

Anticipation, Uncertainty, and Time Inconsistency

Andrew Caplin and John Leahy*

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Abstract

We extend expected utility theory to situations in which agents experience feelings of anticipation at dates prior to the resolution of uncertainty. We show how these anticipatory feelings may result in time inconsistency. We provide examples to show the impact of anticipation on decision making.

Key Words: anticipation, expected utility, gambling.

JEL Classification: D81

1 Introduction

We all experience feelings related to our uncertainty about the future, such as hopefulness, fear, suspense, and anxiety. The economic theory of decision making under uncertainty has largely ignored these anticipatory emotions. This is unfortunate, since these emotions can have a profound impact on decision making. Individuals may try to avoid risky situations that trigger fear, or they may seek out risk in order to enhance feelings of excitement.

*New York University and Boston University respectively. We thank Vince Crawford, Charles Freeman, Simon Gilchrist, Mark Machina, Yaw Nyarko, Efe Ok, Chuck Wilson, and Ruth Wyatt for valuable comments. Caplin thanks the C.V. Starr Center at NYU and Leahy thanks the National Science Foundation and the Sloan Foundation for financial support. Send correspondence to Andrew Caplin at Dept. of Econ., NYU, 269 Mercer St, NY, NY 10003 or e-mail to caplina@fasecon.econ.nyu.edu.

They may try to gather information to reduce anxiety, or avoid information to maintain suspense.

Psychological theory has long recognized the importance of anticipation. For example Lazarus [1966] provides a fascinating discussion of the anticipatory nature of anxiety and stress. There is now a massive theoretical and experimental literature on the determinants and the effects of anxiety.¹ In a typical experiment, Averill and Rosenn [1972] found that a substantial group of subjects preferred to receive a painful electric shock immediately rather than wait and possibly receive a shock in the future. While difficult to reconcile with standard expected utility theory, such behavior is easily explained by a theory that allows a role for anxiety associated with future uncertainty.

In economics, Jevons [1905] discusses the impact of future events on current utility. His insights were largely forgotten until the work of Loewenstein [1987] and Elster and Loewenstein [1992] rekindled economists' interest in anticipatory feelings. Loewenstein models the impact of anticipation on an individual's discount factor in a deterministic setting. He shows experimentally that most people prefer to slightly delay a pleasant experience in order to savor it, and to bring forward an unpleasant experience to shorten the period of dread.

In this paper we present a general model of decision making with anticipatory feelings. We extend a standard two period model of choice under uncertainty to include the possible effects of future lotteries on current utility. To capture the wide variety of possible anticipatory feelings, we impose minimal a priori structure on the nature of these feelings.

While our general theory involves a minimum of structure, it nevertheless delivers novel implications due to the time inconsistency of preferences over lotteries. As time passes, so do the anticipatory emotions associated with the presence of uncertainty. Once these feelings have passed, they lose their earlier importance in determining preferences, and preferences may change as a result. Anticipatory feelings affect agents' plans for the future, but are sunk when it comes time to carry out these plans.

To illustrate the impact of anticipatory feelings and time inconsistency on decision making, we present two examples to illustrate the kinds of behavior that we might expect out of a such a model. Our first example is based on a suggestion of Jevons [1905], and concerns a family contemplating a future vacation to one of two locations. The family is indifferent between

¹For a survey of some of the more recent literature, see Öhman [1993].

the two locations given the information that they possess. However they enjoy thinking about the impending trip, and find it is easier to anticipate the vacation if they know which vacation they will take. This means that the household's preferences over possible vacation lotteries change over time. In order to facilitate anticipation, they would like to know their destination some time prior to departure. At the point of departure, however, all that matters is which vacation looks more promising at that date. Their choice at this date may or may not coincide with the choice that they would have made earlier. Given this potential change in attitude, we show that the family may wish to commit to one of the locations at an early stage.

The example illustrates not only the use of commitment devices to overcome time inconsistency, but also the possibility that information may have negative value. In the absence of full commitment, the family may wish to avoid information prior to departure, since the knowledge that they will be unable to resist using new information to change their decision may reduce their ability to anticipate the vacation. In contrast, classical expected utility maximizers would prefer to delay the decision in order to make the best use of any new information concerning the quality of a vacation in one of the two locations. The possible desire to avoid information in the presence of anticipatory emotions is exactly what is studied in the psychological and medical literature on anxiety.²

Our second example concerns suspense and gambling on sporting events. We hypothesize that suspense is positively related to the amount that is at stake on the outcome of an event. This provides a simple reason for agents to bet that their favorite team will win. By betting on their emotional favorite, agents increase their stake in the outcome, thereby heightening feelings of suspense. Our prediction that agents will prefer to bet on their emotional favorites finds strong support in the empirical analysis of Babad and Katz [1991], who study betting on soccer matches in Israel.³

This example clarifies the difference between an anticipatory emotion such as suspense and a conventional attributes of utility functions such as risk aversion. In the example, agents place bets in the first period, and the outcome of the bets are realized in the second. They place the bets in order

²Grant, Kajii, and Polak [1996] demonstrate the difficulty of allowing for “preference for less information” in existing models of choice under uncertainty.

³Rather than interpret this in the rational choice framework that we use, Babad and Katz view this betting pattern as evidence of an emotional influence on beliefs, which they refer to as wishful thinking.

to enhance feelings of suspense in the first period, despite their risk aversion with respect to final wealth lotteries in the second. Again this illustrates the time inconsistency. In the second period, the suspense has passed, and agents prefer to lower the volatility of final wealth. Risk aversion puts limits on the amount the agent is willing to bet in order to enhance suspense.

We view our general model as a natural extension of the classical expected utility model. In the classical theory, the acceptance of the substitution axiom rules out any preferences over the timing of the resolution of uncertainty. In this way, the theory implicitly rules out all feelings that might arise in periods prior to the resolution of uncertainty.⁴ Our approach is to expand the state space over which lotteries are defined to include relevant anticipatory feelings, and to apply the expected utility approach on this richer state space. In this respect we choose a very different approach than that followed by proponents of non-expected utility theory. The argument against the substitution axiom is that people’s emotions respond to uncertainty. Since we have encoded these emotional responses into the state space, it is reasonable to assume that the substitution axiom holds.

Our approach of expanding the prize space and retaining the substitution axiom is the natural follow up to some suggestions of Samuelson [1952] and Machina [1989]. Machina wrote:

“For my part, I will grant that separability⁵ may be rational provided the descriptions of consequences are sufficiently deep to incorporate any relevant emotional states, such as disappointment (e.g. at having won \$0 when you might have won \$5 million), regret and so on.” (Machina [1989] p.1662).

Our approach also builds on the work of Kreps and Porteus [1978, 1979a, 1979b] on preferences concerning the date of resolution of uncertainty. We follow their lead by employing the space of “temporal lotteries” as a basic building block in our theory. However, we reject their axiom of time consistency, since it rules out the anticipatory feelings that we wish to model.

In section 2 we provide our general two period decision problem with anticipatory feelings, and prove that optimal strategies exist. In section 3, we apply the model to the example of the upcoming vacation. In section 4, we

⁴The inability to consider feelings in prior periods forms the heart of the provocative critique of Pope [1985].

⁵Separability is the same as the substitution axiom.

explore the connection between suspense and gambling. Section 5 discusses our work in relation to various connected literatures in decision theory, in particular the work of Kreps and Porteus. Section 6 contains concluding remarks.

2 A Model of Anticipatory Feelings

The model has three components. The first is a definition of the relevant prize space, which we take to be a very abstract space of personal mental states. We impose minimal structure on individual preferences over mental states. The second component is a formal description of the space of lotteries in the physical world and of the manner in which uncertainty about these lotteries resolves over time. The third component is a mapping that connects physical lotteries with mental states. We refer to the complete model as the **psychological expected utility** model. After presenting the basic model, we develop the corresponding decision-theoretic framework and prove that optimal strategies exist.

2.1 States, Lotteries, and Utility

We replace the standard prize space with a more personal space of “psychological states,” comprising a complete description of the individual’s state of mind. An individual can experience many different mental states such as anxiety or excitement, and it is these that we connect to the agent’s level of utility and well-being. We define the agent’s preferences on lotteries over sequences of states of mind.

We model an agent’s state of mind as a vector of real numbers. Formally, there are two periods and two spaces $X_t \subseteq R^{n_t}$, $t = \{1, 2\}$, that represent the possible psychological states in the two periods. Let X denote the product space $X_1 \times X_2$. We define the space of psychological lotteries $P(X)$ to be the space of all Borel probability distributions on X together with the topology of weak convergence.⁶

We assume that the decision maker has a time-separable expected utility function defined on $P(X)$. In light of the standard theorems on choice under

⁶We endow X with the product topology and $P(X)$ with the topology of weak convergence. Since each space X_t is separable, X is separable and $P(X)$ is separable and metrizable by the Prohorov metric.

uncertainty, this is equivalent to assuming preferences over psychological lotteries that satisfy standard axioms for choice under uncertainty, including a substitution axiom.⁷ Let $E_r f$ denote the expectation of the random variable f with respect to the measure r .

Assumption 1 There exists a bounded, continuous function $U : X \rightarrow R$ such that for $p, q \in P(X)$, $p \succsim q$ if and only if $E_p U \geq E_q U$. In addition the function $U : X \rightarrow R$ has a time additive representation,

$$U(x) = u_1(x_1) + u_2(x_2).$$

where $u_t : X_t \rightarrow R$, for $t \in \{1, 2\}$.

Turning to the physical lotteries, the basic data are two spaces describing all of the physical prizes that the agent may receive in each period, $Z_t \in R^{m_t}$, $t \in \{1, 2\}$. Since our concern is anticipation, the timing of physical and psychological lotteries and utility is important. We take as our convention that the period t psychological state is realized at the end of period t , immediately after the physical state for the period is realized. Hence there is no time for anticipation within a period. Given the two period set up, this means that the agent will only experience feelings of anticipation in the first period. Those feelings will only concern the second period uncertainty that remains unresolved after the outcome of the first period lottery has been realized. Since there are no anticipatory feelings in the second period, we make the simplifying assumption that the physical and psychological reward spaces are the same in the second period, $Z_2 = X_2$. This is equivalent to assuming that only the actual physical prize in period 2 influences that period's psychological state.

The first period is more subtle. To capture anticipatory emotions, we allow the psychological prize in period 1 to depend not only on the physical prize received in the first period, but also on the remaining uncertainty

⁷ $P(X)$ is a mixture space: given $p, q \in P(X)$, and $\lambda \in [0, 1]$, $p\lambda q \in P(X)$ assigns to Borel sets $A \subset X$ the probability $\lambda p(A) + (1 - \lambda)q(A)$. Given $p \in P(X)$, let (p_1, p_2) denote the corresponding marginal distribution on the spaces X_t . Theorem 10.1 in Fishburn [1982] proves that assumption 1 is equivalent to assuming a complete, transitive, and continuous preference order on $P(X)$ such that

- (i) $p, q, r \in P(X)$ and $\lambda \in [0, 1]$, $p \succeq q$ implies $p\lambda r \succeq q\lambda r$.
- (ii) $p, q, r \in P(X)$ such that $p \succeq q$ and $q \succeq r$, there exist $\lambda_1, \lambda_2 \in [0, 1]$ such that $p\lambda_1 r \succeq q$ and $p\lambda_2 r \preceq q$.
- (iii) For all $p, q \in P(X)$, $(p_1, p_2) = (q_1, q_2)$ implies $p \sim q$.

concerning the physical prize that will be realized in the second period. To capture this, we follow the methods of Kreps and Porteus [1978] to formalize evolving uncertainty. Let Z_1 be the space of physical prizes in period 1, and L_2 the space of Borel probability distributions over period two prizes. Define $Y_1 = Z_1 \times L_2$ and L_1 to be the space of Borel probability measures over Y_1 .⁸

The elements of Y_1 can be thought of as the pure outcomes in the first period; these pure outcomes include a prize from Z_1 and a lottery over future prizes from L_2 . Each such outcome determines just how much uncertainty regarding the second period prize remains to be resolved in the second period. Thus a lottery $l_1 \in L_1$ specifies the period 1 view of the likely state of knowledge at the end of the period. In this way, a lottery $l_1 \in L_1$ encodes the timing of the resolution of uncertainty. Following Kreps and Porteus, we refer to each element $l_1 \in L_1$ as a **temporal lottery**.

To complete our model of preferences, we define the function $\phi : Y_1 \rightarrow X_1$ which gives the psychological state that results from an agent facing the outcome $y_1 \in Y_1$. We assume that ϕ is continuous.

We are now in a position to define the utility function over temporal lotteries. This utility function is induced by the mapping ϕ and the utility function over psychological prizes. Given $y_1 = (z_1, l_2) \in Y_1$,

$$V_1(y_1) = u_1(\phi(y_1)) + E_{l_2}[u_2(x_2)].$$

V_1 looks like a standard time-separable expected utility function except for the presence of l_2 in first-period utility.

As we will see in the examples in sections 4 and 5, it is ϕ that gives the theory structure. It is in this mapping that we capture different psychological attitudes towards uncertainty.⁹

2.2 A General Decision Problem

The decision problems that we consider all have the same general structure. Given an initial state $s_1 \in S_1$, the agent chooses an action $\alpha \in A_1$ from a feasible set $\Gamma_1(s_1) \subseteq A_1$. The initial state and action determine the physical

⁸We endow Y_1 with the product topology, and L_1 with the topology of weak convergence. It can be shown that both L_1 and Y_1 are separable, metrizable spaces.

⁹The nature of the model as a formulation of anticipatory feelings is captured in the assumption that the period t feelings are based purely on the current physical prize and uncertainty that is as yet unresolved. The past prizes and past psychological states play no role in determining current feelings.

payoff in the first period according to a payoff function, $\eta : S_1 \times A_1 \longrightarrow Z_1$. Between the first and the second period, a new exogenous state $s_2 \in S_2$ is realized according to the Markov transition function $Q(B, s)$,

$$Q(B, s) = \Pr\{s_2 \in B | s_1 = s\}.$$

Finally in the second period, the agent chooses a lottery over second period prizes from a feasible set which may depend both upon the earlier action choice and the new information, $\Gamma_2(\alpha_1, s_2) \subseteq L_2$.¹⁰ After this, all remaining uncertainty is resolved and the second period payoff is realized. We make standard simplifying assumptions on the various data of the problem.

Assumption 2 The spaces A_1, S_1 , and S_2 are subsets of finite-dimensional Euclidean spaces. The choice correspondences Γ_1 and Γ_2 are compact-valued and continuous. The function $\eta : S_1 \times A_1 \longrightarrow Z_1$ is continuous in the first argument.

Strategies are defined in the standard fashion. A period two policy is a measurable function $\pi_2 : A_1 \times S_2 \longrightarrow L_2$ with $\pi_2(\alpha_1, s_2) \in \Gamma_2(\alpha_1, s_2)$. We let Π_2 denote the set of such policies. An overall strategy is a combination of an initial choice $\alpha_1 \in \Gamma_1(s_1)$ and a second period policy $\pi_2 \in \Pi_2$.

The lottery over second period prizes that is anticipated at the end of period 1 depends both on the distribution of second period exogenous shocks and on the strategy selected in the second period. We denote this lottery by $\lambda(\alpha_1, \pi_2 | s_1) \in L_2$,

$$\lambda(\alpha_1, \pi_2 | s_1) \equiv \int \pi_2(\alpha_1, s_2) Q(ds_2, s_1).$$

Here π_2 gives the second period prize distribution conditional on the second period state. Integrating over second-period states yields λ .

The dependence of the first period temporal lottery on the second period strategy gives rise to the time inconsistency of optimal choices, and requires us to take a stand on the appropriate definition of optimal strategies in the presence of time inconsistency.

¹⁰Whether or not mixed strategies are allowed in period 2 is determined by the nature of the correspondence $\Gamma_2(\alpha_1, s_2)$.

2.3 Optimal Strategies: Definition and Existence

Our approach to the definition of optimal strategies in the presence of time inconsistency is to exploit the recursive structure of the optimization problem. The appropriate standard of rationality in period 2 is straightforward: we know that the agent in the second period will make a decision that is optimal for that period alone. Let $J_2(\alpha_1, s_2)$ denote the value of an optimal policy conditional on the second period state and action. In the second period the agent chooses a lottery from Γ_2 that maximizes expected utility:

$$J_2(\alpha_1, s_2) = \max_{l_2 \in \Gamma_2(\alpha_1, s_2)} E_{l_2}[u_2(z_2)]. \quad (1)$$

We define the second period choice correspondence in the natural manner as:

$$G_2(\alpha_1, s_2) = \{l_2 \in \Gamma_2(\alpha_1, s_2) \mid J_2(\alpha_1, s_2) = E_{l_2}[u_2(z_2)]\}.$$

An optimal policy in period 2 is a measurable selection from $G_2(\alpha_1, s_2)$. Let Π_2^* denote the set of optimal period 2 policies. Note that with our assumptions, the theorem of the maximum and a standard measurable selection theorem guarantee that Π_2^* is non-empty (Hinderer [1970]).

The subtle point in the definition of optimal strategies concerns the nature of the period 1 choice. There may be more than one optimal policy in the second period, and the method of selecting among such indifferent policies may impact the payoff in the first period. Without time consistency, there is no presumption that indifference extends back from period 2 to period 1. Consider a simple example in which the agent has only one choice to make, which is a choice between two distinct lotteries, A and B , in period 2. Suppose that from a second period perspective the lotteries are indifferent, but that from a first period perspective the agent is happiest knowing that he will choose lottery A in the second period. Can the agent tie his hands in period 2 by insisting on the selection of lottery A ?

The suggestion of Strotz [1955] was that indeed such a selection can be made. The agent in each period restricts attention to the constraints on future choices implied by future optimality, and then picks the best current strategy in light of these constraints. There are those who feel otherwise, and the suggestion of Peleg and Yaari [1973] was to re-cast the decision problem as a game between the self in different periods, and to look for perfect equilibria of the game. According to this approach, selection of lottery B is an equilibrium, and there is no presumption that agents can commit

themselves to the better choice from the current viewpoint. Variants of the game theoretic approach have been followed in most of the recent literature (e.g. Laibson [1997]). The work of Caillaud, Cohen, and Jullien [1994] is a notable exception.

In this paper, we take the side of Strotz, since we do not feel that all perfect equilibria of a game are candidate optimal solutions. We regard second period indifference as revealing that either decision can be committed to in the first period without loss in terms of second period utility. This makes it feasible for the agent in period 1 to commit to the choice that maximizes first period utility, just as they would in any other decision problem. The complexity that is introduced by time inconsistency lies more in the determination of the feasible set than in the nature of the decision-making criterion.¹¹ Our approach to the breaking of indifference is thus to return to the suggestion of Strotz, and to allow agents in earlier periods to commit their later selves to a particular strategy, provided that there is never a strictly positive gain to deviating from the proposed strategy.

Given an initial state $s_1 \in S_1$, we refer to strategies $\pi = (\alpha_1, \pi_2)$ with $\pi_2 \in \Pi_2^*$ as **consistent** strategies. The (non-empty) set of such strategies is denoted $\Pi^C(s_1)$. We solve for the first period value function in two steps. Given s_1 and α_1 , let $K(s_1, \alpha_1)$ denote the value of the second period policy in Π_2^* that maximizes first period utility:

$$K_1(s_1, \alpha_1) = \sup_{\pi_2 \in \Pi_2^*} [u_1(\phi(\eta(s_1, \alpha_1), \lambda(\alpha_1, \pi_2|s_1))) + E_{\lambda(\alpha_1, \pi_2|s_1)}[u_2(z_2)]] . \quad (2)$$

Now define the period 1 value function in terms of the first period choice that maximizes K ,

$$J_1(s_1) = \sup_{\alpha_1 \in \Gamma(s_1)} K_1(s_1, \alpha_1) \quad (3)$$

An optimal strategy is a choice of π_2 that maximizes (2) and a choice of α_1 that maximizes (3). A slight adaptation of a theorem of Harris [1985] shows that an optimal strategy exists.

Proposition 1 *An optimal strategy exists.*

¹¹It is of interest that the most recent advances in the analysis of renegotiation proof equilibria of games also support the original vision of Strotz. His recursive solution concept is equivalent to the renegotiation proof equilibrium of the game of perfect information played between the self in different periods (e.g. Benoit and Krishna [1993] and Farrell and Maskin [1989]).

Proof: If we consider the decision maker in period 1 and in period 2 as two separate individuals, our solution concept is equivalent to selecting the subgame perfect equilibrium that is best from the period 1 perspective. It is easy to confirm that our model satisfies all of the assumptions of Theorem 1 in Harris [1985]. This Theorem states that the set of subgame perfect equilibria is compact. Since our objectives are continuous, an optimum exists. \square

One interesting issue concerns the extension of the model to a multi-period framework. Given the time inconsistency of preferences, there is no general theorem on the existence of recursively optimal choices in such settings. In this light, our discussion of the existence of an optimal consistent strategy in the two-period decision problems of section 3 merely scratches the surface of a very intricate set of questions concerning the formulation and solution of decision problems when preferences are not time consistent. We are actively researching this area, and are confident that progress can be made.

3 Example 1: Alaska or Hawaii? Anticipation and Commitment

In our first example we extend some early ideas of Jevons [1905] on the feelings concerning an anticipated vacation to a stochastic environment. We allow the state of prior knowledge to be a key determinant of anticipatory pleasure. We show that this leads to a theory of planning.

3.1 The Model

There are two periods. An agent is considering whether or not to take a vacation in the second period, and if so, whether to go to Alaska or to Hawaii. To keep matters simple, we assume that the only exogenous uncertainty concerns the weather in Alaska. From a first period viewpoint, the probability of good weather in Alaska is 0.5. At the start of the second period, the agent will get a signal, s' , gives a revised probability that the weather in Alaska will be good. This signal is distributed uniformly on $[0, 1]$.

The second period pure prize space, Z_2 , consists of the final level of wealth and the nature of the final vacation,

$$Z_2 = \{(h, w) : h \in \{G, B, H, \emptyset\}, 0 \leq w \leq W_0\},$$

where H denotes the vacation to Hawaii, G and B denote the vacations to Alaska in good and bad weather respectively, \emptyset denotes no vacation, and $w \in R_+$ denotes the final level of wealth in dollars.

We place the following utility function $u_2 : Z_2 \rightarrow R$ on second period prizes:

$$u_2(h, w) = \begin{cases} w, & \text{if } h \in \{B, \emptyset\}; \\ 4000 + w & \text{if } h = H; \\ 6,000 + w, & \text{if } h = G. \end{cases}$$

A vacation in Alaska in bad weather is equivalent to no vacation in terms of its level of utility, while the trip to Hawaii yields utility equivalent to \$4,000, and the trip to Alaska in good weather is worth \$6,000 in monetary terms.

Each vacation costs \$2,000 and the agent is endowed with an initial level of wealth W_0 which we assume is greater than \$4000 so that the agent can afford both vacations.

The only decision that the agent makes during period 1 is how much money to put down as a deposit on the trip to Hawaii. This deposit may be any proportion of the total cost of the vacation, $\alpha \in A_1 = [0, 1]$, where $\alpha = 1$ corresponds to a full down-payment of \$2,000.¹² At the beginning of the second period, armed with the new signal s' , the agent may complete the payment and take the trip to Hawaii, pay the full cost of the trip to Alaska, or stay at home.¹³ The latter two choices involve losing the deposit. Any vacation that is to be taken occurs immediately after making this second period decision.

In classical expected utility theory, the solution to the model is transparent. The optimal deposit is $\alpha = 0$, and the decision depends on final beliefs about the weather in Alaska. The agent chooses Alaska if $s' > \frac{2}{3}$, and chooses Hawaii otherwise. With this, we know that from an ex ante viewpoint, the probability of going to Alaska is $\frac{1}{3}$, and the probability of going to Hawaii is $\frac{2}{3}$.

Our model allows for the probability distribution over second period vacations to impact first period feelings. In particular, any method for increasing the certainty of taking the trip to Hawaii will turn out to have value. The

¹²We preclude deposits on Alaska for simplicity.

¹³For simplicity of exposition, we ignore mixed strategies in presenting the model. It is easy to confirm that the results are unchanged when such strategies are admitted.

end result may be a decision by the agent to put down a deposit to tie their hands in favor of the Hawaii trip. Note that if the agent puts down the full deposit on Hawaii in period 1, then they can be sure that they will go to Hawaii in the final period, and this choice will sometimes be optimal when one takes account of the anticipatory pleasure.

In terms of the example, we assume that the period 1 feeling of anticipation can be represented by a non-negative scalar, with higher values corresponding to greater levels of anticipatory pleasure,

$$X_1 = \{a \in R : a \geq 0\}.$$

We assume that utility is increasing and linear in the level of anticipation,

$$u_1(a) = \beta a. \tag{4}$$

The parameter $\beta > 0$ scales the importance of anticipatory utility relative to the utility of the vacations themselves.

What remains is to specify which features of a given lottery over second period outcomes determines the level of anticipatory pleasure in period 1, $\phi : Y_1 \rightarrow X_1$. We tie anticipation to the taking of a vacation: if there is no vacation, there will be no anticipation. We also tie anticipation to beliefs about the pleasure that will be derived in the vacation. Finally, anticipatory pleasure is greater, the greater is the agent's certainty that an outcome will occur. The following functional form satisfies these conditions:

$$\phi(y_1) = \max[6000p_G(y_1), 4000p_H(y_1)].$$

3.2 The Solution

We solve the model recursively. Given a downpayment of α , the optimal choice is to go to Alaska rather than Hawaii if the signal is sufficiently high for the utility difference to exceed the cost difference of 2000α ,

$$s' > \frac{2 + \alpha}{3}.$$

The two vacations are indifferent at the cut-off value, $s' = \frac{2+\alpha}{3}$. It is never optimal to stay home.

This policy us to solve for the second period value function as,

$$J_2(s', \alpha) = \begin{cases} 6000s' + W_0 - 2000(1 + \alpha), & \text{if } s' \geq \frac{2+\alpha}{3}; \\ 2000 + W_0, & \text{if } s' < \frac{2+\alpha}{3}. \end{cases}$$

We can now compute the expected payoff to any strategy α in the first period. Given $\alpha \in [0, 1]$ and the period 2 policy, the probability of going to Hawaii is $p_H(\alpha) = \frac{2+\alpha}{3}$. Conditional on choosing Alaska, the probability that the weather is good depends on the signal s' , which in this case ranges uniformly over the range $[\frac{2+\alpha}{3}, 1]$. The probability of going to Alaska and experiencing good weather is therefore,

$$p_G(\alpha) = \frac{5 - 4\alpha - \alpha^2}{18}$$

With this information, we have identified the complete lottery $y(\alpha) \in Y_1$ associated with any first period strategy choice $\alpha \in [0, 1]$. The value of choosing α in the first period is therefore,

$$K_1(\alpha) = \beta \max[6000p_G(\alpha), 4000p_H(\alpha)] + 4000p_H(\alpha) + 6000p_G(\alpha) + E_{y(\alpha)}[w].$$

Since $4000p_H(\alpha) \geq 6,000p_G(\alpha)$ for all $\alpha \in [0, 1]$, a little algebra shows that $K_1(\alpha)$ is proportional to:

$$7 + \frac{3W_0}{1,000} + 8\beta - 2\alpha + 4\alpha\beta + \alpha^2$$

Since $K(\alpha)$ is convex, the optimum is at one of the two corners and depends on β in the natural way. With $\beta = \frac{1}{4}$ the agent is indifferent between a full deposit and no deposit, with $\beta > \frac{1}{4}$ the agent places a full deposit, and with $\beta < \frac{1}{4}$ the agent places no deposit.

3.3 Implications and Extensions

The example shows that an individual may find it worthwhile to put money down in the first period in order to raise the probability of visiting Hawaii in the second period. This desire to commit is a response to the time inconsistency of the preferences over lotteries. In the second period, the down-payment looks like a very bad idea, since it makes it more costly to respond to new information concerning the desirability of an Alaskan vacation. From a first-period perspective, this inflexibility is desirable, since it makes it possible to more intensely savor the vacation in Hawaii.

It should be noted that there are parameterizations of the problem in which the agent would like to avoid learning the signal s' . If the utility of Alaska in good weather exceeded \$6000, then even a full downpayment on Hawaii would be insufficient to ensure that the second period decision would be to visit Hawaii. In this case the agent may prefer not to view the updated signal on the weather in Alaska. In this manner, anticipatory emotions may rationalize the deliberate avoidance of information.

There are many ways to generalize the example. Consider cases in which there is a potentially unpleasant experience in period 2, in which case the relevant emotional response may be dread. In this case one might expect the individual to undertake actions in period 1 designed to make the period 2 outcome less readily predictable, in order to dilute the focus on the negative outcome. This connects to other interesting issues that would arise if we were to endogenize the timing of the lottery. Just as in the experiments reported in Loewenstein [1987], one might expect to find a greater incentive to delay an experience that carries anticipatory pleasure than one that brings anticipatory pain.

4 Example 2: Ali or Frazier? Suspense and Gambling

For many sports fans, part of the pleasure of watching an event derives from feelings of suspense. They get pleasure from not knowing the outcome of an important event and watching the uncertainty resolve in front of their eyes.

An example from one of our personal histories exemplifies the importance of suspense. In the United Kingdom, the second Ali-Frazier fight was shown with a tape delay. A close friend spent the whole day in almost complete isolation to avoid learning the outcome of the fight. He clearly believed that it would be more enjoyable to watch the uncertainty unfold without knowing the winner in advance. Unfortunately for him, half an hour before the fight was to be shown, his father walked into the room and said, “What are you so worried about? Ali won.” While this was the preferred outcome, he was crushed to have the joy taken out of watching the fight. This simple example suggests that decisions such as which sporting events to watch are best understood inside a framework that allows for enjoyment of suspense.

4.1 The Model

There are two periods. At the beginning of the first period nature draws a probability that Ali will win the fight: for simplicity we take this probability to be fixed at $1/2$ (recall that Frazier was the reigning world champion). There are two decisions to be taken in period 1. One concerns how much money to bet on the outcome of the fight. We make the simple assumption that the bets are fair: if the agent bets $(b^A, b^F) \geq 0$ on Ali and Frazier respectively, then the payoff in period 2 is $2b^i$ if agent i wins. The total amount that the agent bets is constrained to be no more than the level of initial wealth, $W_0 > 0$. The second decision involves whether or not to watch the fight at the end of period 1.

In period 2, the agent learns the outcome of the fight whether or not the fight was watched, and all bets are settled. We rule out draws, so that either Ali wins and becomes champion or Frazier wins and remains champion. The second period prize space includes the final outcome of the fight and the final level of wealth. Let A denote a victory by Ali, F a victory by Frazier, and $w \in R_+$ the final level of wealth,

$$Z_2 = X_2 = \{(c, w) : c \in \{A, F\}, 0 \leq w \leq 2W_0\}.$$

We assume that second period utility is given by,

$$u_2(c, w) = \begin{cases} w - \beta w^2 & \text{if } c = F; \\ u_A + w - \beta w^2 & \text{if } c = A. \end{cases}$$

The agent favors an Ali victory which we capture by the term $u_A > 0$.

Since the second period utility function is concave in wealth, the agent is risk averse to wealth gambles and from a second period perspective betting on the fight is a bad idea. The impact of the bet, however, can spill over onto period one. In period 1, there are two types of psychological prize. One prize is a feeling of suspense. We assume that suspense can be represented as a simple non-negative scalar, with higher values corresponding to greater levels of suspense. The second prize derives from some activity other than watching the fight. For concreteness, we shall take this activity to be a day of gardening. We treat this as a discrete prize, with a dummy variable $\gamma = \{1, 0\}$ recording the presence or absence of this prize respectively,

$$X_1 = \{(a, \gamma) | a \in R_+, \gamma \in \{0, 1\}\}.$$

We assume that utility is a linear function of these two states,

$$u_1(a, \gamma) = sa + g\gamma$$

where the parameter s and g reflect the pleasure of suspense and the pleasure of gardening respectively.

The physical prize in the first period is either to watch the match on television (T) or to garden (\emptyset),

$$Z_1 = \{T, \emptyset\}.$$

This implies that any element $y_1 \in Y_1$ must specify the corresponding first period prize, z_1 , and a probability distribution over outcomes in the set Z_2 . Given the even odds that Ali wins, this comes down to specifying the (deterministic) final wealth levels contingent on Ali winning and on Frazier winning, $W_A(y_1)$ and $W_F(y_1)$ respectively.

What remains is to specify which features of a given lottery over second period outcomes determines the level of anticipatory feelings in period 1, $\phi : Y_1 \rightarrow X_1$. We make the simple assumption on the nature of suspense that it is increasing in the stakes. Specifically, it is increasing in the absolute difference between utility should Ali win and utility should Frazier win,

$$\Delta(y_1) = |u_A + W_A(y_1) - W_A(y_1)^2 - W_F(y_1) + W_F(y_1)^2|.$$

The specific assumption on functional form is,

$$\phi(y_1) = \begin{cases} (\Delta(y_1), 0), & \text{if } z_1(y_1) = T. \\ (0, 1); & \text{if } z_1(y_1) = \emptyset \end{cases}$$

Recall that the first argument in the period 1 psychological state space is the level of suspense, and the second is the gardening prize. The simple assumption is made that if the fight is not watched, there are no feelings of suspense.

4.2 The Solution

A simplifying feature of the decision problem is that the agent does not make a choice in the second period. In the first period the agent decides how much to bet on each fighter and whether or not to watch the fight. It is immediate that if the decision is made not to watch the fight, then the optimal bet

is zero, since in this case the bet leaves first period utility unaffected and lowers second period utility through risk aversion.¹⁴ Even if the decision is made to watch the fight, it is clear that betting on both fighters cannot improve utility. The shared component of any bet does not increase the level of excitement in the first period and does not affect final wealth since the odds are fair. We therefore look for a solution in which the agent bets only on one fighter. In this case, it is clearly preferable to place money on Ali rather than Frazier. The distribution of final wealth does not depend on which fighter the agent supports, but the level of excitement in period 1 increases if the bet is placed on the favored fighter. This means that we can simplify notation and consider only bets of amount $b \in [0, W_0]$ placed on Ali.

Conditional on watching the fight, the optimal bet balances the gains in first period excitement against the loss in second period utility. Given that the probability of Ali winning is $1/2$, a bet $b \geq 0$ on Ali leads to a loss in second period utility of βb^2 . The first period utility associated with betting b and watching the fight is $s(u_A + b - \beta b^2)$. The optimal bet therefore solves:

$$\max_{b \geq 0} [su_A + sb - (1 + s)\beta b^2].$$

The optimal bet b^* follows from the first order condition:

$$b^* = \frac{s}{2(1 + s)\beta} > 0.$$

Whether or not it is worthwhile to watch the fight depends on whether the gain to watching the fight is greater than the pleasure derived from gardening. The unique optimum is to watch the fight and to bet amount b^* if and only if,

$$g < s \left(u_A + \frac{s}{4(1 + s)\beta} \right).$$

This solution has the natural comparative statics. Watching television and betting is more attractive the more the individual cares who wins, the higher the suspense parameter s , the lower the level of risk aversion as measured by β , and the smaller is the pleasure involved in the alternative activity g .

¹⁴This simple result follows from the assumption that there is no suspense associated with gambling unless the event is watched. A more realistic case would allow for a non-zero impact of gambling on suspense even if the fight could not be watched.

4.3 Implications and Extensions

The example shows that agents may gamble even if gambling has adverse implications for wealth and final utility. They do so because it heightens feelings of suspense in periods prior to the resolution of uncertainty. This form of behavior is strongly tied to the time inconsistency of optimal plans. In the final period, the gamble is not a good idea, since the individual is risk averse. The bet is undertaken because of its interaction with the unfolding uncertainty, not because of its impact on the ultimate outcome of the lottery. In this sense our theory of gambling ties strongly with some early methodological comments of Samuelson:

“...I am satisfied that a large fraction of the sociology of gambling and of risk taking will never significantly be discernible in terms of the money prizes alone, as distinct from elements of suspense....” (Samuelson [1952], p.676-77)

The broader theory of gambling implicit in the example has a number of testable implications. The theory predicts a preference for betting on the (emotionally) favored outcome, and that the decision to gamble is linked with the decision on watching the event. This leads to the hypothesis that broadcasting an event live will increase the amount of gambling on the outcome. We believe that practitioners of the art of design of lotteries and other avenues for gambling are exploiting these results in an intuitive manner already. After all, off-track betting outlets typically show the races on television, and even lottery selections are commonly televised. The desire to heighten suspense may also explain why agents appear to be risk loving when confronted with small bets and risk averse when confronted with large bets.

In contrast to our model, previous theories of gambling are essentially static, and are based on the properties of the utility function over final prizes. This is most transparent in the model of Friedman and Savage [1948], where the final prize is simply the level of wealth. It is also true of the non-expected utility model of Conlisk [1993], discussed in section 5.1 below.

There are many ways to generalize the example. One simple extension would be to allow the probability of Ali winning the fight to differ from $1/2$ and to let the level of suspense depend on this probability. It seems reasonable that the level of suspense would fall as this probability approached zero or one; the greater is the uncertainty concerning the winner, the greater the feeling of suspense. An immediate implication would be that both the level

of gambling and the size of the audience would be increasing functions of the prior uncertainty concerning the outcome.

Another extension would be to allow for multiple periods. If agents believe that the outcome will not to be determined until the later periods, then the audience and the level of gambling may grow as the fateful period approaches. If instead the outcome is determined in the early periods, the audience will most likely taper off.

5 Literature: Past, Present, and Future

Is our model really needed to cover anticipatory feelings, or can they be adequately treated in existing models? To answer this question, we discuss our ideas in the context of static non-expected utility theory in section 5.1, and the dynamic theory of Kreps and Porteus in section 5.2. In section 5.3 we discuss prospects for further development of the psychological expected utility theory.

5.1 Static Non-expected Utility Theory

In contrast with our approach, much recent progress in the theory of choice under uncertainty has involved the rejection of the substitution axiom for lotteries, and the development of various non-expected utility theories. Many of these theories have been developed in response to the observed violations of the substitution axiom, such as the Allais paradox. In fact, the chief goal of many non-expected utility theories is to relax the substitution axiom as little as possible, while nevertheless allowing for some limited class of violations.

Given this goal, most non-expected utility theories give little explicit consideration to the psychological origins of the implied departures from the classical substitution axiom.¹⁵ As an example, consider the rank-dependent expected utility model of preferences over monetary lotteries (see Quiggin [1982]). For any monetary lottery X with associated cumulative distribution function F_X , the rank-dependent expected utility $U^{rd}(X)$ can be expressed as:

$$U^{rd}(X) = \int u(m)d[f \circ F_X(m)],$$

¹⁵Exceptions include the work of Bell [1985] and Loomes and Sugden [1986].

where $u : R_+ \rightarrow R$ is the utility of the pure prize m , and the function $f : [0, 1] \rightarrow [0, 1]$ acts to distort the assignment of probabilities to prizes. This class of non-expected utility models has been further analyzed by Allais [1988], Green and Jullien [1989], and Grant and Kajii [1994], among others. As Green and Jullien point out, the model can be interpreted as arising from a version of expected utility theory in which the entire context of the lottery impacts the utility of an individual prize. For example, it allows an individual to value the prospect of \$100 at the 80th percentile more than they value this same prize at the 40th percentile.

What are the proposed psychological origins of this way of viewing lottery outcomes? On this point there is no consensus. In fact there is no consensus on whether the feelings that account for the probability transformation are anticipatory or retrospective. In his original article, Quiggin [1982] referred to the model as the anticipatory utility model, while Grant and Kajii [1994] define their special case of the model as a theory of disappointment aversion. It is not surprising that there can be disagreement on this point, since the model is silent on the connection between lotteries and psychological states.

This points to one vital difference between these classes of non-expected utility model and our psychological expected utility model. In these non-expected utility theories, rather than model the psychology of preferences directly, any such psychological forces that may be at work are left to be implicitly defined by the properties of the final utility function. This is recognized by Machina in his survey article on non-expected utility theory. After his comment (quoted at length in the introduction) on the appeal of the substitution axiom “provided the descriptions of consequences are sufficiently deep to incorporate any relevant emotional states”, he goes on to add:

“...preferences over observables such as monetary outcome levels could legitimately be non-separable. In other words, the various non-expected utility models of Table 1¹⁶ could legitimately represent risk preferences when the consequences consist of monetary outcome levels.”(Machina, [1989], p.1663).

Given the lack of explicit psychological rationale, it is not easy to apply non-expected utility theories to settings such as the anticipated vacation and the big fight introduced in Sections 3 and 4 above. Application is all the

¹⁶In Table 1 Machina lists a wide variety of non-expected utility models, including the rank- dependent expected utility model.

more difficult in light of the second major qualitative difference between our model and non-expected utility theory, which is that we allow for dynamic phenomena, including time inconsistency of preferences. The static nature of non-expected utility theories is especially limiting in any application in which the evolution of personal feelings toward the lottery space is critical.

The model of gambling due to Conlisk [1993] illustrates some of the limitations inherent in attempts to apply static non-expected utility theory to such problems. For a given monetary lottery X with associated cumulative distribution function F_X , Conlisk introduces a function $G(X)$ to summarize the “gambling utility” associated with the lottery. In order to get the overall utility of the gamble for the agent with initial wealth W_0 , Conlisk adds this gambling utility to a standard expected utility function:

$$U(X : W_0) = \int u(W_0 + m)dF_X(m) + \varepsilon V(X)$$

Conlisk argues that $G(X)$ may depend on the standard deviation of the lottery winnings, or some such measure related to the possible dispersion of outcomes.

It is clear that there are some close analogies between the Conlisk model of gambling and our model of suspense in section 5. Indeed, our first period induced expected utility function can take precisely the same form as $G(X)$, and for exactly the same reason:

“The term $[G(X)]$ might be thought of as the utility of the excitement and suspense felt between the time he accepts the prospect, and the time that the uncertainty is resolved.” [Conlisk [1993], p.262]

Despite these good intentions, the static nature of the Conlisk model makes it impossible to capture the importance of time. The result is a theory of gambling that is not as rich as the theory that we presented in section 5. There is no connection to other activities that heighten suspense, such as watching the fight; there is no theory that suggests which events will be worth gambling on; and there is no theory of which fighter to on which to bet.

More broadly, we believe that one of the advantages of our formulation over static non-expected utility models is that the latter theories attempt to telescope a dynamic pattern of feelings into a single static utility function.

We believe that many subtle phenomena in the psychology of risk-taking will require explicitly dynamic formulations. This applies not only to the issue of suspense and gambling, but also to such phenomena as disappointment, as modelled by Bell [1985] and Loomes and Sugden [1986]. While their models are explicit concerning the way in which a given lottery may produce feelings of disappointment, they remain static. Among other things, this means that the models do not include the anticipatory phase. Have you ever felt disappointed about an outcome without having experienced prior feelings of hopefulness?¹⁷

5.2 The Kreps-Porteus Model

Our approach to the dynamic choice problem has borrowed heavily from the dynamic non-expected utility model introduced by Kreps and Porteus ([1978], [1979a], [1979b]). They provided the original definition of the space of temporal lotteries, and we have followed their lead in using this as the domain for our model of evolving uncertainty. Another important similarity is that they characterize preferences over these temporal lotteries that, within any given period, satisfy a substitution axiom and that therefore admit of an expected utility representation, just as do the preferences that we analyze. A third point of similarity is that the Kreps-Porteus model allows for preferences over the date of resolution of uncertainty, as does our model. In intuitive terms, it is clear that the example of the anticipated vacation in section 4 involves a preference for early resolution of uncertainty, since the agent prefers to have all information about where they would travel realized as soon as possible. Conversely, the example of suspense and gambling in section 5 involves a preference for late resolution of uncertainty, since the agent prefers to remain uninformed about the outcome of the fight for as long as possible.

Despite these similarities, our model differs in fundamental ways from that of Kreps and Porteus. One significant difference is that they link together the expected utility functions in distinct periods with the assumption of time consistency. What this means is that the agent's current preferences over future lotteries must remain unchanged as time passes. In the two examples that we have presented, the preferences over lotteries are time

¹⁷Psychological recognition of the link between these two emotions can be found in the very interesting book of Ortony, Clore, and Foss [1988].

inconsistent, and it was precisely the time inconsistency that made the examples interesting. It is hardly surprising that time consistency is highly restrictive in settings with anticipatory feelings, since earlier feelings become bygones as the period of anticipation shrinks, and thereby lose their direct role in determining preferences.

The second difference between our model and that of Kreps and Porteus lies in the fact that, for us, the expected utility function on the space of temporal lotteries is induced by a more fundamental description of the psychological response to uncertainty. By adding in the additional data that is necessary to encode this psychological response, we believe that we are providing applied theorists with a highly flexible tool, which can be used in different ways depending on the underlying context. Of course the theory will need to be enriched to handle the wide array of possible extensions.

5.3 Prospects for Psychological Expected Utility Theory

In this section we offer preliminary comments on some of the most promising avenues for further exploration of the psychological expected utility theory, and also some of the thorny theoretical issues that will have to be resolved in order for the theory to achieve its full potential.

5.3.1 Other Anticipatory and Retrospective Feelings, and Moods

In addition to enriching the theory of suspense and of anticipatory excitement sketched out in sections 4 and 5 above, there are other anticipatory emotions that can be analyzed, including fear, anxiety, worry, and nervousness. Each of these will have its own set of associated decision-theoretic phenomena, just as feelings of suspense connect to decisions on gambling. In this manner, we believe the theory opens the door to considering some of the issues raised by Elster [1996] in his plea for theoretical consideration of the emotions.

There are other important emotional responses to uncertainty that occur in the periods following the resolution of uncertainty. These include disappointment, relief, pleasant surprise, and satisfaction. These are also candidates for study, and we believe that the model can be usefully extended to provide insight into the nature and implications of these feelings. Of even greater interest are examples in which there are both anticipatory and retrospective feelings, as when anticipation turns to disappointment if things

do not turn out as hoped, or when feelings of fear give rise to feelings of relief if the feared outcome does not happen. One interesting possibility is that extending the theory in this direction will offer a dynamic insight into the construction of reference points. In this manner, our approach may ultimately offer an alternative to existing static versions of prospect theory (see Kahnemann and Tversky [1979]). However, it is important to note that the development of a general theory in this direction will not be trivial, since a number of subtle issues are raised by the simultaneity involved in determining feelings when there are both forward and backward linkages. What exactly happens when feelings today depend upon anticipated future feelings, and vice versa?

5.3.2 Feelings about Feelings and the Substitution Axiom

With respect to the psychological state space, our model makes the substitution axiom very close to an axiomatization of deliberative personal rationality. Suppose you list all of your possible sequences of personal psychological states, and describe them in a detailed enough manner to pin down just how pleasurable each such sequence would be to you in any future period. Now you are asked to compare two lotteries over such sequences of personal states, and are asked whether or not your preferences can be altered when you mix each lottery probabilistically with a third lottery over sequences of psychological states, i.e. you are asked whether your preferences obey the substitution axiom. If you answer that such a substitution could indeed alter your preferences, then it would be legitimate to ask you on which personal feelings such a reversal of preference was based. Any answer that you gave would necessarily reveal an incompleteness in your original description of your psychological state: the feelings upon which you differentiate between these two lotteries should already have been encoded in the list of psychological states, and therefore should not “spill over” and impact your preferences over composite lotteries.

In our view, the real difficulty that this discussion points to is that the mapping from external lotteries to psychological states may be extremely complex. Issues of this sort were foreshadowed in the early comments of Samuelson [1952]. He noted that the substitution axiom must always be applied to a fixed set of prizes, such as monetary lotteries plus feelings of suspense, and that it need not impose restrictions on preferences over simpler entities, such as monetary prizes alone. He went on to worry about how to

stop the state space from expanding out of all control.

“If every time you find my axiom falsified, I tell you to go to a space of still higher dimensions, you can legitimately regard my theories as irrefutable and meaningless...” (Samuelson [1952], p.677.)

We believe that the issue of just how far one must go in elaborating these feelings is a pragmatic one that will vary from application to application. In the thorniest of problems, one can imagine the kind of situation hinted at by Samuelson, in which lotteries over any fixed set of prizes necessarily add feelings that were ignored in the last iteration. But we believe that there is much fruitful ground to be covered before we run into this form of infinite regress.

6 Concluding Remarks

In this paper we have introduced the psychological expected utility model, and used it to analyze the impact on decision-making of anticipatory feelings, such as suspense and anticipatory excitement. The broader goal of our research agenda is to open up a variety of psychologically interesting phenomena to rational analysis, and in this respect our work has just begun.¹⁸

There are two specific areas in which we are actively pursuing further theoretical insights. The first concerns the nature of the mapping between physical lotteries and psychological states. Here the important questions concern the properties of the mapping that capture given emotions, and the nature of the behavior that can be captured by a given class of mappings.

The second concerns the analysis of decision problems. While optimal strategies exist for the two period model, there are examples of non-existence when there are more periods. Surprisingly, there is no acceptable general theory of optimal decision making in multi-period problems with time inconsistency.¹⁹ In the longer run, advances in our understanding of the logic of anticipatory emotions will have to go hand-in-hand with advances in our understanding of optimal decisions with time inconsistent preferences.

¹⁸The survey article of Rabin [1998] maps out many of the areas in which economic analysis intersects with psychological theory.

¹⁹See Caplin and Leahy [1998] for examples of non-existence and for sufficient conditions for existence.

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