[1] Give an example of a three-player bargaining game with discounting in which history-dependent strategies can be used to support an inefficient payoff division of the cake (i.e., some amount in actually thrown away in equilibrium).

[2] Prove that the random-proposer version of the n-person bargaining model has a unique stationary equilibrium. Examine its history-dependent equilibria just as we did in class for the “rejector-proposes” model.

[3] Consider a variant of the bargaining game with no discounting but with a fixed cost paid by both parties whenever there is a disagreement. Find the equilibria of this game.

[4] Consider two-person bargaining in which there is no discounting, but with a fixed probability $p$ that the game will exogenously end following a rejection, with exogenous payoffs $a$ and $b$ ($a + b < 1$). Describe the equilibria of this game.

[5] Prove that $v(n)/n \geq v(s)/s$ for all $s$ is equivalent to balancedness for a symmetric characteristic function.

[6] The core has an interesting asymmetry. It demands that allocations not be blocked for the grand coalition, but does not test the credibility of the blocking allocations themselves. One way to get around this is to define the credible core as follows:

The credible core of any coalition $S$, $C(S)$, is the set of all allocations in $V(S)$ that are unblocked by any subcoalition $T$ using only allocations from its credible core $C(T)$.

[a] Why is this informal definition not a formally correct definition? Reformulate it using recursion.

[b] Prove that the credible core equals the traditional core.

[c] Speculate on how you would proceed if the number of players were (countably) infinite.

[7] Give an example of a superadditive game (in the sense that every coalition is superadditive, not just the grand coalition) which fails condition [M], discussed in class. Give another example of a non-superadditive game which satisfies condition [M].

[8] Prove that the limit of efficient bargaining equilibria must lie in the core of the associated characteristic function.

[9] In the employer-employee game studied in class, compute $m$ explicitly for all sets of active players and discount factors, and use this to examine weak efficiency.

[10] Prove that if the discount factor is close enough to zero, then a bargaining equilibrium must be efficient. Provide some intuition for this.
[11] Go over the algorithmic derivation of $m^*$ in the class notes.

[12] We show in the notes that the condition $\sum_{i=1}^{n} m_i^* > v(N)$ is sufficient for the inefficiency of bargaining equilibrium, no matter who proposes. Show by means of an example (satisfying condition [M]) that it is not necessary. [Such an example appears in Chatterjee et al (1993, RES), but try it yourself before you look at this paper.]

[13] Here are a couple of examples that extend the characteristic function form described in Chatterjee et al. Both are taken from Ray and Vohra (Games and Economic Behavior 1999). The idea is that characteristic functions are inadequate in many situations because they do not capture interactions across players. Often, a better and more general tool is the (transferable utility) partition function, which describes the aggregate payoff to very coalition, but that could vary depending on which coalition structure that coalition lives in.

Formally, let $N$ be a set of players. A partition of $N$ is a division of $N$ into disjoint coalitions $\pi = \{S_1, \ldots, S_m\}$ which exhaust the entire space. A partition function assigns to every partition $\pi$ and every coalition $S \in \pi$ a number $v(S, \pi)$.

(a) Suppose that three individuals face a linear demand curve for their homogeneous product, $A - bx$, where $x$ is the aggregate quantity sold and $A$, $b$ are positive. Each has a common marginal cost of production, $c$. Construct the partition function for all coalition structures.

(b) Now consider a very special bargaining model in which each player can make an offer to a coalition, just as in class, but must restrict herself (for simplicity) to equal division of that coalition’s payoffs. What is the equilibrium coalition structure?

(c) Can you re-do this example for 4 or 5 players?

(d) As another example of a partition function game, consider the provision of a public good by three symmetric agents. Suppose that the three players together can get a total payoff of 3. If one player stands alone and the other two are together, the standalone player gets 2 and the remainder get a total of 0.5. Finally, assume that if all are separate, each get 0. Describe the partition function and the equilibrium of the bargaining game.

(e) What happens in the above examples (as $\delta \to 1$) if players are not constrained to equal division but can make arbitrary offers, as in the characteristic function bargaining model studied in class?

(f) For more on this sort of model, see the Ray-Vohra paper. For the public goods model in particular, see Ray and Vohra (Journal of Political Economy, 2001).