1. Say that you have a regression model of the form

\[ y = \beta x + \varepsilon, \]

where \( \varepsilon \) is independently and identically distributed according to the uniform distribution \( U[-0.5, 0.5] \), and \( x \) has a mean of 0 in the population.

1. Show that the OLS estimator of \( \beta \) is unbiased and consistent.

2. Say that you have access to a random sample of \( N \) observations on \( x_i \) and \( d_i \), where

\[ d_i = \begin{cases} 
1 & \iff y_i > 0 \\
0 & \iff y_i \leq 0 
\end{cases}, \quad i = 1, \ldots, N. \]

Define an unbiased estimator for \( \beta \), if one exists.

3. Derive the conditional expectation of \( y \) given \( x \) and given \( y > 0 \).

4. The data set described above has been augmented to include information on the value of \( y \), not only the sign, but only for those observations for which \( d_i = 1 \). That is, you observe \( \tilde{y}_i \), where

\[ \tilde{y}_i = \begin{cases} 
 y_i & \iff y_i > 0 \\
0 & \iff y_i \leq 0 
\end{cases}, \quad i = 1, \ldots, N. \]

Using only the sample of observations for which \( \tilde{y}_i > 0 \), define a consistent least squares estimator for \( \beta \) based on your answer to (c). (Hint: This will be a nonlinear least squares estimator).

2. We talked about estimating the parameter of the negative exponential distribution using the method of maximum likelihood. The negative exponential p.d.f. and c.d.f. are:

\[ f(t) = \alpha \exp(-\alpha t) \]
\[ F(t) = 1 - \exp(-\alpha t), \]
where the parameter $\alpha$ is positive.

Assume that $t$ is the duration of unemployment of a labor market participant. You have access to data collected as follows. A large number of individuals are observed to become unemployed in a particular month, and then are followed for 12 months. Individuals who leave unemployment prior to the end of the 12 month observation period have their total length of time in unemployment recorded as $t_i$. For those who are still unemployed at the end of the observation period, all we know is that their completed spell of unemployment will be greater than 12 months.

1. Let the subsample of individuals for whom $t_i < 12$ be denoted by $C$. We showed that if we had access to a random sample of unemployment spells, the maximum likelihood estimator of $\alpha$ was the inverse of the sample mean. Show that this estimator applied to group $C$ only is inconsistent.

2. Derive a consistent estimator for $\alpha$ that uses information from $C$ and from the “censored” unemployment spells, i.e., those that last for over 12 months.

3. 10.2

4. 10.6