Intro to Economic analysis

Alberto Bisin - NYU

1 Rational Choice

- The central figure of economics theory is the individual *decision-maker* (DM). The typical example of a DM is the *consumer*.
- DM has well defined *preferences* over a *choice set*.
- DM is *rational*, and hence her preferences can be represented by a *utility function*.
- DM chooses her preferred element in the choice set, that is she *maximizes* her utility function in the choice set.

1.1 Preferences

- The choice set is represented by X - some sets of objects; e.g., some set of *consumption bundles*. An example with two commodities is given in Figure 1.

Figure 1

- Preferences are represented by a *weak preference relation* by $\succsim$ (note that it is not $\geq$), which allows us to compare pairs of alternatives x, y in the set of objects X.

  Definition 1 *We read $x \succsim y$ as "x is at least as good as y" or "x is weakly preferred to y".*

  – In words, $x \succsim y$ means that the consumer thinks that the bundle x is at least as good as the bundle y; we also say "x is better than y." From $\succsim$ we can derive two other important relations on X:

Definition 2 Strict preference relation, $\succ$, *defined by*

$$x \succ y \Leftrightarrow x \succsim y \text{ but not } y \succsim x$$

where $\Leftrightarrow$ means if and only if, and read x is strictly preferred to y.

Definition 3 Indifference relation, $\sim$, *defined by*

$$x \sim y \Leftrightarrow x \succsim y \text{ and } y \succsim x$$

and read x is indifferent to y.

- *A simple illustration* - imagine that we present a consumer with any pairs of alternatives, x and y, and ask how she compares them: Is either x or y better than the other in your eyes? For each pair x and y, we can imagine four possible responses to our question:
  - (i) x is better than y, but y is not better than x,
  - (ii) y is better than x, but x is not better than y,
  - (iii) neither is better,
  - (iv) I am unwilling to commit to a judgment.
- Is the answer that x is better than y *and* y is better than x logically possible? [see discussion in class]

1.2 Rationality

- The hypothesis of rationality is embodied in two assumptions about the weak preference relation $\succsim$.

Definition 4 Completeness - *for all* $x, y \in X$, *either* $x \succsim y$ *or* $y \succsim x$ *or both.*

- Completeness simply requires that any two elements of the choice set can be compared by the DM. As such it is hardly objectionable as a first order assumption. But can you think of situations in which it fails ? Try introspection.

Definition 5 Transitivity - *for all* $x, y, z \in X$*, if* $x \succsim y$ *and* $y \succsim z$ *then* $x \succsim z$*.*

- Transitivity implies that a DM never faces a sequence of pairwise choices in which her preferences cycle. Is it clear that transitivity of preferences is necessarily a property that preferences would have to have? Try introspection again.

- Completeness and transitivity are assumptions about people's choice behavior and not statement of logical necessity. The point is whether or not completeness and transitivity are a reasonably accurate description of how DMs' preferences.

  - Lack of transitivity implies the possibility for *money pumps*. Here's an example of a money pump.

    Consider an economy with 3 goods, x, y, z, and 2 agents, 1, 2.. Agent 1 is endowed with one unit of z, one unit of y, and some money. Agent 2 is endowed only with one unit of good x. Agent 1 has non transitive preferences $\succsim$ such that: $x \succsim y \succsim z$ and $z \succ x$. We do not specify any preferences for agent 2 [make sure that at the end of the example you understand why we do not need to]. Consider the following sequence of trades between agent 1 and 2: Agent 2 gives agent 1 a unit of good x in exchange for a unit of good y. - Agent 2 gives agent 1 a unit of good y in exchange for a unit of good z. - Agent 2 gives agent 1 a unit of good z in exchange for a unit of good x plus a (small but positive) sum of money. Note that the sequence of trades is constructed so that it is feasible (possible) from the given endowments, and so that agent 2 is willing to enter each of these trades.

    At the end of the sequence of trades agent 2 has the unit of x he was endowed with, plus a sum of money. Agent 1 has instead the units of y and z he was endowed with, but he has lost a sum of money. This trade cycle can be repeated indefinitely until agent 1 lost all his money.

  - Experimental evidence is not always consistent with transitivity. The following experimental behavior is often documented.

A student if shown two objects, a pen and a mug. She is asked to say which object she likes better and she answers "the pen." She is then asked how much she is willing to spend to buy the pen, and she answers "$1." In the course of the experiment the student is given the mug as a present. At the end of the experiment she is asked if she wants to give back the mug in exchange for $1, and *she refuses.* This is a violation of transitivity:

$$\$1 \succsim \text{ pen } \succsim \text{ mug } \textit{and} \text{ mug } \succ \$1$$

.

– Is transitivity a natural property of preferences of a group ? Here is how you can argue negatively.

Consider an economy with 3 goods, $x,\ y,\ z$ and 3 agents, $1, 2, 3$. Suppose that each agent has complete transitive preferences over the 3 goods, and that no agents is ever indifferent between any of the possible binary comparisons. We can then represent their preferences by rankings. Agent 1's rank (from best to worst) is x, y, z; agent 2's is y, z, x; and agent 3's is z, x, y. Suppose that the preferences of the group of agents are formed by majority voting from the preferences of the 3 agents; that is, the groups prefers x to y if and only if at at least 2 of the agents do. Construct now the preferences of the group, $\succsim$, and note that:

$$x \succsim y \succsim z \text{ and } z \succ x$$

That is, the group has non-transitive preferences.

1.3 Utility representation

- Rationality of DM (completeness and transitivity of preferences) allows us to represent the preference relation, $\succsim$, by a utility function.

Definition 6 *A function $u : X \to R$ is a utility function representing preference relation $\succsim$ if for all $x, y \in X$, $x \succsim y \Leftrightarrow u(x) \geq u(y)$.*

Proposition 1 *A preference relation $\succsim$ can be represented by a utility function if and only if it is complete and transitive.*

Proof. Assume the set X contains a finite number N of elements, $(x_1, \ldots, x_N)$. [the general proof is much more complicated and is omitted]

[only if] By contradiction.

Suppose $\succsim$ is not complete. Then there exist $x, y \in X$ such that neither $x \succsim y$ nor $y \succsim x$. But if $\succsim$ can be represented by some $u(.)$, then either $u(x) \geq u(y)$ or $u(y) \geq u(x)$ or both. This is a contradiction.

Suppose $\succsim$ is not transitive. Then ... continue by analogy with the argument used above for completeness.

[if] By induction (proof the statement is true for $N = 1$; then assume it is true for any arbitrary $N - 1$ and show it is true for N).

The case $N = 1$ is obvious. Assume the statement for $N - 1$. Rank (from best to worst) in terms of preferences $\succsim$ all elements $(x_1, \ldots, x_{N-1})$. The utility representation then exist, by the induction assumption: $u(x_n)$, for any $n = 1, \ldots, N - 1$. We just need to construct $u(x_N)$ so that the statement is true. Check that this construction satisfy the requirement: If there exist a $n = 1, \ldots, N - 1$ such that $x_n \sim x_N$ then $u(x_N) = u(x_n)$. If $x_N \succ x_n$ for any $n = 1, \ldots N - 1$ then $u(x_N) = max_{n=1,\ldots,N-1}\ u(x_n) + 1$. If $x_n \succ x_N$ for any $n = 1, \ldots N - 1$ then $u(x_N) = min_{n=1,\ldots,N-1}\ u(x_n) - 1$. If x_N is between x_n and x_{n+1} in the ranking then $u(x_N) = \frac{1}{2}u(x_n) + \frac{1}{2}u(x_{n+1})$.

[where did we use completeness and transitivity in the "if" part of the proof?] ■

- The only property of utility function that is important is how to order the bundles. The size of the utility difference between any two bundles doesn't matter.

Proposition 2 *If a utility function $u(x)$ represents the preference relation $\succcurlyeq$, any* monotonic strictly increasing transformation *of $u(x)$, $f(u(x))$, also represents the same preference relation.*

Proof. Suppose u represents some particular preference relation, $\succcurlyeq$. Then, by definition,

$$u(x) \geq u(y) \text{ if and only if } x \succcurlyeq y$$

But if $f(u)$ is a monotonic strictly increasing transformation of u, then

$$u(x) \geq u(y) \text{ if and only if } f(u(x)) \geq f(u(y))$$

Thus,

$$f(u(x)) \geq f(u(y)) \text{ if and only if } x \succcurlyeq y$$

and the function $f(u)$ represents the preference relation, $\succcurlyeq$ in the same way as the function u. ■

1.4 Choice

- A rational agent chooses the element x of the choice set X to which is associated the highest utility $u(x)$. Formally, a rational agent choice problem is written:

$$max_{x \in X} \; u(x)$$

- Example: consumer problem.

  - Let m be a fixed amount of money available to the consumer and let $p = (p_1, ..., p_k)$ be the vector of prices for goods $1, ..., k$.

    Definition 7 *The set of affordable objects, is given by*

    $$B = \{x \in X : p_1 x_1 + \cdots + p_k x_k \leq m\}$$

    and is called the *budget set*. Note that we also write $p_1x_1+\cdots+p_kx_k$ more compactly in vector notation as px.

  - Thus, the problem of preference maximization can then be written as:

    $$\text{choose } x \text{ to maximize } u(x) \text{ subject to } x \in B$$

Or equivalently,

choose x to maximize $u(x)$ subject to x is in X and $px \leq m$

- The most commonly used utility function in economics is the *Cobb-Douglas* utility function:

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

where $\alpha, \beta > 0$.

Any monotonic transformation of a *Cobb-Douglas* utility function represent the same preferences. Moreover, you may find some monotonic transformation very useful. For example,

$$v(x_1, x_2) = \ln(x_1^{\alpha} x_2^{\beta}) = \alpha \ln x_1 + \beta \ln x_2$$

- The *demand function* is a function that relates the optimal bundle - the quantities demanded - to the different levels of prices and income. That is, $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. Clearly, for each different set of prices and income, there will be a different bundle which is the optimal choice for the consumer. The demand functions for the *Cobb-Douglas* utility function is the solution of the following maximization problem:

$$max_{x_1,x_2}\, u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

subject to

$$p_1 x_1 + p_2 x_2 = m$$

Can you solve this maximization problem ? Try substituting the constraint into the utility function. Alternatively, try constructing the Lagrangian.

1.5 References

H. Varian, *Intermediate Microeconmics - A Modern Approach*, fifth edition, Norton. (33-36; see also 2-2.5, 3.3-3.5).