

Cultural Transmission, Socialization and the Population Dynamics of Multiple State Traits Distributions

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Abstract

This paper studies the population dynamics of preference traits in a model of intergenerational cultural transmission. Parents socialize and transmit their preferences to their offspring with endogenous intensities.

Populations concentrated on a single cultural group are in general not stable. There is a unique stable stationary distribution, and it supports two or more cultural groups (those with greater intolerance). The larger the heterogeneity of intolerance levels across cultural groups, the smaller the number of traits which are supported at the stable stationary distribution.

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1 Introduction

The view that preferences, norms, and, more generally, cultural attitudes should be considered as endogenous with respect to socio-economic systems has been now extensively motivated in the social sciences.¹ In this paper we study a specific model of preference formation, involving intergenerational cultural transmission. We build on the analysis of transmission and adoption of cultural traits developed by Cavalli Sforza-Feldman [1981], and Boyd-Richerson [1985]. In these papers, and in most of the ensuing literature, the intergenerational transmission mechanism is independent of any of the parents' decision. We study instead the population dynamics of the distribution of preferences or cultural traits in a model in which preference traits are determined by a process of parental socialization: preferences of children are acquired through an adaptation and imitation process which depends on their parents' socialization actions, and on the cultural and social environment in which children live. In such models, the effort parents deploy in socializing their offspring (vertical transmission; Cavalli Sforza-Feldman (1981)) is chosen optimally and depends on the cultural environment of the parents and the children. In particular, parental effort depends on the distribution of the population with respect to the relevant trait, which affects the socialization of children through teachers and role models (oblique transmission; Cavalli Sforza-Feldman (1981)).

The vertical and oblique transmission processes, in this context, induce families to socialize children more intensely whenever the set of cultural traits they wish to transmit is common only to a minority of the population. On the contrary, families which belong to a cultural majority do not spend many resources directly socializing their children, since their children adopt or imitate with high probability the cultural trait most predominant in society at large, which is the one their parents desire for them. Intergenerational cultural transmission therefore can explain, e.g., the persistence of various ethnic and religious traits which has been extensively documented and has recurrently surprised social scientists. For instance, the view that immigrants in the U.S. naturally assimilated in a melting pot process has proved empirically fallacious; see Herberg [1955], and Glazer and Moynihan [1963] for early realizations of this, and Mayer [1979] who, studying Orthodox Jewish communities in New York in the 70s, concluded that they were facing a 'cultural Renaissance' rather than the complete assimilation considered inevitable by much of the previous sociological literature on the subject. Also, Borjas [1995] studied the assimilation of immigrants' 'ethnic capital' in the United States, finding quite slow rates of cultural convergence. More generally, outside the United States, we can find many examples of the striking persistence of ethnic and religious minorities: Jews of the diaspora, Basques, Catalans, Corsicans, Irish Catholics, in Europe, Quebecois in Canada, Orthodox Christian Albanians in Italy, 'Blancs Matignons' in the French Carribean islands, and many others.

¹See Duesenberry (1949); Kapteyn-Wansbeek-Buyze (1980); Iannaccone (1990); Leibenstein (1950); Pollak (1976).

In this paper we formally study the stability properties of our model of cultural transmission with vertical and oblique socialization. We show that the implications of the model regarding the limit distribution of the population over different preference or cultural traits change critically when we allow family socialization to be the result of an intentional action by parents. We show that with multiple state traits populations, not *all* cultural groups are supported by a stationary distribution which is locally stable. The distribution of the population might in fact tend to converge to one in which only a subset of dimension ≥ 2 of traits is sustained. For any $1 < k \leq N$ we derive conditions which guarantee that the unique stable stationary distribution of the population is concentrated on k traits. We show that k is inversely related to a measure of the heterogeneity of the intolerance levels across traits. In the limit, for $N \rightarrow \infty$, the distribution with full support over traits is sustained if and only if all traits have symmetric intolerance levels.

2 Model

Consider a population with N possible cultural traits, indexed by $i \in \{1, \dots, N\}$. The N -dimensional vector $q = [q^i]_{i \in \{1, \dots, N\}}$ represents the distribution of the cultural traits in the population, and satisfies $\sum_{i=1}^N q^i = 1$. Let S^N denote the N -dimensional simplex. We have then $q \in S^N$.

Families are composed of one parent and a child, and hence reproduction is asexual.² All children are born without defined preferences or cultural traits, and are first exposed to their parent's trait. Vertical socialization to the parent's trait, say i , occurs with probability d^i . If a child from a family with trait i is not vertically socialized, which occurs with probability $1 - d^i$, she picks the trait of a role model chosen randomly in her parent's population (i.e., she picks trait i with probability q^i and trait $j \neq i$ with probability q^j). In other words, oblique transmission operates by random matching within society at large, with intensity measured by q^i .

Let P^{ij} denote the probability that a child from a family with trait i is socialized to trait j . P^{ij} also denotes the fraction of children with a type i parent who acquire preferences of type j . The socialization mechanism just introduced is then characterized by the following transition probabilities, for all i, j :

$$P^{ii} = d^i + (1 - d^i)q^i \tag{1}$$

$$P^{ij} = (1 - d^i)q^j \tag{2}$$

For vertical socialization choices d^i , $i \in \{1, \dots, N\}$, the dynamical system for the distribution of traits in the population is, in continuous time³:

²In Bisin-Verdier (2000) we study a related cultural transmission model with sexual reproduction and endogenous marriage, but single state traits.

³The dynamic evolution equations can be obtained from a population model with non overlapping

$$\dot{q}^i = q^i \left[\sum_{j \neq i} q^j (d^i - d^j) \right], \quad \forall i \in \{1, \dots, N\}. \quad (3)$$

The system satisfies $\sum_{i=1}^N \dot{q}^i = 0$, and hence $\sum_{i=1}^N q^i(t) = 1$, for all t , if and only if $\sum_{i=1}^N q_0^i = 1$. As a consequence, we can restrict ourselves to the dynamical system which consists of:

$$\dot{q}^i = q^i \left[\sum_{j \neq i} q^j (d^i - d^j) \right], \quad \text{for } i = 1, \dots, N-1 \quad (4)$$

$$q^N = 1 - \sum_{i=1}^{N-1} q^i \quad (5)$$

and the initial conditions q_0^i , $i \in \{1, \dots, N\}$, such that $\sum_{i \in \{1, \dots, N\}} q_0^i = 1$.

We now study a cultural transmission mechanism in which parents take costly actions to socialize their children, and hence endogenously determine vertical socialization, d^i , for all i .

Let V^{ij} denote the utility to a type i parent of a type j child, $i, j \in \{1, \dots, N\}$. The expected lifetime utility (abstracting from socialization costs) of a family of type i is then:

$$P^{ii}V^{ii} + \sum_{j \neq i} P^{ij}V^{ij}$$

where P^{ii} and P^{ij} are the transition probability defined in (1-2).

We assume that, for all $i, j \in \{1, \dots, N\}$, with $i \neq j$, $V^{ii} > V^{ij}$. Such assumption can be rationalized as a form of myopic or paternalistic altruism: Parents, while altruistic, prefer children to adopt their own cultural trait and hence try to socialize them to this trait.⁴ The intensity of the parents of type i 's preferences for having children with their

generations in which a fraction dt of the population dies between t and $t + dt$ and just before dying socializes offsprings according to the cultural transmission process described in the text. Then the evolution of the fraction of cultural trait i is ruled by:

$$q^i(t + dt) = q^i(t)(1 - dt) + dt \left[q^i(t)P^{ii}(t) + \sum_{j \neq i} q^j(t)P^{ji}(t) \right]$$

or after substitution of $P^{ii}(t)$ and $P^{ji}(t)$ and manipulation, provides:

$$q^i(t + dt) - q^i(t) = dt \left[-(1 - q^i(t))(1 - d^i) + \sum_{j \neq i} q^j(t) ((1 - d^j)q^i(t)) \right]$$

This at the limit of $dt \rightarrow 0$, gives equations (3) in the text.

⁴See Bisin-Verdier (2001b) for an evolutionary justification of paternalistic altruism.

own cultural trait is measured by $\frac{1}{N-1} \sum_{j \neq i} \Delta V^{ij}$.⁵

We also assume that socialization is costly. Let $H(d^i)$ denote socialization costs: For any $i \in \{1, \dots, N\}$: the map $H : [0, 1] \rightarrow \mathfrak{R}_+$ is \mathcal{C}^2 , strictly increasing and strictly quasi-convex; moreover $H(0) = 0$, $\frac{\partial H(0)}{\partial d^i} = 0$, and $\lim_{d^i \rightarrow 1} \frac{\partial H(d^i)}{\partial d^i} = \infty$.

Parents of type i choose $d^i \in [0, 1]$ to maximize:

$$-H(d^i) + P^{ii}V^{ii} + \sum_{j \neq i} P^{ij}V^{ij} \text{ s. t. (1-2)} \quad (6)$$

Under our assumptions the socialization choice problem satisfies the following necessary and sufficient first order conditions, for all $i \neq j$:

$$H'(d^i) = \sum_{j \neq i} q^j (V^{ii} - V^{ij}) = \sum_{j \neq i} q^j \Delta V^{ij} \quad (7)$$

For any i , let $\Delta V^i = [\Delta V^{ij}]_{j=1}^N$. Let $d(q, \Delta V^i)$ denote the solution to (7). It follows that $d(q, \Delta V^i)$ is increasing in each element of ΔV^i : naturally, the more parents prefer having children with their own cultural trait, the larger are their incentives to socialize their children to their own trait. The dynamics of the fraction of the population with cultural trait i is then determined by equations (4-5) evaluated at $d^i(q) = d(q, \Delta V^i)$.

If $N = 2$ the dynamics is:

$$\dot{q}^i = q^i (1 - q^i) (d^i(q^i) - d^j(1 - q^i)), \quad i \neq j$$

which is a logistic with an added non-linear term ($d^i(q^i) - d^j(1 - q^i)$). In this case it is immediate to see that if, as in most of the previous literature, we assume that vertical transmission is exogenously determined, then d^i and d^j are independent of q^i , and in the limit one cultural group will generically dominate (the group with higher vertical socialization rate; that is, group i if $d^i > d^j$). If instead vertical socialization results from the parents' rational effort choice, under our assumption d^i is decreasing in q^i and the dynamics of cultural traits will robustly look as in Figure 1: a unique stable steady state of the population dynamics, q^* , appears in which both traits are represented, while the dominant steady states, $q^i = 0$, $q^i = 1$, are not stable.

Characterizing the dynamic behavior of the distribution of traits in the population is more complicated in the multiple state traits case ($N > 2$), and we will make extra assumptions. But even in the general case it is easy to see that any homogeneous population constitutes an unstable stationary state of the dynamics of (4)-(5) evaluated at $d^i = d(q, \Delta V^i)$.

Proposition 1. *Under Assumptions 1 and 2, any degenerate distribution, i.e., any*

⁵The preference parameters ΔV^i , while un-observable, can be structurally estimated from socialization data; see Bisin-Topa-Verdier (2004) for an empirical analysis of religious traits in the U.S.

distribution q such that, for some $i \in \{1, \dots, N\}$, $q^i = 1$ (and hence $q^j = 0$, $\forall j \neq i$), is a locally unstable stationary distribution.

Proof. Pick an arbitrary $i \in \{1, \dots, N\}$. Differentiating (4)-(5) at the stationary state q such that $q^i = 1$, $q^j = 0$, $\forall j \neq i$, gives:

$$\left(\frac{\partial \dot{q}^i}{\partial q^i} \right)_q = -[d(q, \Delta V^i) - d(q, \Delta V^N)] = (H')^{-1}(\Delta V^{N1}) > 0 \quad (8)$$

as $d(q, \Delta V^i) = (H')^{-1}(0) = 0$. Also, $\left(\frac{\partial \dot{q}^i}{\partial q^k} \right)_q =$

$$= [d(q, \Delta V^i) - d(q, \Delta V^k) - (d(q, \Delta V^i) - d(q, \Delta V^N))] = (H')^{-1}(\Delta V^{N1}) - (H')^{-1}(\Delta V^{k1})$$

and

$$\left(\frac{\partial \dot{q}^k}{\partial q^k} \right)_q = [d(q, \Delta V^k) - d(q, \Delta V^i)] = (H')^{-1}(\Delta V^{k1}) > 0 \text{ for } k \neq i, N \quad (9)$$

$$\left(\frac{\partial \dot{q}^k}{\partial q^h} \right)_q = 0 \text{ for } k \neq i, N \text{ and } h \neq N;$$

(8) and (9) then readily imply local instability of q . ■

Thus, degenerate homogeneous distribution of traits are unstable. We will next characterize (Propositions 2, 3) and study the stability properties (Proposition 4) of heterogeneous distributions of preferences for N state traits populations. The following assumptions greatly simplify the algebra: For any $i \in \{1, \dots, N\}$: *i*) $\Delta V^{ij} = \Delta V^{ik}$, $\forall j, k \neq i$ (abusing notation, we let $\Delta V^{ij} \equiv \Delta V^i$); *ii*) $H(d^i) = \frac{1}{2}(d^i)^2$.

Under these conditions (7) becomes:

$$d(q^i, \Delta V^i) \equiv (1 - q^i)\Delta V^i \quad (10)$$

and the dynamic system (4)-(5), evaluated at (10), can be written as:

$$\dot{q}^i = q^i \left[(1 - q^i)\Delta V^i - \sum_{j=1}^N q^j(1 - q^j)\Delta V^j \right] \text{ for all } i \quad (11)$$

Let \mathcal{F}_k , with $1 \leq k \leq N$, denote the set of all k -dimensional subsets of $\{1, \dots, N\}$; \mathcal{F}_k contains $\binom{N}{N-k} \equiv \frac{N!}{(N-k)!}$ different subsets of $\{1, \dots, N\}$. We say that a stationary distribution supports $F_k \in \mathcal{F}_k$, and we denote it $q(F_k)$, if it is contained in the appropriate simplex:

$$q(F_k) \in S^{F_k} \equiv \{q \in S^N \mid q^i = 0, \text{ for } i \notin F_k\}$$

Without loss of generality we order the cultural groups so that

$$\Delta V^1 > \Delta V^2 > \dots > \Delta V^N. \quad (12)$$

Strict ranking is obviously generic.

Proposition 2. *Under our assumptions, a stationary distribution which supports F_k exists iff*

$$\Delta V^i > [k - 1]G^{F_k}, \forall i \in F_k \quad (13)$$

where $\frac{1}{G^{F_k}} \equiv \sum_{i \in F_k} \frac{1}{\Delta V^i}$. Moreover, a stationary distribution $q(F_k)$, which supports F_k , is defined by:

$$q^i(F_k) = 1 - \frac{k-1}{\Delta V^i} G^{F_k} \text{ for } i \in F_k \text{ and } q^j = 0 \text{ for } j \notin (F_k). \quad (14)$$

Proposition 2 should be interpreted to imply that a cultural group i is not supported by a stationary state if it is not intolerant enough relatively to the other groups; G^{F_k} can be in fact considered a measure of the cultural intolerance of the traits belonging to F_k ; e.g., if $\Delta V^i = \Delta V$ for all $i \in F_k$, $G^{F_k} = \frac{\Delta V}{k}$. Moreover, $q^i(F_k)$ increases in ΔV^i and decreases in ΔV^j , for $j \neq i$.

Proof. (If) All $F_1 \in \mathcal{F}_1$ and all $F_2 \in \mathcal{F}_2$ are supported by a stationary distribution. The fact that all $F_1 \in \mathcal{F}_1$ belong to C is trivial. As for all $F_2 \in \mathcal{F}_2$, we need to show that for any arbitrary $i, j \in F_2$, and any arbitrary $F_2 \in \mathcal{F}_2$, $\Delta V^i > G^{F_k}$. But $G^{F_2} = \frac{\Delta V^i \Delta V^j}{\Delta V^i + \Delta V^j}$, and, $\Delta V^i > \frac{\Delta V^i \Delta V^j}{\Delta V^i + \Delta V^j} = G^{F_2}$.

Let

$$C \equiv \bigcup_{k=1}^N \{F_k \in \mathcal{F}_k : \Delta V^i > [k - 1]G^{F_k}, \forall i \in F_k\}$$

Note that $C \neq \emptyset$ since \mathcal{F}_1 and all \mathcal{F}_2 belong to C .

For any $F_k \in C$, with $k > 1$, any q which solves (14) belongs to the interior of S^{F_k} , and hence it satisfies $q^i = 0$ for $i \notin F_k$ and

$$(1 - q^i)\Delta V^i - \sum_{j \in F_k} q^j(1 - q^j)\Delta V^j = 0 \quad \text{for } i \in F_k$$

which implies: $(1 - q^i)\Delta V^i = (1 - q^j)\Delta V^j$, for $i \neq j \in F_k$; i.e., q is a stationary distribution.

(Only if) For any arbitrary $F_k \in \mathcal{F}_k$, a stationary state, q , in the interior of S^{F_k} satisfies: $q^i = 0$, for $i \notin F_k$, and

$$(1 - q^i)\Delta V^i - \sum_{j \in F_k} q^j(1 - q^j)\Delta V^j = 0 \quad \text{for } i \in F_k$$

or:

$$\frac{1 - q^i}{\frac{1}{\Delta V^i}} = \frac{1 - q^j}{\frac{1}{\Delta V^j}} = \frac{\sum_{h \in F_k} (1 - q^h)}{\sum_{h \in F_k} \frac{1}{\Delta V^h}} = (k - 1)G^{F_k}, \quad \text{for } i \neq j \in F_k$$

and thus:

$$q^i = 1 - \frac{k - 1}{\Delta V^i} G^{F_k}, \quad \text{for } i \in F_k \quad (15)$$

i.e., q satisfies (14) for F_k .

Obviously, (15) can be satisfied only if $F_k \in C$. If $F_k \notin C$, then no stationary state exist in the interior of S^{F_k} . Finally, a stationary state on the boundary of S^{F_k} is a stationary state in the interior of $S_{F_{k-1}}$ for some F_{k-1} , and hence satisfies (14) for such F_{k-1} . ■

Proposition 3. *Under our assumptions, there exist a $k^* \geq 2$ such that:*

A unique $F_{k^} \in \mathcal{F}_{k^*}$ is supported by a stationary distribution.*

All $F_k \in \mathcal{F}_k$, for $k < k^$ are supported by a stationary distribution.*

No $F_k \in \mathcal{F}_k$, for $k > k^$ is supported by a stationary distribution.*

F_{k^*} is the largest subset of cultural groups $\{1, \dots, N\}$ which is supported by a stationary distribution. Since $\Delta V^1 > \Delta V^2 > \dots > \Delta V^N$, for an arbitrary k , the unique $F_k \in \mathcal{F}_k$ which is possibly supported by a stationary distribution is

$$F_k = \{1, \dots, k\}.$$

In other words, the cultural groups with highest intolerance are supported.

Proof. Without loss of generality, order groups so that

$$\Delta V^1 > \Delta V^2 > \dots > \Delta V^N.$$

For any $k \in \{1, \dots, N\}$, let $\hat{F}_k \equiv \{1, \dots, k\}$. Recall that $\Delta V^1 > \Delta V^2 > \dots > \Delta V^N$, and construct k^* has follows:

$$k^* \equiv \max k \in \{1, \dots, N\} \text{ such that } \Delta V^k > (k - 1)G^{\hat{F}_k}.$$

Let $q(\hat{F}_k)$ denote a stationary state in the interior of $S^{\hat{F}_k}$. By construction of \hat{F}_k , and using (12), if such a stationary state exists, it is unique. For $k = k^*$, existence follows from Proposition 2 and the construction of k^* . Finally, $\hat{F}_{k^*} = F_{k^*}$, by (12).

By the ordering (12), $\Delta V^k > (k - 1)G^{\hat{F}_k}$ implies that,

$$\Delta V^{k'} > (k' - 1)G^{\hat{F}_{k'}}, \quad \text{for any } k' < k, \quad (16)$$

$$\Delta V^{k''} < (k'' - 1)G^{\hat{F}_{k''}}, \text{ for any } k'' > k, \quad (17)$$

As a consequence: for $k > k^*$, no stationary state exists in the interior of $S^{\hat{F}_k}$ (from (16)); while, for any $k \leq \hat{k}$, there exists a stationary state q_k in the interior of $S^{\hat{F}_k}$ (from (17)). ■

Proposition 4. *Under our assumption the stationary distribution $q(F_{k^*})$, which supports F_{k^*} is locally stable. Moreover, any stationary distribution $q(F_k)$, which supports F_k , for $k < k^*$ is locally unstable.*

Proof. See the Appendix.

The population dynamics in the case of 3 state locally stable traits ($k^* = N = 3$) is illustrated by figure 2.

We obtain the following simple corollary.

Corollary. *If*

$$\sum_{i=1}^N \frac{1}{\Delta V^i} > \frac{N-1}{\text{Min}_i\{\Delta V^i\}},$$

there is a unique stationary state in the interior of S^N :

$$q^i = 1 - \frac{N-1}{\Delta V^i} \left[\sum_{i=1}^N \frac{1}{\Delta V^i} \right]^{-1}, \quad \forall i \in \{1, \dots, N\}$$

Moreover such stationary state is locally stable.

Note that condition (18) is stricter for larger N (in the limit, for $N \rightarrow \infty$, it requires symmetric preferences for children of their own type across cultural groups: ΔV^i independent of i). This Corollary then identifies symmetry of the parents' preferences for children as a factor which facilitates the stability of heterogeneous stationary distributions of traits in the population.

3 Discussion

Since our theoretical analysis of population dynamics only produces local stability results, we proceed to simulate the dynamical system, with the objective of gaining a better understanding of the global stability properties of $q(F_{k^*})$ under our assumptions. Starting with $N = 3$, we simulate the process choosing values for ΔV^i that satisfy condition (13), $\forall i \in F_N$. The simulations are performed in discrete time for 500 iterations, which are more than sufficient to reach the stationary distribution (see Figure 3).

As initial conditions, we choose a set A_0 of points in the interior of (but close to) the simplex S^N , starting with the vertices $(1 - 2\varepsilon, \varepsilon, \varepsilon)$, $(\varepsilon, 1 - 2\varepsilon, \varepsilon)$, $(\varepsilon, \varepsilon, 1 - 2\varepsilon)$ and moving along the segments that join these points.⁶

The result of the simulations is that for every possible initial condition in this set, the process converges to the stationary distribution $q(F_{k^*})$, defined in (14). A few typical simulation runs are reported in Figure 3, using different initial conditions. Essentially, in the simulations, we check that the vector field of the system is inward pointing on the boundary of A_0 ; and moreover that the dynamical system does not converge to a limit cycle from any initial condition on the boundary of A_0 . This is strong indication that the basin of attraction of $q(F_{k^*})$ is in fact the whole interior of the simplex S^N ; and therefore that the stationary distribution $q(F_{k^*})$ is indeed globally stable.

The same results hold with $N = 4$ state traits population. In this case we study both the case in which the values for ΔV^i are such that *i*) condition (13) is satisfied for $k^* = 4 = N$; see Figure 4; and *ii*) and condition (13) is satisfied for $k^* = 3 < N$; see Figure 5.

In this paper we have studied a simple stylized model of intergenerational cultural transmission of multiple state traits. Cultural transmission is the result of vertical and oblique transmission, and the intensity of vertical transmission is determined by the rational choice of parents. Many simplifying assumptions have made the formal analysis tractable: e.g., asexual reproduction, no horizontal transmission (through peers), quadratic socialization costs, parental preferences for children of their own trait independent of the cultural environment. All these assumption can be relaxed without changing the nature of the analysis.

Most importantly, we have studied the population dynamics of traits determined by cultural transmission in isolation, with no attempt to analyze the genetic and cultural co-evolution of traits. Several authors have built on the work of Cavalli Sforza-Feldman (1981) and Boyd-Richerson to study co-evolutionary models; see for instance Bowles-Gintis (2003) and Gintis (2003a,b). The integration of co-evolution with rational parental choice has yet to be developed; but, for a first attempt, see Bisin-Verdier (2001a).

⁶We use $\varepsilon = 0.001$ and a step size equal to 0.001 in performing this grid search.

Appendix

Proof of Proposition 4. Without loss of generality, one can order groups so that

$$\Delta V^1 > \Delta V^2 > \dots > \Delta V^N.$$

Suppose $k^* = N$. Then, necessarily, by (13),

$$\sum_{i=1}^N \frac{1}{\Delta V^i} > \frac{N-1}{\text{Min}_i\{\Delta V^i\}}, \quad (18)$$

By (12), $\Delta V^N = \min_{i \in \{1, \dots, N\}} \Delta V^i$. Consider then the system

$$\dot{q}^i = q^i \left[(1 - q^i) \Delta V^i - \sum_{j=1}^N q^j (1 - q^j) \Delta V^j \right], \quad i = 1, \dots, N-1 \quad (19)$$

$$q^N = \sum_{i=1}^{N-1} q^i \quad (20)$$

which is equivalent to (11). The Jacobian matrix of this dynamical system evaluated at q is given by $[a^{ik}]_{i,k \in \{1, N-1\}}$, where:

$$a^{ii} \equiv \left(\frac{\partial \dot{q}^i}{\partial q^i} \right)_q = - \left(1 - \frac{N-1}{\Delta V^i} G^N \right) \Delta V^N < 0 \quad \text{for } i \neq N$$

$$a^{ik} \equiv \left(\frac{\partial \dot{q}^i}{\partial q^k} \right)_q = \left(1 - \frac{N-1}{\Delta V^i} G^N \right) (\Delta V^k - \Delta V^N) \geq 0 \quad \text{for } i, k \neq N \text{ and } i \neq k$$

We now introduce the following result.

Lemma; Tambs-Lyche, 1928. *Suppose the $n \times n$ -dimensional (real) matrix $A \equiv [a^{ik}]$ satisfies the following conditions:*

$$a^{ik} \geq 0 \text{ for all } i, k \text{ (} i \neq k \text{),}$$

there exists positive numbers t_1, \dots, t_n such that $\sum_{j=1}^n t_j a^{ij} < 0$, for $i = 1, \dots, n$

Then the real parts of all the characteristic roots of A are non positive.

For a proof of this result, see Marcus-Minc [1964], pp. 158-59.

Using the Lemma, we then need to find positive numbers t_1, \dots, t_{N-1} such that

$$\sum_{k=1}^{N-1} t_k \left(\frac{\partial q^i}{\partial q^k} \right) < 0, \quad \text{for } i = 1, \dots, N-1$$

Let $F_{N-1} = \{1, \dots, N-1\}$, and consider $t_k = \frac{G^{F_{N-1}}}{\Delta V^k} > 0$.

Then, $\sum_{k=1}^{N-1} t_k \left(\frac{\partial q^i}{\partial q^k} \right) =$

$$= \sum_{k=1, k \neq i}^{N-1} \frac{G^{F_{N-1}}}{\Delta V^k} \left(1 - \frac{N-1}{\Delta V^i} G^N \right) (\Delta V^k - \Delta V^N) - \frac{G^{F_{N-1}}}{\Delta V^i} \left(1 - \frac{N-1}{\Delta V^i} G^N \right) \Delta V^N$$

which has the sign of

$$\begin{aligned} \sum_{k=1, k \neq i}^{N-1} \frac{G^{F_{N-1}}}{\Delta V^k} (\Delta V^k - \Delta V^N) - \frac{G^{F_{N-1}}}{\Delta V^i} \Delta V^N &= (N-2)G^{F_{N-1}} - \Delta V^N \sum_{k=1}^{N-1} \frac{G^{F_{N-1}}}{\Delta V^k} \\ &= (N-2)G^{F_{N-1}} - \Delta V^N \end{aligned}$$

(the last equality follows from $\frac{1}{G^{F_{N-1}}} = \sum_{k=1}^{N-1} \frac{1}{\Delta V^k}$). We need to show then that

$$(N-2)G^{F_{N-1}} < \Delta V^N \quad (21)$$

From (18),

$$\frac{1}{G^{F_N}} = \sum_{i=1}^N \frac{1}{\Delta V^i} > \frac{N-1}{\text{Min}_i \{\Delta V^i\}} = \frac{N-1}{\Delta V^N}.$$

But, also, $\frac{1}{G^{F_N}} = \frac{1}{G^{F_{N-1}}} + \frac{1}{\Delta V^N}$. As a consequence, $\frac{1}{G^{F_{N-1}}} + \frac{1}{\Delta V^N} > \frac{N-1}{\Delta V^N}$, which implies $\Delta V^N > (N-2)G^{F_{N-1}}$.

This proves the local stability of the $q(F_N)$ (note that $F_N = \{1, \dots, N\}$); moreover $q(F_N)$ is uniquely defined by

$$q^i = 1 - \frac{N-1}{\Delta V^i} \left[\sum_{i=1}^N \frac{1}{\Delta V^i} \right]^{-1}, \quad \forall i \in \{1, \dots, N\} \quad (22)$$

As noted, $q(F_N)$ exists if condition (18) is satisfied.

We next prove the local stability result for heterogeneous distributions of preferences whenever condition (18) does not hold.

Note that

$$\Delta V^k > (\text{ resp. } <) (k-1)G^{\hat{F}_k}$$

implies that

$$\Delta V^k > (\text{ resp. } <) (1 - q_k^i) \Delta V^i, \quad \forall i \in \hat{F}_k;$$

and, from (10), $\Delta V^i = d(0, \Delta V^i)$. In particular,

$$\Delta V^k < (1 - q_k^i) \Delta V^i, \quad \forall i \in \hat{F}_k, \quad k > \hat{k} \tag{23}$$

and

$$\Delta V^k > (1 - q_k^i) \Delta V^i, \quad \forall i \in \hat{F}_k, \quad k \leq k^* \tag{24}$$

(23) then implies that the same argument used to prove local uniqueness of the stationary state in the interior of S^N , under condition (18), in Lemma 1, proves then local uniqueness of q_{k^*} ; while (24) implies that any stationary state in the interior of S_{F_k} , for $k < k^*$, is locally unstable; see Figure 1 for the phase diagram in the case $k^* = 3$.

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Figure 1: N=2 Traits

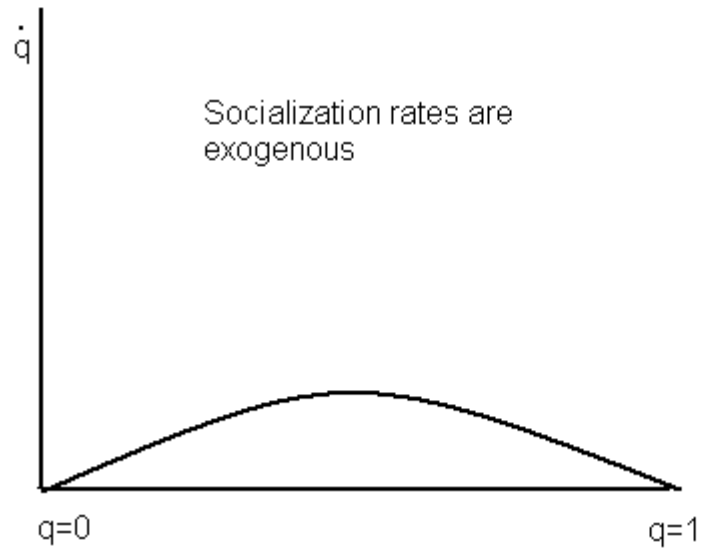
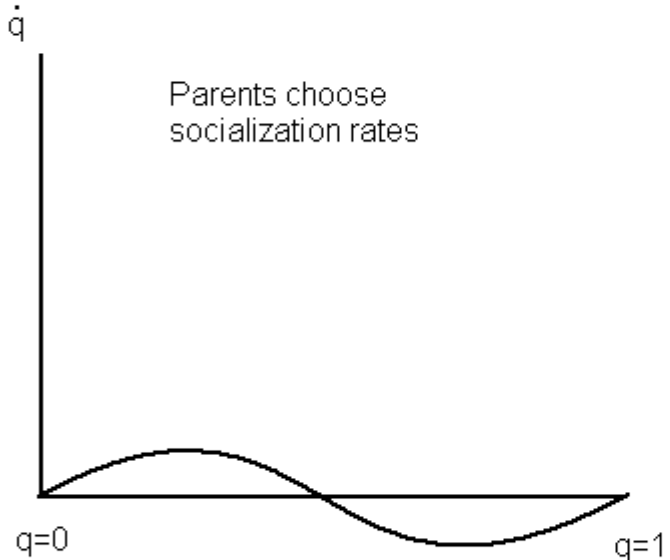
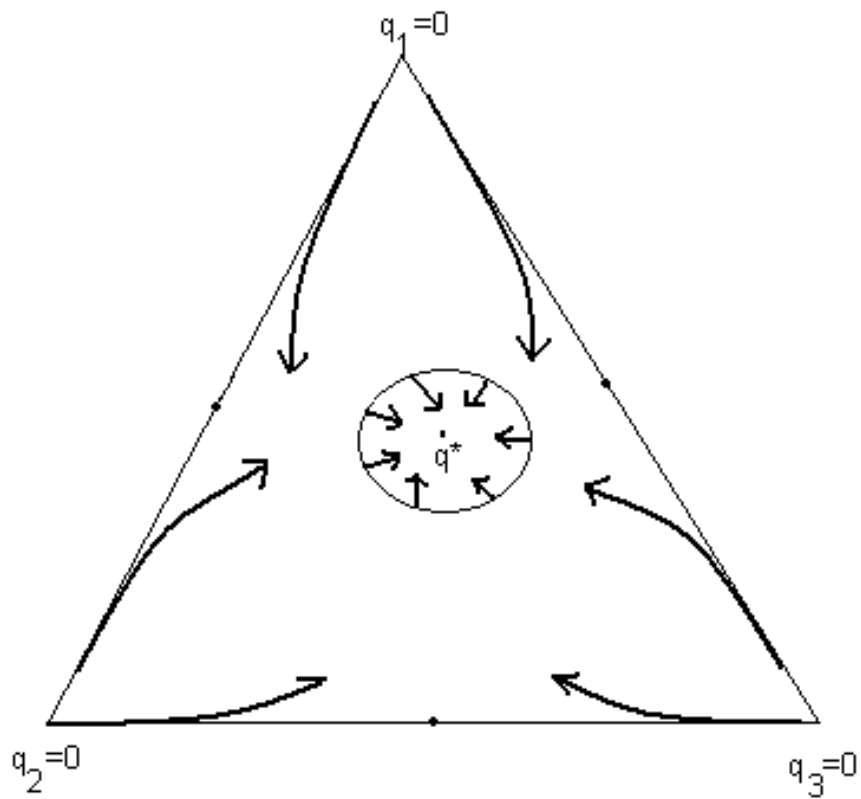


Figure 2: The Dynamics of 3-Trait Populations
with a stable stationary state q^* in S^3



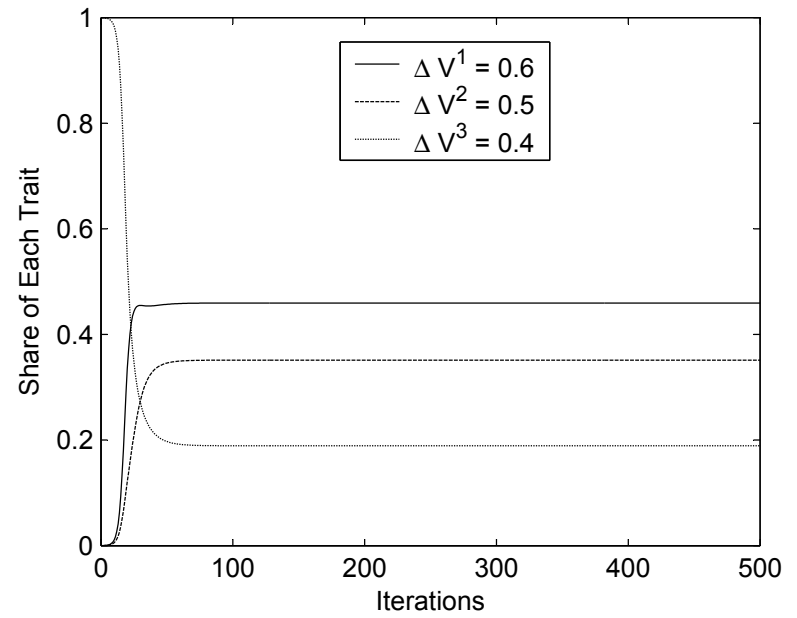
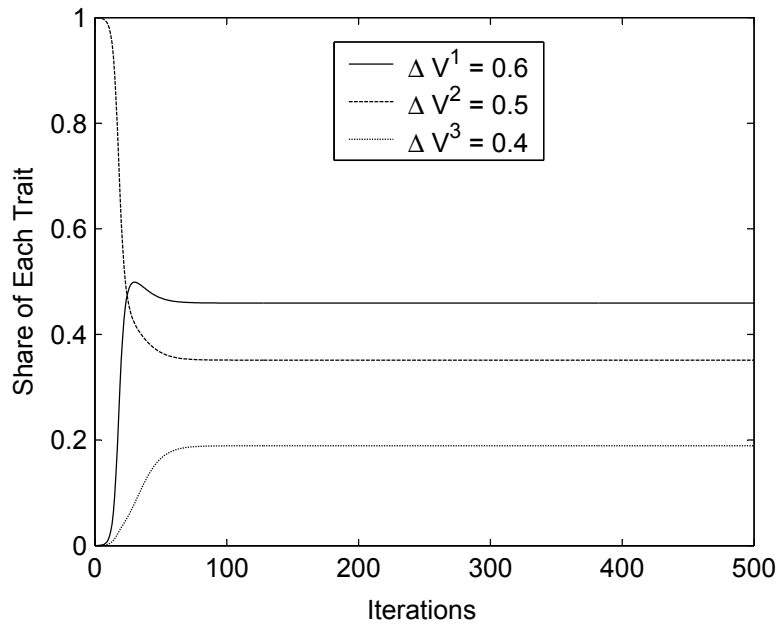
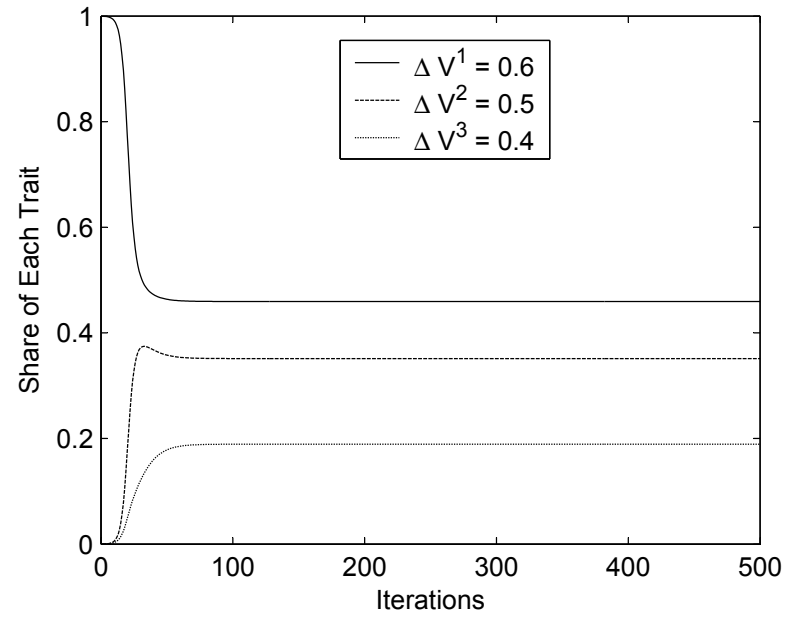
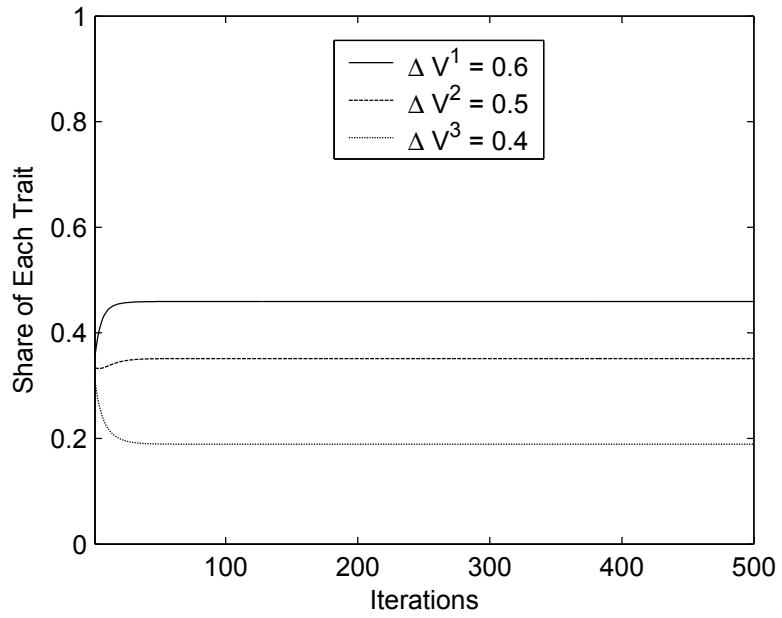


FIGURE 3 - Simulations with $N = 3$; ΔV^i Such That Condition (13) is Satisfied

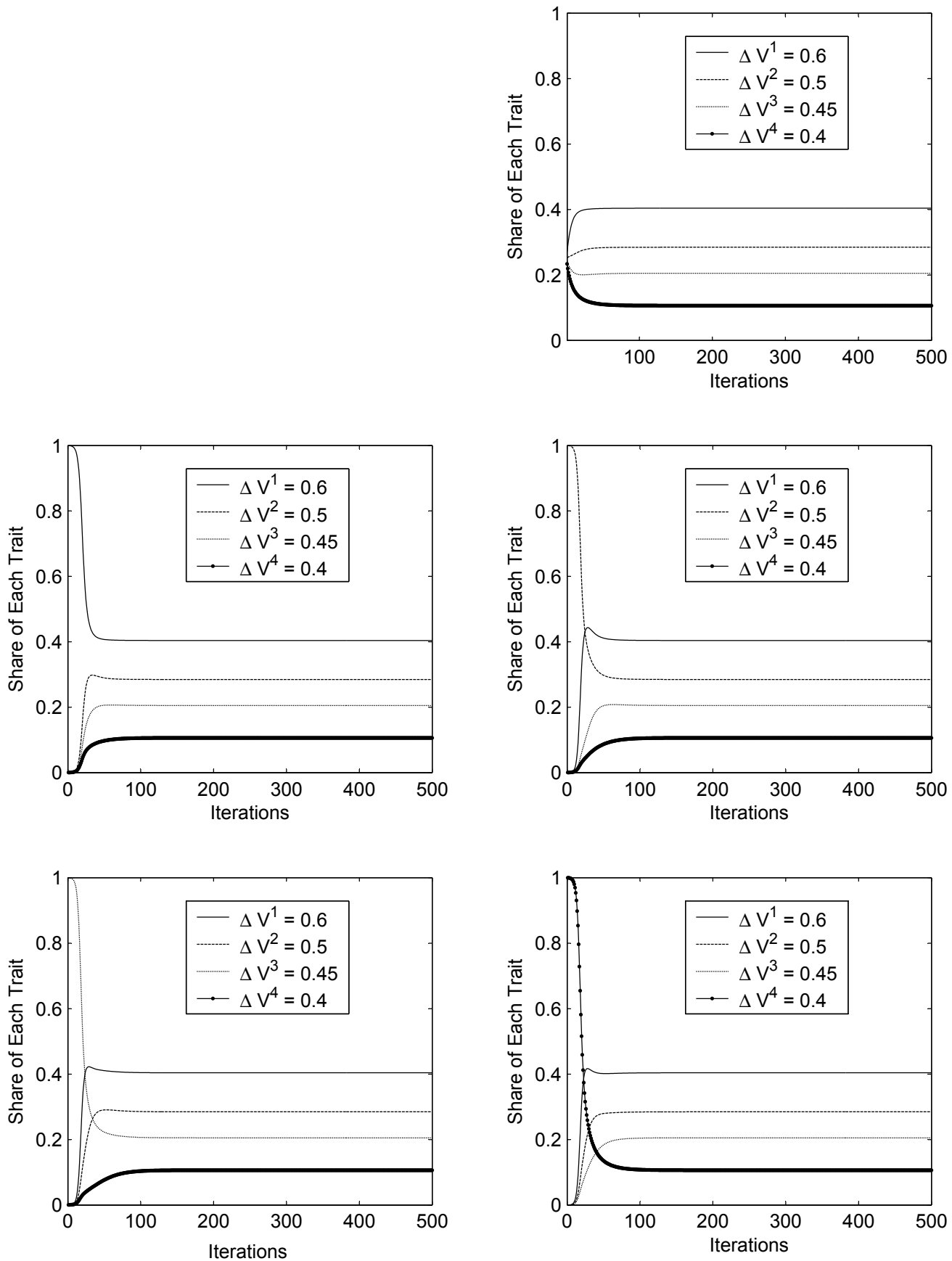


FIGURE 4 - Simulations with $N = 4$; ΔV^i Such That $k^* = 4$

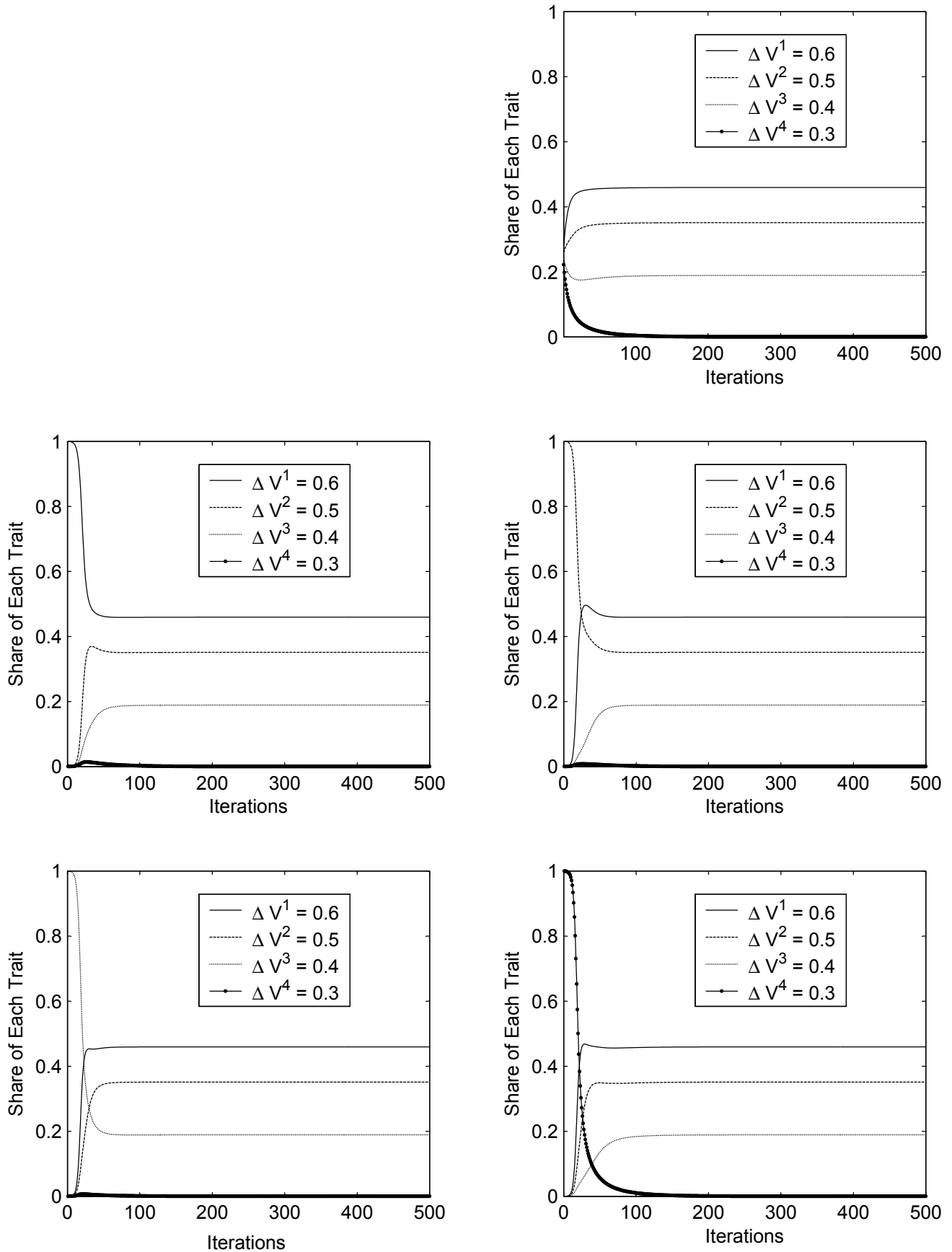


FIGURE 5 - Simulations with $N = 4$; ΔV^i Such That $k^* = 3$