1. In a population, each individual has preferences given by

\[ u_i(l, c) = \alpha_i l + (1 - \alpha_i)c, \]  

(1)

where \( l \) is leisure, \( c \) is consumption of a market good, and \( \alpha \) is a preference parameter. Total time available to each individual is given by \( T \), and all time is spent either in leisure or working in the market, with labor supply \( h = T - l \). Total consumption is

\[ c = wh + Y, \]

where \( w \) is the wage rate and \( Y \) is nonlabor income. You have access to a random sample of \( N \) observations drawn from this population, and the data contain the information \( \{P_i, w_i\}^N_{i=1} \), where

\[ P_i = \begin{cases} 1 & \text{iff } h_i > 0 \\ 0 & \text{iff } h_i = 0 \end{cases} \]

(a) Assume that all agents have the same preference parameter, \( \alpha_i = \bar{\alpha}, \forall i \). On the basis of the sample information available to you, suggest a consistent estimator of \( \alpha \). For your estimator to be well-defined, what condition must be satisfied by the sample observations?

(b) Assume that each population member’s value of \( \alpha_i \) is an independently and identically distributed (i.i.d.) draw from the distribution \( F(\alpha) \), where

\[ F(\alpha) = \alpha^\delta, \alpha \in (0, 1), \delta > 0. \]

Define a consistent estimator of \( \delta \) based on the data available to you. Does the consistency of your estimator depend on their being population variability in wages? Why or why not?

2. (Continuation of 1) There exists a different data set, also a random sample from the population, in which information on hours worked, \( h_i \), and nonlabor income \( Y_i \) is also available, so you have access to \( \{h_i, w_i, Y_i\}^M_{i=1} \). Assume that \( T \) is the same for all population members.
(a) If \( h_i \not\in \{0,T\} \) for all \( i \), state why preferences cannot be represented by (1).

(b) Consider an alternative representation of preferences:

\[
\begin{align*}
   u_i &= \alpha_i \ln(l) + (1 - \alpha_i) \ln(c).
\end{align*}
\]

As in (1.b), assume that the preference parameters \( \alpha_i \) are i.i.d. draws from the power distribution \( F(\alpha) \). Write down the log likelihood function of the sample as a function of \( \delta \).

(c) Derive a (consistent) maximum likelihood estimator of \( \delta \) that only uses sample observations for which \( h_i > 0 \).

(d) You divide your (total) sample in half, with \( M/2 \) observations (\( M \) even) in subsample 1 and \( M/2 \) observations in subsample 2. In subsample 1 you implement the estimator you (implicitly) defined in (2.b). You obtain an estimate of \( \delta \) equal to 1.1 with an estimated (asymptotic) standard error equal to 0.10. Using subsample 2 you implement the estimator defined (2.c). Your estimate of \( \delta \) is equal to 0.80, with an associated standard error estimate of 0.2. Based on these estimates, what can you conclude about the appropriateness of (2) as a representation of preferences in the population, if anything?

3. In the Flinn (2006) analysis of minimum wages in a search and bargaining model, he examined identification and estimation of the model under the assumption that there was no measurement error in wages. However, in the data there did exist several observed wages that were lower than the minimum wage. To create a data generating process that could produce such wage observations, consider introducing a measurement error process as follows:

\[
\begin{align*}
   \ln(\tilde{w}) &= \begin{cases} 
   \ln(w) & \text{with probability } \delta \\
   \ln(w) + \varepsilon & \text{with probability } 1 - \delta 
\end{cases}
\end{align*}
\]

where \( \varepsilon \) is independently and identically distributed as a normal random variable with mean 0 and variance \( \sigma^2_\varepsilon \), \( \delta \) is an unknown parameter contained in \((0,1)\), and \( \tilde{w} \) is the observed wage rate in the data.

Assume that you know that the bargaining power parameter \((\alpha)\) is equal to 0.5, and that \( \rho = 0.05 \). Using Current Population Survey (CPS) data (with information on the length of on-going unemployment spells and accepted wages), determine whether the parameters \( \{b,G,\lambda,\delta,\sigma^2_\varepsilon\} \) are identified. Assume that the matching distribution, \( G \), is lognormal, as in Flinn (2006). [HINT: Write down the log likelihood function for the data. What features of the data, if any, allow you to separate the measurement error component of wages from the “structural” one?]
4. Most “point process” stationary models of individual or aggregate behavior imply a
distribution of times between discrete events that is given by the negative exponential
distribution, with

\[ f(t; \alpha) = \alpha \exp(-\alpha t), \ \alpha > 0, \ t > 0, \]
\[ F(t; \alpha) = 1 - \exp(-\alpha t). \]

You believe that there may exist two types of individuals in the population, type \( \alpha_1 \)
and type \( \alpha_2 \), where the type is characterized by its distributional parameter. Types
are not observable by the econometrician.

You have access to a random sample of durations drawn from the population distri-
bution of spell lengths, \( \{t_i\}_{i=1}^N \), where \( N \) is reasonably large. You find that the mean
duration of spells in the sample is 4, and that the sample variance is 16. Based on
these estimates, can you conclude anything about the probability distribution of \( \alpha \)
in the population? If so, what? If not, why not?