“Dipak Banerjee . . . will be remembered as one of the most famous teachers of economics at Presidency College.

“. . . [He was] an extraordinarily erudite person in areas he developed a liking for. It turned out that he had decided chemistry was not going to be one of those areas.”

Two areas (among many) he did not give up on:

1. Economic theory: the more abstract, the better.
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1. Economic theory: the more abstract, the better.

Choice and Order: or First Things First

By D. Banerjee

'There is certainly no obvious kind of market behaviour which can be called indifferent. How long must a person dither before he is pronounced indifferent?'—I. M. D. Little, Oxford Economic Papers, N.S., vol. 1, 1949.

There is some reason to think that the great simplifications in utility theory—achieved in the past dozen years or so—have passed a number of economists by. This is possibly due to the axiomatic presentation favoured by most workers in the field. The excellent survey of consumption theory by Professor H. S. Houthakker has not altogether solved the problem of communication: breadth of scope is often the enemy of emphasis. Moreover, this remarkably lucid survey makes heavy demands on the reader, that he see through a newly-corrected pair of lenses and focus exactly and immediately. The sort of person who can do this easily is, in all probability, one who has been there before, and possesses a fair knowledge of the terrain.
2. The *London Times* crossword:

Crooked course of a Cockney courtship [7]
Status, Intertemporal Choice and Risk

Professor Dipak Banerjee Memorial Lecture

Debraj Ray

New York University

based on joint research with Arthur Robson, Simon Fraser University
Outline

- Study payoffs that depend on relative consumption or status.
- Embed these preferences into standard growth model.
- Describe the equilibrium of such a model.
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- Study payoffs that depend on relative consumption or status.

- Embed these preferences into standard growth model.

- Describe the equilibrium of such a model.

- Particular focus today: emergence of risk-taking lotteries, occupational choice, financial markets, property speculation ...
Friedman (1953) directly addressed the efficiency of risk-taking:

“Differences in tastes [for risk] will dictate different choices from the same alternatives.

“These will be reflected most clearly [in the] allocation of resources to activities devoted to manufacturing the kind of risk attractive to individuals . . .

“The inequality of income in a society may be regarded in much the same way as the kinds of goods that are produced, as at least in part — and perhaps in major part — a reflection of deliberate choice in accordance with the tastes and preferences of the members of the society . . .”

For Friedman, all risk-taking was “deliberate” and therefore efficient

Including asymmetric treatment of downside/upside risk.
The famous Friedman-Savage (1948) utility function:

\[ u(c) \]

no risk-taking here

risk-taking here

no risk-taking here
This pushed Friedman and Savage into a different corner:

- *Increasing* marginal utility?
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**Increasing** marginal utility?

“Diminishing marginal utility plus maximization of expected utility would imply that individuals would always have to be paid to induce them to bear risk. But this implication is clearly contradicted by actual behavior. People not only engage in fair games of chance, they engage freely and often eagerly in such unfair games as lotteries.”

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This pushed Friedman and Savage into a different corner:

*Increasing* marginal utility?

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(Risky occupations and investments included, of course.)

In their paper, they launch an incisive attack on believers in diminishing marginal utility.
The Friedman-Savage rationalization for increasing marginal utility:

“A possible interpretation of the utility function . . . is to regard the [concave] segments as corresponding to qualitatively different socioeconomic levels, and the [convex] segment to the transition between the two levels.

“On this interpretation, increases in income that . . . shift [the consumer] into a new class, that give [him] a new social and economic status, yield increasing marginal utility.”

There is also the “revealed preference” defence (a bit tricky here).
Ideological agenda is clear:

- explain risk-taking as *maximizing* and so efficient behavior.
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Their justification of $u$ motivates a related agenda:

derive Friedman-Savage-like outcomes from a more primitive model in which utility depends on status;

reexamine efficient risk-taking.
This research has intrinsic value for us anyway:

- Robson’s interests in relative income and risk-taking;

- My interest in endogenous evolution of inequality;

- My interest in socially conditioned aspirations.
Utility from relative status is an old idea (see Veblen (1899) and Duesenberry (1949)).


“In the light of abundant evidence that context matters, it seems fair to say that Mr. Duesenberry’s theory rests on a more realistic model of human nature than Mr. Friedman’s. It has also been more successful in tracking actual spending. And yet, as noted, it is no longer even mentioned in leading textbooks.”
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A recent literature has begun to develop on the subject.

Intertemporal Choice with Status Payoffs
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- Continuum of dynasties of unit measure.

- Typical dynasty has initial wealth \( w \).

- Wealth divided between lifetime consumption and bequests:
  \[
  w_t = c_t + k_t.
  \]
Intertemporal Choice with Status Payoffs

- Continuum of dynasties of unit measure.
- Typical dynasty has initial wealth $w$.
- Wealth divided between lifetime consumption and bequests:
  \[ w_t = c_t + k_t. \]
- Bequests create new wealth via production function (human + physical investments):
  \[ w_{t+1} = f(k_t). \]
- Same production function for every agent.
- Assume $f$ increasing and differentiable, with $f(0) = 0$. 
$F_t = \text{cdf}$ of consumption in society at generation $t$
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\( \bar{F}_t(c) \)

\( F_t(c) \)

\( \bar{F}_t(c_2) \)

\( \bar{F}_t(c_1) \)

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\( c_2 \)
\[ F_t = \text{cdf of consumption in society at generation } t \]
Within-generation lifetime utility $u(c, s)$, where

- $c = \text{consumption and}$

- $s = \tilde{F}_t(c) = \text{status}$. 
Within-generation lifetime utility $u(c, s)$, where

- $c =$ consumption and
- $s = \bar{F}_t(c) =$ status.

Agent's objective: choose a policy to max

$$\sum_{t=0}^{\infty} \delta^t E u(c_t, \bar{F}_t(c_t))$$

where $\delta \in (0, 1)$ is the discount factor (or cross-generational tie).

(what's the $E$ for?)
All **fair randomizations** available for any wealth.

Zero profit “lottery creators” can create a fair gamble from any fixed upfront payment.

Setting up a business with risky outcomes, playing the stock market, etc.
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Setting up a business with risky outcomes, playing the stock market, etc.

No exogenous uncertainty: risk-taking decisions made endogenously.

Agent policy: a (possibly time-dependent) map from starting wealth to a randomization over wealth, then division of realizations into consumption and bequests.
Equilibrium

- Fix initial distribution of wealth $G = G_0$. 
Equilibrium

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- An equilibrium is a sequence of cdfs \( F \equiv \{F_t\} \) for consumption and \( G \equiv \{G_t\} \) for wealth, and a policy for each individual such that

  - (i) Each individual policy maximizes expected utility given \( F \).
  
  - (ii) Given \( G_t \), individual policies together generate \( F_t \) and \( G_{t+1} \).
Two worlds

- Production function is convex.

- Nonconvergent. Deterministic equilibrium.
Two worlds

- Production function is **convex**.

- Nonconvergent. Deterministic equilibrium.

- Production function is **strictly concave**.


- Paper studies both cases. Here we emphasize the second.
Convergence and Randomization

Assumption on production function (Solow, Ramsey, Loury):

\[ f \text{ smooth, increasing, strictly concave, with } \delta f'(0) > 1, \text{ and } f(k) < k \text{ for all } k \text{ large.} \]
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Assumption on utility function:

\[ u \text{ smooth, nondecreasing and concave in } c, \text{ increasing in } s \text{ with } u_s(c, s) > 0. \]

If \( u_c(c, s) > 0 \), then \( u_c(c, s) \downarrow \text{ in } c, \text{ with } u(c, 1)/c \to 0 \text{ as } c \to \infty. \)

Note 1: no assumption on curvature of \( u \) in status.

Note 2: allow for \( u \) to depend on status alone (pure status).
Quick comment on randomization:

- Will never randomize on capital bequests, only on within-generation activities that affect lifetime consumption.
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Without loss of any generality, then, work with

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where \( b_t \) is lifetime consumption “budget”.
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\[ f(k_t) = b_{t+1} + k_{t+1}, \]

where \( b_t \) is lifetime consumption “budget”.

This budget can be used by the individual to buy any fair bet.

The outcomes \( c_t \) are “realized consumption”.

Distribution of \( c_t \) over everyone is \( F_t \) at date \( t \).
A steady state is an equilibrium with $F_t$ unchanged over time.
Steady State

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- (B) The distribution of realized lifetime consumption is a particular randomization $F^*$ of $b^* \equiv f(k^*) - k^*$ (to be described).
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- (B) The distribution of realized lifetime consumption is a particular randomization $F^*$ of $b^* \equiv f(k^*) - k^*$ (to be described).

- (C) This steady state is unique in the class of all steady states in which almost all individuals have positive wealth.
Observation 1. In equilibrium, \( u(c, \bar{F}_t(c)) \) is concave for all \( t \).
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Positive measure of consumption realizations
Observation 1. In equilibrium, $u(c, \bar{F}_t(c))$ is concave for all $t$. 

Additional fair bet to be composed with original bet.
Observation 2. In a steady state with positive wealth, everyone must make the investment \( k^* \) that solves \( \delta f'(k^*) = 1 \).
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Proof.

Define $\mu(b) = u(b, F(b))$. By Observation 1, $\mu(b)$ is lifetime utility if consumption budget is $b$. 
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- Define \( \mu(b) = u(b, F(b)) \). By Observation 1, \( \mu(b) \) is lifetime utility if consumption budget is \( b \).
- Use a version of the turnpike theorem (Mitra and Ray (1984)) to show that \( k_t \) converges for everyone; say to \( k^* \).
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- First-order condition (or Euler-Ramsey equation) tells us:

  $$\mu'(b_t) = \delta f'(k_t)\mu'(b_{t+1})$$
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Convergence

- Must every equilibrium with positive wealth converge to our steady state?
Convergence

Must every equilibrium with positive wealth converge to our steady state?

Proposition. Make Assumptions [fconc] and [ugen].

Fix any initial wealth distribution, bounded, infimum wealth positive.

Then any equilibrium sequence of consumption distributions must converge over time to our unique steady state distribution.
Proof

Step 1. \( \sup_{i,j} |k_t(i) - k_t(j)| \to 0 \) and \( \sup_{i,j} |b_t(i) - b_t(j)| \to 0 \) as \( t \to \infty \).
Proof

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Step 2. Let $b_t =$ average of $b_t(i)$. Then $\sigma \equiv \limsup_t b_t > 0$. 
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Step 3. For every $\epsilon > 0$, there is $T$ such that $b_t \in [\sigma - \epsilon, \sigma + \epsilon]$ for all $t \geq T$ and all individuals.
Step 4. For $T$ large all consumption budgets fall in the same linear segment of $\mu_T(c) \equiv u(c, F_T(c))$. 
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But $V_T$ is strictly concave, so $k_T$ pinned down if LHS is the same.
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Step 6. The common value of $k_t$, $t \geq T$, must converge.

Proof slippery; omitted.
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Proof slippery; omitted.

Step 7. The limit value is $k^*$. 
Friedman-Savage (1948) revisited.

\[ F^*(c) \]

\[ u(c, F^*(c)) \]
Friedman-Savage (1948) revisited.

Indifferent? Not really.

Exogenous uncertainty.
Three Differences from Friedman-Savage

1. Phenomenon arises “naturally” when utility depends on status.

- No dependence on *ad hoc* description of preferences with varying curvature.
Three Differences from Friedman-Savage

1. Phenomenon arises “naturally” when utility depends on status.

   No dependence on *ad hoc* description of preferences with varying curvature.

2. The argument is *scale-neutral*.

   Two insulated societies two different production technologies will generally settle into two different steady states.

   *Both* the steady states will generally exhibit the Friedman-Savage property, even though overall wealths are different.
Three Differences from Friedman-Savage

3. Risk-taking is *inefficient*

(despite the fact that it is willingly taken, as in Friedman).
Three Differences from Friedman-Savage

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- Normalization: \( s = 1/2 \) for everyone under full consumption equality.
Three Differences from Friedman-Savage

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Normalization: \( s = 1/2 \) for everyone under full consumption equality.

**Proposition.** Make Assumptions \([fconc]\) and \([ugen]\), and suppose that \( u \) is strictly concave in \((c, s)\).

Then our unique steady state is Pareto-inefficient.
Three Differences from Friedman-Savage

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**Proposition.** Make Assumptions [fconc] and [ugen], and suppose that $u$ is strictly concave in $(c, s)$.

Then our unique steady state is Pareto-inefficient.

**Proof.** In steady state, lifetime expected utility is given by

$$
\int u(c, F^*(c))dF^*(c) < u \left( \int cdF^*(c), \int F^*(c)dF^*(c) \right) = u(c^*, 1/2).
$$
Final Remarks

- Concern for status in a conventional model of economic growth.

- Equilibrium involves persistent randomization and ex-post lifetime inequality.

- Generates Friedman-Savage with no reliance on changing utility curvature, and is scale-neutral.

- Friedman-Savage informally justified their utility function using relative status.

- That reformulation leads to *inefficient* risk-taking (in contrast to the main agenda of Friedman).
Situation different if the production function is *convex*

(as in Romer, Lucas, new growth theory).

Inequality does not have to be recreated. Equilibrium is deterministic in the pure-status model

(independent of curvature in $u$ or $f$!)

A theme in common with the endogenous inequality literature:

“If inequality didn’t exist, it would have to be invented.”
The End

P.S. Crooked course of a Cockney courtship [7]

Me/and/’er
Non-Convergence and Deterministic Equilibrium

Proposition.

Make the following assumptions:

(i) $u$ is pure-status (only depends on $s$).

(ii) $f$ is convex.

(iii) $G$ has full support and $u(G(w))$ is strictly concave.
Non-Convergence and Deterministic Equilibrium

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(i) \( u \) is pure-status (only depends on \( s \)).

(ii) \( f \) is convex.

(iii) \( G \) has full support and \( u(G(w)) \) is strictly concave.

Then there exists an equilibrium with deterministic policy

\[
c = (1 - \delta)w \quad \text{and} \quad k = \delta w,
\]

and equilibrium status for every individual constant over time.
Idea of Proof

- Suppose everyone uses suggested policy.

- Say one individual deviates *only* at date $t$ (Blackwell).

- Status for ever after is $G_{t+1}(w')$, where $w' =$ wealth at $t+1$.

- Then, for every initial $w$ at date $t$, chooses $k \in [0, w]$ to max .
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$$u(F_t(w - k)) + \delta V_{t+1}(f(k)) = u(F_t(w - k)) + \frac{\delta}{1 - \delta} u(G_{t+1}(f(k)))$$
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\[
\begin{align*}
 u(F_t(w - k)) + \delta V_{t+1}(f(k)) &= u(F_t(w - k)) + \frac{\delta}{1 - \delta} u(G_{t+1}(f(k))) \\
 &= u \left( G_t \left( \frac{w - k}{1 - \delta} \right) \right) + \frac{\delta}{1 - \delta} u(G_{t+1}(f(k)))
\end{align*}
\]
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$$= u\left(G_t\left(\frac{w - k}{1 - \delta}\right)\right) + \frac{\delta}{1 - \delta} u(G_t(k/\delta)).$$
Idea of Proof, contd.

\[ u(F_t(w - k)) + \delta V_{t+1}(f(k)) = u\left(G_t\left(\frac{w - k}{1 - \delta}\right)\right) + \frac{\delta}{1 - \delta} u\left(G_t\left(\frac{k}{\delta}\right)\right) \]
Idea of Proof, contd.

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Lemma. \( u(G_t(x)) \) is strictly concave for all \( t \).
Idea of Proof, contd.

\[ u(F_t(w - k)) + \delta V_{t+1}(f(k)) = u \left( G_t \left( \frac{w - k}{1 - \delta} \right) \right) + \frac{\delta}{1 - \delta} u \left( G_t \left( \frac{k}{\delta} \right) \right) \]

- **Lemma.** \( u(G_t(x)) \) is strictly concave for all \( t \).

- **Proof.** If everyone uses suggested policy, then for all \( w \),
  \[ G_{t+1}(w) = G_t \left( f^{-1}(w) / \delta \right), \]

- Use recursion and convex \( f \).
Idea of Proof, contd.

\[ u(F_t(w - k)) + \delta V_{t+1}(f(k)) = u \left( G_t \left( \frac{w - k}{1 - \delta} \right) \right) + \frac{\delta}{1 - \delta} u \left( G_t \left( k / \delta \right) \right) \]

Lemma. \( u(G_t(x)) \) is strictly concave for all \( t \).

Proof. If everyone uses suggested policy, then for all \( w \),

\[ G_{t+1}(w) = G_t \left( f^{-1}(w) / \delta \right) , \]

Use recursion and convex \( f \).

Now maximize \( k = \delta w \).
Remarks

1. Equilibrium induces *strictly concave* optimization problem
despite convexity in $f$ and no assumption on curvature of $u$.

Intuition: wealth distribution stays dispersed, no bunching.
Remarks

1. Equilibrium induces *strictly concave* optimization problem despite convexity in $f$ and no assumption on curvature of $u$.

   Intuition: wealth distribution stays dispersed, no bunching.

2. Equilibrium policy independent of wealth distribution or $u/f$.

   Same as planner using logarithmic utility and linear production.
The policy we’ve identified is (under some conditions), the *only* deterministic equilibrium.
A Converse

- The policy we’ve identified is (under some conditions), the *only* deterministic equilibrium.

- A deterministic equilibrium is

  - *regular* if at each date, some person uses a strict best response at all but possibly a countable number of wealths;
A Converse

- The policy we've identified is (under some conditions), the only deterministic equilibrium.

- A deterministic equilibrium is
  - regular if at each date, some person uses a strict best response at all but possibly a countable number of wealths;
  - smooth if every $i$ uses a sequence of differentiable policies $\{c_t^i\}$, with $\epsilon^i \leq c_t^i(w) < 1$ at all $w$ and $t$, for some $\epsilon^i > 0$. 
Converse, contd.

Proposition.

Make the following assumptions:

(i) $u$ is pure-status.

(ii) $f$ arbitrary (but smooth and increasing).

(iii) $G$ has full support.
Converse, contd.

Proposition.

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(i) $u$ is pure-status.

(ii) $f$ arbitrary (but smooth and increasing).

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Then any regular, smooth equilibrium must display the policy

$$c = (1 - \delta)w \text{ and } k = \delta w,$$

(common to all individuals and time stationary).