Poverty and Self-Control

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Alternative Approaches to the Study of Poverty

Constraints:

- absence of credit: low investments
- absence of insurance: vulnerability to stochastic shocks
- nonconvexity in feasible set (nutrition, health, education)
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Constraints:

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Psychology

- failed aspirations
- lack of or biases in information
- temptation, lack of self-control, inability to commit

Poverty / self-control trap? **poverty ⇒ limited self-control.**
Two Examples from Developing Countries

- Investments
  - Poor forego profitable small investments
  - de Mel-McKenzie-Woodruff (2008): Sri Lankan microenterprise
  - Survey in Banerjee-Duffo (2011)
Two Examples from Developing Countries, contd.

- Public Distribution Debate
  - Public food distribution system in India
  - Huge debate on food versus cash transfers
- Similar issues elsewhere:
  - e.g. conditional transfers, Progresa/Oportunidades
  - microfinance: lending to women
Self-Control or Just Present Bias?

- Use of commitment products in LDCs.
  - Ashraf et al (2003) review
  - Ashraf-Karlan-Yin (2006) field experiment on commitment savings in the Philippines
  - (see also theory in Ambec and Treich (2007) and Basu (2010)).
Poverty and Self-Control:

- If self-control is a fixed trait, policy outlook not good.

- Another possibility: poverty *per se* may damage self-control.

- Source of poverty traps that complements nonconvexities or aspirations failure.

- Policies that help the poor begin to accumulate assets may be highly effective, even if they are temporary.
Self-Control

- Self-control is an intuitive idea:
  - Ability or inability to follow through on an intended plan
  - Operationally, to match a choice made with full precommitment.
Self-Control

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- More specifically:

  - **External** versus **internal** devices.

  - **External**: locked savings, retirement plans, Roscas etc.

  - **Internal**: the use of psychological private rules (Ainslee).

- See Strotz (1956), Phelps-Pollak (1968), or Laibson (1997).
Other Possibilities:

- Costly will-power, e.g., dual self models (Thaler-Shefrin 1981, Fudenberg-Levine 2006)
- Resisting tempting alternatives (Gul and Pesendorfer 2003)
- Ainslee private rules as self-discovery (Ali 2011)

Theoretical literature on the approach pursued here:

- Bernheim-Ray-Yeltekin (1999)
- Banerjee-Mullainathan (2010)
Assets and Incomes

- Asset equation

\[ W_t + y = c_t + \frac{W_{t+1}}{\alpha}. \]
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- Define present value of income:

\[ P \equiv \frac{\alpha}{\alpha - 1} y. \]

- Add to get total assets: \( A_t \equiv W_t + P \), so that

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- Credit Constraint:
  \[ A_t \geq B = \Psi(P) > 0. \]
Preferences \( u(c) = c^{1-\sigma} / (1 - \sigma) \), for \( \sigma > 0 \).

\[
u(c_0) + \sum_{t=1}^{\infty} \delta^t u(c_t)\]
Preferences \[ u(c) = c^{1-\sigma}/(1-\sigma), \text{ for } \sigma > 0. \]

\[ u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t), \quad 0 < \beta < 1. \]
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\[ u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t), \quad 0 < \beta < 1. \]

- Standard model: \( \beta = 1. \)
- If \( \delta \alpha > 1 \) [growth] and \( \mu \equiv \frac{1}{\alpha}(\delta \alpha)^{1/\sigma} < 1 \) [discounting], then

\[ A_{t+1} = (\delta \alpha)^{1/\sigma} A_t \]

\[ c_t = (1-\mu)A_t. \]

- \( \rightarrow \) Ramsey policy.
Policies and Values

A policy $\phi$ specifies continuation asset $A_{t+1}$ after every history.
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\[
V(h_t) \equiv u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \ldots
\]

\[
P(h_t) \equiv u(c_t) + \beta [\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \ldots] = u(c_t) + \beta \delta V(h_t, \phi(h_t))
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  \]

- No self-starvation: $c \geq \nu A$ for some $\nu$ tiny but positive.
Equilibrium Policy

Following the policy is better than trying something else.

\[ P(h_t) \geq u(A(h_t) - \frac{x}{\alpha}) + \beta \delta V(h_t, x) \text{ for every } x \in [B, \alpha(1 - \nu)A(h_t)]. \]
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Generating Equilibrium Values

- Lower bound on infimum values:

\[ L_0(A) \equiv u \left( A - \frac{B}{\alpha} \right) + \frac{\delta}{1 - \delta} u \left( \frac{\alpha - 1}{\alpha} B \right). \]
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- Recursive sequence of correspondences on \([B, \infty), \{\mathcal{V}_k\}:

  - \(\mathcal{V}_0(A) = [L_0(A), \text{Ramsey}(A)]\).

  - \(\mathcal{V}_k\) generates \(\mathcal{V}_{k+1}\) for all \(k \geq 0\). Then \(\mathcal{V}(A) = \bigcap_{t=0}^{\infty} \mathcal{V}_t(A)\).
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- **Proposition 1.** An equilibrium exists: \(\mathcal{V}(A) \neq \emptyset\) for all \(A\).

- \(\mathcal{V}\) compact-valued closed graph; \(\max H(A), \min L(A).\)
Self-Control Definition

- **Self-control at \(A\):**

  \[ \Rightarrow \text{Accumulation at } A \text{ in } \textbf{some} \text{ equilibrium.} \]

- **Strong self-control at \(A\):**

  \[ \Rightarrow A_t \rightarrow \infty \text{ from } A, \text{ in } \textbf{some} \text{ equilibrium.} \]
Self-Control Definition

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- Strong self-control at $A$:
  \[ \Rightarrow A_t \to \infty \text{ from } A, \text{ in some equilibrium.} \]

- No self-control at $A$:
  \[ \Rightarrow \text{No accumulation at } A \text{ in any equilibrium.} \]

- Poverty trap at $A$:
  \[ \Rightarrow \text{Slide to credit limit } B \text{ from } A \text{ in every equilibrium.} \]
Self-Control and No Self-Control

The diagram illustrates the relationship between self-control and no self-control. The graph shows two distinct paths:

- The solid blue line represents self-control, indicating a path that values higher outcomes at higher levels of self-control.
- The dashed black line represents no self-control, showing a less desirable path with lower outcomes.

The graph is plotted on a coordinate system with axes labeled $A'$ and $A$, and points $B$ marked along the axes.

The text within the diagram reads:

- Self control
- No self control
- No self control
- No self control
Uniformity and Nonuniformity

Uniform case:

Self control at every $A$, or its absence at every $A$.

Nonuniform case:

Self-control at $A$, no self-control at $A'$. 
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Uniformity and Nonuniformity

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Proposition 2. Suppose no credit constraints, so that $B = 0$.

- Then every case is uniform.

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- Poverty bias not built in; contrast Banerjee and Mullainathan (2010).
Credit Constraints and Non-Uniformity

- $B > 0$ destroys scale-neutrality (in $A$), but how exactly?
Credit Constraints and Non-Uniformity

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Some intuition:

- Self-control depends on the severity of the consequences of a lapse in self-control.
- Consequences more severe when the individual has more assets; hence more to lose.

Problem:

- Severity (suitably normalized) isn’t monotonic in assets.
The Structure of Lowest Values
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\begin{align*}
V(A) &\quad B \\
H(A) & \\
L(A) &
\end{align*}
The Structure of Lowest Values

$V(A)$$B$$H(A)$$L(A)$$A^*$
The Structure of Lowest Values

$V(A)$

$H(A)$

$L(A)$

$B$  $A'$  $A_*$  $A$
Proposition 3. If $A' > B$ is continuation for $A_*$ under lowest value at $A_*$, then $A'$ is followed by value $H^-(A')$. 

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Proposition 3. If $A' > B$ is continuation for $A_*$ under lowest value at $A_*$, then $A'$ is followed by value $H^-(A')$.

$$u(c''_t) + \beta \delta \text{Blue} = u(c'_t) + \beta \delta \text{Orange} \Rightarrow u(c''_t) + \delta \text{Blue} < u(c'_t) + \delta \text{Orange}.$$
Lowest Values

- Structure is remarkably simple. Following a deviation:
- One more binge, followed by highest-value program.
- Like Abreu penal codes, but for entirely different reasons.
- But argument also reveals why $L(A)$ jumps up occasionally.
maximize \( u(A - x/\alpha) + \beta\delta L(x) \), say max at \( x = \hat{A} \).
maximize $u(A - x/\alpha) + \beta \delta L(x)$, say max at $x = \hat{A}$.

Not possible; get a contradiction:

$$u(\hat{c}_t) + \beta \delta \text{Blue} \leq u(c'_t) + \beta \delta \text{Orange} \Rightarrow u(\hat{c}_t) + \delta \text{Blue} < u(c'_t) + \delta \text{Orange}.$$
maximize \( u(A - x/\alpha) + \beta \delta L(x) \), say max at \( x = \hat{A} \).

So \( \hat{A} > A' \), and \( u(\hat{c}_t) + \beta \delta \text{Blue} = u(c'_t) + \beta \delta \text{Orange} \).
maximize $u(A - x/\alpha) + \beta \delta L(x)$, say max at $x = \hat{A}$.

So $\hat{A} > A'$, and $u(\hat{c}_t) + \beta \delta \text{Blue} = u(c'_t) + \beta \delta \text{Orange}$.

By concavity of $u$, $A'$ may need to jump up, so $L(A)$ jumps too.
Argument So Far

- The problem of internal self-control is both simple and complex.

- Simple: what happens after lapse of control is easy to describe.

- Lapse followed by one round of high $c$, then back to best path.
The problem of internal self-control is both simple and complex.

- **Simple**: what happens after lapse of control is easy to describe.
  - Lapse followed by *one* round of high $c$, then back to best path.

- **Complex**: jump in worst values makes comparative statics hard.
  - As wealth goes up, can get cycles of control / failure of control.
Markov Equilibrium: Values and Continuations

\[ V \]

\[ B \]

\[ A \]

\[ A' \]

\[ B \]

\[ A \]
Markov Equilibrium: Values and Continuations
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Markov Equilibrium: Values and Continuations
Markov Perfect Equilibria: Savings Function, $\beta=0.75$, $\alpha=1.28$, $\delta=0.8$
Illustration of the nonuniform case:

Markov continuation asset
Illustration of the nonuniform case:

Markov continuation asset

Maximal continuation $X(A)$

But the simulations suggest otherwise...
Illustration of the nonuniform case:

Markov continuation asset

Maximal continuation $X(A)$

But the simulations suggest otherwise...
Savings, $\beta=0.75$

- 45° line
- Highest saving
- Best SPE
- Ramsey
- Markov
- Worst SPE

Graph showing the savings with different models and parameters.
Savings, $\beta=0.75$

- 45° line
- Highest saving
- Best SPE
- Ramsey
- Markov
Equilibrium Values, $\beta=0.75$

- SPE best
- SPE worst
- Ramsey
- Markov
Proposition 4 [Central Result]. In the non-uniform case,

- There is $A_1 > B$, such that every $A \in [B, A_1)$ has a poverty trap.
- There is $A_2 \geq A_1$ such that all $A \geq A_2$ exhibit strong self-control.
Proposition 4 [Central Result]. In the non-uniform case,

- There is $A_1 > B$, such that every $A \in [B, A_1)$ has a poverty trap.
- There is $A_2 \geq A_1$ such that all $A \geq A_2$ exhibit strong self-control.
Outline I. The Poverty Trap

- $X(A)$: maximum wealth choice. Then $X(A) < A$ close to $B$. 
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- $X(A)$: maximum wealth choice. Then $X(A) < A$ close to $B$. 

\[\begin{array}{c}
\text{A'} \\
\text{B} \\
\text{M} \\
\text{A}
\end{array}\]
Outline I. The Poverty Trap

- $X(A)$: maximum wealth choice. Then $X(A) < A$ close to $B$. 

![Diagram showing the concept of maximum wealth choice](image)
Outline I. The Poverty Trap

- $X(A)$: maximum wealth choice. Then $X(A) \leq A$ close to $B$. 

![Diagram showing the relationship between $A$, $A'$, $X(A)$, and the points $B$, $A_1$, $M$, and $A_2$.]
Outline II. Strong Self-Control

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Outline II. Strong Self-Control, contd.

$X(A)$

$A^{**}$ $A^{***}$
Outline II. Strong Self-Control, contd.

\[ X(A) \]

\[ \mu_1 = \frac{A^{**}}{B}, \quad \mu_2 = \frac{A^{***}}{B} \]
Outline II. Strong Self-Control, contd.

\[ \mu_1 = A^{**}/B, \quad \mu_2 = A^{***}/B \]

\[ \left[ (\mu_1)^k A^{**}, (\mu_2)^k A^{***} \right] \]

\[ \left[ (\mu_1)^{k+1} A^{**}, (\mu_2)^{k+1} A^{***} \right] \]
Outline II. Strong Self-Control, contd.

\[ X(A) \]

\[ \mu_1 = A^{**}/B, \quad \mu_2 = A^{***}/B \]

\[(\mu_1)^m(\mu_2)^nA\]

\[ [(\mu_1)^kA^{**}, (\mu_2)^kA^{***}] \]

\[ [(\mu_1)^{k+1}A^{**}, (\mu_2)^{k+1}A^{***}] \]
Some Implications of the Model
Some Implications of the Model

1. Link Between Credit Limit and Self-Control

- Modified neutrality: only $B/A$ matters.

- Easier credit (lower $B$) reduces $A_1$ and $A_2$ thresholds:
  - More individuals successfully exercise self-control

- Offsetting effect: those who fall into the poverty trap will fall further.

- Summary: ambiguous effects, depending on where you start.
2. **Asset-Specific MPCs**

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- Jump in financial assets $W$.
- Nonuniform case: decumulation to accumulation.
- So low MPC from financial assets.
2. Asset-Specific MPCs


- \( B/A = B/(W + \text{permanent income}) \).
- Jump in financial assets \( W \).
- Nonuniform case: decumulation to accumulation.
- So low MPC from financial assets.
- Jump in income. If \( B/(\text{perm inc}) \) constant, \( B/A \uparrow \).
- High MPC in non-uniform case.
- At best \( B \) unchanged; then identical MPCs.
3. The Demand For External Commitment Devices:

- Why isn’t all savings done through external commitment?
  - Obvious answer: uncertainty creates the need for flexibility.
  - But external commitments undermine internal self-control:
    - E.g., locking up money in inaccessible account increases $B$. 
3. The Demand For External Commitment Devices:

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- E.g., locking up money in inaccessible account increases $B$.

- Implication for institutional design:

- External commitment needed to escape poverty trap, but . . .

- To keep people saving once out of the poverty trap, we need the commitments removed.

- Offer targeted lockboxes: once target achieved, funds are transferred into a standard account
4. Policy Experiments:

- Can compare accounts with different features
- lock/unlock principal/interest combinations.

Examples:

- Standard account
- Lock-box with threshold balance, unlocked fully afterwards
- Lock-box with minimum balance, unlocked excess balance
- Lock-box with principal always locked, interest never locked

- Need extended model with taste shocks to utility in every period.
Values and saving in the stochastic model with two taste shocks:
Value functions, low and high thresholds, with full unlocking:

Lockbox regime: High max balance
Standard prob
Lockbox regime: Low max balance
Value functions for the minimum balance problem:

- Lockbox Regime 1: High max balance
- Standard prob
- Lockbox Regime 2: High max balance
Summary

- We know that a failure of self-control can lead to poverty.
- Is the opposite implication true?
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- Is the opposite implication true?
- Model constructed for scale-neutrality:
  - The result isn’t effectively “assumed”, say, by positing that the poor are more prone to temptation.
- Ainslee’s personal rules as history-dependent equilibria
Summary

- We know that a failure of self-control can lead to poverty.
- Is the opposite implication true?
- Model constructed for scale-neutrality:
  - The result isn’t effectively “assumed”, say, by positing that the poor are more prone to temptation.
- Ainslee’s personal rules as history-dependent equilibria
- Structure of optimal personal rules is surprisingly simple:
  - Deviations entail further “falling off” the wagon, followed by “climbing back on”.
The ability to impose self-control rises with wealth.

The self-control problems that keep people in poverty may be a consequence of poverty.
The ability to impose self-control rises with wealth.

The self-control problems that keep people in poverty may be a consequence of poverty.

Novel policy implications, among them, for interplay between external and internal commitments:

- External self-control devices can undermine internal self-control

- Lock-box savings accounts with self-established targets and unlocking of principal may be particularly effective devices for increasing saving