ABSTRACT. We study a model in which lifetime individual utility is derived from both present and past consumption streams. Each of these streams is discounted, the former forward in the usual way, the latter backward. We assume that an individual at date $t$ evaluates consumption programs according to some weighted average of his own felicity (as perceived at date $t$) and that of a “future self” at some date $T > t$. This simple model shows that individuals may exhibit impatience across alternatives that are positioned in periods adjacent to the present, but patience across similar choices positioned in the more distant future, that such impatience is attenuated as an individual grows older, and that lifetime choice plans are generally time-inconsistent. The model is used to capture the notion of parental influence and investigate its impact on equilibrium savings. The paper also examines other applications of “backward discounting.”

1. INTRODUCTION

We investigate some implications of “backward discounting.” Traditional “forward discounting” presumes that individuals place more weight on future payoffs relative to current payoffs. “Backward discounting” captures the idea that people put more weight on their current payoffs relative to their past payoffs. Presumably, lifetime individual happiness at any date is a combination of current experience, anticipation and memory,

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This paper revives an earlier draft, dated March 2000, with updated references and discussion; see [http://www.econ.nyu.edu/user/debraj/Papers/Backward.pdf](http://www.econ.nyu.edu/user/debraj/Papers/Backward.pdf). In particular, Caplin and Leahy (2004) discuss backward discounting in the context of the social discount rate, though social discounting is not our focus here. Gollier and Weitzmann (2010), Zuber (2011) and Jackson and Yariv (2015) discuss time-inconsistency in the context of preference aggregation; our work has some connection with this literature in that we also aggregate preferences, but across selves. Ray acknowledges financial support under a John Simon Guggenheim Fellowship for 1997-98, when these ideas were first developed. He acknowledges more recent support under National Science Foundation Grant SES-1261560. Wang acknowledges financial support from SSHRC of Canada. We thank Gregor Smith, George Loewenstein, seminar participants at CEA, Queen’s, McGill, and the SITE Summer Conference, and especially Matt Jackson and Leeat Yariv who encouraged us to revive this paper.

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with both the future and the past discounted relative to the present. We discuss some implications of this psychological postulate.

Whether or not past utilities are discounted has received little attention. The ostensible reason is simple: the past is sunk and does not influence current decision-making. But there are several reasons to hold off on such an assertion.

First, the perception of past experience may well influence current evaluation. For instance, suppose that you hire an agent to look after your business affairs over some years, and allocate a stream of consumable dividends to you. At the end of it all you write the agent a letter of recommendation. Might your letter not depend on your utility evaluation of the agent’s performance, as it appears to you at the end of this period? And if so, will that not affect the way in which the agent allocates consumption to you?

A leading example of such an agent is an elected government, which might deliberately increase populist expenditure in the run-up to an election, in the hope of favorably influencing voter decisions.

Second, when one agent has a say in decision-making for another, it might lead to conflict even though both agents fundamentally have the same preferences. Think of parenting: typically, a parent wants the child to be more forward-looking than the parent thinks the child to be. One possibility is that such disagreements emanate from distinct preferences, but this explanation begs the question of why those preferences are distinct to begin with.\(^1\) Indeed, why are they invariably distinct in just the way that makes a parent more forward-looking than the child? Backward discounting hints at a compelling yet parsimonious explanation: a 40 year old parent may want her 15 year old son to be more forward-looking because she, in effect, is maximizing the utility of her child’s 40 year old self. Situated as she is at the age of 40, she therefore discounts her child’s per-period utility backwards, while the child discounts it forwards. There is no need to assume any fundamental differences in preferences across parents and children, only a natural difference on where they are located in the life cycle.

A third reason for studying backward discounting is that an agent may place weight on her future selves, and the fact that those future selves discount their past utility will imply that current decisions are affected. Consider a 30 year old individual saving for retirement at age 70. What is she saving for? In part, she saves for the pleasures of retirement (consumption, status, charitable contributions, bequests) that are currently discounted by a factor of \(\delta^{40}\) or thereabouts, where \(\delta\) is her annual discount factor. But

\(^1\)See, e.g., Doepke and Zilibotti (2014) on models of parenting where parents are children are assumed to have different preferences.
in part, the question is not what she is saving for but whom she is saving for: she is saving for her 70 year old self, and she may place a separate weight on the happiness of that self.

Now this appears to be semantic jugglery. Suppose that an individual has some “intrinsic” discount factor for future consumptions, and additional concern for future selves. Why not simply combine the two to arrive at some “effective” discount factor that evaluates future consumption? Of course, this argument is correct. The point is, however, that such discounting is never geometric, assuming that the “intrinsic” discount factors are geometric and that there is both forward and backward discounting. Indeed, we shall argue that our model gives rise naturally to some of the experimental observations that motivate models of hyperbolic discounting (see, e.g., Ainslie (1991), Loewenstein and Prelec (1992), Loewenstein and Thaler (1989), Laibson (1997, 1998), and O’Donoghue and Rabin (1999)). But our results are “minimal” in the sense we ask for no departures from the widely-used notion of geometric discounting. It is also in line with recent models of collective decision-making that give rise to time-inconsistency, such as Gollier and Weitzman (2010), Zuber (2011), and Jackson and Yariv (2015).

In short, we presume that an individual needs to balance his notions of happiness (as evaluated by his current self) with the corresponding pleasures that he knows he will value when he comes to some natural ending point — say, retirement. He knows that at that latter stage he will look back at his life. The consequent “reverse discounting” involved in looking back conflicts with the “forward discounting” that his current self uses.

Why might a 30 year old also place weight on her 70 year old self? One possible answer is that the 70 year old self is the imprint of parental influence, the result of many years of using a parent as a role model. When intergenerational links are strong (for instance in societies in which aged parents live with their grown children), there may be stronger empathy for the aged self. The theory we develop below suggests that societies with stronger parental influence or closer family ties will have higher savings rates and assign higher value to investment in human capital.²

²East Asia, where family ties are known to be relatively strong, is perhaps the region that would provide strong support for this point of view. For instance, despite much lower interest rates (up to around 2000), Japan’s savings rate is much higher than the rest of the industrialized countries. According to International Monetary Fund (2001a), for the two decades leading to 2000, the interest rates (central bank discount rates) for Japan range from 0.5 to 6%, while the rates for the US range from 4.5 to 8.5%, and for Europe from around 3 to around 11%. For the same period, the gross national savings rates for Japan range from 30 to 35%, but range from 15 to 20% for the US, and 20 to 24 % in Europe. (See International Monetary Fund (2001b).) In the recent years of economic downturn, the interest rates of most industrialized countries have
A fourth reason for studying the backward discounting model is that matters for policy, and for welfare economics more generally. As Caplin and Leahy (2004) observe, backward discounting has implications for the choice of the social discount rate. We do not pursue the welfare implications of backward discounting in this paper.

Backward discounting is a parsimonious idea that goes naturally with the usual forward discounting, and the two in conjunction have testable implications that other theories (such as hyperbolic discounting) don’t deliver. For instance, several of the predictions in Proposition 1 seem novel and are not generated from any other model we know. Or, as already discussed, our model suggests that people may prefer an improving sequence over a worsening sequence over time, where such a sequence might refer to diverse outcomes, such as relief from pain, emotional episodes, and monetary payments. These predictions are supported by numerous surveys and experiments. Section 4 discusses a number of diverse applications in which backward discounting might play a crucial role, such as case-based decision theory, elections, and evolutionary selection.

In summary, what makes our model compelling is that one simple, plausible postulate of backward discounting simultaneously generates a number of different findings and predictions in different economic and cultural contexts. We hope that the reader will view the exercises below as a first step to a deeper exploration of these matters.

2. BACKWARD DISCOUNTING AND DIFFERENT “SELVES”

2.1. Preliminaries. Our model is set in discrete time. Indices such as $t$, $s$, $T$ and $N$ designate integer periods. A person lives from period 0 to period $N$, and $\{c_s\}_{s=0}^{N}$ is a consumption sequence over the person’s lifetime. Let $u$ be a one-period utility function defined on consumption at every date.

Our main postulate is that a person at date $t$ discounts both the past and the future. We use a common discount factor $\delta < 1$ for both directions of discounting. This is not at all necessary for the results that follow, but it helps in reducing notational clutter.
Consider a person currently alive at date $t$. How might he evaluate lifetime utility starting from some sequence of consumption $\{c_s\}_{s=0}^{N}$? He discounts future utilities by the discount factor of $\delta$ per period, and, he discounts past utilities by $\delta$ per period as well. This forward and backward discounting system gives us the present value of the lifetime utility of his “current self” at date $t$ as follows

$$\sum_{s=0}^{N} \delta^{t-s} u(c_s).$$

2.2. Different Selves. The discounting system in the above formula is obviously not geometric in $s$, implying that there could be time inconsistency in the person’s time preference. For example, a “current self” at date $t+1$ calculates the present value of his lifetime utility at

$$\sum_{s=0}^{N} \delta^{t+1-s} u(c_s),$$

which puts a weight of $\delta$ on $u(c_t)$ and 1 on $u(c_{t+1})$. In contrast, a “current self” at date $t$ puts a weight of 1 on $u(c_t)$ and $\delta$ on $u(c_{t+1})$. Thus, the “current self” at date $t+1$ may regret the consumption choices made by the “current self” at date $t$.

Such regret may be partly anticipated by the person when he makes decisions. To capture this anticipation, the person may put some direct weight on the preferences of his “future selves.” Assume, then, that at date $t$, the person puts a weight of $\alpha$ on the preference of his “current self” and $1 - \alpha$ on the preference of his “self” at some salient future date $T(t)$. This $T(t)$ may or may not be persistently sensitive to $t$. For instance, $T(t)$ may simply be a fixed number that’s independent of $t$. The essential idea is that $T(t)$ represents a “rest-point” for a person of age $t$: a special age at which he might feel comfortable evaluating his life. It is for this reason that $T(t)$ will be re-interpreted below as the age of a “shadow parent,” a parental role model who provides a natural age for stock-taking of one’s own life.

The adjusted present value of the individual’s lifetime utility is now

$$\alpha \sum_{s=0}^{N} \delta^{t-s} u(c_s) + (1 - \alpha) \sum_{s=0}^{N} \delta^{T(t)-s} u(c_s).$$

Combining these two summations, we obtain an expression for lifetime utility at date $t$:

$$\sum_{s=0}^{N} d(t, s) u(c_s),$$

(1)
where \( d(t, s) \) can be thought of as the effective discount factor applied by a person at date \( t \) to consumption at date \( s \), and is given by

\[
d(t, s) = \alpha \delta^{[s-t]} + (1 - \alpha) \delta^{[T(t) - s]}
\]

2.3. Time Preference and Time Consistency. This simple model generates certain observed features in a natural way. First, unless \( T(t) \) is fixed and the person puts the entire weight on the “future self” at \( T(t) \), it creates time-inconsistency in individual decisions. This is almost immediate from examining the sequence of effective discount factors given by (2). They are not geometric.

As an example, suppose that \( t = 30 \) and \( T(t) = 65 \), with weights \( \alpha \) and \( 1 - \alpha \). Notice that preferences over the very near future (say years 30 and 31) are governed largely by the (exponential) discount factor of the current self — the comparison \( \delta^{35} : \delta^{34} \) matters far more than the comparison \( \delta^{34} : \delta^{35} \), the latter being the weights applied by the 65 year old shadow parent. However, as the delay is pushed into the future — say to a consumption comparison across ages 50 and 51 — the bias towards the present exerted by the current self is increasingly compensated for by the bias towards the future exerted by the future self, and the two effects cancel, from the vantage point of the thirty-year old. Finally, it is possible that our thirty-year old current self may indeed exhibit negative discounting for ages close to 65, as the preferences of the future self now outweigh the increasingly fragile comparisons of his current self over these distant time periods.

Notice that the time-inconsistency is of a particular kind. There is a preference for current consumption (once the decision time comes around). This well-documented phenomenon has often been captured by postulating non-geometric preferences (Strotz, 1956). The best-known of these is the hyperbolic class of discount factors (see Laibson (1997, 1998) and Harris and Laibson (2001)). The hyperbolic class, indeed, accounts for the following empirical regularity: discounting seems to be more active for time delays that are situated in the immediate future, whereas delays situated at a more distant date are viewed more neutrally (see Thaler (1981), Ainslie (1991), Loewenstein and Prelec (1992), Laibson (1997) and O’Donoghue and Rabin (1999)). Our model exhibits this phenomenon by combining forward and backward discounting, generating a sequence of nongeometric “effective” discount factors.

But the model makes more predictions. Here are three examples. First, as already discussed, it suggests that there is negative discounting near the end of the horizon. We haven’t tested this, but here is a proposed example. Consider, for instance, the question of whether a 30 year old might entertain a consumption sacrifice of $100,000 at the age
of 50, so that his old self (at, say 70), might obtain that added consumption, or slightly less. If the answer is yes, we have a case of negative discounting.

Second, it predicts that present bias tends to disappear with advancing age. A fifty-year old who places weight on his 65 year old future self will assign greater weight to the future self for the comparison 55–56 than the 30 year old will assign to the comparison 35–36. This makes the fifty-year old appear less prone to present bias.

Third, the moderating influence of backward discounting disappears once we consider choices made at adjacent periods very late in life, well past the “retirement self.” Such periods are discounted “forward” by all concerned selves. Thus impatience should arise again for choices offered (to younger individuals) at hypothetical late adjacent periods. These are predictions that are qualitatively different from hyperbolic discounting. We now proceed to formalize these ideas.

Recall the effective discount factor $d(t,s)$ from (2). For each $t$ and $s \geq t$, let

$$i(t,s) \equiv \frac{d(t,s)}{d(t,s+1)}$$

measure the degree of “one-period impatience at $s$” that person $t$ feels regarding adjacent choices at date $s$. This is essentially the reciprocal of the discount factor; e.g., in the standard model, $i(t,s) = 1/\delta$. We may summarize our discussion so far in the following way.

**Proposition 1.** [1] For each $t$, one-period impatience at $s$ declines with $s$ for all $s \in \{t, t + 1, \ldots, T(t) - 1\}$. Relatedly, for each $s < T(t)$, one-period impatience increases with both the individual’s age $t$ and the age of the shadow parent $T$; in fact, it increases in the joint variable $T(t) + t$.

[2] For each $t$, and for $s < T(t)$, one-period impatience at $s$ falls below 1 — that is, discounting “ultimately” turns negative — if $s$ is close enough to $T(t)$ and $T(t) - t$ is large enough.

[3] On the other hand, for each $t$, and for $s \geq T(t)$, one-period impatience reverts to its maximum value of $1/\delta$.

[4] If $T(t) - t$ does not increase in $t$, current one-period impatience $i(t,t)$ is a nonincreasing function of $t$, strictly decreasing whenever $T(t) - t$ decreases in $t$.

The first part of the proposition states that from the vantage point of a person at date $t$, the relative impatience across adjacent dates in the future declines as the future is made more distant, being highest for choices between “today” and “tomorrow”. This
translates precisely into what Loewenstein and Prelec (1992) call the common difference principle: “if a person is indifferent between receiving \(x > 0\) and \(y > x\) at some later time, ... then he or she will strictly prefer the better outcome if both outcomes are postponed by a common amount [of time].”

The second statement in this first part of the proposition fixes the date \(s\) and makes a similar argument as a person gets older: effectively, he becomes more impatient as he approaches closer to \(s\). This is essentially a restatement of the earlier assertion except that we say something more about impatience varies with both \(t\) and the age \(T(t)\) of the shadow parent.

To get a sense of the “end points”, consider both \(i(t, t)\) (impatience regarding current delays) and \(i(t, T(t) - t)\) (impatience concerning future delays). It is easy to see that

\[
(3) \quad i(t, t) > 1 \text{ provided that } \frac{\alpha}{1 - \alpha} > \delta^{T-t-1},
\]

which will always be true if the current self and the future self are sufficiently separated in time (or if \(\alpha > 1/2\) and there is some separation between current self and the future self). On the other hand,

\[
(4) \quad i(t, T(t)) < 1 \text{ provided that } \frac{\alpha}{1 - \alpha} < \frac{1}{\delta^{T-t-1}},
\]

which will also be true if there is sufficient separation between the current self and the future self. Taken together, (3) and (4) suggest the following possibility: a person at date \(t\) may be impatient (in a perfectly standard way) over adjacent periods in the vicinity of the present, while he actually exhibits negative discounting for future periods. This is the assertion in the second part of the proposition.

However, the third part of the proposition states that as the date \(s\) crosses the date of the shadow self \(T(t)\), then discounting again reverts to its old ways: after all, both \(t\) and \(T(t)\) will discount adjacent dates at \(s\) (which exceeds both \(T(t)\) and \(t\)) in exactly the same way.

The fourth and final part of the proposition states that the proclivity of a person to be impatient over current and immediately adjacent future choices is attenuated as that person grows older. Specifically, this tendency towards moderation is observed as long as the future self “grows” more slowly than the current self. In particular, if the age of the future self is fixed, observed patience must always increase in age. It would be of interest to examine empirical evidence that bears on this prediction.
Proof of Proposition 1. Note that for \( s \in \{ t, t+1, \ldots, T(t) - 1 \} \),
\[
i(t, s) = \frac{\alpha \delta^{s-t} + (1-\alpha)\delta^{T(t)-s}}{\alpha \delta^{s+1-t} + (1-\alpha)\delta^{T(t)-s-1}} = w(t, s)(1/\delta) + [1 - w(t, s)]\delta,
\]
where
\[
w(t, s) = \frac{\alpha \delta^{s+1-t}}{\alpha \delta^{s+1-t} + (1-\alpha)\delta^{T(t)-s-1}} = \frac{\alpha}{\alpha + (1-\alpha)\delta^{T(t)-s-1}}.
\]
It is easy enough to see from this expression that \( w(t, s) \) is declining in \( s \). It follows from (5) that \( i(t, s) \) is less than \( 1/\delta \), and is in addition a declining function of \( s \), for fixed \( t \), as long as \( s \in \{ t, t+1, \ldots, T(t) - 1 \} \). Also note that \( w(t, s) \) (and thus \( i(t, s) \)) is increasing in \( T(t) + t \) for \( s < T(t) \). That completes the proof of part (i).

To prove part (ii), set \( s = T(t) - 1 \) in (6) and note that as \( T(t) - t \) becomes large, \( w(t, T(t) - 1) \) can be made arbitrarily small. Consequently, if \( T(t) - t \) is large enough, \( i(t, T(t) - 1) < 1 \), using (5).

Next, to prove part (iii), note that for \( s \geq T(t) \),
\[
i(t, s) = \frac{\alpha \delta^{s-t} + (1-\alpha)\delta^{T(t)}}{\alpha \delta^{s+1-t} + (1-\alpha)\delta^{T(t)-1}} = 1/\delta,
\]
which establishes that part.

Finally, to prove part (iv), notice that that
\[
w(t, t) = \frac{\alpha}{\alpha + (1-\alpha)\delta^{T(t)-t-2}},
\]
which shows that \( w(t, t) \) is declining in \( t \) as long as the conditions of part (iv) hold. Using this information in (5), we see that \( i(t, t) \) must decline as well.

2.4. Backward Discounting and Hyperbolic Discounting. There are some similarities and differences between a backward discounting model and a hyperbolic discounting model. A generalized hyperbolic discount function is given by
\[
d(t, s) = [1 + \beta(s - t)]^{-\gamma/\beta}.
\]
(See Loewenstein and Prelec (1992) for an axiomatic derivation.) This discount function gives us
\[
i(t, s) = \left( \frac{d(t, s)}{d(t, s + 1)} \right)^{-\gamma/\beta} = \left( 1 + \frac{\beta}{1 + \beta(s - t)} \right)^{-\gamma/\beta}.
\]
In line with Proposition 1, one-period impatience in a generalized hyperbolic discounting model, \( i(t, s) \), is decreasing in \( s \) and increasing in \( t \). In contrast to that Proposition, however, \( i(t, s) \) does not jump up later, as it does in the case of backward discounting.
That is, a person is more patient when a trade-off occurs in a more distant future, as long as the trade-off does not occur after the date of his future self. Also in contrast with Proposition 1, \( i(t, t) \) is constant under generalized hyperbolic discounting, while it is typically decreasing in the backward discounting model. An individual becomes more and more patient in a backward discounting model, while her current impatience remains the same in a hyperbolic discounting model.

3. Equilibrium Consumption Plans

In the previous section, we have remarked on the inherent time-inconsistency present in a model of backward discounting. Now we study actual consumption decisions. A person at date \( t \), far removed from date \( T(t) \), may currently engage in high consumption, and then plan on saving higher and higher fractions of his income as he gets into middle age and approaches the age of \( T(t) \). Indeed, such plans are perfectly consistent with his preferences at date \( t \). The problem is, he may want to revise these decisions as he gets older.

Thus it is to be expected that while a calculation of optimal intertemporal consumption and savings may show initial periods of high planned consumption and intermediate periods of high planned savings, these features may not manifest themselves in observed behavior once time-inconsistency is taken into account.

The purpose of this section is to examine these matters by looking at both “intended” and “equilibrium” behavior. Beginning with the work of Strotz (1956) and Phelps and Pollak (1968), such behavior has been studied in models with “changing tastes,” especially models of hyperbolic discounting; see, e.g., Laibson (1994), Harris and Laibson (2001) and Bernheim, Ray and Yeltekin (2015). We exploit the structure of the backward discounting setup here, and relate equilibrium savings behavior to the existence of a shadow parent.

Specifically, suppose that a person lives for \( N \) periods and places weight on just two selves: his current self at date \( t \), and a “shadow parent age \( T(t) \leq N \). In this section, we fix \( T(t) = T \) and view it as some unchanging age; say, the age at retirement. The per-period utility indicator is taken to be logarithmic, though this assumption is made for simplicity in exposition: the analysis that follows can, in principle, be conducted for all constant-elasticity utility functions. But the special case of logarithmic utility yields particularly sharp predictions that are amenable to easy computation, and so we stick to this particular formulation.
We also keep the background model of asset accumulation as simple as possible. There is a constant per-period interest rate of \( r \) on both borrowing and lending, and a constant geometric discount factor of \( \delta \) for both forward and backward discounting. Assets are \( A_0 \) to begin with, and the agent receives an exogenous income stream \( \{y_s\} \) over the dates \( 0, 1, \ldots, N \). Let \( A_s \) denote assets in period \( s \). Then, if \( c_s \) is consumed at that date,

\[
A_{s+1} = (1 + r)(A_s + y_s - c_s).
\]

An agent located at date \( t \) seeks to choose \( \{c_s\} \) over the dates \( t, \ldots, N \) to maximize the expression in (1). Noting that the decisions up to \( t - 1 \) have already been made, this is tantamount to maximizing

\[
\alpha \sum_{\tau=t}^{N} \delta^{\tau-t} \ln c_{\tau} + (1 - \alpha) \sum_{\tau=t}^{N} \delta^{|\tau-T|} \ln c_{\tau}.
\]

### 3.1. Planned Consumption and Savings

First, ignore the time consistency problem and simply map out the profile of optimal consumption and savings, viewed from the vantage point of a person at date 0. Even though it isn’t hard to solve this problem by exploiting the usual Euler equations, we will use an approach based on value functions. This has two advantages. First, it will give us optimal policy functions for each period (not just consumption and savings values), making it easier for us to compare changing attitudes across time periods. Second, this is the approach that will be necessitated when we solve the equilibrium problem (the one that takes the possibility of dynamic inconsistency into account).

It will be useful to keep track of two different “value functions”. The first, denoted by \( V_t \), records the utility experienced by the current self from date \( t \) onwards; we write this discounted back to date \( t \). That is,

\[
V_t(A) = \sum_{\tau=t}^{N} \delta^{\tau-t} \ln c_{\tau},
\]

where the \( c \)'s will refer to optimal consumption decisions planned by the agent at date 0. The second value function, \( W_t \), tracks the utility experienced from date \( t \) onwards, but discounted backward and forward to date \( T \), which is the fixed “retirement age” that our agent is presumed to place separate weight on:

\[
W_t(A) = \sum_{\tau=t}^{N} \delta^{|\tau-T|} \ln c_{\tau}.
\]

(Because \( T \) is fixed, we do not carry it in the notation.) Notice that at date \( N \), both these functions are trivially logarithmic, and are of the form

\[
V_N(A) = a_N \ln (A + y_N) + b_N,
\]

3.1. Planned Consumption and Savings.
and

\[ W_N(A) = p_N \ln(A + y_N) + q_N, \]

where \( a_N = 1,\ p_N = \delta^{N-T}, \) and \( b_N = q_N = 0.\) Using the forms deliberately suggested by (7) and (8), we conjecture that at any date \( t < N,\)

\[ V_{t+1}(A) = a_{t+1} \ln(A + M_{t+1}) + b_{t+1}, \]

and

\[ W_{t+1}(A) = p_{t+1} \ln(A + M_{t+1}) + q_{t+1}, \]

where the expression \( M_s \) records the present value of flow income from any date \( s: \)

\[ M_s = y_s + \frac{y_{s+1}}{1+r} + \cdots + \frac{y_N}{(1+r)^{N-s}}. \]

This conjecture helps us to set up the optimization problem (viewed from time 0) that will be solved at date \( t: \) for any asset level \( A \) at date \( t, \) choose \( c \) to maximize

\[ \alpha[\delta^t \ln c + \delta^{t+1} V_{t+1}(A')] + (1 - \alpha)[\delta^{t-T} \ln c + W_{t+1}(A')], \]

where \( A' = (1+r)(A - c + y_t). \)

It is easy to see that the solution to this problem is given by

\[ c = \frac{a\delta^t + (1 - \alpha)\delta^{t-T} [A + M_t]}{a\delta^t + (1 - \alpha)\delta^{t-T} + aa_{t+1}\delta^{t+1} + (1 - \alpha)p_{t+1}} [A + M_t] \equiv \lambda_t [A + M_t]. \]

Using this rule, we obtain corresponding forms for \( V_t \) and \( W_t. \) First,

\[ V_t(A) = a_t \ln(A + M_t) + b_t, \]

where

\[ a_t = 1 + \delta a_{t+1} \]

and

\[ b_t = \ln(\lambda_t) + \delta a_{t+1} \ln(1 + r) + \ln(1 - \lambda_t)] + \delta b_{t+1}. \]

Similarly,

\[ W_t(A) = p_t \ln(A + M_t) + q_t, \]

where

\[ p_t = \delta^{t-T} + p_{t+1} \]

and

\[ q_t = \delta^{t-T} \ln(\lambda_t) + p_{t+1} \ln(1 + r) + \ln(1 - \lambda_t)] + q_{t+1}. \]
This completes (and justifies) the inductive step. Combining (10), (11), and (13), we can simplify the optimal consumption ratio \( \lambda_t \) to

\[
\lambda_t = \frac{\alpha \delta^t + (1 - \alpha) \delta^{t-T}}{\alpha \delta^t a_t + (1 - \alpha) p_t}.
\]

This expression still contains the (endogenous) terms \( a_t \) and \( p_t \), but (11) and (13) can be used to arrive at closed-form solutions for them. Doing so, we see that

\[
a_t = \frac{1 - \delta^{N-t+1}}{1 - \delta},
\]

while

\[
p_t = \frac{\delta [1 - \delta^{T-t}] + 1 - \delta^{N-T+1}}{1 - \delta} \quad \text{for} \ t < T,
\]

\[
p_t = \frac{\delta^{t-T} [1 - \delta^{N-t+1}]}{1 - \delta} \quad \text{for} \ t \geq T.
\]

With the help of these closed forms (16) and (18), we can explore the planned consumption ratio \( \lambda_t \). Notice first that there is an intrinsic tendency for the ratio to drift upwards simply by virtue of the finite horizon nature of the problem. For instance, in the last period, all of permanent income will be consumed. We can “benchmark” this drift by setting \( \alpha = 1 \) in the problem above, whereupon the situation reduces to a perfectly standard life cycle problem. The consumption ratio sequence for this problem, which we denote by \( \bar{\lambda}_t \), can be separately computed or arrived at by putting \( \alpha = 1 \) in (15) above and then using (16):

\[
\bar{\lambda}_t = \frac{1 - \delta}{1 - \delta^{N-t+1}} = \frac{1}{a_t}.
\]

This benchmark helps us get a handle on the extent to which our model departs from the standard formulation (in terms of predicting additional savings out of permanent income). Using (15) and (19), form the ratio

\[
\theta_t = \frac{\lambda_t}{\bar{\lambda}_t} = \frac{\alpha + (1 - \alpha) \delta^{t-T} \delta^{-t}}{\alpha + (1 - \alpha) (p_t/a_t \delta^t)},
\]

and observe that a value of \( \theta_t = 1 \) implies that (at date \( t \)) there is no divergence between the consumption ratios predicted by the two models. On the other hand, if \( \theta_t < 1 \) then the model of backward discounting predicts a higher savings rate, and this effect is directly related to the amount by which \( \theta \) falls below unity.

To match the algebra with intuition, note that at any date \( t \geq T \), there is no difference between the discounting exhibited by the current self and the future self. So consumption behavior should be quite independent of the weights placed on these two selves,
and in particular should coincide with that of the standard model. Indeed, if we substitute the value of \( a_t \) (from (16) and the value of \( p_t \) (from (18) for \( t \geq T \)) in (20), we see that \( \theta = 1 \). For such time periods, there is no discrepancy (in consumption ratios) between our model and the standard formulation: \( \theta_t \) equals 1 for all \( t \geq T \).

The more interesting comparison is for dates that are less than \( T \). The following observation is critical: for all \( 0 \leq t < T \),

\[
\delta^{T-t} < \frac{p_t}{a_t}.
\]

This is easy enough to establish by direct computation, using (16) and (18).

Combining (21) with (20), it is easy to see that consumption ratios are lowered for all periods up to the age of the future self. [Later, once we describe the equilibrium version, we shall attempt to provide quantitative estimates of the magnitude of this effect.]

It should also be clear that if the agent increases the weight on his future self, then the savings rate increases at each date.

Finally, what is the relationship between the extent of divergence and time? At what point over the agent’s lifetime do we observe maximal (planned) divergence from the standard model? It turns out, not surprisingly, that the answer to this question depends on the weight that the agent attaches to the future self. If this weight is high enough, the agent wants future versions of himself to carry out the bulk of the savings. Formally, the following is true:

**Observation 1.** There exists \( \hat{\alpha} \in (0,1] \) such that if \( \alpha \leq \hat{\alpha} \), \( \theta_t \) always increases in \( t \); while if \( \alpha > \hat{\alpha} \), \( \theta_t \) first decreases and then increases in \( t \).

The proof of this observation is relegated to the Appendix. But a numerical example may be useful: say a person starts at age 30. If this is normalized to zero, we may take \( N = 50 \) and \( T = 35 \). If \( \delta = 0.8 \) and equal weight is put on the future self so that \( \alpha = 0.5 \), it is easy to compute that \( \theta_t \) begins at 0.36, reaches its minimum of 0.02 at \( t = 16 \), and goes back up to 1 at \( t = 35 \). For this value of \( \delta \), \( \hat{\alpha} \approx 0.0005 \). However, for \( \delta = 0.95 \), \( \hat{\alpha} \approx 0.19 \).

Notice that (apart from the intended illustration of Observation 1), the example shows that the effects on savings are extremely large. However, because these outcomes are not time-consistent, we revisit the issue of magnitude when we study the equilibrium problem.
3.2. Equilibrium Consumption and Savings. Next, we take into account the fact that planned paths will not be honored by future selves. This is precisely an instance of the general problem posed by Strotz (1956), and analyzed by numerous authors (see, e.g., Phelps and Pollak (1968), Peleg and Yaari (1973), Bernheim and Ray (1987), Laibson and Harris (2001), O’Donoghue and Rabin (1999), and Bernheim, Ray and Yeltekin (2015)). The idea is to treat each self as a separate player in a noncooperative game, and analyze the subgame perfect equilibria of such a game.

With an open-ended time horizon, there could be several equilibria of this game (for the usual reasons). In the logarithmic case — and with constant elasticity utility functions more generally — we can solve this problem completely for the finite horizon model. It turns out that there is a unique subgame perfect equilibrium, which can be described using backward recursion. We have already laid the groundwork for this in the previous section. To retain comparability, it will actually help to abuse notation and use exactly the same notation to describe value functions, consumption ratios, etc. We begin as before by observing that in the last period $N$, the value functions are exactly in the form given by (7) and (8), with the restrictions $a_N = 1$, $p_N = \delta N - T$, and $b_N = q_N = 0$. Therefore conjecture that at some date $t$,

$$V_{t+1}(A) = a_{t+1} \ln(A + M_{t+1}) + b_{t+1},$$

and

$$W_{t+1}(A) = p_{t+1} \ln(A + M_{t+1}) + q_{t+1},$$

Now we can state the problem that faces the agent at date $t$: for any asset level $A$,

$$\max_c \alpha \left[ \ln c + \delta V'_{t+1}(A') \right] + (1 - \alpha) \left[ \delta |t-T| \ln c + W_{t+1}(A') \right],$$

where $A'$ is $(1 + r)[A - c + y_t]$, as before. Notice how in this problem the effective discount factors are different from those in the previous section; compare the maximand here with its counterpart (9). Solving this problem, we see that

(22)  \[ c = \frac{a + (1 - a)\delta|t-T|}{a + (1 - a)\delta|t-T| + aa_{t+1}\delta + (1 - a)p_{t+1}}[A + M_t] \equiv \lambda_t[A + M_t]. \]

This gives us solutions for $V_t$ and $W_t$. First,

$$V_t(A) = a_t \ln(A + M_t) + b_t,$$

where

(23)  \[ a_t = 1 + \delta a_{t+1} \]

and

(24)  \[ b_t = \ln(\lambda_t) + \delta a_{t+1}[\ln(1 + r) + \ln(1 - \lambda_t)] + \delta b_{t+1}. \]
Similarly,

\[ W_t(A) = p_t \ln(A + M_t) + q_t, \]

where

\begin{equation}
(25) \quad p_t = \delta^{t-T} + p_{t+1}
\end{equation}

and

\begin{equation}
(26) \quad q_t = \delta^{t-T} \ln(\lambda_t) + p_{t+1} [\ln(1 + r) + \ln(1 - \lambda_t)] + q_{t+1}.
\end{equation}

This completes the inductive step. Notice that these forms look no different from those in the previous problem (though the quantitative magnitudes of \( b_t \) and \( q_t \) will indeed be different because of the new values of \( \lambda_t \) for the equilibrium problem).

As before, (23) and (25) yield closed-form solutions for \( a_t \) and \( p_t \):

\begin{equation}
(27) \quad a_t = \frac{1 - \delta^{N-t+1}}{1 - \delta},
\end{equation}

and

\begin{equation}
(28) \quad p_t = \frac{\delta[1 - \delta^{T-t}]}{1 - \delta} + \frac{1 - \delta^{N-T+1}}{1 - \delta} \quad \text{for } t < T,
\end{equation}

\begin{equation}
(29) \quad = \frac{\delta^{t-T}[1 - \delta^{N-t+1}]}{1 - \delta} \quad \text{for } t \geq T.
\end{equation}

Combining (22), (23), and (25), we can simplify the equilibrium consumption ratio \( \lambda_t \) to

\begin{equation}
(30) \quad \lambda_t = \frac{\alpha + (1 - \alpha)\delta^{t-T}}{\alpha a_t + (1 - \alpha)p_t}.
\end{equation}

Notice that the solution to the \( a_t \)'s and the \( p_t \)'s are precisely what they were in the planned model (this gives the logarithmic case some extra sharpness). But the consumption ratios are different (compare (30) with its predecessor (15)). Recalling that consumption ratios in the standard model are given by \( \bar{\lambda}_t = 1/a_t \), we can form our measure of relative consumption ratios, just as we did before, by

\begin{equation}
(31) \quad \theta_t \equiv \frac{\lambda_t}{\bar{\lambda}_t} = \frac{\alpha + (1 - \alpha)\delta^{t-T}}{\alpha + (1 - \alpha)(p_t/a_t)}.
\end{equation}

Note that a value of \( \theta_t = 1 \) implies that (at date \( t \)) there is no divergence between the consumption ratios predicted by the two models. As in the case of planned consumption paths, it is easy to check that there is indeed no divergence between the two ratios when \( t \geq T \).
For dates that are less than $T$, exactly the same observation as in (21) applies here: for all $0 \leq t < T$,

\[ \delta^{T-t} < \frac{p_t}{a_t}. \]

Combining (32) with (31), it is easy to see that consumption ratios are lowered for all periods up to the age of the future self. But we can say a bit more now than we could in the planning exercise: as long as $t < T$, the extent of the divergence between equilibrium consumption ratios and the ratios in the standard model must monotonically decline over time. That is, the greatest impetus to savings comes at the earliest dates (controlling for income, of course).

To check this, it is sufficient to see that $\delta^{T-t}$ increases over time, while $p_t/a_t$ declines, and then apply these findings to (31).\footnote{It is immediate that $\delta^{T-t}$ increases with $t$ for $t < T$. To check the claim for $p_t/a_t$, note that this expression equals $\frac{\delta^{1-x_t+t+1}}{1-\delta^{T-t+1}} = \frac{\delta^{1-x_t+t+1}}{1-\delta^{x_t+t+1}}$, where $x = \delta^{T-t}$ and $D = \delta^{N-T+1}$. It is easy to check (e.g., by differentiation) that this last expression is a monotonically decreasing function of $x$, while $x$ is itself an increasing function of $t$.}

As we have argued (see the planned case), a monotonically decreasing divergence of consumption ratios is not necessarily what an agent might want. In general, he would like future incarnations of himself to do the bulk of the savings. In fact, it is easy to check that the divergence ratios are larger under the planning problem than under the equilibrium problem for every date $t < T$. But given time-inconsistency, he knows that these large desired divergences are not going to be honored. This realization is built into the equilibrium problem.

We should note, however, that this result does not rule out a situation in which equilibrium savings rates rise and then fall over time. See, for example, Table 1 in the next section.

It will be useful to summarize the foregoing analysis (including our findings in the planned case) in the form of a proposition:

**Proposition 2.** Suppose that an agent faces an accumulation problem, and places weights $\alpha$ on himself and $1 - \alpha$ on a future self of fixed age $T$. Then

[1] In both the planning and equilibrium versions of the problem, consumption ratios (out of permanent income) are lower relative to those obtained for the standard problem, at every date $t < T$. 

18 DEBRAJ RAY AND RUQU WANG

<table>
<thead>
<tr>
<th>Age</th>
<th>Benchmark Model</th>
<th>Weight on Shadow Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>14</td>
<td>25 34 43 52 59</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>24 32 39 46 53 59</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>25 29 34 39 43 48</td>
</tr>
</tbody>
</table>

TABLE 1. SAVINGS RATES (%) IN THE BENCHMARK AND EQUILIBRIUM MODELS.

[2] For dates $t \geq T$, there is no discrepancy between the consumption ratios.

[3] A larger weight on the future self depresses consumption ratios even further (at each date), in both the planning and equilibrium versions.

[4] The planned divergence ratios are larger (at each date $t$ such that $0 < t < T$) than the equilibrium divergence ratios. Furthermore, the equilibrium divergence ratios monotonically decline over time, while no such presumption can be entertained for the planned divergence ratios (as described fully in Observation 1).

3.3. A Numerical Example. Just how large are the equilibrium effects in Proposition 2? We have already seen a numerical example for the planned case, but we know that such planned divergences will not be observed. Hence it is the equilibrium case that merits a more careful study. Table 1 provides some numerical magnitudes for a particular parametric configuration. As our results are all analytical and as the numbers are quite robust to changes in the parameters, we did not feel it necessary to report on a wide variety of cases.

In what follows, we look at equilibrium savings rates for individuals of age 30, 40 and 50, when lifetime is taken to be 80 and the age of the shadow parent is set to a “retirement age” of 65. The interest rate on wealth is 7%, and we use a discount factor of 0.95. Our model individual receives a wage income of 1 unit per period until retirement, and begins life at age 30 with an asset level of 2 units.5

To facilitate the use of everyday empirical observations, the savings rates we report are not out of permanent income, but out of current income (which is wage income plus any interest income on assets). The results are fairly strong. The first column of the table reports savings rates at various ages in a standard life-cycle “benchmark model” (which is just our model with $\alpha$ set equal to unity). The second column does the same

5 Alternative specifications are available on request from the authors.
for our equilibrium model when a low weight of 0.1 (that is, $\alpha = 0.9$) is assigned to the shadow parent. The effect on equilibrium savings, as Table 1 shows, is extremely strong. Savings rates nearly double at age 30 and the positive effects at later ages, while not as strong, are still significant.

As stated in Proposition 2, the bulk of the impact of parental influence takes place in the earlier phases of life (in the equilibrium model). This is clearly borne out by the table. Notice, however, that this does not preclude an increase in the savings rate over time, followed even by a decline. Under the weight of a large accumulation of assets, the natural tendency will be to curb savings somewhat in later years. This is obvious in the post-retirement phase, of course. But our numerical computations suggest that this phenomenon may be heightened (even at pre-retirement ages) in the presence of parental influence.

Finally, observe that further weight placed on the shadow parent lead to even more dramatic effects on the savings rate. These numbers are provided, not necessarily for the sake of realism, but to suggest that socio-cultural phenomena such as upbringing and influence may have effects on savings rates that simply swamp any changes that might be brought about by the usual economic policies.

3.4. Remarks on the Shadow-Parent Assumption. It is well known that a parent’s influence on the behavior of their children is very significant (see, e.g., Hess and Torney (1967), Bandera (1977) and Moschis (1987), among many others). Several aspects of this influence have been studied. Among other things, parents have a significant impact on such outcomes as their children’s choice of career (Dryler (1998)), their focus on academic excellence (Salili (1994)), their perception of leadership (Harris and Hartman (1992)), their political attitudes (Hess and Torney (1967)), and on characteristics such as home ownership (Henretta (1984)). Bregman and Killen (1999) show that parental influence was judged by adolescents and young adults to be most important when their decisions focused on short-term goals. A literature in economics (Becker and Mulligan (1997), Bisin and Verdier (1998, 2000), Doepke and Zilibotti (2014)) incorporates parental influence and upbringing as fundamental features of intergenerational economic interaction. Indeed, Weinberg (2001) found that there is a correlation between children’s outcomes and their parents’ child-rearing practices. Parents with higher incomes are more able to mold their children’s behavior through pecuniary incentives, while parents with lower incomes have to rely on (less effective) nonpecuniary mechanisms, such as corporal punishment.
A particularly important target for parental influence — and this is what we emphasize here — is the attitude to consumption and savings. Moschis (1987, p. 77) summarizes the literature thus: “[T]here appears to be reasonably good supportive evidence that the family is instrumental in teaching young people basic rational aspects of consumption. It influences the development of rational consumption orientations related to a hierarchy of economic decisions delineated by previous writers...: spending and saving, expenditure allocation, and product decisions, including some evaluative criteria.”

However, in our opinion, this form of influence extends far beyond deliberate attempts by parents to inculcate rudimentary notions of financial budgeting in their children, through the use of an allowance for instance. The appropriate channel of influence may be more akin to what Hess and Torney (1967) have termed anticipatory socialization: the acquisition of attitudes and values about adult roles that have only limited relevance for the child but serves as a basis for subsequent adult behavior. In an interesting and provocative essay, Brim (1966) views the socialization of an individual as a series of complex interpersonal relationships embedded in that individual. At the cost of some simplification, we might interpret this as stating that a particular personality is nothing more than the weighted combination of other personalities in the “cognitive neighborhood” of the individual in question.

While staying away from the provocative analogies with interactive systems, we do notice that our model of backward discounting can be re-interpreted as a special case of this formulation. We can interpret a person’s future self at date $T(t)$ as representative of a “shadow parent’s” preference. Therefore, the weight $1 - \alpha$ on this “future self” can be interpreted as a parent’s influence on the child’s consumption planning.

Turning, now, to the specifics of our formulation, the backward discounting model is actually one of the simplest ways in which a shadow parent situated at age 65, say, might disagree with the current self situated at age 30. We assume that all selves have the same preference functional, but that they discount both their future and their past. This gives rise to a minimal but (in our opinion) cogent form of disagreement: the two selves will surely disagree on intertemporal decisions to be taken at intermediate ages. Parental influence refers to a state of affairs in which the shadow parent’s preferences have been partly internalized by the current self.

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6 As Ward (1974) describes it, such influence might be embodied in “implicit often unconscious learning for roles which will be assumed sometime in the future”.

7 Our assumption that these two discount factors are the same is purely for convenience and affects none of the substantial points of the paper.
The shadow parent in this model can also be replaced by “role models.” People look up to their role models, who presumably are older in most cases. We argued earlier that people discount past utilities. Hence, these role models most likely prescribe plans which are consistent with preferences arising from backward discounting. Therefore, these preferences can served as the “future self” in the model of the previous discussion, and the weight of $1 - \alpha$ on the “future self” measures the influence of these role models.

4. Other Applications of Backward Discounting

A basic premise of our model is that economic agents discount time both forward and backward. The former phenomenon is familiar, and is often interpreted as the degree of impatience. The latter is less familiar, or at least little-used in economics. It may be interpreted as the weakness of memory.

It is obvious why backward discounting does not (and should not) occupy the same central location in economics that forward discounting does. The latter applies to decisions that are to be made; the former (often) applies to the evaluation of choices which have been made in the past. The past cannot be changed, so that backward-discounting is accordingly less central in the theory of decision-making.

This argument is problematic from two points of view. First, even if the past cannot be changed, it still affects perceptions of happiness. Backward discounting suggests that the past will always be viewed through the opposite end of the telescope. This suggests that feelings of regret may be more endemic than we think. Indeed, if this subsequent view from the “opposite end” is not fully internalized at the time of decision-making, the model consistently predicts regret — an observation that may be of some use in psychology.

Second, future valuations of experience may influence the way in which we make current decisions. When we explore the implications of this hypothesis, the issue of backward discounting acquires new importance.

There are several situations in which “backward valuations” might influence “forward decisions”. This paper is about one such situation, in which a future self’s utility from a sequence of decisions is taken into account, knowing that this future self will be looking back on these decisions, and hence discounting backward. But there are other situations, and the purpose of this section is to summarize some of them.
4.1. **Social Decision-Making.** As Caplin and Leahy (2004) have observed, the phenomenon of backward discounting can give rise quite naturally to an argument for greater patience on the part of a social planner, compared to the discount factor of the agents in the economy. Suppose that generation \( \tau \) derives lifetime utility \( V_\tau \) from a consumption stream \( \{c_t\} \). A social planner might want to choose \( \{c_t\} \) to maximize some weighted sum of these generational utilities,

\[
\lambda_\tau V_\tau,
\]

where the \( \lambda \)'s are positive weights that sum to unity. Assume that each generation exhibits a mixture of backward and forward discounting, so that:

\[
V_\tau(\{c_t\}) = \sum_{s=\tau}^{\infty} \beta_1^{s-\tau} u(c_s) + \sum_{s=0}^{\tau-1} \beta_2^{s-\tau} u(c_s),
\]

where \( \beta_1 \) is the forward rate and \( \beta_2 \) is the backward rate. The it is easy to see that the planner will exhibit a degree of patience that exceeds the patience implied in \( \beta_1 \). [To be sure, there are time-consistency problems as well, but that isn’t the main point here.]

4.2. **Case-Based Decision Theory.** Gilboa and Schmeidler (1995) have introduced case-based decision theory, in which new situations of decision-making must be studied by studying parallels within the realm of one’s own experience. They propose a framework in which such parallel cases are taken as primitives, and seek to derive (axiomatically) decision rules which prescribe a best course of action depending on past performance in parallel cases. They note that both the degree of “similarity” to the case at hand, as well as the utility derived from the similar case, play a role in this axiomatization.

This is a setting in which — for decision problems that involve time — backward discounting arises in a natural measure of “utility from parallel cases”. Because past experience is used as a guide to decision-making for the future, case-based decision theory is a natural setting for backward discounting to play a prominent role.

One implication of this kind of reasoning is that we may actually behave more patiently in real life than our “natural inclinations” would have us behave, an observation that goes well with the rest of the theme of this paper.

4.3. **Evolutionary Models.** The last observation also applies to evolutionary situations in which “success” or “fitness” is measured by some discounted sum of intertemporal rewards, where measurement takes place at the “end of the day”. This might be particularly apt in the context of evolutionary biology, in which fitness (over some intertemporal setting) is determined by the end-state value of some variable. Concentration on
such end-state outcomes is equivalent to a very strong form of backward discounting in which the discount factor is set equal to zero.

Alternatively, in social or cultural contexts, “success” may be defined by the perceived lifetime rewards to an individual (or a role model, or a way of life) at the end of the process. If these perceptions form the basis for cultural selection, then agents (or behavior patterns) that assign heavier weight to future consumption would be the winners; in other words, agents (or cultural modes) that promote harder work and more savings when young are more likely to survive.

To be sure, there is no guarantee of such an outcome. It would all depend on which age groups (or which life stages) are considered natural points of success evaluation. These “natural” points are themselves subject to evolutionary or cultural pressures, leading to a deeper level of recursive analysis that is beyond the scope of this paper.

4.4. Elections. We end with a provocative example, not one we necessarily believe in fully, but one that is interesting and far from being obviously false. Consider elections and re-elections.

Suppose that a person can stay in office for at most two terms, and each term lasts for $N$ periods. To simplify matters, assume that an elected officer’s task is to provide a plan of allocating consumption over these $N$ periods and commit to it. Consider the first term. Let the total assets be $A$, the interest rate be $r$, $c_i$, $i = 1, ..., N$, be period $i$’s consumption, and $u(\cdot)$ be a concave utility function. Let $V = \sum_{n=1}^{N} \delta^{n-1} u(c_n)$ be the conventionally discounted total utility. A candidate’s probability of being elected is then a function of $V$.

It is reasonable to assume that the probability of being re-elected for a second term depends on the incumbent’s history. At the re-election date, voters may recall their past consumption using backward discounting. Let $V_b = \sum_{n=1}^{N} \delta^{N-n} u(c_n)$ be the backward discounted total utility of the first term. A candidate then maximizes the probability of elected and re-elected given by the hypothetical function $P(V, V_b)$, which is increasing in both of its arguments. For simplicity, assume that $\delta = \frac{1}{1+r}$.

It is straight-forward to verify that $c_1 = \cdots = c_N = A / (1 + \delta + \cdots + \delta^{N-1})$ maximizes $V$, subject to budget constraint $\sum_{n=1}^{N} \delta^{n-1} c_n = A$. That is, if a candidate does not aim for re-election, or, if the voters discount utilities in the conventional way, a plan of constant consumption over time would be proposed.
Maximizing $P(V, V_b)$ subject to the same budget constraint implies the following equation: $P_1 \delta^{n-1} u'(c_n) + P_2 \delta^{N-n} u'(c_n) = \lambda \delta^{n-1}$, where $P_1 = \frac{\partial P(V, V_b)}{\partial V} > 0$, $P_2 = \frac{\partial P(V, V_b)}{\partial V_b} > 0$, and $\lambda$ is the Lagrange multiplier. Therefore,

$$u'(c_n) = \frac{\lambda}{P_1 + P_2 \delta^{N-2n+1}}.$$

As the right hand side of the above equation is decreasing in $n$, and $u(\cdot)$ is a concave utility function, we have the following relationship: $c_1 < c_2 < \cdots < c_N$. That is, consumption are increasing over time, since later consumption matter more to the voters at time of re-election.

If an elected candidate does not need to keep his promise, he would propose a plan of constant consumption to maximize $V$. But once elected, he would maximize $V_b$ by reducing the consumption of early periods and increasing the consumption of later periods, so that his probability of being re-elected is maximized. The exact numbers can be calculated by setting $P_1 = 0$ in the above derivations. We can easily see that it makes the consumption profile even steeper, and thus reinforces the effects of re-election. This cannot happen in a model with conventional discounting.

5. Summary

In this paper, we introduce a framework in which lifetime individual felicities are derived from both present and past consumption streams. Each of these streams is discounted, the former forward in the usual way, the latter backward. Backward discounting is generally experienced at the end of an experience, with an individual “looking back” on a sequence of consumptions. Forward discounting is applied to anticipated consumption in the future.

We have argued that these two notions of discounting can interact in novel and interesting ways. For instance, a parent and a child may have exactly the same preference structure, disagreeing only on account of the time period where they are located. In effect, a parent takes actions to maximize the utility of her child viewed from the vantage point of the parent’s own age, thereby discounting more heavily, say, the efforts that go into learning, while discounting more lightly the later rewards that might be reaped on the labor market. (The child, who looks forward, does just the opposite.)

Similar considerations might arise for governments that make populist expenditures at the end of an election cycle. They know that the public might be “looking backward” to evaluate the government, discounting distant experiences more heavily while valuing
recent outcomes. More generally, such considerations will arise whenever an agent (organization, individual) is evaluated by another: the latter may place greater weight on the agent’s more recent activities. We have argued for similar effects in situations where case studies are used to inform future decisions, or in situations of cultural selection in which end states are viewed as ultimate measures of evolutionary success.

One application plays a prominent role in this paper, and this concerns an individual who makes consumption-savings decisions in a standard life-cycle model. That individual is assumed to have preferences that place weight on his current self and also some weight on some “terminal self,” say a self located at retirement age and looking back his life. Thus consumption programs are evaluated according to some weighted average of his own felicity (as perceived at date $t$) and that of a “future self” at some date $T > t$. This simple model can be used, among other things, to show that parental influence (a form of “future self”) may have a positive impact on savings, that individuals may exhibit impatience across alternatives that are positioned in periods adjacent to the present, but patience across similar choices positioned in the more distant future, that such impatience is attenuated as an individual grows older, and that lifetime choice plans are generally time-inconsistent.

Our paper lies quite far from a general and comprehensive study of backward discounting. But we hope that it takes a small step in introducing the reader to the potentially important aspects of such a study.

6. APPENDIX

Here is the proof of Observation 1. Let $D = \delta^{N-T+1}$, $x(t) = \delta^{T-t}$, $f(x) = x^2/\delta^T$, and

$$g(x) = \frac{\delta(1-x) + 1-D|x}{(1-Dx)\delta^T} = \frac{x}{D\delta^{T-1}} + \frac{(1-D+\delta-\frac{\delta}{D})x}{(1-Dx)\delta^T}.$$ 

Then

$$\theta_t = \frac{\alpha + (1-\alpha)f(x(t))}{\alpha + (1-\alpha)g(x(t))} = h(x(t)).$$

It is straightforward to verify that $1-D+\delta-D = (1-\delta^{N-T})(\delta-\delta^{T-N}) < 0$. Therefore, $g''(x) < 0$. Furthermore, $h'(x) = 0$ implies that $f'(x)/g'(x) = h(x)$, which in turn implies that $g'(x) > 0$ at $h'(x) = 0$. Making use of all of the above, and evaluating $h''(x)$ at $h'(x) = 0$, we obtain $h''(x) > 0$. This indicates that $h(x)$ has at most one stationary point, and if that exists, $h(x)$ is decreasing on the left-hand side of that point and increasing on the right-hand side.
Tedious calculations show that at $x = \delta T$ (i.e., at $t = 0$), $f(x) < g(x)$; at $x = 1$ (i.e., at $t = T$), $f(x) = g(x)$. Therefore, $h(x(0)) < h(x(T)) = 1$. Furthermore, $f'(x)g(x) - f(x)g'(x) > 0$ at $x = \delta T$. Hence,

$$h'(x) = \frac{1 - \alpha}{[\alpha + (1 - \alpha)g(x)]^2} \{f'(x)g(x) - f(x)g'(x) + \alpha[f'(x) - g'(x) - f'(x)g(x) + f(x)g'(x)]\}$$

is positive at $\alpha = 0$, and possibly turns negative for larger $\alpha$’s if $f'(x) - g'(x) - f'(x)g(x) + f(x)g'(x) < 0$. The initial slope of $h(x)$ determines whether $h(x)$ is U-shaped or always increasing. If the initial slope is negative, then it is U-shaped. Otherwise, it is always increasing.

Since $x = x(t)$ is an increasing function of $t$, the same property holds for $\theta_t$. 

References


INTERNATIONAL MONETARY FUND (2001b), *World Economic Outlook, October 2001*.


