Hyperbolic Discounting: An Experimental Analysis

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Abstract

In this paper we elicit preferences for discounting via experimental techniques. We then estimate a general specification of discounting that nests exponential and hyperbolic discounting, as well as various forms of *present bias*, including quasi-hyperbolic discounting. The data strongly favor a specification with a small present bias in the form of a fixed cost, of the order of $4 on average across subjects. In this specification, the present bias tends to vanish for large rewards (outside of our sample).

Finally we report some evidence about framing. While point estimates do depend on the question used to elicit discounting preferences, the statistical evidence is rather inconclusive.

1 Introduction

A vast literature in experimental psychology has documented various behavioral regularities that cast doubts on exponential discounting. The most important of such anomalies, called "reversal of preferences," has been interpreted to suggest that agents have a preference for present consumption not consistent with exponential discounting. Psychologists (e.g., Herrnstein, 1961, de Villiers-Herrnstein, 1976, Ainsle-Herrnstein, 1981; see also Ainsle, 1992, 2001) and, most recently, behavioral economists (e.g., Elster, 1979, Laibson, 1997, Loewenstein-Prelec, 1992, O'Donoghue-Rabin, 1999) have noted that the evidence is consistent with a declining rate of time preference, and have consequently

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suggested various specifications of discounting with this property, notably *hyperbolic discounting* and *quasi-hyperbolic discounting*.[1]

Such specifications of discounting introduce a fundamental paradigm change in economic theory: preferences with hyperbolic (or quasi-hyperbolic) discounting, unlike those with exponential discounting, lack time-consistency (see Strotz, 1954, for an early discussion of such issues). When his preferences are time-inconsistent, an agent's preference ordering changes over time. Dynamic choice problems are therefore not determined by the solution of a simple maximization problem, and require the agent to form expectations regarding his own decisions in the future. Moreover, when preference orderings change over time, the scope for normative statements and welfare analysis is of course greatly limited.

While experimental psychologists have collected an impressive amount of data on time preference in support of declining discount rates, some of this data is not without problems. Often experiments have been conducted with hypothetical rewards, or with "points" redeemable at the end of the experiments (thereby eliminating any rationale for time preference); the design of the experiments is seldom immune to issues of strategic manipulability, or of framing effects. Most importantly, rarely have the data been analyzed with proper econometric instruments. To our knowledge the hypothesis of hyperbolic discounting has never been tested statistically against the alternative of exponential discounting. (See Frederick-Loewenstein-O'Donoghue, 2002, for a comprehensive survey of the empirical literature on time preference.) Formal statistical procedures are necessary to identify behavioral regularities with noisy data; this is important in the context of experimental data on discounting which, the literature has repeatedly observed, are particularly noisy (Kirby-Hernnstein, 1995, Kirby, 1997; see also Frederick-Loewenstein-O'Donoghue, 2002).

In this paper we elicit preferences for discounting via experimental techniques. We then estimate a general specification of discounting which nests exponential and hyperbolic discounting, as well as various forms of *present bias* in discounting. We call present bias the psychological phenomenon whereby subjects associate a discrete cost to any future, as opposed to present, reward. The classic example of present bias is then quasi-hyperbolic discounting, as adopted e.g., by Laibson, 1997, and O'Donoghue-Rabin, 1999, in which the cost associated with future rewards is variable, that is proportional to the reward. The specification of present bias we estimate in this paper nests quasi-hyperbolic discounting as well as another specification in which a fixed rather than a variable cost

[1] Of course, a declining rate of time preference is not the only possible explanation of such anomalies of time preference. Rubinstein, 2003, shows how reversal of preferences might be induced by a specific form of procedural rationality. Also, most of the documented anomalies are consistent in principle with preferences over sets of actions, under standard rationality assumptions; see Gul-Pesendorfer, 2001. Finally, various specifications of psychological models of strategic interactions between multiple selves at each time period may rationalize such anomalies; see e.g., Thaler-Shefrin, 1981, Bernheim-Rangel, 2004, Benhabib-Bisin, 2004.

is associated to future rewards. Our methodology allows us to statistically test the exponential against the hyperbolic specifications. It also allows us to identify present bias in preferences for discounting and to test the fixed versus variable cost specification.

Distinguishing empirically between fixed and variable costs has important implications. If the present bias takes the form of a variable cost, that is it contains a quasi-hyperbolic component, then it affects all intertemporal choices; if instead the present bias takes the form of a fixed cost its effects tend to vanish for large rewards.

We find that exponential discounting is rejected by the data. A hyperbolic specification fits our discounting data better; and discount rates decline with delay. If we impose no present bias in the specification, however, estimated instantaneous discount rates appear implausibly high, implying for most agents yearly discount rates of the order of 500 percent. The specification with present bias therefore fares better on this and on other dimensions. Regarding the form of the present bias, the data do not seem to support a quasi-hyperbolic specification: a positive variable cost is not present for most of our subjects and, when it is, it is very small. The data strongly favor a specification with a present bias in the form of a fixed cost, significantly different from 0 and estimated as of the order of \$4 on average across agents.

Finally we report some evidence about framing. The evidence is rather inconclusive: eliciting discount rates by means of different questions produces different point estimates, but for most subjects these estimates are not statistically distinct, that is, confidence intervals do overlap. While this weak evidence for framing contrasts with most results in the literature (surveyed by Frederick-Loewenstein-O'Donoghue, 2002), it does not appear a peculiar feature of our data but rather a consequence of using formal statistical procedure.

2 Discounting Curves

Consider evaluating a dollar amount, say y, t periods from now: t is the delay at which y is received. An arbitrary discount function $D(y,t)$ is such that the value of y with delay t is:

$$yD(y,t).$$

Note that we allow the discount factor $D(y,t)$ to depend on delay t as well as on the amount to be discounted y. The associated subjective discount rate at y is:

$$\frac{\frac{\partial}{\partial t} D(y,t)}{D(y,t)}$$

The function $D(y,t)$ represents *exponential discounting* if

$$D(y,t) = \exp\{-rt\}, \; r > 0; \tag{1}$$

and it represents *hyperbolic discounting* if

$$D(y,t) = \frac{1}{1+rt}, \;\; r > 0 \tag{2}$$

Both exponential and hyperbolic discounting are independent of the amount to be discounted, y. But, in contrast to exponential discounting, preferences that display hyperbolic discounting induce declining subjective interest rates. In particular, the subjective interest rate associated with exponential discounting is $-r$, a constant, while the subjective interest rate associated with hyperbolic discounting is $\frac{-r}{1+rt}$, and hence it is declining in the delay t. Consequently, preferences displaying hyperbolic discounting give rise to the phenomenon of *preference reversals*, see Figure 1.

The other specification of discounting that has been studied in psychology and economics is *quasi-hyperbolic discounting*:

$$yD(y,t) = \left\{ \begin{array}{ll} y & \text{if } t = 0 \\ \alpha \exp\{-rt\} y & \text{if } t > 0 \end{array} \right. , \;\; \alpha < 1. \tag{3}$$

Discounting is quasi-hyperbolic if it displays a present bias, the larger the bias the smaller the parameter α.

Note that the present bias in quasi-hyperbolic discounting takes the form (the interpretation) of a *variable cost* associated to future payoffs: any payoff y is valued at most $y - (1-\alpha)y$ when received in the future. The cost $(1-\alpha)y$ is variable in the sense that it increases linearly with the amount y.[2] This interpretation induces us to consider another possible specification of discounting, in which the present bias is represented by a *fixed cost* b rather than by a variable cost. In this case, the amount y with delay t is valued

$$yD(y,t) = \left\{ \begin{array}{ll} y & \text{if } t = 0 \\ \exp\{-rt\} \left(y + b\, e^{-mt}\right) - b & \text{if } t > 0 \end{array} \right. \tag{4}$$

The parameter m controls the distribution of costs over time. This specification reduces to

$$yD(y,t) = \left\{ \begin{array}{ll} y & \text{if } t = 0 \\ \exp\{-rt\} y - b & \text{if } t > 0 \end{array} \right. \tag{5}$$

for $m = \infty$, where the cost b is not discounted. For $m = 0$, where the cost is discounted, it reduces instead to:

$$yD(y,t) = \left\{ \begin{array}{ll} y & \text{if } t = 0 \\ y \exp\{-rt\} - (1 - \exp\{-rt\})\, b & \text{if } t > 0 \end{array} \right. \tag{6}$$

[2] It is immediate to see that quasi-hyperbolic discounting implies time inconsistency, as well as reversal of preferences. This formulation has been introduced by Phelps-Pollak, 1968, and has been adopted by behavioral economists over the hyperbolic specification for its tractability; see Laibson, 1997, O'Donaghue-Rabin, 1999.

Note that, in contrast with the all other specifications of discounting we have considered, when present bias is represented by a fixed cost the discount factor $D(y,t)$ declines with the amount y. This is the property that we can exploit in the data to identify the fixed cost from the variable cost (quasi-hyperbolic discounting) specification of present bias.

3 Revealed Preferences Experiments: Is Discounting Hyperbolic ?

As noted in the Introduction, the experimental literature which documents preference reversals cannot provide a statistical test of the hyperbolic against the exponential specifications of discounting.

While under the assumption that "reversals" are due to hyperbolic preferences, the delay at which a reversal occurs contains some information about r in equation (2) or (3), individual discount rates cannot be estimated with the data generated by preference reversal experiments. It is then impossible to evaluate results statistically, e.g., to formally distinguish consistent empirical regularities from the effects of noise. In this paper we instead elicit preferences for discounting directly via experimental techniques.

3.1 The Experimental Design

A total of 27 inexperienced subjects were recruited from the undergraduate population of New York University to engage in our experiment. Each subject did the experiment on two different days. During both sessions they were asked a set of questions whose aim was to elicit their discount rates for money using a version of the Becker-DeGroot-Marschak mechanism to be described later. The two sessions differed by the types of questions asked which we will also describe in more detail later in this section.[3]

The paper-and-pencil experiment took place at the Center for Experimental Social Science (C.E.S.S) at New York University. When subjects arrived in the lab they were seated at tables and separated from each other for the duration of the experiment. They were then given a set of instructions which were read out loud to them after they had had a chance to read them individually. Each experimental session lasted about 1/2 an hour and subjects earned on average approximately \$28 in each session. (These are undiscounted amounts; some of these earnings were paid to the subjects at later times).

In Session 1 subjects were asked to reply to a set of 30 questions of the following form:

[3] A set of experimental instructions are provided at the end of the paper. We modified the notation to make it consistent with the one used in the paper.

What amount of money, $\$y$, if paid to you today would make you indifferent to $\$x$ paid to you in t days. $[Q - present]$

In the actual experiment x and t were specified so a typical question would be:

What amount of money, $\$y$, if paid to you today would make you indifferent to $10 paid to you in 1 month.

The amounts $\$x$ varied from $10, to $20, $30, $50, to $100 while t varied from 3 days to 1 week, 2 weeks, 1 month, 3 months, to 6 months. So for each amount we asked six questions involving the six different time frames; with five different amounts this totalled 30 questions.

To give the subjects an incentive to answer these questions truthfully we used a version of the Becker-DeGroot-Marschak mechanism to determine what amount would be paid to the subjects and when. This mechanism was employed on one of the thirty questions drawn at random. For example, say that at the end of the experiment we drew the question that asks the subject what amount, $\$y$, he would require today to make him indifferent between that amount today and $50 to be paid to him in one month. Assume he says $y = \$40$. In that case we would draw a random number uniformly from the interval $[0, \$50]$. If the number drawn was less than the $40 indifference amount stated by the subjects then he or she would have to wait for one month at which time the $50 would be paid. If the number drawn was greater than the $40 indifference amount stated, that amount would be paid immediately. To insure waiting the subject need only state an indifference amount of $50 while to insure receiving some money today the subjects need only state an indifference amount of $0. It is a dominant strategy to report your true indifference amount in this procedure (assuming risk neutrality) and this fact was explained to the subjects. We had no doubt that the subjects understood the incentive properties of the mechanism.

In this experimental session, therefore, subjects received either money today or money in the future. If money today were to be paid subjects were handed a check. If future money were to be paid subjects were asked to supply their mailing address and were told that on the day promised a check would arrive at their campus mailboxes with the promised amount. This was done to minimize any possible transaction costs involved in waiting, i.e., when paid in the future no subject had to travel to the lab to pick up his money etc., it would just arrive at their door.

Session 2 was identical to Session 1 except the question asked was reversed. here we asked:

What amount of money, $\$y$, would make you indifferent between $\$x$ today and $\$y$ t days from now. $(y^{Large} = w)$ $[Q - future]$

Note that in this question instead of asking what amount of money they need today to make them indifferent to a given amount of money in the future, we ask them what amount of money if given at a specific time in the future would make them indifferent to a fixed amount today. In this question they were not allowed to state an amount larger than some pre-determined quantity, y^{Large}. Here y^{Large} varied from \$10, to \$20, to \$30, to \$50 and finally to \$100 while the time horizons varied form 3 days to 1 week, 1 month, to 3 months and finally to 6 months, just as in Session 1. The $\$x$ amounts given them were derived from the answers to questions received in Session 1 and were the minimum of the amounts stated there.[4]

In summation, we employed a within-subject design using 27 subjects and two treatments where the treatments varied according to the type of question asked.

3.2 Estimates of discounting

Kirby, 1997, uses econometric methods to fit discount curves.[5] In his main experiment, the hyperbolic discount specification fits better that the exponential, in the sense that the R^2 is higher, for 19 out of 23 participants. What is missing from his analysis is a formal statistical test of the hyperbolic specification against the exponential alternative. The comparison of R^2 across specifications, while illustrative, cannot be considered sufficient statistical evidence.

To this end we introduce a two-parameter class of discount factor specifications nesting hyperbolic and exponential discounting:

$$D(y, t; \theta, r) = (1 - (1 - \theta)rt)^{\frac{1}{1-\theta}} \tag{7}$$

It is immediate to see that:

$$\begin{aligned} D(y, t; \theta = 1, r) &= \exp\{-rt\} \\ D(y, t; \theta = 2, r) &= \frac{1}{1 + rt}; \end{aligned}$$

and hence estimating the parameter θ from experimental data will possibly allow us to distinguish hyperbolic discounting, that is $\theta = 2$, from exponential discounting, $\theta = 1$.

[4]For example, one question was, "What amount of money, $\$y$, would make you indifferent between \$14 today and $\$y$ 3days from now? ($y^{Large} = 20$)". Here the subject was forced to give an answerof \$20 or less. Note that in answers for Session 1 we had not observed an amount less than\$14 being asked for \$20 in 3 days. In Session 2, seven of the subjects consistently chose the amounts y^{Large} for the full set of 30 questions. Compared to Session 1, a strategy of consistently chosing y^{Large} would favor present payments over the future ones. The choices of these seven agents may reflect a framing effect that affects the estimates of the parameters of their discounting curves for Session 2 . However overall, we fail to identify significant framing effects between the results of Session 1and Session 2; see Section 3.3 below.

[5]See also Green-Marakovic, 1995, Myerson-Green, 1995, Rachlin-Raineri-Cross, 1991.

More generally, it is straightforward to extend the two-parameter specification (7) to a four-parameter specification which also nests quasi-hyperbolic discounting and fixed costs:

$$yD(y,t;\theta,r,\alpha,b)=\begin{cases} y & \text{if } t=0 \\ \alpha\left(1-(1-\theta)rt\right)^{\frac{1}{1-\theta}}y-b & \text{if } t>0 \end{cases}, \quad \alpha<1 \tag{8}$$

Our data consists, for each subject $h=1,2,....$, of answers to a battery of questions such as $[Q-present]$ and $[Q-future]$, for different values of x and t. We present first our analysis of data regarding $[Q-future]$. Qualitative results are the same for $[Q-present]$; we discuss framing in Section 3.3.

Let $y^h(x,t)$ denote the answer given by subject h to question $[Q-future]$ for amount x and delay t. We start by estimating (7), to statistically document declining subjective discount rates. We then estimate (8) to better characterize the functional form of discounting, and the present bias.

To estimate (7) we assume that $y^h(x,t)$, the data is generated by

$$y^h(x,t)=x\left(1-(1-\theta^h)r^ht\right)^{\frac{1}{1-\theta^h}}\varepsilon^h(x,t)$$

where the error $\varepsilon^h(x,t)$ is i.i.d. with respect to subjects h and questions (x,t). Moreover, we assume $\varepsilon^h(x,t)$ is lognormally distributed. Note that we allow the parameters of the discount curve, (θ^h,r^h), to be indexed by the subject. We estimate individual discount curves, independently across subjects, $(\theta^h,r^h)_{h=1,...,25}$, by *non-linear least squares.*

Results are collected in Table 1[6] and are somewhat consistent with Kirby's, 1997, conclusions.In fact, for 23 of the 27 agents the exponential specification, $\theta=1$, is rejected by the data. Nonetheless the estimates do not appear particularly appealing, essentially because the point estimates for r are extremely high, in 17 cases of the order of thousands of percentage points. Even though when discounting is not exponential r does not represent the discount rate, it is still the case that one dollar with no delay is worth more than 5 in a year, for more than half of the agents in the sample at the point estimates. The point estimates of r are in only 1 case less than 100%.[7]

We turn then to a second specification. In this formulation discounting is allowed to be hyperbolic, as in the previous specification. But we include a fixed cost component to the preference for the present, that is we estimate (8) under the restriction that $\alpha=1$. Results are reported in Table 2. The fixed cost b is estimated to be significantly different than 0 for all the subjects (except subject 19 for which the estimate does not converge). It is, on average, about \$4 (with a minimum value of \$.31 and a maximum value of \$5.38). The estimates of r are also more reasonable when we include fixed costs. For

[6] In this and in the following tables estimates for individual subjects are not reported when the non-linear least square algorithm did not converge.

[7] This is by no means only a property of our data. Similar discount rates have been generally imputed from experimental data; see Frederick-Loewenstein-O'Donoghue, 2002, Table 1.1.

instance, the point estimates of r are less than 100% for 15 subjects and less than 30% for 9 subjects. The estimates of θ, the curvature of the discounting function at delays t different than zero, are however very imprecise when a fixed cost is added. For 5 subjects the hypothesis of exponential discounting is not rejected at the 95% confidence interval, and for 1 of these neither is the hypothesis of hyperbolic discounting . For 9 subjects the confidence interval of θ lies in the region smaller than 2, while for 16 subjects it is in the region greater than 1. In other words, the data seem to be clearly consistent with a present bias which we postulated in the form of a fixed cost, but do not have much power to distinguish exponential from hyperbolic discounting.

The final specification we estimate is (8), where both fixed and variable costs are allowed for, that is, where we allow for a quasi-hyperbolic component of present bias. Note that we can identify fixed versus variable costs components since we have data for different amounts x.[8] Results are striking, and are reported in Table 3. For 16 subjects we estimate a value of α greater than one. For the remaining 11 subjects, α is estimated significantly smaller than 1 (that is, we reject $\alpha \geq 1$) only in 5 cases. Finally, the point estimates of α are never smaller than .92, and the lower bound of the 95% interval never lower than .84. These estimates stand in sharp contrast to the much lower (around .6) imputed value of α obtained in a consumption-saving model by Laibson-Repetto-Tobacman (2004). The estimates we obtain for the fixed cost b in this specification are not much varied from those obtained in the previous specification, and still around \$4 on average across agents. The estimates for the curvature of discounting, θ, are still quite imprecise but seem now to favor, at the margin, the exponential specification: for 15 subjects the hypothesis of exponential discounting is not rejected at the 95% confidence interval, and for 11 of these neither is the hypothesis of hyperbolic discounting; but for all 15 subjects exponential discounting is not rejected at the 97% level, while for 12 of these the hypothesis of hyperbolic discounting is rejected at the 97%.

We conclude that the data do not seem to significantly support the quasi-hyperbolic specification, while they do support a fixed cost specification. In fact, if the fixed cost is correctly estimated at about \$4, it would appear to be negligible in most economic applications of interest. Of course strong caution in interpreting our results is necessary, because our estimates are derived from a sample which do not include amounts greater than \$100.

[8] We have also estimated the different functional forms for present bias discussed in Section 2. In particular we have estimated

$$D(y, t; \theta, r, b, \alpha, m) = \begin{cases} y & \text{if } t = 0 \\ \alpha \left((1-\theta)\, rt\right)^{\frac{1}{1-\theta}} \left(y + b\, e^{-mt}\right) - b & \text{if } t > 0 \end{cases} \tag{9}$$

Results (not reported) are not significantly different from those obtained with specification (8) illustrated in Table 3. The estimates of m are large enough so that (8) appears to represent a good approximation, with one less parameter.

3.3 Framing

Do results depend on how the question is posed? Frederick-Loewenstein-O'Donoghue, 2002, survey an extensive literature in experimental psychology documenting framing in discounting experiments.[9] Once again rarely has this literature adopted formal statistical methods to substantiate their claim. To address the issue of framing, in this paper we estimate the same specifications of discounting with the data obtained from question $[Q - present]$, and compare the results with those just discussed, that is, with the estimates with the data from $[Q - future]$.

The results are reported in Table 4, 5, 6 for our three specification of the discounting curves, respectively.[10] To identify and measure framing effects we proceed as follows: for each of the three specifications of discounting, *i)* we produce estimates of the parameters with data from questions of the form $[Q - future]$ and, in addition, of the form $[Q - present]$, *ii)* we construct confidence intervals on parameters' estimates, and *iii)* we check, parameter by parameter, if the confidence intervals obtained with the data generated by different questions overlap.

It is not easy to find clear statistical evidence for framing in our data. The first specification (equation 7), without fixed costs and without a quasi-hyperbolic component, has 2 parameters. For 14 out of the 25 subjects for which we could obtain estimates, the 95% confidence intervals for both parameters overlap; and for 9 of them the confidence intervals of one of the parameter overlap. For only one subject the estimates obtained with the different question are statistically distinct. The second specification (equation 8 with $\alpha = 1$), with fixed costs but no quasi-hyperbolic component, has 3 parameters. We obtain 3 overlapping confidence intervals for 7 out of 24 subjects, 2 overlapping intervals for 9 subjects, 1 overlapping interval for 7 subjects, and finally distinct estimates for only 1 subject. The results for the third specification (equation 8), which has 4 parameters, are as follows: 4 overlapping confidence intervals for 6 out of 17 subjects, 3 overlapping intervals for 5 subjects, 2 overlapping intervals for 3 subjects, 1 overlapping interval for 2 subjects, and finally distinct estimates for only 1 subject.

4 Conclusions

This paper provides the first experimental study of discounting in which data are analyzed with formal statistical methods and the various hypothesis regarding the specification of discounting preferences adopted in the behavioral economics literature are tested. We find clear experimental evidence against exponential discounting. The data

[9] See also Frederick, 2003.

[10] It should be noted that subjects 1, 9, 16, 18 possibly misunderstood this question (for instance, they claimed to be willing to accept an amount y in the present to avoid waiting 30 days for \$15, but to require an amount $y' > y$ in the present to avoid waiting 60 days for the same \$15.; subjects 3, 17, 19 and 23 essentially did not discount

favors a specification of discounting which contains a present bias in the form of a fixed cost, and no quasi-hyperbolic component. The curvature of discounting (exponential vs. hyperbolic), in the fixed cost specification, is not precisely estimated with our data, and is consistent for most subjects with exponential discounting. This finding implies, when extrapolated outside the sample, that present bias vanishes with large rewards. Experiments with relatively large rewards are needed to confirm the fixed cost representation of present bias that we identify in the experimental data.

We hope to soon extend our experiments in two directions. First, to explore how present bias behaves as rewards become large, we want to conduct experiments in countries where wealth and income are low. Second, to better understand discount curves and interest rates under alternative framing, we want to conduct experiments where subjects are paid a lump sum, but are then expected to choose between repaying a sum immediately, or a different sum at a different point in the future.

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Figure 1: Reversal of Preferences

[Figure 1 here]

Consider the following rewards: \$$x$ at time t and \$$x' > x$ at time $t' = t + d$. An agent with hyperbolic discounting, at time 0, would evaluate the first as

$$\frac{x}{1+rt}$$

and the second as

$$\frac{x'}{1+rt'}$$

If $t' - t = d > x' - x$, then at $t = 0$ any agent with hyperbolic discounting would prefer the smaller immediate amount x to the larger delayed amount x'. Note though that, as the joint delay t is increased, the agent's preference for the earlier reward will decline, until the agent will in fact "reverse his preferences" and prefer x'. This is the phenomenon of preference reversal, represented in the figure by crossing discount curves. It is straightforward to show that preference reversals never happen with exponential preferences: in fact, in this case if at $t = 0$ the earlier reward if preferred, that is, if $x > x'e^{-rd}$, then the earlier reward is preferred for any joint delay $t > 0$, as then $xe^{-rt} > x'e^{-rt'} = x'e^{-r(t+d)}$ for any t.

Figure 1

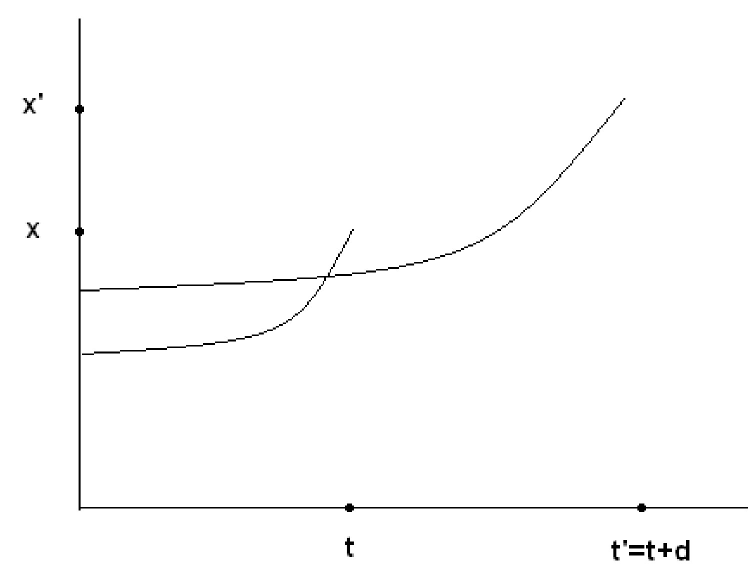

Table 1: Question $[Q - future]$; Specification with No Present Bias

Person	θ	se(θ)	r	se(r)
1	3.88	0.61	33.64	14.50
2	-2.00	4.40	0.50	0.37
3	2.85	1.52	4.16	2.69
4	2.26	0.48	4.99	1.34
5	1.96	0.83	2.88	1.05
6	2.65	0.41	7.24	1.78
7	4.67	3.55	2.66	2.29
8	4.25	0.86	14.22	6.19
9	3.88	0.61	33.64	14.50
10	3.15	1.16	18.36	15.41
11	16.62	5.27	93.63	125.12
12	4.14	0.84	16.42	7.45
13	3.88	0.61	33.64	14.50
14	3.55	0.92	13.06	6.93
15	2.46	0.56	6.37	2.12
16	2.33	0.91	4.08	1.80
17	1.88	2.72	1.20	0.73
18	3.88	0.61	33.64	14.50
19	4.61	2.42	10.45	10.93
20	4.86	1.20	13.95	7.47
21	3.92	0.61	33.81	14.44
22	2.62	0.49	7.03	2.03
23	3.88	0.61	33.64	14.50
24	6.53	1.77	66.44	60.55
25	3.62	0.60	26.63	11.19
26	3.88	0.61	33.64	14.50
27	3.91	0.91	20.25	11.14

Table 2: Question $[Q - future]$; Specification with Fixed Cost

person	θ	se(θ)	r	se(r)	b	se(b)
1	6.27	0.75	14.27	4.30	5.38	0.96
2	-22.74	16.10	0.08	0.04	0.44	0.15
3	-4.94	3.72	0.28	0.11	2.38	0.44
4	-1.02	1.14	0.73	0.22	4.46	0.83
5	-1.39	1.30	0.56	0.14	2.26	0.47
6	1.81	0.52	2.07	0.42	3.51	0.69
7	-6.10	24.75	0.12	0.09	1.33	0.23
8	2.69	1.24	1.53	0.48	4.81	0.71
9	6.27	0.75	14.27	4.30	5.38	0.96
10	3.45	1.31	2.37	0.93	4.68	1.00
11	-0.90	1.41	0.66	0.20	2.17	0.65
12	5.46	1.11	3.00	0.85	4.51	0.59
13	6.27	0.75	14.27	4.30	5.38	0.96
14	4.77	0.69	3.74	0.79	3.40	0.51
15	-1.24	2.22	0.62	0.30	5.31	1.14
16	-0.13	0.83	0.78	0.15	2.45	0.42
17	-4.95	5.83	0.23	0.10	0.52	0.31
18	6.27	0.75	14.27	4.30	5.38	0.96
19						
20	30.78	18.44	309.07	979.21	3.30	1.43
21	6.28	0.75	14.44	4.37	5.32	0.96
22	1.92	0.62	2.01	0.46	3.78	0.74
23	6.27	0.75	14.27	4.30	5.38	0.96
24	41.34	23.05	9370.38	45832.09	4.16	1.07
25	4.35	0.55	6.49	1.56	5.37	0.83
26	6.27	0.75	14.27	4.30	5.38	0.96
27	8.47	1.47	9.80	3.92	4.42	0.89

Table 3: Question $[Q - future]$; Specification with Fixed Cost and Quasi-Hyperbolic Component

person	θ	se(θ)	r	se(r)	b	se(b)	α	se(α)
1	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
2	-8.13	9.66	0.13	0.05	0.73	0.17	0.98	0.01
3	-0.92	2.19	0.48	0.15	3.51	0.47	0.94	0.01
4	-1.28	1.38	0.68	0.24	4.22	1.02	1.01	0.03
5	-0.62	1.31	0.66	0.18	2.62	0.58	0.98	0.02
6	1.70	0.64	1.97	0.52	3.42	0.80	1.01	0.03
7	21.74	24.80	0.41	0.48	1.68	0.28	0.98	0.01
8	-0.51	1.28	0.68	0.18	3.41	0.60	1.09	0.02
9	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
10	4.39	1.70	3.43	2.28	4.94	1.16	0.97	0.05
11	-3.21	2.39	0.42	0.17	1.27	0.69	1.06	0.03
12	5.60	1.52	3.15	1.55	4.53	0.66	1.00	0.03
13	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
14	6.21	0.78	8.90	4.12	3.87	0.52	0.92	0.04
15	-0.06	2.06	0.83	0.43	6.17	1.52	0.96	0.04
16	0.21	0.92	0.85	0.19	2.66	0.52	0.99	0.02
17	-2.66	5.84	0.27	0.13	0.72	0.39	0.99	0.01
18	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
19	-11.59	19.25	0.15	0.21	6.17	1.43	0.93	0.03
20	-48.75		0.04	0.00	3.04	1.28	1.20	0.04
21	1.21	0.80	1.35	0.35	4.62	0.77	1.21	0.03
22	4.57	0.75	1.74	0.50	3.54	0.85	1.02	0.03
23	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
24	41.33	23.02	17944.30		4.16	1.07	0.97	0.13
25	3.95	0.79	5.11	2.20	5.29	0.89	1.02	0.05
26	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
27	-0.02	1.44	0.74	0.23	3.79	0.70	1.16	0.03

Table 4: Question $[Q - present]$; Specification with No Present Bias

Person	θ	se(θ)	r	se(r)
1				
2	3.20	10.24	0.10	0.05
3	-80.90	156.93	0.00	0.05
4	3.90	0.98	7.10	2.91
5	6.90	2.49	1.00	0.27
6	7.40	0.53	31.10	6.24
7	-4.40	8.24	0.30	0.19
8	16.80	3.34	16.50	9.11
9	48.70	24.78	9.10	14.05
10	14.60	4.59	7.90	5.54
11	2.70	0.55	6.30	1.90
12	11.50	1.85	20.30	8.86
13	3.00	0.83	12.10	6.61
14	9.10	3.29	3.10	1.69
15	1.30	3.58	0.70	0.35
16	24.00	5.77	18.80	14.00
17	334.40	4.72E+08	0.00	0.00
18	177.70	433.96	2000.00	44794.41
19				
20	7.70	1.13	4.40	1.05
21	-0.60	5.32	0.30	0.16
22	2.70	4.58	0.80	0.51
23	334.40	4.72E+08	0.00	0.00
24	2.00	0.65	6.70	3.10
25	73.20	45.04	1.20	1.48
26	-0.80	1.66	0.60	0.18
27	10.40	2.00	48.90	31.27

Table 5: Question $[Q - present]$; Specification with Fixed Cost

person	θ	se(θ)	r	se(r)	b	se(b)
1						
2	4.06	12.35	0.14	0.06	-0.02	0.16
3	crazy					
4	5.58	1.29	2.55	0.76	2.80	0.71
5	4.33	1.57	0.62	0.11	0.22	0.22
6	8.83	0.69	24.47	5.35	1.34	0.61
7	-17.04	24.08	0.09	0.06	0.69	0.37
8	18.31	3.07	11.76	5.76	0.55	0.69
9	161.48	626.71	0.25	1.81	1.99	0.89
10	2.27	4.45	0.40	0.16	1.91	0.44
11	1.39	0.61	1.64	0.34	3.28	0.74
12	16.75	2.43	115.38	72.81	-0.86	0.81
13	1.27	0.71	1.75	0.45	3.31	1.01
14	17.51	6.44	0.95	0.48	1.66	0.41
15	-42.61	60.27	0.05	0.06	1.11	0.49
16	21.57	3.57	2.41	0.83	1.46	0.33
17	8.17		0.00	0.00	0.00	0.00
18	-192.27	164.16	-6.12	21.76	2.14	0.50
19						
20	7.48	1.19	3.36	0.87	0.26	0.57
21	0.12	17.10	0.14	0.09	0.41	0.29
22	4.28	2.70	0.84	0.31	-0.74	0.58
23	29.37		0.00	0.00	0.00	0.00
24	2.28	0.57	2.49	0.52	1.95	0.78
25	57.46	17.25	2.56	2.11	-0.31	0.37
26	-41.53		0.05	0.00	0.99	0.46
27	8.92	1.15	30.11	11.27	-0.78	1.03

Table 6: Question [$Q - present$]; Specification with Fixed Cost and Quasi-Hyperbolic Component

person	θ	se(θ)	r	se(r)	b	se(b)		
1							α	se(α)
2	10.80	12.29	0.20	0.09	0.18	0.20		
3							0.99	0.01
4	8.01	1.27	7.72	4.47	3.64	0.69		
5	4.52	1.87	0.63	0.14	0.24	0.27	0.91	0.04
6							1.00	0.01
7	-3.45	17.53	0.15	0.11	1.15	0.48		
8							0.98	0.01
9	163.32	865.87	0.26	4.36	2.00	0.96		
10	7.37	4.04	0.78	0.38	2.69	0.51	1.00	0.05
11	1.87	0.67	1.99	0.49	4.24	0.88	0.97	0.01
12	1.94	2.90	0.68	0.30	-0.89	0.64	0.96	0.02
13	2.47	0.57	3.05	0.75	5.91	0.91	1.24	0.02
14							0.89	0.02
15	-12.57	13.95	0.12	0.08	2.09	0.61		
16	25.68	4.69	8.04	13.09	1.56	0.34	0.97	0.01
17	9.62		0.00	0.00	0.00	0.00	0.97	0.04
18							1.00	0.00
19								
20	9.57	1.17	10.40	6.33	0.92	0.52		
21	11.29	12.75	0.28	0.18	1.02	0.34	0.92	0.04
22	5.38	2.84	1.09	0.52	-0.12	0.68	0.98	0.01
23	9.62		0.00	0.00	0.00	0.00	0.97	0.02
24	3.57	0.48	4.84	1.11	3.61	0.68	1.00	0.00
25							0.90	0.02
26	-16.31	10.87	0.12	0.07	2.05	0.69		
27							0.97	0.01