Wealth distribution and social mobility: A quantitative analysis of U.S. data

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The wealth distribution debate

Which factors drive quantitatively the cross-sectional distribution of wealth?

Which factors drive, most notably, its skewed, thick, right tail (in the U.S. as well as essentially everywhere)?
The wealth distribution debate - cont.ed

A few possible driving factors include:

- Skewed/persistent earnings, non-homogeneous bequests, differential savings, stochastic length of life/dynasty, the infamous $r > g$, (persistent) capital income risk, stochastic discount rates, . . .
The wealth distribution debate - cont.ed

Which factors drive the recent increase in inequality?

Is the distribution losing stationarity?

Figure: Trend in top 1% wealth share

Not quite ready to tackle this, yet!
Literature: A few historical notes on Pareto’s Law

Vilfredo Pareto

introduced in the *Cours d’Economie Politique* (1897) the distribution which takes his name

\[ f(w) \sim w^{-\beta}, \quad x \geq w > 0 \]

to represent empirical wealth distributions, characterized by thick right tails:

\[ \lim_{w \to \infty} e^{\lambda w} Pr(W > w) = \infty, \quad \text{for all } \lambda > 0 \]
"Pareto’s Law," enunciated e.g., by Samuelson (1965):

In all places and all times, the distribution of income remains the same. Neither institutional change nor egalitarian taxation can alter this fundamental constant of social sciences.
Literature: From the “Law” to stable empirical regularities

Distributions of income and wealth which are very concentrated with thick right tails have been well documented over time and across countries:

• U.K.- Atkinson (2001),
• Japan - Moriguchi-Saez (2005),
• France - Piketty (2001),
• U.S. - Piketty-Saez (2003),
• Canada - Saez-Veall (2003),
• Italy - Clementi-Gallegati (2004),
• Norway - Dagsvik-Vatne (1999)
Literature: Dynamic models of Pareto distributions


Literature: From dynamic to dynamic economic models

- The characteristic feature of the previous literature is that the stochastic processes which generate power laws are essentially exogenous.
- The same can be said for the large recent literature on this topic in Econophysics.
Explanatory factors

What does it take to fit the distribution of wealth (that is, to obtain Pareto tails) in a standard macro model (that is, micro-founded):


• Factor 2: Stochastic length of life/dynasty - Diaz Gimenez, Quadrini, and Rios Rull (1997); Benhabib and Bisin (2006).


Explanatory factors - cont.ed

We shall argue that

- Factor 1 - earnings - is empirically insufficient by itself.
- Factor 2 - length of life - amounts to demographic absurdity.
- Factor 3 - saving rates across wealth levels is empirically insufficient by itself (and it leads to empirically untenable non-stationarities when interpreted a’ la Piketty).
- Factor 4 - capital income - is necessary and does well especially when combined with 1 and 3.
Capital income risk - what is it?

Two components of capital income are particularly subject to idiosyncratic risk: ownership of principal residence and private business equity, which account for, respectively, 28.2% and 27% of household wealth in the United States according to the 2001 Survey of Consumer Finances (SCF).

- Case and Shiller (1989) documented a 15% standard deviation of yearly capital gains or losses on owner-occupied housing; Flavin and Yamashita (2002) find a 14% standard deviation of the return on housing, at the level of individual houses, from the 1968-92 waves of the Panel Study of Income Dynamics.

- In the 1989 SCF studied by Moskowitz and Vissing-Jorgensen (2002), both the capital gains and earnings on private equity exhibit very substantial variation, as does excess returns to private over public equity investment, even conditional on survival (private equity is highly concentrated: 75% owned by households for which it constitutes at least 50% of their total net worth).
To be explained as well: Social mobility

• Most studies of the wealth distribution center on the tail - hence on measures of inequality in the cross sectional distribution.

• But an advantage of working with formal macro models is that - once we allow for an explicit demographic structure - we obtain implications for social mobility.
Model: life-cycle and bequests

- Consumers choose consumption $c$ and savings every period, subject to a no-borrowing constraint; per-period utility from consumption is CRRA; wealth $a$ accumulates.
- Life span is $T = 30$ years and certain.
- Consumers leave a bequest at the end of life and get a warm-glow utility.
- Idiosyncratic rates of returns $r$ and labor income $w$: drawn from a distribution at birth, possibly correlated with those of the parent, deterministic within each generation life.
Life-cycle consumption-saving problem

Each agent of generation $n$, given $(r, w)$ faces the following deterministic problem, in recursive form, with $0 \leq t < T$:

$$V_t(a) = \max_{c,a'} [u(c) + \beta V_{t+1}(a')]$$

s.t. \quad a' = (1 + r)(a - c) + w

\begin{align*}
&c \leq a \\
&c \geq 0
\end{align*}

$$V_T(a') = \frac{1}{\beta} e(a')$$

and with functional forms:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad e(a) = A \frac{a^{1-\mu}}{1-\mu}.$$
Life-cycle consumption-saving problem - cont.ed

The solution of the recursive problem can be represented by a map

\[ a_T = g(a_0; r, w). \]

Furthermore:

The map \( g \) satisfies the following:

If \( \mu = \sigma \), \( g(a_0; r, w) = \alpha(r, w)a_0 + \beta(r, w). \)

If \( \mu < \sigma \), \( \frac{\partial g}{\partial a_0} (a_0; r, w) > 0. \)
Wealth dynamics across generations

- Let apex \( n \) denote the generation.
- The process for the rate of return of wealth and earnings processes over generation \( n \), \((r^n, w^n)\) is a finite irreducible Markov Chain with transition \( P \left( r^n, w^n \mid r^{n-1}, w^{n-1} \right) \) such that (abusing notation):

\[
P \left( r^n \mid r^{n-1}, w^{n-1} \right) = P \left( r^n \mid r^{n-1} \right),
\]

\[
P \left( w^n \mid r^{n-1}, w^{n-1} \right) = P \left( w^n \mid w^{n-1} \right)
\]

- The life-cycle structure of the model implies that the initial wealth of the \( n \)’th generation coincides with the final wealth of the \( n - 1 \)’th generation:

\[a^n = a^n_0 = a^{n-1}_T.\]
Wealth dynamics across generations - cont.ed

- We can construct then a stochastic difference equation for the initial wealth of dynasties, induced by the (forcing) stochastic process for \((r^n, w^n)\), and mapping \(a^{n-1}\) into \(a^n\):

\[
    a^n = g \left( a^{n-1}; r^n, w^n \right),
\]

where the map \(g(.)\) represents the solution of the life-cycle consumption-saving problem.
Wealth dynamics across generations - cont.ed

Furthermore (see Saporta, 2005):

If \( \mu = \sigma \) and \( (\alpha(r^n, w^n), \beta(r^n, w^n)) \) satisfy the restrictions of a reflective process (Benhabib, Bisin, and Zhu 2011), the tail of the stationary distribution of \( a^n \) is asymptotic to a Pareto law

\[
Pr(a^n > a) \sim ca^{-\gamma},
\]

where \( \lim_{N \to \infty} E \left( \prod_{n=0}^{N-1} (\alpha(r^{-n}, w^{-n}))^{\gamma} \right)^{\frac{1}{N}} = 1. \)

If instead, keeping \( \sigma \) constant, \( \mu < \sigma \), a stationary distribution might not exist; but if it does,

\[
Pr(a^n > a) \geq ca^{-\gamma}.
\]
Wealth dynamics across generations - cont.ed

• If $\mu = \sigma$, to induce a limit stationary distribution of $a_n$ it is required that the contractive and expansive components of the effective rate of return tend to balance, i.e., that the distribution of $\alpha(r^n, w^n)$ display enough mass on $\alpha(r^n, w^n) < 1$ as well some as on $\alpha(r^n, w^n) > 1$; and that effective earnings $\beta(r^n, w^n)$ be positive and bounded, hence acting as a reflecting barrier (these are the restrictions for a reflective process).

• In the general case, $\mu < \sigma$, saving rates and bequests tend to increase with initial wealth; as a consequence the model displays a distinct expansive tendency acting against the stationarity of $a_n$. 
Wealth dynamics across generations - cont.ed

• The stochastic properties of labor income risk, $\beta(r^n, w^n)$, have no effect on the tail stationary distribution of wealth if it exists.

• Heavy tails in the stationary distribution require that the economy has sufficient capital income risk: if $\mu = \sigma$, for instance, an economy with limited capital income risk, where $\alpha(r^n, w^n) \leq \tilde{\alpha} < 1$ and where $\tilde{\beta}$ is the upper bound of $\beta(r^n, w^n)$, has a stationary distribution of wealth bounded above by $\frac{\tilde{\beta}}{1-\tilde{\alpha}}$.

• As long as a stationary distribution exists, wealth inequality (e.g., the Gini coefficient of the tail) increases with the capital income risk agents face in the economy, as measured by a "mean preserving spread" on the distribution of $\alpha(r^n, w^n)$, the bequest motive $A$, smaller $\mu$. 
Quantitative exercise: Method of simulated moments

Objective: Which of the explanatory factors drives what of the distribution of wealth and of social mobility?

Main assumption: Wealth and social mobility data are generated by a stationary distribution.

Specifics:

• Fixed parameters: $\sigma = 2$, $T = 30$, $\beta = 0.97$ per annum.

• Estimated parameters: $\mu, A$, 4-state Markov Chain grid for $r^n$ and probabilities on the diagonal of the transition matrix (imposing equal probabilities off diagonal).

• Moments to match (data and model’s stationary distribution): wealth quintiles (8) and diagonal probabilities in the wealth transition matrix (7)
Input data


- Originally a 100-state Markov chain: each percentile of income distribution

- Reduce that to a 10-state Markov chain: each decile

- [.898] 0.001, 7.3, 14.96, 22.51, 30.68, 39.93, 51.41, 66.41, 87.18, 161.21
Input data: Labor income

Transition matrix

\[
T_{30} = \begin{bmatrix}
0.209 & 0.157 & 0.133 & 0.111 & 0.093 & 0.077 & 0.065 & 0.057 & 0.051 & 0.048 \\
0.176 & 0.150 & 0.131 & 0.112 & 0.098 & 0.085 & 0.074 & 0.065 & 0.057 & 0.052 \\
0.162 & 0.150 & 0.131 & 0.114 & 0.100 & 0.089 & 0.078 & 0.068 & 0.059 & 0.049 \\
0.121 & 0.128 & 0.124 & 0.116 & 0.108 & 0.100 & 0.092 & 0.082 & 0.072 & 0.058 \\
0.095 & 0.106 & 0.113 & 0.114 & 0.111 & 0.108 & 0.102 & 0.095 & 0.085 & 0.068 \\
0.076 & 0.089 & 0.099 & 0.107 & 0.111 & 0.112 & 0.112 & 0.108 & 0.101 & 0.085 \\
0.061 & 0.075 & 0.087 & 0.098 & 0.108 & 0.114 & 0.117 & 0.119 & 0.116 & 0.106 \\
0.049 & 0.063 & 0.076 & 0.090 & 0.104 & 0.116 & 0.124 & 0.129 & 0.129 & 0.122 \\
0.038 & 0.050 & 0.063 & 0.079 & 0.095 & 0.110 & 0.126 & 0.139 & 0.151 & 0.149 \\
0.028 & 0.035 & 0.046 & 0.059 & 0.072 & 0.088 & 0.107 & 0.135 & 0.175 & 0.256
\end{bmatrix}
\]
Output data

- Cross-sectional wealth distribution: shares in bottom 20%, 20-40%, 40-60%, 60-80%, 80-90%, 90-95%, 95-99%, and top 1% of net worth holdings in the 2007 SCF.

- Wealth transition across generations: six-year transition matrix (1983-1989) in Kennickell and Starr-McCluer (1997) with the SCF (states are bottom 25%, 25-49%, 50-74%, 75-89%, 90-94%, top 2-5%, and top 1%; then raised to the power of 5.
Calibration: Social mobility

Wealth Transition Matrix

\[
T_{30} = \begin{bmatrix}
0.341 & 0.286 & 0.211 & 0.107 & 0.032 & 0.020 & 0.003 \\
0.285 & 0.269 & 0.236 & 0.132 & 0.042 & 0.029 & 0.005 \\
0.212 & 0.239 & 0.271 & 0.169 & 0.056 & 0.042 & 0.009 \\
0.176 & 0.221 & 0.285 & 0.187 & 0.065 & 0.064 & 0.013 \\
0.156 & 0.207 & 0.284 & 0.192 & 0.072 & 0.068 & 0.023 \\
0.123 & 0.180 & 0.273 & 0.193 & 0.082 & 0.098 & 0.051 \\
0.084 & 0.142 & 0.237 & 0.180 & 0.092 & 0.149 & 0.118
\end{bmatrix}
\]
# Estimates

## Table: Parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>Markov Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(1)</strong> Preferences</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>[2]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.4563</td>
</tr>
<tr>
<td>$A$</td>
<td>0.3591</td>
</tr>
<tr>
<td>$\beta$</td>
<td>[0.97]</td>
</tr>
<tr>
<td>$T$</td>
<td>[30]</td>
</tr>
<tr>
<td><strong>(2)</strong> Rate of return</td>
<td></td>
</tr>
<tr>
<td>$r$ grid (six-year)</td>
<td>0.0118</td>
</tr>
<tr>
<td>prob. grid</td>
<td>0.2848</td>
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<tr>
<td>Stationary distr.</td>
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<td>0.1060</td>
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<td></td>
<td>0.2540</td>
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<td>0.2537</td>
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<td></td>
<td>0.1866</td>
</tr>
<tr>
<td></td>
<td>0.2361</td>
</tr>
<tr>
<td></td>
<td>0.2365</td>
</tr>
<tr>
<td></td>
<td>0.3775</td>
</tr>
<tr>
<td></td>
<td>0.2250</td>
</tr>
<tr>
<td></td>
<td>0.2253</td>
</tr>
</tbody>
</table>

Notes: $r$ is real, post-tax, detrended for growth. Annual mean is 2.5%, standard deviation 31.2%. Consistent with earlier estimates by Campbell and Vissing-Jørgensen.
Cross-sectional distribution of wealth

Table: Wealth quintiles

<table>
<thead>
<tr>
<th>Moments</th>
<th>Share of wealth</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-90</th>
<th>90-95</th>
<th>95-99</th>
<th>99-100</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>SCF 2007</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.045</td>
<td>0.112</td>
<td>0.120</td>
<td>0.111</td>
<td>0.267</td>
<td>0.336</td>
<td>0.816</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td>0.011</td>
<td>0.039</td>
<td>0.083</td>
<td>0.132</td>
<td>0.115</td>
<td>0.121</td>
<td>0.166</td>
<td>0.333</td>
<td>0.799</td>
</tr>
</tbody>
</table>
Fit of benchmark

Data
Benchmark
Social mobility - Selected aspects

Table: Transition matrix

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Our Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of wealth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-24</td>
<td>25-49</td>
<td>50-74</td>
</tr>
<tr>
<td>Diagonal</td>
<td>0.341</td>
<td>0.269</td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.084</td>
<td>0.142</td>
</tr>
<tr>
<td>Shorrock</td>
<td>0.941</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Benhabib & Bisin & Luo
Social mobility - Full transition matrix

Data

\[
T_{30} = \begin{bmatrix}
0.341 & 0.286 & 0.211 & 0.107 & 0.032 & 0.020 & 0.003 \\
0.285 & 0.269 & 0.236 & 0.132 & 0.042 & 0.029 & 0.005 \\
0.212 & 0.239 & 0.271 & 0.169 & 0.056 & 0.042 & 0.009 \\
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0.123 & 0.180 & 0.273 & 0.193 & 0.082 & 0.098 & 0.051 \\
0.084 & 0.142 & 0.237 & 0.180 & 0.092 & 0.149 & 0.118 \\
\end{bmatrix}
\]

Our Simulation

\[
T_{30} = \begin{bmatrix}
0.316 & 0.217 & 0.235 & 0.159 & 0.039 & 0.034 & 0 \\
0.272 & 0.254 & 0.244 & 0.152 & 0.043 & 0.027 & 0.009 \\
0.246 & 0.240 & 0.262 & 0.167 & 0.050 & 0.036 & 0 \\
0.201 & 0.263 & 0.263 & 0.165 & 0.060 & 0.038 & 0.009 \\
0.199 & 0.239 & 0.272 & 0.087 & 0.096 & 0.088 & 0.018 \\
0.174 & 0.279 & 0.248 & 0.038 & 0.064 & 0.113 & 0.085 \\
0.131 & 0.292 & 0.196 & 0.014 & 0.030 & 0.140 & 0.192 \\
\end{bmatrix}
\]
Diagonal of benchmark

- Data
- Benchmark

Wealth share vs Wealth percentile
Savings rates

Data

Figure: Synthetic saving rates by wealth group - Data

Synthetic saving rates: $s_t^p = \frac{w_t^p - w_{t+1}^p}{y_t^p}$, $p$-th fractile
Savings rates - cont.ed

Table: Synthetic saving rates by wealth group

<table>
<thead>
<tr>
<th>Moments</th>
<th>Share of wealth</th>
<th>Bottom 90</th>
<th>90-99</th>
<th>99-100</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data 2000-2009</strong></td>
<td>-4</td>
<td>9</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>-6.5</td>
<td>0.0</td>
<td>25.7</td>
<td></td>
</tr>
</tbody>
</table>

Age distribution is assumed to be uniform.
Bequests

Simulation: an example

Figure: Bequests out of initial wealth
Counterfactuals

To identify explanatory factors,

Re-estimate the model by restricting:

1. Constant \( r \) [Results still very preliminary!]
2. \( \mu = 2 \) [Results still very preliminary!]
3. Constant \( w \) [Results not ready altogether!]

Simulate the estimated model under the same restrictions.
Re-estimation with restrictions

Table : Parameter estimates

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Markov Chain</th>
<th>σ</th>
<th>μ</th>
<th>A</th>
<th>β</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\mu = 2$</td>
<td></td>
<td>[2]</td>
<td>[2]</td>
<td>0.5430</td>
<td>[0.97]</td>
<td>[30]</td>
</tr>
<tr>
<td>(2) Const. r</td>
<td></td>
<td>[2]</td>
<td>1.5010</td>
<td>0.5543</td>
<td>[0.97]</td>
<td>[30]</td>
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<td>Rate of return (6-year)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) $\mu = 2$</td>
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<td>0.1133</td>
<td>0.3676</td>
<td>0.5992</td>
<td>0.8207</td>
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<td>(2) Const. r</td>
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</table>
Re-estimation

Table: Wealth quintiles

<table>
<thead>
<tr>
<th>Moments</th>
<th>Share of wealth</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-90</th>
<th>90-95</th>
<th>95-99</th>
<th>99-100</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td>-0.002</td>
<td>0.001</td>
<td>0.045</td>
<td>0.112</td>
<td>0.120</td>
<td>0.111</td>
<td>0.267</td>
<td>0.336</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>(1) $\mu = 2$</td>
<td>0.029</td>
<td>0.082</td>
<td>0.147</td>
<td>0.203</td>
<td>0.149</td>
<td>0.125</td>
<td>0.178</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>(2) Const. $r$</td>
<td>0.013</td>
<td>0.067</td>
<td>0.107</td>
<td>0.186</td>
<td>0.238</td>
<td>0.167</td>
<td>0.167</td>
<td>0.054</td>
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</table>
Re-estimation

Table: Diagonal of transition matrix

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-24</th>
<th>25-49</th>
<th>50-74</th>
<th>75-89</th>
<th>90-94</th>
<th>95-99</th>
<th>99-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of wealth Data</td>
<td>0.341</td>
<td>0.269</td>
<td>0.271</td>
<td>0.187</td>
<td>0.072</td>
<td>0.098</td>
<td>0.118</td>
</tr>
<tr>
<td>Simulation (1) $\mu = 2$</td>
<td>0.300</td>
<td>0.272</td>
<td>0.267</td>
<td>0.153</td>
<td>0.052</td>
<td>0.065</td>
<td>0.160</td>
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<tr>
<td>Simulation (2) Const. r</td>
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<td>0.256</td>
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<td>0.310</td>
<td>0.0</td>
<td>0.210</td>
<td>0.250</td>
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</table>
Earnings are not enough; Kindermann and Krueger (2014)

Estimate earning process and its transition to match the moments of the wealth distribution:

Table 7: Wealth Distribution in Benchmark Economy

<table>
<thead>
<tr>
<th></th>
<th>Share of total sample (in %)</th>
<th></th>
<th></th>
<th></th>
<th>Top (%)</th>
<th></th>
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<td></td>
<td></td>
<td>Quintiles</td>
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<td>Top (90-95)</td>
<td>Top (95-99)</td>
<td>Top (99-100)</td>
<td>Gini</td>
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<tr>
<td></td>
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<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>90-95</td>
<td>95-99</td>
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<tr>
<td>Model</td>
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<td>0.0</td>
<td>0.8</td>
<td>4.1</td>
<td>11.6</td>
<td>83.6</td>
<td>14.6</td>
<td>23.3</td>
</tr>
<tr>
<td>US Data</td>
<td></td>
<td>-0.2</td>
<td>1.1</td>
<td>4.5</td>
<td>11.2</td>
<td>83.4</td>
<td>11.1</td>
<td>26.7</td>
</tr>
</tbody>
</table>

Great fit!
Earnings are not enough; Kindermann and Krueger (2014) - cont.ed

But earning process is way way off, empirically:

Seven states - first five are roughly from data, top two are estimated to fit wealth distribution

| Earnings categories, median | 1 0.1159 0.3405 1.0000 2.9369 8.6255 15.8180 1284.3139 |

Top state has ratio to the median = 1284 (or at least 400 – 500 depending on interpretation); and
At the stationary distribution, the top state has 0.25% of population.
Earnings are not enough; Kindermann and Krueger (2014) - cont.ed

On average top .1% in U.S. makes about 2 mil; and the median earnings is about 40K; that is, top .1% has ratio to the median about 50 and top .25% even smaller.
Earnings are not enough; Castaneda et al. (2002)

Estimate earning process and its transition to match the moments of the wealth distribution:

4 state process

hourly wages of households in top state is about 1,000 times larger than those of households in bottom state;
present values of the life-time earnings of households in top state is about 120 times those in bottom state;
extra kick from perpetual youth - stochastic length of life/dynasty.
Counterfactual simulation A: Shut down capital income risk

Table: Counterfactual A: Wealth quintiles

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-90</th>
<th>90-95</th>
<th>95-99</th>
<th>99-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of wealth Data</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.045</td>
<td>0.112</td>
<td>0.120</td>
<td>0.111</td>
<td>0.267</td>
<td>0.336</td>
</tr>
<tr>
<td>Simulation</td>
<td>(1) Benchmark</td>
<td>0.011</td>
<td>0.039</td>
<td>0.083</td>
<td>0.132</td>
<td>0.115</td>
<td>0.121</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>(2) Const. $r$</td>
<td>0.018</td>
<td>0.047</td>
<td>0.112</td>
<td>0.196</td>
<td>0.159</td>
<td>0.166</td>
<td>0.229</td>
</tr>
</tbody>
</table>
Fit of counterfactual: const. r

- **Data**
- **Const. r**
Counterfactual simulation A - cont. ed

Table: Counterfactual A: Diagonal of transition matrix

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-24</th>
<th>25-49</th>
<th>50-74</th>
<th>75-89</th>
<th>90-94</th>
<th>95-99</th>
<th>99-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of wealth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.341</td>
<td>0.269</td>
<td>0.271</td>
<td>0.187</td>
<td>0.072</td>
<td>0.098</td>
<td>0.118</td>
</tr>
<tr>
<td>Simulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Benchmark</td>
<td>0.316</td>
<td>0.254</td>
<td>0.262</td>
<td>0.165</td>
<td>0.096</td>
<td>0.113</td>
<td>0.192</td>
</tr>
<tr>
<td>(2) Const. $r$</td>
<td>0.342</td>
<td>0.234</td>
<td>0.289</td>
<td>0.147</td>
<td>0.142</td>
<td>0.001</td>
<td>0.256</td>
</tr>
</tbody>
</table>
Diagonal of counterfactual: const. r

Wealth share

Wealth percentile
Counterfactual simulation B: Shutting down labor income

Table: Counterfactual B: Wealth quintiles

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.1) Low $w$</td>
<td>0.153</td>
<td>0.180</td>
<td>0.204</td>
<td>0.187</td>
<td>0.111</td>
<td>0.074</td>
<td>0.070</td>
<td>0.021</td>
</tr>
<tr>
<td>(4.2) Medium $w$</td>
<td>0.146</td>
<td>0.146</td>
<td>0.147</td>
<td>0.151</td>
<td>0.091</td>
<td>0.071</td>
<td>0.106</td>
<td>0.140</td>
</tr>
<tr>
<td>(4.3) High $w$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Non-stationary</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fit of counterfactual: const. low w

Data
Const. low w

Wealth share

Wealth percentile

0
100
Counterfactual simulation B - cont.ed

Table : Counterfactual B: Diagonal of transition matrix

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-24</th>
<th>25-49</th>
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<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of wealth</td>
<td>0.341</td>
<td>0.269</td>
<td>0.271</td>
<td>0.187</td>
<td>0.072</td>
<td>0.098</td>
<td>0.118</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.1) Low $w$</td>
<td>0.180</td>
<td>0.194</td>
<td>0.162</td>
<td>0.069</td>
<td>0</td>
<td>0</td>
<td>0.190</td>
</tr>
<tr>
<td>(4.2) Medium $w$</td>
<td>0.238</td>
<td>0.248</td>
<td>0.252</td>
<td>0.196</td>
<td>0.119</td>
<td>0.464</td>
<td>0.550</td>
</tr>
<tr>
<td>(4.3) High $w$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Non-stationary</td>
</tr>
</tbody>
</table>
Diagonal of counterfactual: const. medium w

Wealth share

Wealth percentile
Counterfactual simulation C: Shutting down differential savings

Table: Counterfactual C: Wealth quintiles

<table>
<thead>
<tr>
<th>Moments</th>
<th>0-20</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-90</th>
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<td>0.045</td>
<td>0.112</td>
<td>0.120</td>
<td>0.111</td>
<td>0.267</td>
<td>0.336</td>
</tr>
<tr>
<td>Simulation (5) ( \mu = 2 )</td>
<td>0.026</td>
<td>0.072</td>
<td>0.163</td>
<td>0.259</td>
<td>0.179</td>
<td>0.113</td>
<td>0.141</td>
<td>0.048</td>
</tr>
</tbody>
</table>
### Counterfactual simulation C - cont.ed

#### Table: Counterfactual C: Diagonal of transition matrix

<table>
<thead>
<tr>
<th>Moments</th>
<th>Share of wealth</th>
<th>0-24</th>
<th>25-49</th>
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<tr>
<td><strong>Data</strong></td>
<td></td>
<td>0.341</td>
<td>0.269</td>
<td>0.271</td>
<td>0.187</td>
<td>0.072</td>
<td>0.098</td>
<td>0.118</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>(5) $\mu = 2$</td>
<td>0.314</td>
<td>0.242</td>
<td>0.255</td>
<td>0.158</td>
<td>0.059</td>
<td>0.049</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Conclusion I: Results

Capital income risk and differential savings are fundamental factors in explaining wealth distribution and social mobility (in the U.S.)

Earnings by themselves are not enough

Capital income risk estimates are roughly consistent with observations regarding return on real estate and private business equity

Estimate of inter-generational correlation on returns on wealth is about zero

To do:

more on the mechanisms associated to different factors, estimate without requiring stationarity
Conclusion II: The re-birth of socialism