In these notes we provide some supplementary results and discussion not included in the main text of the paper. In particular, we provide a proof of the existence of a Markovian equilibrium, the proof of Proposition 3, some examples for three-player games and further discussion of the efficiency result. We also provide the proof of the four player inefficiency example and construct an equilibrium with an inefficient (globally) absorbing state.

1 Existence of Markovian Equilibrium

**Theorem.** Suppose that the set of states is finite. Then there exists a stationary Markov equilibrium.

**Proof.** For any finite set $A$ let $\Delta(A)$ stand for the set of all probability measures on $A$. For any state $x$, let $H(x)$ be the set of all $(S, y)$ pairs such that $S \in S(x, y)$. Restrict attention to states $x$ such that $H(x) \neq \emptyset$.\footnote{That is, there is some state $y$ and some approval committee $S$ that can approve a transition from $x$ to $y$. For all those $x$ for which $H(x) = \emptyset$, there is no need to specify any strategies for the players.} For any $i$, let $H(x, i)$ be a subset of $H(x)$ constrained by the further requirement that $i \in S$ for every $(S, y) \in H(x, i)$. Define

$$
\Sigma_i \equiv \prod_x \{\Delta(H(x)) \times [0, 1]^{H(x,i)}\}
$$
Then \( \Sigma \) may be viewed as the collection of \( i \)'s Markovian strategies (indeed, those for which \( i \)'s response does not depend on the identity of the proposer). So \( \Sigma \equiv \prod_i \Sigma_i \) is the collection of all Markovian strategy profiles \( \sigma \). Clearly, \( \Sigma \) is identifiable with a compact convex subset of some Euclidean space.

Use the notation \( V_i(x, \sigma) \) instead of \( V_i^\sigma \) for value functions. It is easy to see that \( V_i(x, \sigma) \) is continuous in \( \sigma \). Now we construct a suitable fixed-point mapping.

Fix some strategy profile \( \sigma \). For each person \( i \) and each state \( x \), define

\[
B(i, x, \sigma) \equiv \arg\max_{(T, z) : T \in S(x, z)} \left[ \Lambda_i(x, z, T)V_i(z, \sigma) + (1 - \Lambda_i(x, z, T))V_i(x, \sigma) \right]
\]

where \( \Lambda_i(x, z, T) = \prod_{j \in T \setminus i} \lambda_j(x, z, T) \) is the probability of the proposal being accepted under the profile \( \sigma \). Note that \( \Delta(B(i, x, \sigma)) \) is the set of “best-response” proposals by \( i \) at state \( x \), given \( \sigma \). Define

\[
M_i(\sigma) \equiv \prod_x \Delta(B(i, x, \sigma)).
\]

Now for acceptance-rejection decisions. For every state \( x \) and \( (S, y) \in H(x, i) \), collect all values of \( \lambda_i(x, y, S) \) such that (a) \( \lambda_i(x, y, S) = 1 \) if \( V_i(y, \sigma) > V_i(x, \sigma) \), (b) \( \lambda_i(x, y, S) = 0 \) if \( V_i(y, \sigma) < V_i(x, \sigma) \), and (c) \( \lambda_i(x, y, S) \in [0, 1] \) if \( V_i(y, \sigma) = V_i(x, \sigma) \). Define \( L_i(\sigma) \) to be the product of these collections as \( (y, S) \) goes over all elements of \( H(x, i) \) and then as \( x \) ranges over all states. These are the best-response accept-reject decisions by \( i \), given \( \sigma \).

To complete the construction, define \( R_i(\sigma) \equiv M_i(\sigma) \times L_i(\sigma) \). It is easy to see that for each \( \sigma \), \( R_i(\sigma) \) is a nonempty, compact convex subset of \( \Sigma_i \), and that it is upperhemicontinuous in \( \sigma \). Then the product correspondence \( R(\sigma) \equiv \prod_i R_i(\sigma) \) has all the same properties, and maps from \( \Sigma \) to \( \Sigma \). By Kakutani’s fixed point theorem, the correspondence has a fixed point which can easily be seen to be an equilibrium.

2 Further Discussion of the Efficiency Theorems

2.1 A Compact Set of States

Here we prove that our efficiency result extends to the case of a compact set of states, with the additional restriction that the proposer protocol is deterministic.

**Proposition 3.** Suppose that every individual is benign, the proposer protocol is deterministic and the set of states is compact. Then in characteristic function games with permanently binding agreements, every limit payoff of every pure strategy equilibrium is efficient.

**Proof.** We first provide another proof of Proposition 1. Given the restriction to a deterministic protocol and pure strategies, there is no underlying source of randomness. Therefore,
we may write:

\[ V_i^\sigma(h_t) = (1 - \delta)u_i(x_t) + \delta V_i^\sigma(h_{t+1}) \]

for all \( i \in N \). It is still the case that \( V_i^\sigma(h_{t+1}) \geq u_i(x_t) \), which again implies that \( V_i^\sigma(h_t) \geq u_i(x_t) \). We also claim that

\[ V_i^\sigma(h_{t+1}) \geq V_i^\sigma(h_t) \]

for all histories \( h_t \), for if not, then for some history \( V_i^\sigma(h_{t+1}) < V_i^\sigma(h_t) \), which, given the definition of the value function implies:

\[ V_i^\sigma(h_t) < (1 - \delta)u_i(x_t) + \delta V_i^\sigma(h_t) \]

which cannot be true. Therefore, we know that the equilibrium value function is monotonically increasing.

Of course, since the set of states is compact, equilibrium values must be contained in a compact set. Moreover, since values are nondecreasing over histories, we know that it must converge to some limit \( V_i^\sigma \); but this implies that \( u_i(x_t) \) also converges to \( u_i^\sigma \) and the equilibrium is absorbing. Indeed, it is easy to see that \( V_i^\sigma(h_t), V_i^\sigma(h_{t+1}) \to V_i^\sigma \) so it must be that \( V_i^\sigma = u_i^\sigma \).

We now show that in any equilibrium, convergence must occur in finite time. Notice that \( V_i^\sigma(h_t) \geq \max\{u_i(x_t), V_i^\sigma(h_{t-1})\} \). Therefore, on the equilibrium path we know that

\[ V_i^\sigma(h_t) \geq \max\{u_i(x_t), u_i(x_{t-1}), \ldots, u_i(x_0)\} \tag{1} \]

Given (1) and that the set of payoffs within a given coalition is Pareto efficient with respect to that coalition, we know the following:

(a) If any coalition structure is visited more than once with distinct payoffs for at least two players, it cannot be absorbing.

(b) Convergence to \( u^* \) must be from below. That is, for all \( t \), \( u(x_t) \leq u^* \).

We now show that convergence must occur in finite time. For each coalition structure \( \pi \), let \( U(\pi) \subset \mathbb{R}^N \) denote the set of feasible payoffs, and let \( B(u, \epsilon) \) denote an open ball around \( u \) with radius \( \epsilon \). First suppose that there exists some \( \epsilon > 0 \) small enough such that \( u^* \in B(u^*, \epsilon) \cap U(\pi') \neq \emptyset \) for a unique coalition structure, \( \pi' \). In this case, it is a direct result of (a) that convergence must occur in finite time.

Otherwise, for all \( \epsilon > 0 \), there is a set of coalition structures \( \Pi(\epsilon) = \{\pi_1, \ldots, \pi_K\} \), with \( K > 1 \), such that \( u^* \in B(u^*, \epsilon) \cap \bigcap_{k=1}^{K} U(\pi_k) \neq \emptyset \) and for all \( \pi \in \Pi(\epsilon) \), there exists \( t \geq T(\epsilon) \) such that \( \pi(x_t) = \pi \).\(^2\) Take the largest such set \( \Pi(\epsilon) \) (which surely exists since there are only

\(^2\)That is, along the equilibrium path, the coalition structure \( \pi \) is actually visited, and \( u(x_t) \in B(u^*, \epsilon) \).
By assumption, we have that for all $\pi \in \Pi(\epsilon)$, $u^* \in U(\pi)$ and that there exists a state $x_t$ along the equilibrium path such that $\pi(x_t) = \pi$ and $u(x_t) \leq u^*$. If $u(x_t) = u^*$, then convergence must have occurred (and in finite time). If $u(x_t) < u^*$, it contradicts the Pareto efficiency of the payoff set within a given coalition structure since $u(x_t), u^* \in U(\pi(x_t))$. Thus convergence must be in finite time.

To complete the proof, we must show that $u^*$ is efficient. The proof of this makes use of the same benignness argument as in the main text of paper and is, therefore, omitted.

2.2 Observable Behavior Strategies

Proposition 2 fails to hold if behavior strategies are used by the players with publicly observable randomization devices. Consider the following game with four states.

$$
\begin{align*}
 x_1 : & \quad u(x_1) = (2, 1, 1) \\
 x_2 : & \quad u(x_2) = (2.5, 2, 2) \\
 x_3 : & \quad u(x_3) = (2, 3, 1) \\
 x_4 : & \quad u(x_4) = (2, 1, 3)
\end{align*}
$$

We assume that transitions from any state require the consent of all players, and that player 1 proposes whenever the state is $x_1$. We construct a history dependent equilibrium such that the sample path $\{x_1, x_1, \ldots\}$ has strictly positive probability.

Clearly, since all players’ consent is required for any transition, $x_2$, $x_3$ and $x_4$ must be absorbing. Recall that behavior strategies are observable and can be conditioned upon, and player 1 proposes with probability 1 from $x_1$. Let $p(n) = \frac{1}{n+1}$ and consider a $t$-period history of the form: $h_t = \{(x_1, p(i))\}_{i=1}^t$. That is, in each period $i$, player 1 proposed $x_2$ with probability $p(i)$ and $x_1$ with probability $1 - p(i)$, but the realisation was $x_1$. Consider the following strategies:

- Player 1 uses the randomisation device $p(t)$ in each period to make his proposal, assuming $p(i)$ has been used for all $i < t$, otherwise, he proposes $x_3$ or $x_4$ as required below.

- If player 1 uses $p(t)$ as his randomisation device and $x_2$ is realised, then both players 1 and 2 accept the offer. If $x_1$ is realised, they may either accept or reject. However, if $p(t)$ is not used, player 2 rejects the proposal. If player 2 accepts, then player 3 rejects.

---

3 Once $u^*$ is reached, no player would ever accept a transition giving him/her a lower one-period payoff. This follows because the limit payoff is $u^*$ and so the player will never be compensated for accepting such a transition.

4 To be sure, we need to expand the notion of a history for this to make sense.
In the first history after \( p_t \neq p(t) \) and a rejection by player \( i, i = 2, 3 \), player 1 offers a transition to \( x_{i+1} \) and the offer is accepted by both players. If it is rejected by \( j \neq i \), player 1 continues offering \( x_{i+1} \), while if it is rejected by player \( i \), player 1 switches and offers \( x_{j+1} \).

It is easy to see that these strategies constitute an equilibrium for \( \delta \) large enough. Moreover, since \( \sum_{t=1}^{\infty} \frac{1}{(t+1)^2} < \infty \), we have that \( \prod_{t=1}^{\infty} (1 - p(t)) > 0 \), which means that the path \( \{x_1, x_1, \ldots\} \) has strictly positive probability and that along this path, each player’s expected value converges only in the limit.

## 3 Other Examples For Three-Player Games

In the examples below, if a coalition structure is omitted, it means that either every player obtains an arbitrarily large negative payoff or there is some legal impediment to the formation of that coalition structure. In all of the examples of this section, we assume minimal approval committees; for example, from the singletons, players 1 and 2 can approve a transition to any state \( y \) such that \( \pi(y) = \pi_1 \).

### 3.1 More on Inefficiency

One response to the inefficiency example of Section 4.1.1 in the main text is that the inefficient state described there will never be reached starting from the singletons. Setting the initial state to the singletons has special meaning: presumably this is the state from which all negotiations commence. However, this is wrong on two fronts (at least for Markov equilibria) as we now show.

#### 3.1.1 Coordination Failures

Coordination failures, leading to inefficiency from every initial state, are a distinct possibility, even in three player games. Consider the following:

\[
\begin{align*}
  x_0 : \pi(x_0) &= \pi_0, & u(x_0) &= (2, 2, 2) \\
  x_1 : \pi(x_1) &= \pi_1, & u(x_1) &= (-1, 1, 1) \\
  x_2 : \pi(x_2) &= \pi_2, & u(x_2) &= (1, -1, 1) \\
  x_3 : \pi(x_3) &= \pi_3, & u(x_3) &= (1, 1, -1)
\end{align*}
\]

**Result 1.** Suppose that everyone proposes with equal probability at every date. Then, for \( \delta \in [\frac{3}{5}, 1) \), there is an MPE in which \( x_i \) is absorbing, and from \( x_0 \), there is a transition to \( x_i \) with probability \( \frac{1}{3} \) for \( i = 1, 2, 3 \).
3.1.2 Convergence to Inefficiency From The Singletons

Consider the following example, which is a variation on the “failed partnership” example of Section 4.1.1.

\[
\begin{align*}
  x_0 : & \quad \pi(x_0) = \pi_0, \quad u(x_0) = (5, 5, 5) \\
  x_1 : & \quad \pi(x_1) = \pi_1, \quad u(x_1) = (0, 6, 8) \\
  x_2 : & \quad \pi(x_2) = \pi_2, \quad u(x_2) = (3, 0, 10) \\
  x_3 : & \quad \pi(x_3) = \pi_3, \quad u(x_3) = (4, 4, 0)
\end{align*}
\]

**Result 2.** For any history-independent proposer protocol such that at \( x_0 \) each player has strictly positive probability of proposing, there exists \( \delta \in (0, 1) \) such that if \( \delta \geq \delta \), all stationary Markovian equilibria involve a transition from \( x_0 \) to \( x_3 \) — and full absorption into \( x_3 \) thereafter — with strictly positive probability.

**Proof.** Let \( \alpha = (\alpha_1, \alpha_2, \alpha_3) \in \text{int}(\Delta) \) denote the proposers’ protocol at \( x_0 \). First notice that in every equilibrium \( x_1 \) and \( x_2 \) must be absorbing. The states \( x_1 \) and \( x_2 \) give players 2 and 3, respectively, their unique maximal payoff. Moreover, at \( x_1 \) (resp. \( x_2 \)) player 2 (resp. player 3) has veto power over any transition. Second, in every equilibrium, \( x_0 \) cannot be absorbing. This follows because players 2 and 3 can always initiate a transition to \( x_1 \) and earn a higher payoff.

We now proceed with the rest of the proof. First, we rule out a “cycle” by proving the following: If there is a positive probability transition from \( x_0 \) to \( x_3 \), then \( x_3 \) must be absorbing. Indeed, suppose not. Then for \( i = 1, 2 \), \( V_i(x_0) = V_i(x_3) = 4 \). But then, from \( x_0 \), player 1 will always reject a transition to \( x_2 \), which means that \( V_2(x_0) \geq 5 \), a contradiction.

Next suppose that the probability of reaching \( x_3 \) from the singletons is zero. Observe that \( V_1(x_0) \leq 3 \), for if not, \( x_1 \) is the only absorbing state reachable from \( x_0 \), implying that \( V_1(x_0) \to 0 \) for \( \delta \) sufficiently high, a contradiction. Similarly, \( V_3(x_0) \leq 8 \), for if not, \( x_2 \) is the only absorbing state reachable from the singletons. But then for \( \delta \) sufficiently high, \( V_2(x_0) \leq 4 \), implying that players 1 and 2 would initiate a transition to \( x_3 \), a contradiction. Finally, observe that since \( x_3 \) is not reached with positive probability, it must be that \( V_2(x_0) \geq 4 \), since otherwise, 1 would offer \( x_3 \) and it would be accepted.

Let \( p_i \) denote the probability of a transition from \( x_0 \) to \( x_i \) for \( i = 0, 1, 2 \). By assumption, \( p_3 = 0 \) and we have just shown that \( p_1, p_2 > 0 \). Given \( p_i \), write the equilibrium value functions and take the limit as \( \delta \to 1 \) to obtain:

\[
\begin{align*}
  \bar{v}_1(x_0) &= \frac{3p_2}{1-p_0} \leq 3 \\
  \bar{v}_2(x_0) &= \frac{6p_1}{1-p_0} \geq 4 \\
  \bar{v}_3(x_0) &= \frac{8p_1+10p_2}{1-p_0} \leq 8
\end{align*}
\]
From the third equation in (2), we see that \( p_2 = 0 \), which then implies that the first equation is satisfied with strict inequality. Therefore, player 1 strictly prefers to propose \( x_2 \), and the offer will be accepted by player 3. Hence, \( p_2 > \alpha_1 > 0 \), a contradiction. It then follows that for \( \delta \) sufficiently high the same conclusion may be drawn.

### 3.2 Cyclical Equilibria

Next, equilibrium \textit{cycles} become a distinct possibility:

\[
\begin{align*}
  x_0 & : \pi(x_0) = \pi_0, \quad u(x_0) = (1, 1, 1) \\
  x_1 & : \pi(x_1) = \pi_1, \quad u(x_1) = (0, 2, 1) \\
  x_2 & : \pi(x_2) = \pi_2, \quad u(x_2) = (\frac{1}{2}, 4, 1)
\end{align*}
\]

**Result 3.** Suppose that everyone proposes with equal probability at every date. Then there is an equilibrium with the following transitions:

\[
x_0 \rightarrow_{\frac{2}{3}} x_1 \rightarrow_{1} x_2 \rightarrow_{\frac{2}{3}} x_0
\]

### 3.3 Dynamic Inefficiency In Every Equilibrium

Though we did not formally prove this for characteristic functions, every Markovian equilibrium must exhibit full \textit{dynamic} efficiency from \textit{some} initial state. This is no longer true for games with externalities:

\[
\begin{align*}
  x_0 & : \pi(x_0) = \pi_0, \quad u(x_0) = (1, 1, 1) \\
  x_1 & : \pi(x_1) = \pi_1, \quad u(x_1) = (10, 0, 0) \\
  x_2 & : \pi(x_2) = \pi_2, \quad u(x_2) = (0, 10, 0) \\
  x_3 & : \pi(x_3) = \pi_3, \quad u(x_3) = (0, 0, 10)
\end{align*}
\]

If \( x_i, i = 1, 2, 3 \) were absorbing, then for \( j \neq i \), \( V_j(x_i) = 0 \). However, notice that in every Markovian equilibrium, for all \( i = 1, 2, 3 \), \( V_i(x_0) \geq 1 \). Therefore, \( j \) must accept a proposal from \( x_i \) to \( x_0 \), hence a profitable deviation exists. Finally, it can be shown that any cyclical Markovian equilibrium must necessarily spend time at \( x_0 \). We have therefore proved:

**Result 4.** Suppose that everyone proposes with equal probability at every date. Then every Markovian equilibrium exhibits dynamic inefficiency from every initial state.
4 Strong Inefficiency in Four Player Games

Consider the following four player game:

\[
\begin{align*}
\pi(x_1) &= \{(1,2), (3), (4)\}, \quad u(x_1) = (4, 4, 4, 4) \\
\pi(x_2) &= \{(1), (2,3), (4)\}, \quad u(x_2) = (5, 5, 5, 5) \\
\pi(x_3) &= \{(1,2), (3,4)\}, \quad u(x_3) = (0, 0, 10, 10) \\
\pi(x_4) &= \{(1,2), (3,4)\}, \quad u(x_4) = (2, 2, 2, 2)
\end{align*}
\]

and recall from the main text

**Observation 2.** For \(\delta\) sufficiently high, every stationary Markov equilibrium is inefficient starting from any initial state.

Before proceeding with the proof of Observation 2, we first allay any concerns about existence (or lack thereof) and provide an example displaying inefficiency. Figure 1 graphically depicts one such set of equilibrium transitions. In words, \(x_1\) is absorbing; from state \(x_2\), players 1 and 2 initiate a transition back to \(x_1\), while players 3 and 4 initiate a transition to \(x_3\); from \(x_3\) players 1 and 2 implement a transition to \(x_4\) when they propose, while players 3 and 4 leave the state unchanged when they propose; finally, from \(x_4\), all players propose a return to \(x_1\).

To verify that this description constitutes an equilibrium, begin with state \(x_1\). Obviously players 3 and 4 do not benefit from changing the state to \(x_4\), which is all they can unilaterally do. Players 1 and 2 can (bilaterally) change the state to \(x_2\), by the presumed minimality of approval committees. If they do so, the subsequent trajectory will involves a stochastic path back to \(x_1\).\(^5\) Some fairly obvious but tedious algebra reveals that the Markov value function

\(^5\)We are arguing in the spirit of the one-shot deviation principle, in which the putative equilibrium
$V_i(x, \delta)$ satisfies

$$V_i(x_2) = 5 - 3\delta + \frac{\delta^2(1 + \delta)}{2 \left(1 - \frac{\delta}{2}\right)}$$

for $i = 1, 2$. This value converges (as it must) to that of the absorbing state — 4 — as delta goes to 1, but the important point is that the convergence occurs “from below”, which means that $V_i(x_2, \delta)$ is strictly smaller than $V_i(x_1, \delta) = 4$ for all delta close enough to 1. Perhaps more intuitively but certainly less precisely, the move to state $x_2$ starts off a stochastic cycle through the payoffs 5, 0 and 2 before returning to absorption at 4, which is inferior to being at 4 throughout. This verifies that players 1 and 2 will relinquish the opportunity at $x_1$ to switch the state to $x_2$. It also proves that once at state $x_2$, players 1 and 2 will want to return to the safety of $x_1$ if they get a chance to move.

On the other hand, players 3 and 4 will want to move the state from $x_2$ to $x_3$. Proving this requires more value-function calculation. A second round of tedious algebra reveals that

$$V_i(x_3, \delta) - V_i(x_2, \delta) = 5 - 6\delta + \delta^2$$

for $i = 3, 4$. This difference vanishes (as it must) as $\delta$ approaches 1, but once again the important point is that the difference is strictly positive for all $\delta$ close to 1 (indeed, for all $\delta$), which justifies the move of 3 and 4.

That 1 and 2 must want to move away as quickly as possible from state $x_3$, and 3 and 4 not at all, is self-evident. That leaves $x_4$. At this state players 3 and 4 receive their worst payoffs, and will surely want to move to $x_1$, and indeed, players 1 and 2 will want that as well. Our verification is complete.

With the issue of existence of an inefficient equilibrium now dispelled, we now turn our attention to Observation 2, the proof of which proceeds in a number of steps.

**Step 1: $x_3$ and $x_4$ are not absorbing.**

It is easy to see that $V_i(x_4) \geq 2$ for $i = 1, 2$. Moreover, since players 1 and 2 can initiate a transition from $x_3$ to $x_4$, $x_3$ is easily seen to be not absorbing. Similarly, $V_j(x_1) \geq 4$ for $j = 3, 4$; therefore, since players 3 and 4 can achieve $x_1$ from $x_4$, $x_1$ is not absorbing.

**Step 2: $x_2$ absorbing implies $x_2$ is globally absorbing.**

---

6 We verify this by differentiating $V_i(x_2, \delta)$ with respect to $\delta$ and evaluating the derivative at $\delta = 1$.

7 Because we’ve developed the state space model at some degree of abstraction, we’ve allowed any player to make a proposal to any coalition, whether or not she is a member of that coalition. This is why players 1 and 2 ask 3 and 4 to move along. Nothing of qualitative import hinges on allowing or disallowing this feature. The transition from $x_4$ back to $x_1$ would still happen, but more slowly.

strategies are subsequently followed. Even though the one-shot deviation principle needs to be applied with care when coalitions are involved, there are no such dangers here as all coalitional members have common payoffs.
Suppose that \( x_2 \) is absorbing. Then clearly from \( x_1 \), players 1 and 2 would induce \( x_2 \). Moreover, since \( x_3 \) and \( x_4 \) are not absorbing, if \( x_2 \) is not reached, then \( x_1 \) must be reached infinitely often. But then 1 or 2 would get a chance to propose with probability 1 and would then take the state to \( x_2 \), a contradiction.

**Step 3: \( x_2 \) cannot be globally absorbing.**

If \( x_2 \) is globally absorbing then, from \( x_2 \), players 3 and 4 can get a payoff of 10 for some period of time, by initiating a transition to \( x_3 \), followed by, at worst, 2 for one period and 4 for another period, before returning to \( x_2 \), where it will get 5 forever thereafter.\(^8\) This sequence of events is clearly better for players 3 and 4 than remaining at \( x_2 \).

**Step 4: \( x_1 \) absorbing implies \( x_1 \) globally absorbing.**

Steps 2 and 3 imply that \( x_2 \) cannot be absorbing. Moreover, Step 1 tells us that neither \( x_3 \) nor \( x_4 \) can be absorbing. In particular, from \( x_2 \) players 3 and 4 initiate a transition to \( x_3 \), from \( x_3 \) players 1 and 2 initiate a transition to \( x_4 \) and (at least) players 3 and 4 initiate a transition back to \( x_1 \). Therefore, if \( x_1 \) is absorbing, it is globally absorbing.

**Step 5: Every equilibrium is inefficient.**

First suppose that we had an equilibrium in which \( x_1 \) is not absorbing. Then from the above analysis, nothing is absorbing. Now consider \( x_2 \). If players 1 and 2 always accept an offer of a transition from \( x_1 \) to \( x_2 \), then 3 and 4 will strictly prefer to initiate a transition from \( x_2 \) to \( x_3 \): in so doing, they can achieve an average payoff of at least \( \frac{10+2+4}{3} = \frac{16}{3} > 5 \). However, it is easily seen that players 1 and 2 earn an average payoff strictly less than 4 in this case. Therefore, players 1 and 2 would rather keep the state at \( x_1 \), contradicting the presumption that \( x_1 \) was not absorbing.

The only remaining possibility is one in which players 1 and 2 are indifferent between a \( x_1 \) and \( x_2 \) and players 3 and 4 are indifferent between \( x_2 \) and \( x_3 \). If such an equilibrium were to exist, it must be that \( V_i(x_1) = V_i(x_2) = 4 \) for \( i = 1, 2 \), and \( V_j(x_2) = V_j(x_3) = 5 \) for \( j = 3, 4 \). Therefore, if such an equilibrium were to exist, it would also be inefficient since players spend a non-negligible amount of time at the inefficient states \( x_1 \) and \( x_4 \).

Thus either \( x_1 \) is the unique absorbing state or there is a sequence of inefficient cyclical equilibria depending on \( \delta_n \) such that players 1 and 2 are indifferent between \( x_1 \) and \( x_2 \) and players 3 and 4 are indifferent between \( x_2 \) and \( x_3 \).

---

\(^8\)Surely, players 1 and 2 must initiate a transition to \( x_4 \) with some positive probability; otherwise, \( x_3 \) would be absorbing (which Step 1 shows to be impossible). However, once at \( x_4 \), under the assumption that any player can propose to move to any state, and the fact that (by Step 2) from \( x_1 \) there would be an immediate transition to \( x_2 \), there is no need to even pass through the intermediate state \( x_1 \).