) \sum_{h=2}^H \beta_1^h, \text{ where } K(w) = \max \left[0, \frac{1}{(2H(1-w) - 1)} \left(\frac{A\pi_0(1-w)^2}{2C(1+\pi_0)} + 1 \right) \right] \quad (\text{C.45})$$

Step 2: Next, we prove the constrained efficiency of the choice of the ownership structure w^{**} (Proposition 4). The constrained-efficient choice of w is specified in equation (40). From the proof of the first-best efficiency of the benchmark case (Proposition 3), we know that $\forall w$:

$$\frac{\partial \mu}{\partial \beta_1^f} = \frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f}, \text{ and } \frac{\partial \mu}{\partial w} = \frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial w}. \quad (\text{C.46})$$

It follows that $\forall w$:

$$\frac{d\mu}{dw} = \frac{\partial \mu}{\partial \beta_1^f} \frac{d\beta_1^f(w)}{dw} + \frac{\partial \mu}{\partial w} = \frac{d[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw}, \quad (\text{C.47})$$

where the planner is constrained to choose the same $\frac{d\beta_1^f(w)}{dw}$ as the entrepreneur. The constrained efficiency of w^{**} in the owner-managed firm with moral hazard follows.

The proof that the ownership structure choice is w^{**} under the corporation structure follows readily. The welfare of investors $\{h = 2, \dots, H\}$ is given by $\mu - \mu^1$, where μ is the aggregate compensating welfare and μ^1 is the individual compensating welfare of the manager. In equilibrium, μ^1 is such that $U^{a1}(c_0^{a1} + \mu^1, c^{a1}) \equiv \bar{W}$, the manager's reservation utility (see

equation 36). That is, μ^1 is a constant, which implies that the optimal ownership structure of a corporation simply maximizes μ and, by implication, is also constrained efficient.

Step 3: Finally, we prove that the choice of the risk loading under both of these governance structures, β_1^{**} , is greater than the benchmark case (first-best), β_1^* . Since w^{**} is constrained efficient, it follows that at $w = w^{**}$,

$$\frac{d\mu}{dw} = \frac{\partial\mu}{\partial\beta_1^f} \frac{d\beta_1^f(w)}{dw} + \frac{\partial\mu}{\partial w} = 0 \Rightarrow \frac{\partial\mu}{\partial\beta_1^f} = - \frac{\left(\frac{\partial\mu}{\partial w}\right)}{\left(\frac{d\beta_1^f}{dw}\right)}. \quad (\text{C.48})$$

Now, consider the case where $\beta_1^f(w^*) > \sum_{h=2}^H \beta_1^h$. Then, for the moral-hazard economy, $\frac{d\beta_1^f(w)}{dw} < 0$ at $w = w^{**} > w^*$. By the first-best efficiency of w^* , we have $\frac{\partial\mu}{\partial w} = 0$ and $\frac{\partial^2\mu}{\partial w^2} < 0$ at $w = w^*$. This, in turn, implies that $\frac{\partial\mu}{\partial w} < 0$ at $w = w^{**}$. From equation (C.48), we conclude that $\frac{\partial\mu}{\partial\beta_1^f} < 0$ at $w = w^{**}$. That is, the choice of β_1^f under moral hazard, β_1^{**} , exceeds the first-best (for which $\frac{\partial\mu}{\partial\beta_1^f} = 0$). The proof for the case where $\beta_1^f(w^*) < \sum_{h=2}^H \beta_1^h$ follows analogously. \diamond

References

- Acharya, V. and A. Bisin (2000) Optimal Financial Market Integration and Security Design, Working Paper, London Business School.
- Aggarwal, R. and A. Samwick (2003) Why Do Managers Diversify Their Firms? Agency Reconsidered, *Journal of Finance*, 58(1), 71–118.
- Allen, F. and D. Gale (1988) Optimal Security Design, *Review of Financial Studies*, 1, 229–263.
- Allen, F. and D. Gale (1991) Arbitrage, Short Sales and Financial Innovation, *Econometrica*, 59, 1041–1068.
- Al-Najjar, N. I. (1995) Decomposition and Characterization of Risk with a Continuum of Random Variables, *Econometrica*, 63, 1195–1224.
- Amihud, Y. and B. Lev (1981) Risk Reduction as a Managerial Motive for Conglomerate Mergers, *Bell (RAND) Journal of Economics*, 605–617.
- Bettis, J., Bizjak, J. and M. Lemmon (1999) Insider Trading in Derivative Securities: An Empirical Examination of the Use of Zero-Cost Collars and Equity Swaps by Corporate Insiders, Working Paper, College of Business, Arizona State University.
- Bettis, J., Coles, J. and M. Lemmon (2000) Corporate Policies Restricting Trading by Insiders, *Journal of Financial Economics*, 57, 191–220.
- Bisin, A. and P. Gottardi (1999) Competitive Equilibria with Asymmetric Information, *Journal of Economic Theory*, 87, 1–48.
- Campbell, J., Lettau, M., Malkiel, B. and Y. Xu (2001) Have Individual Stocks Become More Volatile? An Empirical Investigation of Idiosyncratic Risk, *Journal of Finance*, 56(1), 1–43.
- Denis, D., Denis, D. and A. Sarin (1997) Agency Problems, Equity Ownership, and Corporate Diversification, *Journal of Finance*, 52(1), 135–160.
- Diamond, D. and R. Verrechia (1982) Optimal Managerial Contracts and Equilibrium Security Prices, *Journal of Finance*, 37, 275–288.
- Fama, E. and M. Miller, (1972) *The Theory of Finance*, Dryden Press, Hinsdale, IL.
- Garvey, G. and T. Milbourn (2002) Incentive Compensation When Executives Can Hedge the Market: Evidence of RPE in the Cross-Section, *Journal of Finance*, forthcoming.

- Holmstrom, B. and P. Milgrom (1987) Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica*, 55, 303–328.
- Jensen, M. and W. Meckling (1976) Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics*, 3, 305–360.
- Jin, L. (2002) CEO Compensation, Diversification and Incentives: Theory and Empirical Results, *Journal of Financial Economics*, forthcoming.
- Lambert, R. (1986) Executive Effort and Selection of Risky Projects, *Rand Journal of Economics*, 17(1), 77–88.
- Laffont, J. and D. Martimort (2002) *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.
- Magill, M. and M. Quinzii (1998) Incentives and Risk Sharing in a Stock Market Equilibrium, Working Paper, University of Southern California.
- May, P. (1995) Do Managerial Motives Influence Firm Risk Reduction Strategies? *Journal of Finance*, 50(4), 1291–1308.
- Ofek, E. and D. Yermack (2000) Taking Stock: Equity-Based Compensation and the Evolution of Managerial Ownership, *Journal of Finance*, 55(3), 1367–1384.
- Ou-Yan, H. (2002) An Equilibrium Model of Asset Pricing and Moral Hazard, Working Paper, Fuqua School of Business, Duke University.
- Prescott, E. and R. Townsend (1984) Optima and Competitive Equilibria with Adverse Selection and Moral Hazard, *Econometrica*, 52, 21–45.
- Ramakrishnan, R. and A. Thakor (1984) The Valuation of Assets under Moral Hazard, *Journal of Finance*, 39(1), 229–238.
- Willen, P. (1997) Some Notes on CAPM, mimeo, Princeton University.

Captions for Figures

Figure 1: This figure plots $(1-w^{**})$, the equilibrium ownership in the moral-hazard economy of Section 5, as a function of $\bar{\beta}_1^f$, the initial or ‘intrinsic’ aggregate risk loading of the firm. The benchmark ownership in the case of no moral hazard, $(1-w^*)$ is also plotted. The numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^2 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\sum_{h=2}^H \beta_1^h = 2.5 > 0$ and $\sum_{h=2}^H \beta_1^h = 0$, consistent with the assumptions. The initial beta of the firm on aggregate risk, $\bar{\beta}_1^f$, is varied from 0.32 to 5.44. The competitive equilibrium is computed by a numerical fixed-point algorithm using the analytical expressions in Appendix A.

Figure 2: This figure plots $\beta_1^f(w^{**})$, the equilibrium beta of the firm on aggregate risk in the moral-hazard economy of Section 5, as a function of $(1-w^{**})$, the equilibrium ownership in the moral-hazard economy, when $\bar{\beta}_1^f$, the initial or ‘intrinsic’ aggregate risk loading of the firm, is varied. The numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^2 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\sum_{h=2}^H \beta_1^h = 2.5 > 0$ and $\sum_{h=2}^H \beta_1^h = 0$, consistent with the assumptions. The initial beta of the firm on aggregate risk $\bar{\beta}_1^f$ is varied from 0.32 to 5.44. The competitive equilibrium is computed by a numerical fixed-point algorithm using the analytical expressions in Appendix A.

Figure 3: This figure plots the equilibrium beta of the firm, $\beta_1^f(w)$, against the exogenous equity ownership of the manager, $(1-w)$. The initial or ‘intrinsic’ aggregate risk loading of the firm, $\bar{\beta}_1^f$, equals 3.84. At this risk loading, the endogenous equity ownership of the manager, $(1-w^{**})$, is 0.327. The exogenous equity ownership $(1-w)$ is varied such that $(1-w) = (1-w^{**}) + \epsilon$, $\epsilon \in [0, 0.65]$. These numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^2 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\sum_{h=2}^H \beta_1^h = 2.5 > 0$ and $\sum_{h=2}^H \beta_1^h = 0$, consistent with the assumptions. The competitive equilibrium is computed by a numerical fixed-point algorithm using the analytical expressions in Appendix A.

Addendum to the paper – Appendix D: Capital Asset Pricing Model (CAPM) Economy and Its Competitive Equilibrium – General Case

The economy is populated by H agents, F firms. Agent h 's preferences are represented by a Constant Absolute Risk Aversion utility function $u^h(\cdot)$ over the consumptions at date 0 and date 1, denoted as c_0^h and c_1^h , respectively:

$$u^h(c_0^h, c_1^h) \equiv -\frac{1}{A}e^{-Ac_0^h} - \frac{1}{A}e^{-Ac_1^h}. \quad (\text{D.1})$$

The economy has a N -dimensional orthogonal normal factor structure (x_1, \dots, x_n) , which is a multivariate normal with mean 0, and variance–covariance matrix (normalized to) I , the identity matrix. In particular, each agent h 's endowments in period 1, y_1^h , is generated as a linear combination of N underlying normal risk factors, and hence is in general correlated with other agents' endowments:

$$y_1^h - E(y_1^h) \equiv \sum_{n=1}^N \beta_n^h x_n, \quad h = F + 1, \dots, H. \quad (\text{D.2})$$

The first $F < H$ agents are the entrepreneurs. Entrepreneur h owns the firm $f(=h)$.

Each firm's cash flow, y_1^f , is also generated by the N factors. Without loss of generality, we assume that the stock market risk is driven by $C < N$ common orthogonal factors, (x_1, \dots, x_C) , and F orthogonal factors, $(x_{C+1}, \dots, x_{C+F})$, which correspond to the sectoral risk added by each firm's cash flow:

$$y_1^f - E(y_1^f) \equiv \sum_{c=1}^C \beta_c^f x_c + \beta_f^f x_{C+f}, \quad f = 1, \dots, F. \quad (\text{D.3})$$

Purely financial assets have payoff z_i , $i = 1, \dots, I$, which in terms of the factor structure is written as

$$z_i - E(z_i) \equiv \sum_{c=1}^C \beta_c^i x_c + \sum_{j=1}^I \beta_j^i x_{C+F+j}, \quad i = 1, \dots, I, \quad (\text{D.4})$$

where $(x_{C+F+1}, \dots, x_{C+F+I})$ contains the additional risks in the return structure of financial markets.

In our economy, all agents can trade the I financial assets and the C orthogonal factors. Moreover, there exists a stock market for trade of the F firms, although the participation

of entrepreneurs (agents $h \leq F$) is restricted; in particular, entrepreneur f cannot trade his own firm f , $f = 1, \dots, F$. Using the I financial assets, the C markets for factors, and the market for the F stocks, under the assumptions of payoff orthogonality, agents can replicate the payoffs of first $C + F + I$ risk factors. (Participation restrictions in the stock market translate into analogous participation restrictions in the market for the F firms' risk factors. Similarly, the positive net supply of stocks translates into positive net supply of all factors.) Therefore it is formally equivalent, and it turns out to be convenient to represent the competitive equilibrium of our CAPM economy in terms of not only the market but also the prices of the $C + F + I$ risk factors. We do this in the following.

We use a single index for all factors: $j \in \mathcal{J} \equiv \{1, \dots, J\}$, where $J \equiv C + F + I$. In general, $J < N$, and the set of risky assets traded by agent h , denoted as \mathcal{J}^h , may be a proper subset of \mathcal{J} , that is, $\mathcal{J}^h \subset \mathcal{J}$ for some h , but we assume that all agents h are allowed to trade the risk-free bond. H_j denotes the set of agents trading asset j , $|H_j|$ being its size.

The problem of each agent h is to choose a consumption allocation at time 0, c_0^h , portfolio positions in the risk-free bond and in all tradable assets, $[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}}$, and a consumption allocation at time 1, a random variable c_1^h , to maximize the expected utility

$$E \left[u^h(c_0^h, c_1^h) \right] \equiv -\frac{1}{A} e^{-Ac_0^h} + E \left[-\frac{1}{A} e^{-Ac_1^h} \right], \quad (\text{D.5})$$

subject to the budget constraints and the restricted participation constraints:

$$c_0^h = y_0^h - \pi_0 \theta_0^h - \sum_{j \in \mathcal{J}} \pi_j \theta_j^h, \quad h > F \quad (\text{D.6})$$

$$c_0^h = y_0^h + w^h p^h - \pi_0 \theta_0^h - \sum_{j \in \mathcal{J}} \pi_j \theta_j^h, \quad h \in H, \quad h \leq F \quad (\text{D.7})$$

$$c_1^h = y_1^h + \theta_0^h + \sum_{j \in \mathcal{J}} \theta_j^h x_j, \quad h > F \quad (\text{D.8})$$

$$c_1^h = (1 - w^h) y_1^h + \theta_0^h + \sum_{j \in \mathcal{J}} \theta_j^h x_j, \quad h \leq F \quad (\text{D.9})$$

$$\theta_j^h = 0, \quad j \notin \mathcal{J}^h. \quad (\text{D.10})$$

Note that the budget constraint for entrepreneur h includes the time-0 proceeds from the sale of a fraction w^h of his firm amounting to $w^h p^h$. As discussed in the paper, under rational expectations, the price of the firm p^h is given by $p^h = \pi_0 E(y_1^h) + \sum_{1 \leq j \leq J} \pi_j \beta_j^h$. Let s_j^h denote the positive supply of risk factor j provided by the entrepreneur h through the sale of fraction w^h of his firm. Under the factor decomposition (equation D.3) for each firm's cash flows, these positive supplies are given by $s_0^h = w^h E(y_1^h)$ and $s_j^h = w^h \beta_j^h$, $1 \leq j \leq J$, so that the proceeds from sale of the firm, $w^h p^h$, can also be expressed as $w^h p^h = \sum_{0 \leq j \leq J} \pi_j s_j^h$.

Definition 2 A competitive equilibrium is a consumption allocation (c_0^h, c_1^h) , for all agents $h \in \mathcal{H}$, which solves the problem of maximizing (D.5) subject to (D.6–D.10) at prices $\pi \equiv [\pi_0, \pi_j]_{j \in \mathcal{J}}$, and such that consumption and financial markets clear

$$\sum_h (c_0^h - y_0^h) \leq 0, \quad (\text{D.11})$$

$$\sum_h (c_1^h - y_1^h) \leq 0, \text{ with probability 1 over } \Omega, \text{ and} \quad (\text{D.12})$$

$$\sum_h \theta_j^h = s_j, \quad j = 0, 1, \dots, J, \quad (\text{D.13})$$

where s_j is the net supply of factor j , $s_j \equiv \sum_{1 \leq h \leq F} s_j^h$.

Proposition 9 The competitive equilibrium of the two-period CAPM economy, defined by equations (D.5)–(D.10), with the market-clearing condition given by equations (D.11)–(D.13), is characterized by prices of assets (π_j) , portfolio choices (θ_j^h) , and consumption allocations (c_t^h) , given below.

$$\pi_0 = \exp \left\{ A(y_0 - E y_1) + \frac{A^2}{2H} \sum_{h=1}^H \left[(1 - R_h^2) \text{var}(y_1^h) + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2 \right] \right\}, \quad (\text{D.14})$$

where

$$y_0 = \frac{1}{H} \sum_{h=1}^H y_0^h, \quad y_1 = \frac{1}{H} \sum_{h=1}^H y_1^h, \quad (\text{D.15})$$

$$\beta_j = \text{cov} \left[\frac{1}{|H_j|} \left(\sum_{h \in H_j, h \leq F} (1 - w^h) y_1^h + \sum_{h \in H_j, h > F} y_1^h \right), x_j \right], \quad (\text{D.16})$$

$$s_0^h = w^h E(y_1^h), \quad s_j^h = w^h \beta_j^h, \quad 1 \leq j \leq J, \quad s_j = \sum_{1 \leq h \leq F} s_j^h, \quad 0 \leq j \leq J, \quad (\text{D.17})$$

$$\frac{\pi_j}{\pi_0} = E(x_j) - A \left(\beta_j + \frac{1}{|H_j|} s_j \right), \quad (\text{D.18})$$

and for $h > F$ (non-entrepreneurs),

$$R_h^2 \equiv \frac{\sum_{j \in \mathcal{J}^h} (\beta_j^h)^2}{\text{var}(y_1^h)}, \quad (\text{D.19})$$

$$\theta_j^h = \left(\beta_j + \frac{1}{|H_j|} s_j \right) - \beta_j^h, \quad j \in \mathcal{J}^h, \quad \text{and } \theta_j^h = 0, \quad j \in (\mathcal{J}^h)^c, \quad (\text{D.20})$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h - E(y_1^h) - \sum_{j \in \mathcal{J}^h} \pi_j \theta_j^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad (\text{D.21})$$

$$c_1^h = \theta_0^h + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + \left(y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j \right), \quad (\text{D.22})$$

$$\text{var}(c_1^h) = \text{var}(y_1^h) - \sum_{j \in \mathcal{J}^h} (\beta_j^h)^2 + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2, \quad (\text{D.23})$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + E(y_1^h) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h), \quad (\text{D.24})$$

and finally, for $h \leq F$ (entrepreneurs),

$$R_h^2 \equiv \frac{\sum_{j \in \mathcal{J}^h} (1 - w^h)^2 (\beta_j^h)^2}{\text{var}(y_1^h)}, \quad (\text{D.25})$$

$$\theta_j^h = \left(\beta_j + \frac{1}{|H_j|} s_j \right) - (1 - w^h) \beta_j^h, \quad j \in \mathcal{J}^h, \quad \text{and } \theta_j^h = 0, \quad j \in (\mathcal{J}^h)^c, \quad (\text{D.26})$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h + w^h p^h - (1 - w^h) E(y_1^h) - \sum_{j \in \mathcal{J}^h} \pi_j \theta_j^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad (\text{D.27})$$

$$c_1^h = \theta_0^h + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + (1 - w^h) \left(y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j \right), \quad (\text{D.28})$$

$$\text{var}(c_1^h) = (1 - w^h)^2 \text{var}(y_1^h) - \sum_{j \in \mathcal{J}^h} (1 - w^h)^2 (\beta_j^h)^2 + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2, \quad (\text{D.29})$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + (1 - w^h) E(y_1^h) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h). \quad (\text{D.30})$$

This equilibrium, which exhibits a positive supply of assets, is similar to the one without positive supply (see Willen, 1997, and Acharya and Bisin, 2000), but all expressions for the entrepreneurs are modified to reflect the facts that (i) entrepreneur h holds only a fraction $(1 - w^h)$ of his firm; (ii) at time 0, entrepreneur h collects proceeds for the remaining fraction w^h of his firm amounting to $w^h p^h$; and (iii) aggregate beta β_j in the case of zero-supply assets is replaced by $(\beta_j + \frac{1}{H_j} s_j)$ to reflect the positive supply of assets.

Proof: Consider the competitive equilibrium of Definition 2. To determine the equilibrium in closed-form, we derive the first-order conditions for each agent's maximization of utility function and then apply the market-clearing conditions. Note that fractions of firms to be sold have already been determined and hence positive supplies of all assets are taken as given by all agents. Since competitive entrepreneurs cannot affect the prices of bond and risk factors (or their aggregate supplies), it follows that the proceeds collected from sales of firms are also taken as given by the respective entrepreneurs. Finally, the technology choice of each firm – the firm's cash flow betas – are also taken as given by all agents: either the betas are observed and contracted upon, as in the case of owner-managed firms with no moral hazard, or these are unobserved but rationally anticipated, as in the case of owner-managed firms with moral hazard and in the case of corporations.

The maximization problem of agent h in equation (D.5) can be cast in terms of the agent's choice of portfolios, $[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}} \in \mathfrak{R}^{J+1}$, as

$$\max_{[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}}} -\frac{1}{A} e^{-Ac_0^h} + E \left[-\frac{1}{A} e^{-Ac_1^h} \right], \quad (\text{D.31})$$

subject to the constraints (D.6)–(D.10). Since all endowments and risky asset payoffs are normally distributed, this objective simplifies to

$$\max_{[\theta_0^h, \theta_j^h]_{j \in \mathcal{J}}} -\frac{1}{A} e^{-Ac_0^h} - \frac{1}{A} e^{-AE(c_1^h) + \frac{A^2}{2} \text{var}(c_1^h)}. \quad (\text{D.32})$$

Using equations (D.8)–(D.9) and the normalizations $E(x_j) = 0$, $\text{var}(x_j) = 1$, $\forall j \in \mathcal{J}$, we obtain

$$E(c_1^h) = E(y_1^h) + \theta_0^h, \quad h > F \quad (\text{D.33})$$

$$E(c_1^h) = (1 - w^h)E(y_1^h) + \theta_0^h, \quad h \leq F \quad (\text{D.34})$$

$$\text{var}(c_1^h) = \text{var}(y_1^h) + \sum_{j \in \mathcal{J}^h} (\theta_j^h)^2 + 2 \sum_{j \in \mathcal{J}^h} \theta_j^h \text{cov}(y_1^h, x_j), \quad h > F \quad (\text{D.35})$$

$$\text{var}(c_1^h) = (1 - w^h)^2 \text{var}(y_1^h) + \sum_{j \in \mathcal{J}^h} (\theta_j^h)^2 + 2(1 - w^h) \sum_{j \in \mathcal{J}^h} \theta_j^h \text{cov}(y_1^h, x_j), \quad h \leq F. \quad (\text{D.36})$$

Taking the first-order condition with respect to θ_0^h , we get

$$\pi_0 e^{-Ac_0^h} = E \left[e^{-Ac_1^h} \right], \forall h. \quad (\text{D.37})$$

Taking the first-order condition with respect to $\theta_j^h \in \mathcal{J}^h$, we get

$$\pi_j e^{-Ac_0^h} = -A E \left[e^{-Ac_1^h} \right] \left(\theta_j^h + \text{cov}(y_1^h, x_j) \right), \quad h > F \quad (\text{D.38})$$

$$\pi_j e^{-Ac_0^h} = -A E \left[e^{-Ac_1^h} \right] \left(\theta_j^h + (1 - w^h) \text{cov}(y_1^h, x_j) \right), \quad h \leq F. \quad (\text{D.39})$$

Dividing equation (D.37) by equation (D.38) for $h > F$, and by equation (D.39) for $h \leq F$, and summing up for $h \in H_j$, we obtain

$$|H_j| \frac{\pi_j}{\pi_0} = -A \sum_{h \in H_j} \theta_j^h - A \text{cov} \left(\sum_{h \in H_j, h \leq F} (1 - w^h) y_1^h + \sum_{h \in H_j, h > F} y_1^h, x_j \right). \quad (\text{D.40})$$

Dividing throughout by H_j , using the market-clearing condition (D.13), and substituting for β_j from the definition (D.16), yields the CAPM pricing relationship of (D.18):

$$\frac{\pi_j}{\pi_0} = -A \left(\beta_j + \frac{1}{|H_j|} s_j \right). \quad (\text{D.41})$$

Substituting equations (D.37) and (D.41) in equations (D.38) and (D.39) yields the following portfolio choice θ_j^h :

$$\theta_j^h = \left(\beta_j + \frac{1}{|H_j|} s_j \right) - \beta_j^h, \quad j \in \mathcal{J}^h, \quad h > F \quad (\text{D.42})$$

$$\theta_j^h = \left(\beta_j + \frac{1}{|H_j|} s_j \right) - (1 - w^h) \beta_j^h, \quad j \in \mathcal{J}^h, \quad h \leq F, \quad (\text{D.43})$$

where we have used the definition $\beta_1^h = \text{cov}(y_1^h, x_j)$.

In order to obtain the portfolio choice θ_0^h , we rewrite the first-order condition (D.37) as

$$\pi_0 e^{-Ac_0^h} = e^{-AE(c_1^h) + \frac{A^2}{2} \text{var}(c_1^h)}, \quad \forall h. \quad (\text{D.44})$$

Taking the natural log, substituting equations (D.6) and (D.33) for $h > F$, or equations (D.7) and (D.34) for $h \leq F$, and rearranging yields

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h - E(y_1^h) - \sum_{j \in \mathcal{J}^h} \pi_j \theta_j^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad h > F \quad (\text{D.45})$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h + w^h p^h - (1 - w^h) E(y_1^h) - \sum_{j \in \mathcal{J}^h} \pi_j \theta_j^h + \frac{A}{2} \text{var}(c_1^h) - \frac{1}{A} \ln(\pi_0) \right), \quad h \leq F. \quad (\text{D.46})$$

Next, substituting equation (D.42) in equation (D.8) and equation (D.43) in equation (D.9) and rearranging, we obtain the three-fund separation theorem:

$$c_1^h = \theta_0^h + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + \left(y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j \right), \quad h > F \quad (\text{D.47})$$

$$c_1^h = \theta_0^h + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + (1 - w^h) \left(y_1^h - \sum_{j \in \mathcal{J}^h} \beta_j^h x_j \right), \quad h \leq F. \quad (\text{D.48})$$

Taking the variance of these expressions yields

$$\text{var}(c_1^h) = \text{var}(y_1^h) - \sum_{j \in \mathcal{J}^h} (\beta_j^h)^2 + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2, \quad h > F \quad (\text{D.49})$$

$$\text{var}(c_1^h) = (1 - w^h)^2 \text{var}(y_1^h) - \sum_{j \in \mathcal{J}^h} (1 - w^h)^2 (\beta_j^h)^2 + \sum_{j \in \mathcal{J}^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2, \quad h \leq F. \quad (\text{D.50})$$

Finally, to obtain the expressions for c_0^h , we take the natural log of equation (D.44) and substitute expression (D.33) or (D.34) for the respective ranges of h . Rearranging the terms, we get

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + E(y_1^h) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h), \quad h > F \quad (\text{D.51})$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + (1 - w^h) E(y_1^h) + \theta_0^h - \frac{A}{2} \text{var}(c_1^h), \quad h \leq F. \quad (\text{D.52})$$

Now, all equilibrium quantities are determined in terms of the risk-free asset's price, π_0 . To determine this, we take the natural log of equation (D.44) and sum over all agents to obtain

$$H \ln(\pi_0) - A \sum_{h=1}^H y_0^h = -A \sum_{h=1}^H E(y_1^h) + \frac{A^2}{2} \sum_{h=1}^H \text{var}(c_1^h). \quad (\text{D.53})$$

Dividing throughout by H , using the definitions for mean endowments y_0 and y_1 in equation (D.15), and substituting for $\text{var}(c_1^h)$ from equations (D.23) and (D.29), π_0 can be determined in terms of the model's primitive quantities as follows:

$$\pi_0 = \exp \left\{ A(y_0 - Ey_1) + \frac{A^2}{2H} \sum_{h=1}^H \left[(1 - R_h^2) \text{var}(y_1^h) + \sum_{j \in J^h} \left(\beta_j + \frac{1}{|H_j|} s_j \right)^2 \right] \right\}, \quad (\text{D.54})$$

where

$$R_h^2 \equiv \frac{\sum_{j \in J^h} (\beta_j^h)^2}{\text{var}(y_1^h)}, \quad h > F, \quad \text{and} \quad R_h^2 \equiv \frac{\sum_{j \in J^h} (1 - w^h)^2 (\beta_j^h)^2}{\text{var}(y_1^h)}, \quad h \leq F \quad (\text{D.55})$$

represent the variability of agent h 's endowment that is spanned by the risky assets tradable by the agent.

The competitive equilibrium is now fully determined in closed-form once the supply conditions are substituted:

$$s_0^h = w^h E(y_1^h), \quad s_j^h = w^h \beta_j^h, \quad 1 \leq j \leq J, \quad s_j = \sum_{1 \leq h \leq F} s_j^h, \quad 0 \leq j \leq J. \quad \diamond \quad (\text{D.56})$$