1 Economic Growth

• What determines the rate of growth of consumption or income (as measured e.g., by Gross Domestic Product) ?

• What determines the rate of growth of per-capita consumption or per-capita income ?

• Let variables in capital letters denote aggregate quantities, and lower case variables denote per-capita quantities; and let $L$ denote the number of agents in the economy:

$$y = \frac{Y}{L}$$

Let an index $t$ denote time: $y_t$ is $y$ at time $t$. Aggregate growth rates depend on the growth rate of the population, $\frac{L_{t+1}}{L_t}$:

$$\frac{Y_{t+1}}{Y_t} = \frac{y_{t+1}L_{t+1}}{y_tL_t} = \frac{y_{t+1}L_{t+1}}{y_tL_t}$$

As a consequence per-capita growth are a better indicator of economic development. As an example, consider the average rate of growth of income in Mexico and in Italy in the period 1950-2000 (get them from the Penn World Dataset of Summers-Heston, at

http://pwt.econ.upenn.edu/php_site/pwt61_form.php)

While the growth rate of the population is in general related to the growth rate of per-capita consumption and income of an economy (and in interesting ways, when you endogeneize fertility and allow the agents choose how many children to have), in first approximation we consider it instead exogenous and we study the growth rate of per-capita consumption and income per-se.
1.1 Production

- Aggregate production is described by a production function

\[ Y_t = F(K_t, L_t) \]

where \( Y_t \) is aggregate income at time \( t \), \( K_t \) is aggregate capital (in the same units as income) at \( t \), and \( L_t \) is the population of (the total number of agents in) the economy at \( t \); assuming that each agent is the same (no productivity differences, e.g., related to education) and works the same number of hours, we also refer to \( L_t \) as the aggregate labor supply in the economy.

We assume the production function is Cobb-Douglas (this works well with data):

\[ Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}, \quad 0 < \alpha < 1 \]

- The marginal product of capital in the economy is:

\[ MP_K = \frac{\partial Y_t}{\partial K_t} = \alpha A(K_t)^{\alpha-1} (L_t)^{1-\alpha} \]

Note it is decreasing in capital, \( K_t \), and increasing in labor \( L_t \). How do you see this? Compute the derivative of \( MP_K \) with respect to \( K_t \) and \( L_t \), respectively.

- The marginal product of labor in the economy is:

\[ MP_L = \frac{\partial Y_t}{\partial K_t} = (1 - \alpha) A(K_t)^{\alpha} (L_t)^{-\alpha} \]

Note it is increasing in capital, \( K_t \), and decreasing in labor \( L_t \). How do you see this? Compute the derivative of \( MP_L \) with respect to \( K_t \) and \( L_t \), respectively.

- The per-capita production function can be computed as follows (the Cobb-Douglas specification makes it very easy):

Pass to per capita income by dividing by \( L_t \):

\[ y_t = \frac{Y_t}{L_t} = A \frac{(K_t)^\alpha (L_t)^{1-\alpha}}{L_t} = A \frac{(K_t)^\alpha (L_t)^{1-\alpha}}{(L_t)^\alpha (L_t)^{1-\alpha}} \]

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that is

\[ y_t = Ak^\alpha \]

- The marginal product of capital in per-capita terms is:

\[ \frac{\partial y_t}{\partial k_t} = \alpha Ak^{\alpha-1} \]

it is decreasing in per-capita capital \( k_t \); that is:

**rich economies have lower marginal product of capital**

where of course “rich” means with high per-capita capital, and the marginal product of capital is in per-capita terms.

[draw figure]

### 1.2 Growth and Savings

- Consider an economy in which all agents are identical (we are not interested in distributional issues, but only in per-capita variables); and live for 2 periods, \( t \) and \( t + 1 \).

The budget constraint of the representative agent at time \( t \) is:

\[ c_t + (k_{t+1} - k_t) = A(k_t)^\alpha \]

Notice that \( (k_{t+1} - k_t) \) is his savings, and also his investment in capital (he has no other ways of saving, and cannot or will not borrow). The agent’s income is due to production at time \( t \): \( A(k_t)^\alpha \).

The budget constraint of the representative agent at time \( t + 1 \) is:

\[ c_{t+1} = A(k_{t+1})^\alpha \]

and the agent consume all his production, since he will not be around next period.

- The representative agent’s preferences are as usual:

\[ u(c_t) + \beta u(c_{t+1}) \]

where we assume \( 0 < \beta < 1 \); and also \( u(c) = lnc \) for simplicity.
The representative agent’s maximization problem (after substitution his budget constraints) into the specification of preferences) is:

$$\max_{k_{t+1}} \ln (A(k_t) - k_{t+1} + k_t) + \beta \ln (A(k_{t+1})^\alpha)$$

(1)

The first order condition (derive it yourself) can be written as:

$$\frac{c_{t+1}}{c_t} = \alpha A(k_{t+1})^{\alpha-1}$$

Notice\(^1\) that the right-hand-side is the marginal product of capital (in per-capita terms) at time \(t+1\). Notice also that this is the same equation we derived when studying savings, only that the marginal product of capital takes the place of \((1 + r)\). In fact this is the same problem; the agent is saving by investing in capital and receives a return equal to the marginal product of capital.

Finally, notice that the first order condition says that

rich economies have lower rates of growth of consumption

where, again, ”rich” means with high per-capita capital, and the growth rate of consumption is in per-capita terms.

Is this last implication true in the data? Look at the the Penn World Dataset of Summers-Heston, at


It is true in a subset of countries, Asian New Industrialized Countries (South Korea, Hong Kong, Thailand, ..), China which is ”poor” in per-capita capital and has been growing at 8% recently, while Germany (and the rest of Europe), the U.S. have been growing at less than 3%.

\(^1\)By substituting the budget constraint equations into the first order condition you can solve for the growth rate of capital in the economy, \(\frac{k_{t+1}}{k_t}\). Try doing it (when you have some free time, it’s easy but algebraically involved. The solution I got is:

$$\frac{k_{t+1}}{k_t} = \frac{\beta \alpha}{1 - \beta \alpha} \left(1 + A(k_t)^{\alpha-1}\right)$$
It is not true however for African and Latin American countries, who are both "poor" in per capita income (and capital) and have been growing very slowly if at all.

1.3 Other Determinants of Growth

- Write the first order conditions of the maximization problem in (1) in terms of $k_{t+1}$ and $k_t$. You will have an equation of the form:

$$k_{t+1} = constant_1 \ (k_t)^\alpha + constant_2 \ k_t, \ constant_2 < 1$$

(2)

Study this equation mathematically as a difference equation (but be careful here: the equation is derived as the first order condition of the maximization problem of an agent who lives only 2 periods; by considering it a difference equation we are implicitly assuming that the same equation would hold if the agents lived an infinite number of periods; in fact this is true, in the sense that an equation of the form (2) holds in this case, even though the expression for the constants is different). The equation has two steady states: $k = 0$ and $k^* > 0$. The steady state $k^*$ is globally stable: all paths of $k_t$ starting from any $k_0 > 0$ converge to it. We conclude that for $\alpha < 1$ the economy does not grow indefinitely!

- There are two possible modifications in our basic model which are such that the economy grows indefinitely. We can then ask ourselves in these economy what are the determinants of the growth rate and hence attempt some explanations which help account for the low growth rates of Africa and Latin America. Both modifications require changing the production function.

  - Human Capital.

    Suppose the production function is:

    $$Y_t = (AK_t)^\alpha \ (h_tL_t)^{1-\alpha}$$

    where $h_t$ is an index of quality of labor, called human capital, and

    $$h_{t+1} = \gamma h_t, \ \gamma > 1$$
In per capita terms, the production function becomes:

\[ y_t = (k_t)^\alpha (h_t)^{1-\alpha} \]

\[ h_{t+1} = \gamma h_t \]

In this case, by proceeding exactly as in the previous section (do it as an exercise), we can compute the growth rate of consumption:

\[ \frac{c_{t+1}}{c_t} = \beta \alpha A \left( \frac{h_{t+1}}{k_{t+1}} \right)^{1-\alpha} \]

Convince yourself that in this case the economy grows. The argument is as follows:

Suppose not.
Then \( \frac{k_{t+1}}{k_t} \rightarrow 1 \) for \( k_t \) large enough.
\( \frac{h_{t+1}}{h_t} \rightarrow \gamma > 1 \) instead, independently of \( k_t \).

Therefore, for large \( k_t \), \( \frac{h_{t+1}}{k_{t+1}} \) increases over time, and the economy grows.

The important question then is:

What does \( \gamma \) depend on?

Partial answer for discussion:

- The structure of the family
- Fertility and its determinants
- Schooling system
- Urban Development
- Technological Innovations and Knowledge.

Suppose the parameter \( A \) in the production function, which measures total productivity (sometimes we refer to it as the general knowledge of an economy), grows over time,

\[ A_{t+1} = \delta A_t \] \hspace{1cm} (3)

Once again you can show that (think about how you would do this) in this case the economy will grow.

Once again, the important question then is:

What does \( \delta \) depend on?
To develop a listing of partial answers for discussion a comment is very useful:

*General knowledge* can be used by the whole economy without diminishing returns: think of a blueprint to produce a medicine; it can be freely copied and used by many firms. We say general knowledge is *non-rival*.

*General knowledge* can be private or public depending on the institutions: think of the blueprint; we can protect it or not with a system of patents. We say that general knowledge is *excludable*. (Non-excludable goods exist: fishing in the sea has proved very hard to exclude over the years).

What is the difference between institutions which guarantee *exclusion* and those which do not?

Think of the following model of total productivity or knowledge:

\[ A_t = A (j_t)^{1-\alpha} \]

where \( j_t \) is knowledge and

\[ j_t = \begin{cases} 
  k_t & \text{under excludability} \\
  \bar{k}_t & \text{under non-excludability} 
\end{cases} \]

and \( \bar{k}_t \) is a measure of average capital that the representative agent in the economy cannot affect with his capital investment. At equilibrium though all agents choose the same capital (there is a single representative agent), and hence \( \bar{k}_t = k_t \).

Compute now for both models the marginal product of capital in per capita terms:

\[
\frac{\partial}{\partial k_t} A (\bar{k}_t)^{1-\alpha} (k_t)^{\alpha} = \alpha A (\bar{k}_t)^{1-\alpha} (k_t)^{\alpha-1} = \alpha A \text{ at equilibrium } [\text{excludable}]
\]

\[
\frac{\partial}{\partial k_t} A (k_t)^{1-\alpha} (k_t)^{\alpha} = A \quad [\text{non-excludable}]
\]
In the model then $\delta$ is endogenous and depends on the existence of institutions which guarantee excludability of general knowledge. If you are thinking that it might not be that $A_{t+1}$ satisfies (3) in equilibrium, you are right. But this is not so important, and we can rig the model so that it does.

Partial answers to the What does $\delta$ depend on? question for discussion (also look at the papers by Murphy-Shleifer-Vishny (1991) and Acemoglu-Robinson (2004) posted on my website):

- The structure of the legal system
- Property rights
- Industrial organization and the patent system
- Urban Development